Optimal Food Stamp Subsidy Scheme

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Abstract

This paper constructs a simple model to illustrate that the current food stamp subsidy scheme fails to achieve its aim due to the crowding-out effect. In particular, if the household income is sufficiently low, only the corner solution exists and full subsidy is needed. The optimal food stamp subsidy scheme is identified and the subsidy efficiency rate is introduced to measure the impact on food expenditures once the subsidy regime changes.

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1 Introduction

The food stamp program in USA provides benefits to low-income people to improve their diets. It is one type of in-kind transfer, in which the food stamp coupon (or electronic benefit transfer – EBT) can be used to purchase most foods at participating stores, but not for other goods and services.

The subsidy scheme of the current food stamp program, which is called Supplemental Nutrition Assistance Program (SNAP), is linearly regressive as follows.¹

The amount of SNAP benefits you get is the TFP amount for your household minus 30% of your net income, where the Thrifty Food Plan (TFP) is a model for how much money a household should spend on

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food. So if you spend 30% of your net income on food, but you still can’t spend as much as the TFP says you should, SNAP give you the rest.

The main question of inquiry is: is the current subsidy scheme optimal – to help households achieve the TFP amount of food expenditure with the least cost? A simple model is constructed to illustrate that the current subsidy scheme fails to achieve its aim – households’ food expenditure no less than the TFP amount. Intuitively, if the household income is sufficiently low, only the corner solution exists, in which the household only spends the amount of food stamp subsidy, while all its own food expenditure in the absence of subsidy would be crowded out. Due to this crowding-out effect, to have the food expenditure no less than the TFP amount, full subsidy is needed.

The current literature focuses on the issue why governments choose to redistribute in-kind rather than in cash (Currie and Gahvari 2008) and on the empirical studies of the marginal propensity to consume food out of food stamp income and cash income (Hoyes and Schanzenbach 2009). The details of how in-kind transfers, in particular the subsidy scheme, are provided are rarely studied. This paper provides a benchmark of the subsidy scheme – optimal subsidy scheme. Thereafter a subsidy efficiency rate is introduced to measure the impact on food expenditures by the subsidy regime change. The result is that in terms of subsidy efficiency rate, especially for low income households, the optimal subsidy scheme identified is better than the current food stamp subsidy scheme, which is better than cash subsidy.

2 The Model

Suppose households have a common utility function: \( U(X, Y) = X^{\frac{3}{10}} Y^{\frac{7}{10}} \), where \( X \) is the food consumption and \( Y \) the composite good. The current subsidy scheme is:

\[
\begin{align*}
S &= TFP - \frac{3}{10}M \quad \text{if} \quad M < \frac{10}{3}TFP \\
S &= 0 \quad \text{if} \quad M \geq \frac{10}{3}TFP
\end{align*}
\]

where \( M \) is the household income and \( S \) the amount of subsidy. Denote \( P_X \) as the food price and \( P_X X \) the food expenditure. Each household solves the following problem.

\[
\begin{align*}
\max_{X,Y} & \quad U(X, Y) = X^{\frac{3}{10}} Y^{\frac{7}{10}} \\
\text{s.t.} & \quad P_X X + Y = M + S \\
& \quad P_X X \geq S \\
& \quad S = TFP - \frac{3}{10}M \quad \text{if} \quad M < \frac{10}{3}TFP \\
& \quad S = 0 \quad \text{if} \quad M \geq \frac{10}{3}TFP
\end{align*}
\]

\(^2\)Based on the current linearly regressive subsidy scheme, we assume the Cobb-Douglas utility function \( U(X, Y) = X^\alpha Y^{1-\alpha} \) and set \( \alpha = \frac{3}{10} \). In reality, spending on food accounts for a larger share of total spending at very low levels of income. Therefore, \( \alpha \) may increase as \( M \) decreases.
The following proposition shows that the current linearly regressive subsidy scheme fails to achieve its aim.

**Proposition 1** For households with income \(0 < M < \frac{10}{3} TFP\), \(P_X X < TFP\).

**Proof.** See the Appendix.

Figure 1 depicts the food expenditure under the current subsidy scheme. Intuitively, if the household income is sufficiently low, only the corner solution exists, in which the household only spends the amount of food stamp subsidy, while all its own food expenditure in the absence of subsidy would be crowded out. Further, if the household income is high, we have the interior solution. Still, partial of its own food expenditure in the absence of subsidy would be crowded out.

