

A Supplement to

“Bias Correction and Refined Inferences for Fixed Effects Spatial Panel Data Models”

Zhenlin Yang^a, Jihai Yu^b, and Shew Fan Liu^a

^a*School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903.*

^b*Guanghua School of Management, Peking University, Beijing 100871, China.*

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This supplemental material adds the Monte Carlo results unreported in the above paper but necessary in supporting the discussions in the paper. It also presents an empirical application of the proposed methods together with the Matlab codes to facilitate the applied researchers.

1 Additional Monte Carlo Results

Tables 1a, 1b, 2, 3a, 3b, 4, 5a, 5b, 5c, and 6 correspond to the reported results in the paper, except that in Tables 1a, 1b, and 2, the results under $n = 500$ are replaced with the results under $n = 250$. The added results are summarized as follows.

Table 1a-r replicates Case (a) of Table 1a by using the wild bootstrap with perturbation distribution being the simple two-point (1 and -1) distribution with equal probabilities, and Case (b) of Table 1a by using REG1 instead of REG2. These results show that wild bootstrap produces similar results as the iid bootstrap, and that use of iid regressors produces much different results compared with those from the non-iid regressors.

Table 2-r replicates Case (a) Table 2 by (a) changing $\beta = (1 \ 1)'$ to $\beta = (.5 \ .5)'$, and (b) using the wild bootstrap method with perturbation distribution being the two-point distribution with unequal probabilities described in Footnote (11) in the paper. From these, we see that reducing SNR in the SED model does not affect much the QMLEs, and the wild bootstrap produces similar results.

Table 3a-r replicates Table 3a by using the wild bootstrap method with perturbation distribution being the two-point (1 and -1) distribution with equal probabilities, and **Table 3a-rr** replicates Table 3a using $\beta = (.5 \ .5)'$. The results show that wild bootstrap gives similar results, and that SNR affects mainly the performance of spatial lag estimators.

Table 5a-r replicates Table 5a using the iid bootstrap method. The results show that iid method indeed performs better under normality, and not as good under extreme normality when n is large.

Table 6-r replicates Table 6 under a weaker spatial dependence or iid regressors. The results show that a weaker spatial dependence results in a better performance of the regular test, and that the way the regressors being generated has more significant effect on the performance of the tests for the covariate effects.

Table 6-rr further replicates Table 6 using $\lambda = \rho = 0.25$. The result show that the true values of the spatial parameters do not have noticeable effect.

2 An Empirical Application

The effectiveness of the bias and variance correction methods given in this paper is demonstrated in an empirical setting using the well known Munnell (1990) data set on public capital productivity. The dataset gives indicators related to public capital productivity for 48 US states observed over 17 years (1970-1986).^{*,†} In Munnell (1990), the empirical model specifies a Cobb-Douglas production function of the form:

$$\lg(gsp) = \beta_0 + \beta_1 \lg(pcap) + \beta_2 \lg(pc) + \beta_3 \lg(emp) + \beta_4 unemp + \epsilon,$$

with possibly two-way fixed effects, where ‘gsp’ is the gross social product of a given state, ‘pcap’, ‘pc’ and ‘emp’ are the inputs of public capital, private capital, and labor respectively. In order to capture business cycle effects an additional variable ‘unemp’ is also added which indicates the state unemployment rate. The model now is extended by adding the spatial effects. The spatial weight matrix (W_n) is specified using a contiguity form where (i, j) th element is indicated as 1 if state i and j share a common border, otherwise 0. The final W_n is row normalised.

Table 7 gives the QML estimates and the second-order bias-corrected QML estimates of the model parameters for the full dataset spanning over the 17 years fitted using the 2FE-SPD model with five different types of spatial specifications: SARAR, SLD, SED, Durbin-SLD and Durbin-SED. When the full dataset is considered for a period of $T = 17$ and thus $N = (n - 1)(T - 1) = 752$ is relatively large, the difference between the original QMLE-based results and the bias-corrected results is not so much. This is in line with the theoretical results on the consistency of the QMLEs.

Table 8 gives the same results for a shorter time interval concentrating on the years 1982-84, allowing us to see the necessity of bias-correction and the effectiveness of the proposed bias-correction methods, when the sample size is not so large (here, $N = 94$). As can be seen from the estimation results of Table 8, there is a clear difference between the original QMLE-based results and the bias-corrected results. Point estimates of the bias-corrected QMLEs of the spatial parameters can be significantly larger than the corresponding QMLEs, in line with the theoretical results that the QMLEs are downward biased. The bias-corrected t -ratios for the spatial effects and the covariate effects can be noticeably smaller compared to the original t -ratios, showing that the original QMLE-based inferences can be conservative (or over rejection) when sample size is not large, in line with the theoretical results reported in the paper.

*The dataset can be downloaded from <http://pages.stern.nyu.edu/~wgreene/Text/Edition6/tablelist6.htm>

†This dataset was previously used in Millo and Piras (2012) in order to illustrate fixed effects and random effects spatial panel data model estimation in a QMLE and GMM context.

Table 1a. Empirical Mean[rmse](sd) of Estimators of λ , 2FE-SPD Model with SLD, $T = 3$, $\beta = (\mathbf{1}, \mathbf{1})'$, $\sigma = 1$

