AY 2015-16 Term 2 Final Examinations

<table>
<thead>
<tr>
<th>Date / Start Time</th>
<th>16 April 2016 / 8:30 am</th>
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<tbody>
<tr>
<td>Course</td>
<td>STAT203 FINANCIAL MATHEMATICS</td>
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<tr>
<td>Groups</td>
<td>G1</td>
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<tr>
<td>Instructor</td>
<td>Prof Tse Yiu Kuen</td>
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INSTRUCTIONS TO CANDIDATES

1. The time allowed for this examination paper is 2 hours.
2. This examination paper contains a total of fourteen (14) questions and comprises five (5) pages including this instruction sheet.
3. You are required to answer ALL questions.
Section A: Multiple choice questions. Each question carries 5 marks. Write your answers in the answer book.

1. Jesse deposits $X$ in a bank on January 1 to earn a nominal rate of 5% a year compounded semiannually. The bank credits interest to his account every June 30 and December 31, except that it does not credit any pro rata interest on withdrawals made between the dates on which interest is credited. Jesse’s deposit of $X$ is exactly enough to provide for withdrawals of 100 every March 31, June 30, September 30 and December 31 over a 10-year period. Determine $X$. (Note: Treat every month as having 30 days so that March 31, June 30, September 30 and December 31 are, respectively, 0.25, 0.5, 0.75 and 1 year from January 1.)

(A) 2573
(B) 3118
(C) 3137
(D) 3157
(E) 3196

2. Harry deposits 1 at times $t = 1, 2, 3$ and 4 in a fund where the force of interest is given by

$$\delta(t) = \frac{1}{5 - t}, \quad 0 \leq t < 5$$

Just after the 4th deposit, $X$ is in the fund. Determine $X$. (Note: Future payments earn forward rate of interest.)

(A) 2
(B) 5
(C) 10
(D) 10.42
(E) 12.08

3. A company pays 100 for a bond with annual coupons $X$ to get an effective yield rate of 5%. The amount of interest earned in the 5th coupon is 4.85. Determine $X$. (Note: Information for this question is complete.)

(A) 5.22
(B) 5.33
(C) 5.70
(D) 6.12
(E) 6.85
4. George makes an investment of $K$ that earns interest at an effective rate of 4% per annum. The investment is just enough to provide quarterly withdrawals of 10 forever, with the first withdrawal exactly 5 years from the date of investment. What is $K$?

(A) 806  
(B) 822  
(C) 837  
(D) 842  
(E) 912

5. Henry is repaying a loan of 1000 with annual payments of 120 for 10 years followed by annual payments of 100 for as long as necessary. The amount of interest in the first payments is 100 and the amount of interest in the 12th payment is $X$. Determine $X$.

(A) 63  
(B) 65  
(C) 75  
(D) 78  
(E) 80

6. Jenny gives Ted a loan at 4% effective to be repaid by 10 annual payments of 100, followed by 5 annual payments of 200. Just after Ted makes the 5th payment, Jenny and Ted discover that each of the 15 payments should have been 10% higher than they were originally scheduled. They agree that Ted will make increased payments of $K$ in the 6th through 10th years to adjust for the error. The payments of 200 in the 11th through 15th years will not change. Determine $K$.

(A) 113  
(B) 129  
(C) 139  
(D) 145  
(E) 149

7. John buys an annuity at a price $X$ that will give him a yield rate of 5% effective. The annuity consists of 10 payments at 3-year intervals, first payment on date of purchase. The first annuity payment is 100, and each successive payment is 5% greater than the previous one. Determine $X$. 


8. The following table shows the current term structure of interest rates:

<table>
<thead>
<tr>
<th>Length of investment (in years)</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>X + 0.01</td>
</tr>
<tr>
<td>3</td>
<td>3.0%</td>
</tr>
<tr>
<td>4</td>
<td>3.4%</td>
</tr>
<tr>
<td>5</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

The one-year forward rate for commencement one year later implied by these data is 3.5%. Find X.

(A) 0.82%
(B) 0.97%
(C) 1.13%
(D) 1.28%
(E) 1.49%

9. Assume the following interest rate environment:

\[
i = 8\% \quad \text{for} \quad 0 \leq t < 2
\]

\[
\delta(t) = 0.015t \quad \text{for} \quad 2 \leq t < 5
\]

\[
d = 6\% \quad \text{for} \quad 5 \leq t < 8
\]

\[
i^{(4)} = 10\% \quad \text{for} \quad t \geq 8
\]

Find the present value of an asset which will pay you a single cash flow of 1,000 at time \( t = 10 \).

(A) 439
(B) 459
(C) 479
(D) 499
(E) 519
10. Harry invests 1,000 in a fund on January 1. On May 1, the fund is worth 1,100 and 600 is withdrawn. On September 1, the fund is worth 400 and 600 is deposited. On January 1 of the following year, the fund is worth 1,200. Harry’s dollar-weighted rate of return for the year is $X$. His time-weighted rate of return is $Y$. Determine $X - Y$.

(A) –6.5%
(B) 5.0%
(C) 7.0%
(D) 19.4%
(E) 25.0%

Section B: Written problems. Write your answers in the answer book. Show your working carefully.

11. (10 marks) Arrange in order of increasing annual effective rate of interest:

(i) An effective rate of interest of 4% per annum.
(ii) A force of interest of 4% per annum.
(iii) A nominal rate of interest of 4% per annum convertible semiannually.
(iv) A nominal rate of discount of 4% per annum convertible semiannually.

12. (15 marks) Jamie’s annual salary increases at a rate of $p$ every year. The mean of his annual salary for the last 10 years is 1.182 times the mean of his annual salary for the last 20 years. Determine $p$.

13. (15 marks) You are given the following information about Bond A and Bond B:

(i) Both bonds have annual coupons, sell at par and mature at par at the same effective rate $i$.
(ii) The term of Bond A is $n$ years and the term of Bond B is $2n$ years.
(iii) The modified duration of Bond A is 5.9139 and the modified duration of Bond B is 9.2921.

Determine the effective rate of interest $i$.

14. (10 marks) Let $S$ be the net worth of a portfolio (i.e., the net present value of a set of asset cash inflows over liability cash outflows). State the conditions of Redington’s immunization in terms of $S$. 

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