1. Betty: \( \left( 1 + \frac{i}{2} \right)^{14.5} = 2 \Rightarrow i = 9.79\% \).

Patsy: \( e^{7.25\delta} = 2 \Rightarrow \delta = 9.56\% \).

Thus, \( i - \delta = 0.23\% \)

**Ans:** B

2. Tony’s annual effective rate is \( 1.05^2 - 1 = 0.1025 \).

The force of interest is \( \ln(1.1025) = 0.0975803 \).

For Frances, the simple interest \( r \) satisfies

\[
\delta(5) = \frac{r}{1 + 5r} = 0.0975803 \Rightarrow r = 0.190550.
\]

Hence, Frances’ account in five years is \( (1 + 5r)1000 = 1953 \).

**Ans:** B

3. For the first statement,

\[
\left( 1 - v^n - d \right) (1 + i) = \frac{(1 + i) - v^{n-1} - i}{\delta} = \frac{1 - v^{n-1}}{\delta} = a_{\overline{n-1}}
\]

Hence, I is true

For the second statement, the PV of an annuity-immediate of 10 per month for the first 8 months of a year is \( 120a_{\overline{12}}^{(12)} \) at the beginning of the year. Thus, the PV of such payments over ten years is \( 120a_{\overline{12}}^{(12)} \bar{a}_{\overline{10}} \). Hence, II is false.

The correct expression for the last statement is \( \frac{\bar{s}_3}{(i)\overline{8}} \). Hence, III is false.

**Ans:** E

4. First,

\[
P(1.005)^{144} = 2000 \Rightarrow P = 975.25.
\]

Next,

\[
Z = 975.25 \times \exp \int_0^{12} \frac{1}{t + 12} \, dt = 975.25 \left( \frac{24}{12} \right) = 1950.
\]

**Ans:** A
5. First, 
\[ e^\delta = 1.0375^2 \Rightarrow \delta = 0.073628. \]
Next, 
\[ 1.0375^2 = \left( 1 - \frac{d(4)}{4} \right)^{-4} \Rightarrow d(4) = 0.072954. \]
Hence, \[ \delta + d(4) = 0.1466. \]
Ans: C

6. The accumulated value of Fund X is
\[ 100s_{\overline{a}|0.15}(1.06)^8 + 100s_{\overline{a}|0.06} = (6.7424)(1.5938) + 9.8975 = 20.6435, \]
which equates the accumulated value in Fund Y of 100s_{\overline{a}|i}. Hence, \[ i = 7.4\% \text{ (by trial and error)}. \]
Ans: D

7. We have
\[ 1000s_{\overline{a}|0.15}(1 + i)^5 = 1000\ddot{a}_{\overline{a}|0.005}, \]
which implies
\[ 137.63164(1 + i)^5 = 176.25713 \]
so that \[ i = 5.07\% \]
Ans: B

8. First,
\[ a(t) = \exp \left( \int_0^t \frac{2}{10 + r} dr \right) = \exp \left( 2 \ln(10 + r) \right|_0^t) = \left( \frac{10 + t}{10} \right)^2. \]
Thus, 
\[ \ddot{a}_{\overline{a}|1} = \frac{1}{a(1)} + \frac{1}{a(2)} + \frac{1}{a(3)} + \frac{1}{a(4)} = \left( \frac{10}{11} \right)^2 + \left( \frac{10}{12} \right)^2 + \left( \frac{10}{13} \right)^2 + \left( \frac{10}{14} \right)^2 = 2.62. \]
Ans: B

9. Let \( j \) be the monthly effective rate. We have
\[ X = 2(Ia_{\overline{a}|0.007444}. \]
Now \( (1 + j)^3 = 1.0225 \) implies \( j = 0.7444\% \). Hence,
\[ X = 2 \left( \frac{\ddot{a}_{\overline{a}|0.007444} - 60v_{60}^{60}}{0.007444} \right) = 2 \left( \frac{48.6077 - 38.4490}{0.007444} \right) = 2729. \]
Ans: B
10. The PV of Annuity 1 is 
\[(Da)_10 = \frac{10 - a_{10}}{i},\]
and the PV of Annuity 2 is 
\[(Ia)_11 + v^{11} \left( \frac{11}{i} \right) = \frac{\ddot{a}_{11}}{i}.\]

Setting the two equal, we have 
\[\frac{\ddot{a}_{11}}{i} = 2 \left( \frac{10 - a_{10}}{i} \right) \Rightarrow \ddot{a}_{11} = 20 - 2a_{10} = 1 + a_{10} \Rightarrow a_{10} = 6.33.\]

Hence, the PV of a 10-year level annuity-immediate of 5 is 5\(\frac{a_{10}}{i}\) = 31.67.

Ans: B

11. The PV of 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, paid at the end of year 2, 3, \ldots, 12, is equivalent to\(a_{1}\) paid at 1, 2, 3, 4, 5, 6, so that its PV is \((a_{1})^2 = 25\). This implies \(a_{1} = 5\). The second annuity is the first annuity with additional payments of 1 at time 1, 2, \ldots, 6. Thus, the PV of the second annuity is \((a_{1})^2 + a_{1} = 25 + 5 = 30\).

Ans: B

12. We first solve for the forward rates one year from now. First, 
\[i_{1,1}^F = \frac{1.0575^2}{1.05} - 1 = 0.065.\]

Similarly, we have 
\[(1 + i_{1,2}^F)^2 = \frac{1.0625^3}{1.05},\]
and 
\[(1 + i_{1,3}^F)^3 = \frac{1.065^4}{1.05},\]
so that the PV of the annuity one year from now (assuming future spot rates are equal to current forward rates) is 
\[5 \left[ \frac{1}{1 + i_{1,1}^F} + \frac{1}{(1 + i_{1,2}^F)^2} + \frac{1}{(1 + i_{1,3}^F)^3} \right] = 5 \left[ \frac{1.05}{1.0575^2} + \frac{1.05}{(1.0625)^3} + \frac{1.05}{(1.065)^4} \right] = 13.15.\]

Ans: B

13. From Sam’s investment, we have 
\[300s_{20\mid 0.08} = 20(300) + 300i(Is)_{20\mid 0.5i},\]
so that 
\[14826.88 = 6000 + 300i \left( s_{20\mid 0.5} - \frac{21}{0.5i} \right) = 6000 + 600s_{20\mid 0.5i} - 12600,\]

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and \( s_{21|0.5i} = 35.711467 \).

Tom’s accumulated value is \( 10s_{21|0.5i} = 10(s_{21|0.5i} - 1) = 347.11 \)

**Ans:** A

14. We have \( C = 120 - 145 = -25 \) and \( I = 60 - 25 - 75 = 10 \). The DWRR is

\[
\frac{10}{75 + \left( \frac{1}{12} + \cdots + \frac{11}{12} \right)(10) - \left( \frac{10}{12} \right)5 - \left( \frac{6}{12} \right)25 - \left( \frac{2.5}{12} \right)80 - \left( \frac{2}{12} \right)35} = \frac{10}{90.83} = 11\%
\]

**Ans:** E

15. For the DWRR, \( I = 85 + W - 90 = W - 5 \). Hence,

\[
\frac{W - 5}{90 - 0.75W} = 0.2 \Rightarrow W = 20.
\]

Hence, the TWRR is

\[
\frac{X}{90} \times \frac{85}{X - 20} = 1.16 \Rightarrow X = 107.6
\]

**Ans:** D