λ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\lambda}_N^{bc3}$	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\lambda}_N^{bc3}$
	(a) Queen Contiguity, REG1			(b) Group Interaction, REG2		
Normal Error, n=50						
.50	.484[.120](.119)	.502.120	.502.120	.469[.095](.089)	.497.088	.499.088
.25	.234[.142](.141)	.248.143	.250.143	.210[.130](.124)	.250.123	.251.123
.00	-.010.158	.001.161	.002.161	-.049[.167](.159)	-.001.160	.001.160
-.25	-.258.161	-.251.164	-.250.165	-.303[.189](.182)	-.250.184	-.248.184
-.50	-.504.163	-.503.166	-.502.167	-.565[.214](.204)	-.509.208	-.507.208
Normal Mixture, n=50						
.50	.483[.119](.117)	.500.118	.501.118	.470[.091](.086)	.498.084	.499.084
.25	.238.139	.253.141	.254.141	.209[.128](.121)	.248.120	.249.120
.00	-.013[.155](.154)	-.002.157	-.001.157	-.048[.160](.152)	-.001.153	.001.153
-.25	-.257.158	-.251.161	-.250.162	-.301[.188](.181)	-.248.182	-.247.183
-.50	-.504.163	-.503.166	-.503.167	-.556[.206](.199)	-.500.203	-.498.203
Lognormal Error, n=50						
.50	.485[.111](.110)	.501.111	.502.111	.470[.090](.085)	.497.083	.498.083
.25	.239.133	.253.134	.254.134	.212[.122](.116)	.249.115	.251.115
.00	-.010.146	.001.149	.002.149	-.045[.154](.147)	.000.147	.002.147
-.25	-.255.151	-.249.154	-.248.154	-.302[.178](.171)	-.251.173	-.250.173
-.50	-.498.152	-.499.155	-.499.156	-.556[.204](.196)	-.503.200	-.501.200
Normal Error, n=100						
.50	.493[.079](.078)	.502.078	.502.078	.482[.067](.065)	.500.064	.501.064
.25	.243.095	.251.095	.252.095	.222[.096](.092)	.248.092	.248.092
.00	-.007[.110](.109)	.000.110	.000.110	-.031[.123](.119)	.000.120	.001.120
-.25	-.255.114	-.250.115	-.250.115	-.289[.146](.141)	-.254.143	-.253.143
-.50	-.503.117	-.501.118	-.501.118	-.538[.162](.158)	-.503.162	-.503.162
Normal Mixture, n=100						
.50	.490.078	.499.078	.500.078	.482[.067](.065)	.500.065	.500.065
.25	.241.095	.249.095	.250.095	.224[.095](.091)	.250.091	.250.091
.00	-.006.106	.001.107	.002.107	-.034[.122](.117)	-.002.118	-.002.118
-.25	-.255.112	-.250.113	-.250.113	-.286[.144](.140)	-.251.142	-.250.142
-.50	-.502.117	-.499.119	-.499.119	-.535[.160](.156)	-.500.159	-.500.159
Lognormal Error, n=100						
.50	.492.075	.501.075	.501.075	.482[.065](.062)	.500.062	.500.062
.25	.242.091	.250.091	.250.091	.225[.093](.090)	.250.090	.250.090
.00	-.006.102	.001.103	.001.103	-.029[.116](.113)	.001.113	.002.113
-.25	-.255.110	-.250.111	-.250.111	-.283[.138](.134)	-.249.136	-.248.136
-.50	-.503.112	-.500.113	-.500.113	-.526[.157](.154)	-.492.159	-.495.159
Normal Error, n=250						
.50	.497[.047](.046)	.501.047	.501.047	.490[.046](.045)	.499.045	.499.045
.25	.246.060	.249.060	.249.060	.236[.069](.068)	.250.068	.250.068
.00	-.004.066	-.001.066	-.001.066	-.017[.090](.088)	.001.088	.001.088
-.25	-.252.071	-.250.072	-.250.072	-.269[.109](.107)	-.249.107	-.249.107
-.50	-.500.073	-.499.073	-.499.073	-.525[.126](.123)	-.501.124	-.501.124
Normal Mixture, n=250						
.50	.495.047	.499.047	.499.047	.491[.046](.045)	.500.045	.500.045
.25	.248.057	.251.057	.251.057	.238[.067](.066)	.251.066	.251.066
.00	-.003.066	.000.066	.000.066	-.016[.088](.087)	.001.087	.001.087
-.25	-.251.071	-.249.071	-.249.071	-.267[.106](.104)	-.247[.105](.104)	-.247[.105](.104)
-.50	-.501.073	-.500.074	-.500.074	-.522[.125](.123)	-.499.123	-.498.123
Lognormal Error, n=250						
.50	.497.046	.500.046	.500.046	.491[.044](.043)	.500.043	.500.043
.25	.247.056	.251.056	.251.056	.238[.065](.064)	.251.064	.251.064
.00	-.003.064	.000.064	.000.064	-.017[.085](.083)	.000.083	.000.083
-.25	-.252.069	-.250.069	-.250.069	-.268[.104](.103)	-.247.103	-.247.103
-.50	-.499.070	-.499.070	-.499.070	-.522[.125](.123)	-.498.123	-.498.123

Table 1a-r. Replicates of Table 1a

λ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\lambda}_N^{bc3}$	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\lambda}_N^{bc3}$
(a) Wild Bootstrap, Queen, REG1						
Normal Error, n=50						
.50	.484[.121](.120)	.502.121	.502.121	.481[.098](.096)	.500.094	.500.093
.25	.234[.143](.142)	.248.145	.249.145	.221[.137](.134)	.247.131	.248.131
.00	-.011.159	.000.162	.001.162	-.037[.171](.167)	-.005.164	-.004.164
-.25	-.256.163	-.250.168	-.249.168	-.285[.198](.195)	-.251.193	-.249.193
-.50	-.499.161	-.497.166	-.497.166	-.539[.227](.224)	-.502.224	-.500.223
Normal Mixture, n=50						
.50	.480[.119](.117)	.497.120	.498.119	.481[.094](.092)	.501.089	.501.089
.25	.234[.142](.141)	.249.144	.250.144	.225[.132](.130)	.251.127	.252.127
.00	-.008.152	.002.156	.004.156	-.031[.167](.164)	.000.162	.001.161
-.25	-.255.159	-.248.164	-.247.164	-.289[.199](.195)	-.253.193	-.252.193
-.50	-.500.158	-.498.163	-.498.164	-.536[.223](.220)	-.500.220	-.498.219
Lognormal Error, n=50						
.50	.483[.112](.111)	.499.114	.500.113	.485[.086](.085)	.504.083	.505.083
.25	.237[.133](.132)	.251.136	.252.136	.228[.122](.120)	.254[.118](.117)	.255.117
.00	-.011.144	-.001.149	.001.149	-.028[.157](.154)	.004.152	.005.152
-.25	-.258[.147](.146)	-.253.152	-.252.152	-.280[.184](.181)	-.245.180	-.243[.180](.179)
-.50	-.503.152	-.503.158	-.502.158	-.535[.211](.208)	-.499.208	-.496.208
Normal Error, n=100						
.50	.488.080	.498.080	.498.080	.485[.079](.078)	.499.076	.499.076
.25	.239[.097](.096)	.248[.098](.097)	.248.097	.231[.112](.111)	.251.109	.250.109
.00	-.008.108	-.001.109	-.001.109	-.025[.145](.143)	-.001.141	-.001.141
-.25	-.258.112	-.252.114	-.252.114	-.279[.173](.170)	-.251.169	-.252.169
-.50	-.506.120	-.503.122	-.503.122	-.533[.199](.197)	-.504.196	-.504.196
Normal Mixture, n=100						
.50	.489[.079](.078)	.498.079	.499.079	.485[.077](.076)	.500.074	.499.074
.25	.241.094	.250.095	.250.095	.229[.111](.110)	.250.107	.249.107
.00	-.005.106	.002.108	.003.108	-.024[.145](.143)	.001.141	.000.141
-.25	-.251.115	-.246.117	-.246.117	-.280[.173](.171)	-.252.169	-.253.169
-.50	-.501.115	-.499.117	-.498.117	-.526[.192](.190)	-.497.190	-.498.190
Lognormal Error, n=100						
.50	.492.075	.501.075	.502.075	.487[.073](.072)	.502.070	.501.070
.25	.242.090	.250.091	.251.091	.231[.106](.104)	.252.102	.251.102
.00	-.008[.103](.102)	.000.104	.000.104	-.021[.135](.133)	.004.131	.003.131
-.25	-.252.110	-.247.112	-.247.112	-.273[.160](.158)	-.244.157	-.245.157
-.50	-.499.110	-.496.112	-.496.112	-.524[.185](.183)	-.495.183	-.495.183
Normal Error, n=250						
.50	.496[.047](.046)	.500.046	.500.046	.492[.053](.052)	.500.051	.500.051
.25	.246.057	.250.057	.250.057	.237[.077](.076)	.249.075	.248.075
.00	-.004.066	-.001.066	-.001.066	-.012[.100](.099)	.003.098	.003.098
-.25	-.251.072	-.249.073	-.249.073	-.267[.122](.121)	-.250.120	-.251.120
-.50	-.501.072	-.500.073	-.500.073	-.521[.142](.141)	-.502.140	-.502.140
Normal Mixture, n=250						
.50	.496.046	.500.046	.500.046	.492[.052](.051)	.500.050	.500.050
.25	.248.059	.251.059	.251.059	.238[.077](.076)	.249.075	.249.075
.00	-.002.066	.001.066	.001.066	-.016[.099](.097)	-.001.096	-.002.096
-.25	-.252.070	-.250.071	-.250.071	-.265[.119](.118)	-.248.117	-.248.117
-.50	-.500.072	-.499.073	-.499.073	-.521[.143](.142)	-.501[.141](.140)	-.501.141
Lognormal Error, n=250						
.50	.496.046	.500.046	.500.046	.492[.052](.051)	.500.050	.500.050
.25	.247.056	.251.057	.251.057	.238.075	.250.074	.250.074
.00	-.004.065	-.001.066	-.001.066	-.015[.096](.095)	.000.094	-.001.094
-.25	-.252.068	-.250.069	-.250.069	-.266[.118](.117)	-.248.116	-.249.116
-.50	-.503.070	-.502.070	-.502.070	-.521[.143](.141)	-.500.140	-.501.140

