Orienteering Problem

A formulation

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Notation

v_{iw}	variable - indicates selection of attraction i in window w, $i \in [0, N]$, $w \in [1, W]$
a_{ij}	variable - indicates selection of path from i to j
s_i	variable - time of arrival at attraction i
σ_{ij}	variable - amount by which time gap between i and j is defaulted

t_{ij}	time taken between arrivals at i and j
u_i	utility of attraction i
T_{max}	time available for the traversal
O_{iw}, C_{iw}	opening and closing times of window w of attraction i
δ_{ij}	time gap required between attractions i and j
z_{ij}, l_{ijk}	temporary variables used in sequencing

Formulation

$$\max \phi = \sum_{i=1}^{N} u_i \sum_{w=1}^{W} v_{iw} \tag{1}$$

s.t.

$$\sum_{w=1}^{W} v_{iw} \leq 1 \quad \forall i \in [1, N] \tag{2}$$

$$\sum_{w=1}^{W} v_{iw} = \sum_{k=1}^{N} a_{ki} = \sum_{k=1}^{N} a_{ik} \quad \forall i \in [1, N] \tag{3}$$

$$(s_i + t_{ij}) - s_j \leq M(1 - a_{ij}) \quad \forall i, j \in [0, N + 1] \tag{4}$$

Formulation

$$\sum_{w=1}^{W} v_{0w} = 1, \sum_{w=1}^{W} v_{(N+1)w} = 1 \tag{5}$$

$$\sum_{j=1}^{N+1} a_{0j} = 1, \sum_{j=0}^{N} a_{j(N+1)} = 1 \tag{6}$$

$$s_{N+1} \leq T_{max} \tag{7}$$

$$a_{ij}, v_{iw} \in \{0, 1\} \quad \forall i, j, w \tag{8}$$

If T_{max} is relative time (length of stay), we can write

$$s_{N+1} - s_0 \leq T_{max}$$

Time Windows

- We have many windows for the same attraction each window has a opening time and a closing time, given by (O_{iw}, C_{iw})

$$\sum_{w=1}^{W} O_{iw} v_{iw} \leq s_i \leq \sum_{w=1}^{W} C_{iw} v_{iw} \quad \forall i \tag{9}$$

- For attractions with one window, $W - 1$ windows will have start and close times $(0, 0)$

Setting time gaps between attractions

- Need to have 'soft' constraints with penalties if gaps are not met

$$s_i - s_j + \sigma_{ij} \geq \delta_{ij}$$

- However the constraint should apply only if s_i comes after s_j
- Can be re-written as:

$$s_i - s_j - M * g_{ij} \leq 0 \tag{10}$$
$$s_j - s_i - M * g_{ji} \leq 0 \tag{11}$$
$$g_{ij} + g_{ji} \leq 1 \tag{12}$$
$$s_i - s_j + \sigma_{ij} \geq g_{ij}\delta_{ij} \tag{13}$$
$$g_{ij} \in \{0, 1\} \forall i, j \tag{14}$$

- ... and penalize σ_{ij}

Size

- $N = 46, W = 4$
- Number of decision variables $= 8694$
- Number of constraints $= 10950$

Time dependent utilities and traversal times

- Utilities of attractions and time of travelling changes with time between attractions
- Use windows as a way to control utilities and time taken, can be made coarse or fine depending on width of window
- Change $u_i \to u_{iw}$, $t_{ij} \to t_{ijw}$

$$
\begin{aligned}
t_{ijw} = & \text{queue time at } i \\
& + \text{ride time at } i \\
& + \text{travelling time from } i \to j \\
& [\text{in window } w]
\end{aligned}
$$

Changes in formulation

- Objective:

$$\max \phi'' = \sum_{i=1}^{N} \sum_{w=1}^{W} u_{iw} v_{iw} - \sum_{i=0}^{N} \sum_{j=0}^{N} [z_{ij} + P(\sigma_{ij})]$$

- Constraints:

$$(s_i + \sum_{w=1}^{W} t_{ijw} v_{iw}) - s_j \leq M(1 - a_{ij}) \tag{15}$$

- The remaining constraints remain

Improvements

- We do not optimize over ALL visitors available at the time of running
- Static utility and queue length values may not reflect state of the system