Due to this crowding-out effect, to achieve the aim – the food expenditure no less than the TFP amount, more subsidy is prerequisite. The following proposition identifies the optimal subsidy scheme – achieving the aim with the least cost.

**Proposition 2** The optimal subsidy scheme is:

\[
\begin{align*}
S &= TFP & \text{if } M < \frac{7}{3} TFP \\
S &= \frac{10}{3} (TFP - \frac{3}{10} M) & \text{if } \frac{7}{3} TFP \leq M < \frac{10}{3} TFP \\
S &= 0 & \text{if } M \geq \frac{10}{3} TFP
\end{align*}
\]

**Proof.** See the Appendix.

Figure 2 depicts the optimal subsidy scheme and the current subsidy scheme as functions of household income. Intuitively, if the income of the household is below some threshold, only the corner solution exists, in which the household only spends the amount of food stamp subsidy, while all its own food expenditure in the absence of subsidy would be crowded out. Further, if the income of the household is high, we have the interior solution. Only partial of its own food expenditure in the absence of subsidy would be crowded out. To have \(P_X X \geq TFP\), full subsidy is needed: \(S = TFP\). Further, if the income of the household is so high that its own food expenditure would be greater than the TFP amount in the absence of subsidy.
3 Food Stamp vs. Cash Subsidy

Presumably, compared with cash, food stamp has a larger impact on food expenditures to justify the implementation of costly food stamp program instead of providing cash subsidy. Current empirical research mainly studies the marginal propensity (MPC) to consume food out of food stamp and cash income. Hoynes and Schanzenbach (2009) claim that $MPC_{\text{stamp}}$ is close to $MPC_{\text{cash}}$, which contradicts with the so called “cash-out puzzle”, where $MPC_{\text{stamp}}$ and $MPC_{\text{cash}}$ represent MPC with an additional dollar of food stamp income and cash income respectively.

Instead of measuring MPC, we are interested in the impact on food expenditures for the individual household by a full *regime change*: either from food stamp to cash or vice versa. We define the *subsidy efficiency rate* $R$ as the ratio of the added food expenditure to the amount of subsidy.

$$R = \frac{\text{food expenditure with subsidy} - \text{food expenditure without subsidy}}{\text{the amount of subsidy}}$$

Clearly, the added food expenditure is constant at 30 cents per dollar cash subsidy: $R_{\text{cash}} = \frac{3}{10}$. The following proposition shows that the current food stamp subsidy scheme is better than the cash subsidy in terms of subsidy efficiency rate, especially for low income households. Further, the optimal subsidy scheme we identified in proposition 2 is even better.

**Proposition 3**

$$\begin{cases} 
R_{\text{stamp}} > \overline{R}_{\text{stamp}} > R_{\text{cash}} & \text{if } 0 < M < \frac{70}{51} TFP \\
R_{\text{stamp}} > \overline{R}_{\text{stamp}} = R_{\text{cash}} & \text{if } \frac{70}{51} TFP \leq M < \frac{7}{3} TFP \\
R_{\text{stamp}} = \overline{R}_{\text{stamp}} = R_{\text{cash}} & \text{if } M \geq \frac{7}{3} TFP
\end{cases}$$

3 “Cash-out puzzle” says $MPC_{\text{stamp}}$ is much higher than $MPC_{\text{cash}}$.

4 Based on our framework, if the household income is high, we have the interior solution and $MPC_{\text{stamp}} = MPC_{\text{cash}}$. If the household income is low, we have the corner solution and $MPC_{\text{stamp}} = 1$ while $MPC_{\text{cash}} = 0$. Hoynes and Schanzenbach (2009) measures the mean values of MPC across households, which depends on the distribution of households’ income.
\[ R_{\text{stamp}} \text{ and } \tilde{R}_{\text{stamp}} \text{ represent the subsidy efficiency rate under the current subsidy scheme and optimal subsidy scheme respectively.} \]

**Proof.** See the Appendix. ■

Figure 3 depicts the subsidy efficiency rates under the current subsidy scheme, the optimal subsidy scheme, and the cash subsidy scheme respectively. Intuitively, if the household income is low, under both the current food stamp subsidy scheme and the optimal subsidy scheme, only the corner solution exists. \( R_{\text{stamp}} = \frac{S - \frac{3}{10}M}{S} > R_{\text{cash}}, \) where \( \frac{3}{10}M \) is the food expenditure in the absence of subsidy. Further, under the current linearly regressive subsidy scheme the amount of subsidy decreases as income increases, while under the optimal subsidy scheme the subsidy is constant at TFP. Thus, we have \( R_{\text{stamp}} > \tilde{R}_{\text{stamp}} \) for low income households.