Table 1b. Empirical Mean[rmse](sd) of Estimators of λ , 2FE-SPD Model with SLD, $T = 3, \beta = (.5,.5)', \sigma = 1$

λ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\lambda}_N^{bc3}$	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\lambda}_N^{bc3}$
(a) Queen Contiguity, REG1						
Normal Error, n=50						
.50	.477[.133](.132)	.500.133	.500.132	.449[.122](.111)	.498.105	.500.105
.25	.231[.157](.156)	.251.159	.252.158	.179[.171](.156)	.248.150	.250.150
.00	-.015[.176](.175)	.000.180	.002.180	-.086[.214](.196)	-.002.191	.001.191
-.25	-.261.180	-.252.185	-.251.185	-.348[.247](.227)	-.252.224	-.249.224
-.50	-.505[.185](.184)	-.502.190	-.501.190	-.609[.283](.262)	-.504.261	-.502.262
Normal Mixture, n=50						
.50	.478[.133](.132)	.501.133	.500.132	.449[.120](.109)	.498.103	.500.103
.25	.229[.158](.157)	.248.159	.249.159	.180[.168](.153)	.248.147	.250.147
.00	-.017[.174](.173)	-.002.177	.000.177	-.088[.212](.193)	-.003.188	.000.188
-.25	-.260.176	-.251.181	-.250.181	-.346[.247](.227)	-.250.224	-.247.225
-.50	-.502.181	-.499.186	-.499.186	-.608[.281](.260)	-.503.260	-.500.260
Lognormal Error, n=50						
.50	.480[.123](.122)	.502.123	.502.122	.454[.112](.102)	.502.097	.504.097
.25	.229[.148](.147)	.249[.150](.149)	.250.149	.184[.157](.143)	.251.138	.254.138
.00	-.013[.162](.161)	.002.165	.003.165	-.079[.193](.176)	.003.172	.006.172
-.25	-.258[.168](.167)	-.248.172	-.247.172	-.341[.225](.206)	-.247.203	-.244.203
-.50	-.504[.173](.172)	-.501.177	-.501.178	-.598[.258](.239)	-.495.239	-.493.240
Normal Error, n=100						
.50	.490.090	.502.090	.502.089	.469[.087](.081)	.499.079	.500.079
.25	.242.108	.253.109	.253.109	.205[.127](.119)	.248.117	.248.117
.00	-.003.122	.006.123	.006.123	-.058[.166](.155)	-.004.153	-.003.153
-.25	-.256[.130](.129)	-.250.131	-.249.131	-.313[.192](.181)	-.249.179	-.249.179
-.50	-.505.131	-.503.133	-.503.133	-.578[.223](.209)	-.506[.209](.208)	-.506.209
Normal Mixture, n=100						
.50	.491.088	.502.088	.502.088	.470[.087](.082)	.500.080	.500.079
.25	.241.105	.252.106	.252.106	.207[.124](.116)	.249.113	.250.113
.00	-.010.120	-.002.121	-.001.121	-.056[.160](.150)	-.001.148	-.001.148
-.25	-.254.129	-.248.131	-.247.131	-.314[.195](.184)	-.251.182	-.250.182
-.50	-.503.130	-.500.131	-.500.132	-.567[.217](.207)	-.496.206	-.495.206
Lognormal Error, n=100						
.50	.490.084	.502.084	.502.084	.470[.084](.079)	.500.077	.500.077
.25	.235[.102](.101)	.246.102	.246.102	.208[.120](.113)	.250.110	.251.110
.00	-.005.116	.004.117	.004.117	-.050[.151](.143)	.003.141	.004.141
-.25	-.258.121	-.252.123	-.252.123	-.316[.185](.172)	-.253.171	-.253.171
-.50	-.502.125	-.499.126	-.499.126	-.565[.208](.197)	-.495.197	-.495.197
Normal Error, n=250						
.50	.496.055	.501.055	.501.055	.483[.061](.058)	.499.057	.499.057
.25	.245.068	.250.068	.250.068	.225[.091](.088)	.249.087	.249.087
.00	-.002.075	.002.076	.002.076	-.028[.115](.112)	.003.111	.003.111
-.25	-.252.082	-.250.082	-.250.082	-.288[.144](.139)	-.250.138	-.249.138
-.50	-.502.083	-.501.083	-.501.083	-.546[.167](.160)	-.502.159	-.502.159
Normal Mixture, n=250						
.50	.496.054	.501.054	.501.054	.484[.061](.059)	.501.059	.501.059
.25	.246.067	.250.067	.250.067	.223[.092](.087)	.247.086	.247.086
.00	-.003.075	.000.076	.000.076	-.032[.117](.112)	-.001.111	.000.111
-.25	-.255.081	-.252.082	-.252.082	-.286[.140](.135)	-.248.134	-.248.134
-.50	-.501.083	-.501.083	-.500.083	-.544[.171](.165)	-.500.164	-.499.164
Lognormal Error, n=250						
.50	.495[.054](.053)	.500.053	.500.053	.484[.059](.057)	.501.056	.501.056
.25	.246.066	.251.066	.251.066	.224[.088](.084)	.248.083	.248.083
.00	-.003.075	.001.075	.001.075	-.033[.114](.109)	-.002.108	-.002.108
-.25	-.253.079	-.250.079	-.250.079	-.287[.138](.133)	-.249.132	-.249.132
-.50	-.499.081	-.498.081	-.498.081	-.541[.164](.159)	-.497.158	-.497.158

Table 2. Empirical Mean[rmse](sd) of Estimators of ρ - 2FE-SPD Model with SED, $T = 3, \beta = (1, 1)', \sigma = 1$

ρ	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\rho}_N^{bc3}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\rho}_N^{bc3}$
(a) Queen Contiguity, REG1						
Normal Error, n=50						
.50	.481[.144](.142)	.500.143	.500.142	.457[.139](.132)	.503.116	.503.115
.25	.233[.171](.170)	.252.171	.254.171	.177[.202](.188)	.258.167	.260[.167](.166)
.00	-.018[.190](.189)	-.001.190	.001[.191](.190)	-.115[.266](.240)	-.004.221	-.001.220
-.25	-.271[.202](.201)	-.255.203	-.254.204	-.382[.299](.268)	-.250.256	-.249.256
-.50	-.516[.203](.202)	-.503.205	-.502.206	-.637[.321](.290)	-.496.287	-.497.288
Normal Mixture, n=50						
.50	.480[.139](.138)	.500.138	.500.137	.458[.137](.130)	.504.114	.504.113
.25	.233[.166](.165)	.252.166	.251.166	.168[.210](.194)	.251.172	.250.171
.00	-.016[.186](.185)	.002.186	.003.186	-.108[.258](.234)	.004.214	.003.214
-.25	-.267[.195](.194)	-.252.196	-.250.197	-.381[.293](.262)	-.248.251	-.249.251
-.50	-.511[.198](.197)	-.498.200	-.498.201	-.636[.313](.282)	-.493.280	-.495.281
Lognormal Error, n=50						
.50	.483[.135](.133)	.504.134	.503.133	.454[.136](.128)	.502.112	.502.111
.25	.237[.160](.159)	.256[.161](.160)	.255.160	.174[.196](.181)	.257.160	.256.160
.00	-.012.179	.006.180	.005.180	-.105[.242](.218)	.009.199	.002.199
-.25	-.264.186	-.248.188	-.249.188	-.368[.273](.247)	-.233.235	-.239[.236](.235)
-.50	-.512.191	-.499.194	-.499.194	-.632[.305](.275)	-.489.272	-.489[.274](.273)
Normal Error, n=100						
.50	.490[.096](.095)	.500.095	.500.095	.467[.107](.102)	.501.093	.501.093
.25	.241.119	.251.119	.251.118	.196[.152](.142)	.252.132	.251.132
.00	-.011.132	-.001.132	.000.132	-.074[.192](.177)	-.002.171	-.002.171
-.25	-.259[.141](.140)	-.249.141	-.249.141	-.333[.215](.199)	-.255.199	-.255.199
-.50	-.510.142	-.501.143	-.501.143	-.574[.220](.207)	-.500.215	-.500.215
Normal Mixture, n=100						
.50	.489[.095](.094)	.500.094	.500.094	.465[.104](.098)	.500.090	.500.090
.25	.240[.118](.117)	.250.117	.250.117	.196[.149](.139)	.253.130	.253.130
.00	-.010.130	.001.130	.001.130	-.073[.189](.174)	.000.168	.000.168
-.25	-.260.138	-.250.138	-.249.138	-.327[.211](.196)	-.249.197	-.249.197
-.50	-.510.138	-.501.139	-.501.139	-.569[.220](.209)	-.495.219	-.495.219
Lognormal Error, n=100						
.50	.494.088	.505.088	.505.088	.465[.107](.101)	.501.092	.500.092
.25	.240.110	.251.110	.251.110	.198[.145](.135)	.256.126	.256[.126](.125)
.00	-.006.126	.004[.127](.126)	.003[.127](.126)	-.064[.174](.162)	.010.156	.010.156
-.25	-.259.136	-.250.136	-.249.136	-.320[.200](.188)	-.239[.189](.188)	-.239.189
-.50	-.508.135	-.500.136	-.500.136	-.561[.214](.205)	-.485.215	-.486.215
Normal Error, n=250						
.50	.496.059	.500.059	.500.059	.477[.080](.077)	.498.073	.498.073
.25	.246.072	.250.072	.250.072	.215[.117](.111)	.251.105	.251.105
.00	-.003.083	.001.083	.001.083	-.053[.157](.147)	-.003.140	-.003.140
-.25	-.255[.088](.087)	-.251.088	-.251.088	-.313[.186](.175)	-.250.168	-.250.168
-.50	-.505.089	-.501.090	-.501.090	-.577[.212](.198)	-.505.193	-.506.193
Normal Mixture, n=250						
.50	.497.059	.501.059	.501.059	.482[.077](.075)	.503.070	.502.070
.25	.245.072	.249.072	.249.072	.214[.117](.111)	.250.105	.250.105
.00	-.004.081	.000.081	.000.081	-.052[.154](.144)	-.001.137	-.001.137
-.25	-.256.088	-.252.088	-.252.088	-.314[.180](.168)	-.250.161	-.251.161
-.50	-.503.088	-.500.089	-.500.089	-.570[.212](.200)	-.498.195	-.498.195
Lognormal Error, n=250						
.50	.496.057	.500.057	.500.057	.478[.080](.077)	.500.072	.499.072
.25	.248.070	.252.070	.252.070	.216[.114](.109)	.254.103	.253.103
.00	-.003.081	.001.081	.001.081	-.051[.147](.138)	.002.131	.001.131
-.25	-.255.085	-.251.085	-.251.085	-.306[.176](.167)	-.241.160	-.242.160
-.50	-.505.087	-.502.087	-.501.087	-.564[.211](.201)	-.490.196	-.491.196