### 4 Conclusion

This paper emphasizes the food stamp subsidy program’s aim – households’ food expenditure no less than the TFP amount, which may conflict with households’ own interests. On the basis of this consideration, a simple model is constructed to illustrate that the current food stamp subsidy scheme fails to achieve its aim due to the crowding-out effect. The optimal subsidy scheme is identified and the subsidy efficiency rate is introduced to measure the impact on food expenditures once the subsidy regime changes.

**Appendix**

**Proof of Proposition 1** Consider the household’s optimization problem (1) under the current subsidy scheme. To have an interior solution, we must have \( \frac{3}{10}(M + S) \geq S, \)
which implies $M \geq \frac{70}{51} TFP$. Therefore, for $M < \frac{70}{51} TFP$, only the corner solution exists, $P_X X = S = TFP - \frac{3}{10} M < TFP$. For $\frac{10}{3} TFP > M \geq \frac{70}{51} TFP$, we have the interior solution, $P_X X = \frac{3}{10} (M + S) < TFP$. $\blacksquare$

Proof of Proposition 2 The government solves the following problem, subject to the household’s problem.

$$
\min_S S \\
\text{s.t. } P_X X \geq TFP \geq S \\
\left( \begin{align*}
\max_{X,Y} U(X,Y) &= X \frac{3}{10} Y \frac{7}{10} \\
\text{s.t. } &P_X X + Y = M + S \quad \text{if } P_X X > S \\
&Y = M \quad \text{if } P_X X \leq S
\end{align*} \right)
$$

Suppose the household has the interior solution, $P_X X = \frac{3}{10} (M + S)$. To minimize $S$, the constraint $P_X X \geq TFP \geq S$ requires $P_X X = \frac{3}{10} (M + S) = TFP \geq S$, which implies $S = \frac{10}{3} (TFP - \frac{3}{10} M)$ and $\frac{3}{10} TFP \leq M < \frac{10}{3} TFP$. Similarly, suppose the household has the interior solution, $P_X X = S$. To minimize $S$, the constraint $P_X X \geq TFP \geq S$ requires $P_X X = S = TFP$. In this case, the possible interior solution is out of the budget set. That is, $\frac{3}{10} (M + S) = \frac{3}{10} (M + TFP) < S = TFP$, which implies $M < \frac{7}{3} TFP$. $\blacksquare$

Proof of Proposition 3 In the absence of subsidy, $P_X X = \frac{3}{10} M$. Under the current subsidy scheme, by proposition 1, for $M < \frac{70}{51} TFP$, we have the corner solution, $P_X X = S = TFP - \frac{3}{10} M$. Therefore, $R_{\text{stamp}} = \frac{S - \frac{3}{10} M}{S} = 1 - \frac{\frac{3}{10} M}{TFP - \frac{3}{10} M} > \frac{3}{10}$. For $M \geq \frac{70}{51} TFP$, we have the interior solution, $P_X X = \frac{3}{10} (M + S)$. $R_{\text{stamp}} = \frac{\frac{3}{10} (M + S) - \frac{3}{10} M}{S} = \frac{3}{10}$. By proposition 2, under the optimal subsidy scheme, for $M < \frac{7}{3} TFP$, we have the corner solution, $P_X X = S = TFP$. Therefore, $R_{\text{stamp}} = \frac{S - \frac{3}{10} M}{S} = 1 - \frac{\frac{3}{10} M}{TFP} > \frac{3}{10}$. For $M \geq \frac{7}{3} TFP$, we have the interior solution, $P_X X = \frac{3}{10} (M + S)$. $R_{\text{stamp}} = \frac{\frac{3}{10} (M + S) - \frac{3}{10} M}{S} = \frac{3}{10}$. Clearly, $1 - \frac{\frac{3}{10} M}{TFP} > 1 - \frac{\frac{3}{10} M}{TFP - \frac{3}{10} M}$ for $M > 0$. $\blacksquare$

References