Table 2-r. Replicates of Case (a) Table 2: Queen Contiguity, REG1

ρ	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\rho}_N^{bc3}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\rho}_N^{bc3}$
	(a) $\beta = (.5 .5)', iid\ Bootstrap$			(b) $\beta = (1 1)', Wild\ Bootstrap$		
Normal Error, n=50						
.50	.481[.144](.142)	.502.143	.502.142	.480[.144](.142)	.501.145	.501.144
.25	.235[.169](.168)	.255.169	.256.169	.233[.171](.170)	.254.172	.255.172
.00	-.016.188	.003.189	.005.189	-.017[.191](.190)	.002.194	.004.194
-.25	-.267[.203](.202)	-.250.205	-.249.205	-.267[.199](.198)	-.251.204	-.249.204
-.50	-.511.205	-.497.208	-.497.209	-.516.201	-.503.207	-.502.208
Normal Mixture, n=50						
.50	.479[.142](.141)	.499.141	.499.140	.481[.142](.141)	.502.144	.502.143
.25	.228[.169](.167)	.248.168	.249.167	.234.168	.254.172	.256.172
.00	-.016[.187](.186)	.003.187	.005.187	-.015.188	.004.192	.006.192
-.25	-.262[.199](.198)	-.246.200	-.244.201	-.265.199	-.249.205	-.248.205
-.50	-.512.199	-.499.202	-.498.203	-.509.196	-.496.203	-.495[.204](.203)
Lognormal Error, n=50						
.50	.483[.132](.131)	.504.131	.504.130	.482[.132](.130)	.503.135	.503.134
.25	.234[.159](.158)	.254.159	.256.159	.235[.158](.157)	.256[.163](.162)	.257.162
.00	-.015.177	.003.178	.005.178	-.016.177	.003.183	.005.183
-.25	-.262.190	-.245.192	-.244.192	-.265[.188](.187)	-.246.195	-.245.195
-.50	-.513.192	-.500.194	-.499.195	-.510.193	-.496.202	-.495.202
Normal Error, n=100						
.50	.489.096	.500.096	.500.095	.491[.096](.095)	.501.096	.501.095
.25	.242.116	.253.116	.253.116	.241.117	.251.118	.251.118
.00	-.010.133	.000.133	.001.133	-.007.131	.003.132	.004.132
-.25	-.262.142	-.252.142	-.252.142	-.258[.140](.139)	-.248.141	-.248.141
-.50	-.507.139	-.498.140	-.498.140	-.507[.142](.141)	-.498.143	-.498.143
Normal Mixture, n=100						
.50	.491.095	.501.095	.501.095	.492[.095](.094)	.503.096	.503.096
.25	.240.115	.250.115	.251.115	.240.115	.251.116	.251.116
.00	-.010.129	.001.129	.001.129	-.010[.128](.127)	.000.129	.001.129
-.25	-.259[.140](.139)	-.250.140	-.249.140	-.258.140	-.248.142	-.248.142
-.50	-.507.142	-.499.143	-.499.143	-.508.143	-.499.146	-.499.146
Lognormal Error, n=100						
.50	.492.092	.503.092	.503.091	.489.091	.501.092	.501.092
.25	.241.112	.252.112	.252.112	.240.111	.251.113	.252.113
.00	-.012[.126](.125)	-.001.126	-.001.126	-.008.126	.003.129	.003.129
-.25	-.260.134	-.250.134	-.250.134	-.258.132	-.248.135	-.248.135
-.50	-.510.137	-.502.138	-.502.138	-.509.136	-.500.140	-.500.140
Normal Error, n=250						
.50	.496.058	.500.058	.500.058	.495[.059](.058)	.499.059	.499.059
.25	.247.073	.251.073	.251.073	.246.073	.250.073	.250.073
.00	-.004.082	.000.082	.000.082	-.004.083	.000.084	.000.084
-.25	-.254.089	-.251.089	-.250.089	-.254.089	-.250.090	-.250.090
-.50	-.501.090	-.497.091	-.497.091	-.505.090	-.501.090	-.501.090
Normal Mixture, n=250						
.50	.496.059	.500.059	.500.059	.495.059	.499.059	.499.059
.25	.247.071	.251.071	.251.071	.247.072	.251.073	.251.073
.00	-.004.082	.000.082	.000.082	-.002.081	.002.081	.002.081
-.25	-.254.088	-.250.088	-.250.088	-.253.087	-.249.088	-.249.088
-.50	-.504.090	-.501.090	-.501.090	-.503.089	-.500.090	-.500.090
Lognormal Error, n=250						
.50	.496.057	.500.057	.500.057	.497.056	.501.056	.501.056
.25	.246.071	.250.071	.250.071	.246[.071](.070)	.250.071	.250.071
.00	-.003.080	.001.080	.001.080	-.005.078	-.001.079	-.001.079
-.25	-.252.087	-.248.087	-.248.087	-.253.084	-.250.085	-.249.085
-.50	-.504.086	-.501.087	-.501.087	-.503.087	-.500.088	-.500.088

Table 3a. Empirical Mean[rmse](sd) of Estimators of λ and ρ , 2FE-SPD Model with SARAR, $T = 3$, $\beta = (1, 1)'$, $\sigma = 1$, Queen Contiguity, REG-1

λ	ρ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$
(a) Normal Error, $n = 50$									
.50	.50	.484[.116](.115)	.500.116	.483[.143](.142)	.500.143	.486[.105](.104)	.502.105	.484[.131](.130)	.502.131
.25	.484[.119](.117)	.501.118	.226[.176](.174)	.242.175	.485[.114](.113)	.501.113	.233[.162](.161)	.250.161	
.00	.483[.118](.116)	.500.117	-.019[.192](.191)	-.002.192	.486[.110](.109)	.503.110	-.015[.177](.176)	.002.177	
-.25	.482[.124](.122)	.500.123	-.267.202	-.251.203	.487[.112](.111)	.503.112	-.265.193	.249.193	
-.50	.484[.125](.123)	.500.124	-.513.208	-.498.209	.489[.111](.110)	.505.111	-.514[.195](.194)	.499.196	
(b) Lognormal Error, $n = 50$									
-.50	.50	-.502.158	-.500.161	.486[.144](.143)	.504.144	-.502.145	-.500.148	.486[.132](.131)	.504[.132](.131)
.25	-.506.165	-.504.168	.232[.174](.173)	.249.174	-.505[.152](.151)	-.503[.155](.154)	.233[.161](.160)	.250.160	
.00	-.501.163	-.499.167	-.006.187	.010.187	-.499.159	-.497.162	-.018[.180](.179)	-.001.180	
-.25	-.500.164	-.498.168	-.262.209	-.246.210	-.501.152	-.499.155	-.263.197	-.246.197	
-.50	-.506.169	-.505.172	-.518[.207](.206)	-.503.208	-.498.157	-.497.160	-.513.194	-.498.195	
(c) Normal Error, $n = 100$									
.50	.50	.494[.078](.077)	.502.078	.490.096	.499.096	.490.078	.499.078	.493.090	.502.090
.25	.490.080	.499.080	.244[.117](.116)	.253.117	.491[.081](.080)	.500.080	.243.111	.252.111	
.00	.493.083	.502.083	-.011[.132](.131)	-.002.131	.494.079	.503.079	-.009.126	.001.126	
-.25	.491[.084](.083)	.500.083	-.258.142	-.249.142	.490.077	.499.077	-.264[.138](.137)	.254[.138](.137)	
-.50	.490[.079](.078)	.499.078	-.509[.142](.141)	-.499.142	.493.077	.501.077	-.509.137	.499.137	
(d) Lognormal Error, $n = 100$									
-.50	.50	-.494.118	-.493.119	.492.094	.501.094	-.503.106	-.503.107	.491[.089](.088)	.500.088
.25	-.501.119	-.500.121	.242.117	.251.117	-.502.112	-.501.113	.240.111	.249.111	
.00	-.496.115	-.495.117	-.008.133	.001.133	-.498.114	-.498.115	-.007.129	.003.128	
-.25	-.505.118	-.504.120	-.258.143	-.248.143	-.497.112	-.496.113	-.257.136	.248.136	
-.50	-.501.118	-.500.120	-.504.148	-.495.149	-.505.109	-.504.110	-.507.137	.498[.138](.137)	
(e) Normal Error, $n = 500$									
.50	.50	.497.033	.499.033	.499.041	.501.041	.499.030	.501.030	.497.040	.499.040
.25	.497.033	.499.033	.247.052	.249.052	.499.032	.501.032	.249.050	.250.050	
.00	.499.033	.501.033	.001.057	.003[.058](.057)	.498.033	.500.033	-.001.057	.001.057	
-.25	.498[.033](.032)	.499.033	-.254.062	-.252.062	.498.033	.500.033	-.250.061	.248.061	
-.50	.498.032	.500.032	-.503.062	-.501.062	.497.032	.499.032	-.501.062	.499.062	
(f) Lognormal Error, $n = 500$									
-.50	.50	-.502.049	-.501.049	.498.041	.500.041	-.499.049	-.499.049	.498.040	.500.040
.25	-.503.051	-.502.051	.249.051	.250.051	-.500.051	-.499.051	.248.050	.250.050	
.00	-.501.050	-.501.050	-.001.060	.001.060	-.501.051	-.500.052	-.002.058	.000.058	
-.25	-.502[.051](.050)	-.502.051	-.253.061	-.251.061	-.499.051	-.498.051	-.252.062	.250.062	
-.50	-.500.049	-.499.049	-.501.063	-.499.064	-.500.048	-.500.049	-.503.062	.502.062	

Table 3a-r. Replicates of Table 1a using Wild Bootstrap

λ	ρ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$
Normal Error, $n = 50$									
.50	.50	.480[.109](.108)	.498.109	.481[.145](.144)	.499.146	.491[.075](.074)	.499.075	.490.095	.499.096
.25	.481[.113](.111)	.500.113	.234[.175](.174)	.251.177	.491[.076](.075)	.499.075	.240[.118](.117)	.249.118	
.00	.481[.114](.113)	.500.114	-.017[.194](.193)	.001.195	.493.077	.501.077	-.011.136	-.002.136	
-.25	.481[.115](.114)	.499.115	-.269[.211](.210)	-.250.212	.490[.077](.076)	.499.077	-.257.143	-.247.144	
-.50	.482[.113](.112)	.500.113	-.521[.209](.208)	-.503.211	.492[.075](.074)	.500.075	-.512.145	-.503.146	
Normal Error, $n = 100$									
-.50	.50	-.505.157	-.500.162	.487[.143](.142)	.505.144	-.499.105	-.498.107	.491[.096](.095)	.500.096
.25	-.510.165	-.505.170	.230[.173](.171)	.248.173	-.499.112	-.498.114	.242.119	.251.119	
.00	-.505.166	-.499.172	-.013.195	.006.197	-.503.112	-.502.114	-.008.132	.001.133	
-.25	-.507.165	-.501.171	-.268[.210](.209)	-.248.211	-.500.110	-.499.112	-.257.140	-.248.141	
-.50	-.504.159	-.500.164	-.524[.207](.206)	-.504.210	-.501.108	-.500.110	-.509.142	-.500.143	
Normal Mixture, $n = 50$									
.50	.50	.485[.106](.105)	.503.107	.483[.139](.138)	.501.139	.493.071	.501.072	.493[.094](.093)	.502.095
.25	.481[.112](.111)	.500.113	.235.173	.253.176	.492.074	.500.074	.240.116	.249.117	
.00	.478[.115](.113)	.497.115	-.021[.193](.192)	-.002.195	.492.076	.501.077	-.006.131	.004.132	
-.25	.478[.113](.111)	.497.113	-.270[.203](.202)	-.252.207	.491[.076](.075)	.500.076	-.259[.139](.138)	-.250.139	
-.50	.480[.112](.110)	.497[.113](.112)	-.518[.208](.207)	-.500.212	.493[.075](.074)	.500.075	-.511[.141](.140)	-.501.142	
Normal Mixture, $n = 100$									
-.50	.50	-.509.154	-.503.160	.486.142	.503.144	-.499.104	-.498.106	.492.094	.501.095
.25	-.499.159	-.494.166	.234.171	.251.174	-.501.108	-.500.110	.241.114	.251.116	
.00	-.507.160	-.502.166	-.019[.193](.192)	.001.196	-.502.110	-.501.112	-.009.132	.000.133	
-.25	-.505.160	-.499.167	-.269[.206](.205)	-.248.210	-.501.110	-.500.112	-.262.140	-.253.142	
-.50	-.502.157	-.498.164	-.514.205	-.495.210	-.499.106	-.498.108	-.508.143	-.499.145	
Lognormal Error, $n = 50$									
.50	.50	.484[.102](.101)	.501.104	.487[.133](.132)	.505.135	.493.068	.501.069	.492.090	.502.091
.25	.481[.108](.106)	.500.108	.232[.163](.162)	.250.166	.491[.073](.072)	.499.074	.242[.113](.112)	.251.114	
.00	.481[.107](.105)	.500.109	-.018[.183](.182)	.001.186	.492[.074](.073)	.500.075	-.006[.127](.126)	.003.128	
-.25	.482[.108](.106)	.500.111	-.265.193	-.246.200	.491[.072](.071)	.499.072	-.258.136	-.248.138	
-.50	.485[.105](.104)	.502.107	-.515[.201](.200)	-.497.208	.493[.072](.071)	.501.072	-.508.134	-.500.137	
Lognormal Error, $n = 100$									
-.50	.50	-.500.150	-.495.157	.484.135	.503.137	-.497.100	-.497.103	.490[.091](.090)	.500.092
.25	-.506.154	-.500.162	.233[.167](.166)	.252.169	-.501.105	-.500.107	.240.110	.249.111	
.00	-.505.156	-.499.164	-.020[.189](.188)	-.001.193	-.502.105	-.501.107	-.008[.126](.125)	.002.127	
-.25	-.502.153	-.497.161	-.262[.196](.195)	-.244.202	-.502.103	-.502.106	-.258.136	-.248.137	
-.50	-.508.150	-.504.157	-.518[.200](.199)	-.499.206	-.499.102	-.498.105	-.512.135	-.502.138	

Table 3a-rr. Replicates of Table 1a using $\beta = (.5 \ .5)'$

λ	ρ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{\text{bc2}}$	$\hat{\rho}_N$	$\hat{\rho}_N^{\text{bc2}}$		$\hat{\lambda}_N$	$\hat{\lambda}_N^{\text{bc2}}$	$\hat{\rho}_N$	$\hat{\rho}_N^{\text{bc2}}$
Normal Error, $n = 50$										
.50	.50	.475[.130](.128)	.497.128	.481[.146](.145)	.501.145		.491.086	.501.086	.488[.096](.095)	.497.095
.50	.25	.477[.134](.132)	.500.133	.228[.173](.171)	.248.171		.492[.091](.090)	.502.091	.244.117	.252.117
.50	.00	.475[.138](.136)	.498.137	-.022[.194](.192)	-.001.192		.489.092	.500.092	-.012[.134](.133)	-.003.133
.50	-.25	.476[.132](.130)	.498.131	-.272[.203](.202)	-.251.203		.492.091	.502.091	-.256.138	-.247.138
.50	-.50	.475[.134](.131)	.497.132	-.523[.207](.206)	-.503.208		.489[.090](.089)	.498.089	-.509[.143](.142)	-.500.143
-.50	.50	-.501.179	-.497.184	.479[.143](.142)	.500.141		-.501.128	-.501.130	.491.096	.500.096
-.50	.25	-.507.184	-.502.189	.233[.173](.172)	.254[.172](.171)		-.503.130	-.502.132	.242.117	.251.117
-.50	.00	-.502.189	-.497.194	-.026[.195](.194)	-.003.193		-.500.133	-.499.135	-.009[.132](.131)	-.001.132
-.50	-.25	-.504.185	-.500.190	-.272[.204](.203)	-.249.204		-.498.131	-.498.133	-.260.142	-.251.142
-.50	-.50	-.498.180	-.493.185	-.523[.208](.207)	-.502.209		-.499.131	-.499.133	-.505.145	-.497.145
Normal Mixture, $n = 50$										
.50	.50	.478[.125](.123)	.501.124	.480[.142](.141)	.500.140		.488[.085](.084)	.499.084	.490[.095](.094)	.500.094
.50	.25	.474[.133](.131)	.499.131	.227[.176](.174)	.249.173		.489[.090](.089)	.500.089	.239.118	.249.117
.50	.00	.479[.132](.130)	.504.131	-.025[.193](.192)	-.001.191		.490.091	.501.091	-.010[.134](.133)	.001.133
.50	-.25	.473[.136](.133)	.498.134	-.278[.207](.205)	-.253.206		.491.088	.503.088	-.262.137	-.251.137
.50	-.50	.478[.128](.126)	.502.126	-.527[.208](.206)	-.503.208		.488.087	.499.087	-.514[.141](.140)	-.503.141
-.50	.50	-.509.174	-.503.178	.479[.145](.144)	.500.143		-.506.124	-.503.126	.491.094	.500.094
-.50	.25	-.506.179	-.498.184	.229[.173](.172)	.251.171		-.503.130	-.500.132	.240.117	.251.117
-.50	.00	-.504.180	-.496.185	-.022[.190](.189)	.002.188		-.502.129	-.499.131	-.011[.132](.131)	.000.131
-.50	-.25	-.508.182	-.500.187	-.272[.201](.200)	-.247.200		-.501.130	-.499.132	-.262[.140](.139)	-.250.139
-.50	-.50	-.509.177	-.501.182	-.523[.200](.199)	-.499.201		-.501.128	-.498.130	-.511[.141](.140)	-.500.141
Lognormal Error, $n = 50$										
.50	.50	.480[.122](.120)	.501.121	.484[.134](.133)	.503.134		.491.082	.501.082	.491.089	.501.089
.50	.25	.478[.125](.123)	.501.124	.237[.163](.162)	.256.162		.490[.083](.082)	.501.082	.239[.111](.110)	.249.110
.50	.00	.481[.126](.125)	.505.126	-.014.184	.005[.184](.183)		.491.083	.502.083	-.011.126	-.001.125
.50	-.25	.477[.127](.125)	.501.125	-.266[.195](.194)	-.247.194		.491.085	.502.085	-.260.135	-.250.135
.50	-.50	.477[.123](.121)	.501.122	-.513.199	-.495.200		.490[.083](.082)	.500.082	-.510.139	-.499.139
-.50	.50	-.502.169	-.500.174	.483[.135](.134)	.501.134		-.500.121	-.498.122	.495.090	.504.090
-.50	.25	-.505.174	-.502.179	.236[.164](.163)	.255.163		-.501.127	-.499.128	.240[.113](.112)	.250.112
-.50	.00	-.504.176	-.501.182	-.013[.182](.181)	.006.181		-.503.128	-.501.130	-.010.126	.001.125
-.50	-.25	-.505.170	-.501.175	-.269[.194](.193)	-.250.193		-.501.122	-.499.123	-.260.138	-.250.138
-.50	-.50	-.503.168	-.498.173	-.524[.193](.191)	-.506.193		-.501.122	-.499.124	-.511.139	-.501.140

Table 3b. Empirical Mean[rmse](sd) of Estimators of λ and ρ , 2FE-SPD Model with SARAR, $T = 3$, $\beta = (1, 1)'$, $\sigma = 1$, Group Interaction, REG-2

λ	ρ	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$	$\hat{\lambda}_N$	$\hat{\lambda}_N^{bc2}$	$\hat{\rho}_N$	$\hat{\rho}_N^{bc2}$
(a) Normal Error, $n = 50$									
.50	.50	.484[.095](.094)	.499.092	.453[.156](.149)	.500.129	.484[.089](.088)	.500.087	.456[.146](.140)	.505.121
.25	.50	.480[.103](.101)	.497.099	.162[.238](.221)	.248.194	.484[.096](.095)	.501.093	.161[.237](.220)	.251.193
.00	.50	.481[.104](.102)	.498.100	-.120[.298](.272)	.001.243	.486[.097](.096)	.501.093	-.120[.301](.276)	.005.247
-.25	.50	.481[.104](.102)	.496.100	-.408[.362](.326)	-.257.299	.488[.097](.096)	.502.094	-.407[.365](.330)	-.252.306
-.50	.50	.484[.099](.098)	.498.096	-.685[.400](.354)	-.512[.335](.334)	.491.095	.504.093	-.682[.413](.370)	-.506.354
(b) Lognormal Error, $n = 50$									
-.50	.50	-.527[.218](.216)	-.499.218	.453[.158](.150)	.501.130	-.522[.214](.213)	-.494.215	.458[.147](.141)	.507[.123](.122)
.25	.50	-.534[.237](.235)	-.505[.237](.236)	.164[.235](.219)	.251.191	-.524[.226](.225)	-.495.227	.171[.220](.205)	.259.179
.00	.50	-.532[.239](.237)	-.504.239	-.117[.301](.277)	.004.249	-.528[.239](.237)	-.501.239	-.114[.293](.270)	.010.242
-.25	.50	-.530[.237](.235)	-.504.237	-.407[.357](.320)	-.257.295	-.519.240	-.494.241	-.396[.349](.317)	-.243.293
-.50	.50	-.524[.233](.232)	-.500.233	-.689[.403](.355)	-.518[.337](.336)	-.528[.251](.250)	-.505.252	-.661[.399](.364)	-.489.345
(c) Normal Error, $n = 250$									
.50	.50	.497.044	.501.044	.477[.082](.079)	.500.074	.497.043	.500.042	.477[.081](.078)	.500.073
.25	.50	.497.043	.500.043	.209[.124](.117)	.250.110	.497.042	.500.042	.209[.119](.112)	.250.105
.00	.50	.497[.041](.040)	.499.040	-.056[.161](.151)	.001.142	.498.040	.500.039	-.056[.158](.148)	.002.138
-.25	.50	.498.038	.500.038	-.327[.204](.189)	-.253.178	.498.038	.500.038	-.322[.194](.180)	-.247.169
-.50	.50	.499.035	.500.035	-.590[.232](.214)	-.501.203	.500.035	.501.035	-.588[.229](.211)	-.497.200
(d) Lognormal Error, $n = 250$									
-.50	.50	-.508[.123](.122)	-.498.122	.476[.082](.078)	.499.073	-.509[.122](.121)	-.498.121	.476[.081](.078)	.500.073
.25	.50	-.510.118	-.502.118	.213[.121](.115)	.253.108	-.504.118	-.496.118	.210[.120](.113)	.251.106
.00	.50	-.507.116	-.500.116	-.063[.167](.155)	-.005.146	-.509.113	-.502.113	-.058[.161](.150)	.000.140
-.25	.50	-.502.105	-.497.105	-.326[.201](.186)	-.252.175	-.507.105	-.502.105	-.320[.192](.179)	-.245.169
-.50	.50	-.506.099	-.502.099	-.592[.235](.216)	-.503.204	-.503.100	-.499.100	-.589[.234](.217)	-.498.205
(e) Normal Error, $n = 500$									
.50	.50	.498.030	.500.030	.484[.065](.063)	.500.060	.498.030	.500.030	.484[.065](.063)	.501.060
.25	.50	.499.029	.500.029	.220[.098](.093)	.248.089	.498.029	.500.029	.223[.096](.092)	.252.087
.00	.50	.500.027	.501.027	-.040[.128](.122)	.001.116	.500.027	.501.027	-.044[.128](.120)	-.001.114
-.25	.50	.500.025	.501.025	-.303[.160](.151)	-.249.144	.500.025	.501.025	-.305[.158](.148)	-.249.141
-.50	.50	.499.023	.500.023	-.562[.187](.176)	-.496.168	.499.022	.500.022	-.565[.192](.180)	-.497.172
(f) Lognormal Error, $n = 500$									
-.50	.50	-.505.087	-.500.087	.485[.065](.063)	.500.060	-.505.085	-.499.085	.484[.064](.062)	.501.059
.25	.50	-.507.082	-.503.082	.220[.098](.094)	.248.089	-.504.081	-.500.081	.223[.096](.092)	.252.088
.00	.50	-.503.075	-.500.075	-.041[.131](.124)	.000.118	-.502.075	-.499.075	-.044[.127](.119)	-.001.113
-.25	.50	-.504.070	-.502.070	-.303[.161](.152)	-.249.145	-.501.071	-.499.071	-.303[.159](.150)	-.248.143
-.50	.50	-.501.065	-.499.065	-.569[.192](.179)	-.503.171	-.502.065	-.500.065	-.562[.187](.176)	-.494.168

Table 4. Empirical Means of the Non-Spatial Estimators, 2FE-SPD Model with SLDGroup Interaction, REG2, $T = 3$

λ	$\hat{\beta}_{1N}$	$\hat{\beta}_{2N}$	$\hat{\sigma}_N^2$	$\hat{\beta}_{1N}^{bc}$	$\hat{\beta}_{2N}^{bc}$	$\hat{\sigma}_N^{2,bc}$	$\hat{\beta}_{1N}$	$\hat{\beta}_{2N}$	$\hat{\sigma}_N^2$	$\hat{\beta}_{1N}^{bc}$	$\hat{\beta}_{2N}^{bc}$	$\hat{\sigma}_N^{2,bc}$
(a) $\beta = (1, 1)', \sigma = 1$												
Normal Error, n=50												
.50	1.041	1.035	0.984	0.996	0.998	0.992	0.533	0.530	0.985	0.496	0.499	0.991
.25	1.039	1.030	0.982	0.997	0.995	0.992	0.532	0.524	0.981	0.498	0.496	0.991
.00	1.035	1.023	0.980	0.997	0.992	0.992	0.529	0.519	0.978	0.498	0.494	0.991
-.25	1.032	1.023	0.978	0.997	0.995	0.992	0.524	0.519	0.975	0.496	0.496	0.992
-.50	1.030	1.019	0.974	0.999	0.994	0.989	0.527	0.514	0.970	0.501	0.494	0.990
Normal Mixture, n=50												
.50	1.040	1.031	0.975	0.996	0.994	0.982	0.532	0.520	0.981	0.495	0.490	0.988
.25	1.041	1.030	0.973	1.000	0.996	0.982	0.531	0.523	0.973	0.497	0.495	0.983
.00	1.038	1.030	0.973	1.001	0.998	0.984	0.526	0.518	0.973	0.495	0.493	0.986
-.25	1.035	1.025	0.966	1.001	0.997	0.980	0.524	0.515	0.963	0.496	0.492	0.979
-.50	1.028	1.023	0.969	0.997	0.997	0.985	0.521	0.520	0.962	0.496	0.500	0.981
Lognormal Error, n=50												
.50	1.036	1.031	0.944	0.994	0.995	0.951	0.529	0.523	0.946	0.493	0.493	0.952
.25	1.036	1.032	0.947	0.996	0.999	0.957	0.529	0.521	0.946	0.496	0.494	0.956
.00	1.028	1.020	0.936	0.992	0.990	0.947	0.525	0.519	0.944	0.495	0.494	0.957
-.25	1.029	1.019	0.942	0.996	0.992	0.955	0.522	0.517	0.943	0.494	0.494	0.959
-.50	1.026	1.017	0.940	0.996	0.993	0.956	0.518	0.514	0.926	0.494	0.494	0.945
Normal Error, n=100												
.50	1.028	1.023	0.993	1.000	0.999	0.996	0.526	0.521	0.993	0.501	0.499	0.996
.25	1.027	1.019	0.991	1.000	0.996	0.995	0.524	0.517	0.990	0.500	0.496	0.995
.00	1.023	1.020	0.990	0.998	0.999	0.996	0.524	0.516	0.991	0.501	0.496	0.997
-.25	1.020	1.020	0.989	0.996	1.000	0.995	0.521	0.514	0.988	0.499	0.496	0.995
-.50	1.024	1.018	0.988	1.002	0.999	0.995	0.520	0.514	0.986	0.500	0.497	0.994
Normal Mixture, n=100												
.50	1.026	1.022	0.990	0.998	0.998	0.993	0.523	0.518	0.988	0.497	0.497	0.991
.25	1.024	1.019	0.987	0.998	0.996	0.992	0.525	0.519	0.986	0.501	0.498	0.990
.00	1.022	1.018	0.985	0.997	0.996	0.990	0.522	0.515	0.985	0.499	0.496	0.991
-.25	1.023	1.018	0.987	1.000	0.998	0.994	0.523	0.517	0.983	0.501	0.499	0.991
-.50	1.022	1.019	0.982	1.000	1.001	0.989	0.518	0.515	0.983	0.498	0.498	0.992
Lognormal Error, n=100												
.50	1.024	1.021	0.973	0.997	0.998	0.977	0.524	0.518	0.969	0.499	0.497	0.972
.25	1.025	1.023	0.964	1.000	1.002	0.968	0.522	0.516	0.966	0.498	0.496	0.971
.00	1.023	1.015	0.963	0.999	0.995	0.969	0.520	0.514	0.962	0.497	0.495	0.968
-.25	1.022	1.016	0.970	0.999	0.997	0.977	0.520	0.516	0.964	0.499	0.498	0.972
-.50	1.021	1.012	0.960	1.000	0.995	0.966	0.516	0.514	0.958	0.497	0.498	0.967
Normal Error, n=250												
.50	1.011	1.010	0.997	0.999	0.998	0.999	0.512	0.512	0.997	0.499	0.499	0.998
.25	1.010	1.009	0.996	0.998	0.997	0.998	0.512	0.512	0.996	0.500	0.500	0.998
.00	1.009	1.009	0.996	0.998	0.997	0.998	0.509	0.509	0.996	0.497	0.497	0.998
-.25	1.009	1.010	0.996	0.997	0.998	0.999	0.508	0.511	0.995	0.497	0.500	0.998
-.50	1.009	1.010	0.995	0.998	0.999	0.998	0.511	0.510	0.994	0.500	0.499	0.997
Normal Mixture, n=250												
.50	1.014	1.013	0.997	1.002	1.000	0.998	0.513	0.509	0.996	0.500	0.497	0.997
.25	1.012	1.010	0.993	1.000	0.998	0.995	0.512	0.511	0.995	0.500	0.498	0.996
.00	1.010	1.011	0.995	0.998	0.999	0.997	0.510	0.512	0.993	0.498	0.500	0.996
-.25	1.012	1.011	0.996	1.001	1.000	0.998	0.510	0.510	0.997	0.498	0.498	1.000
-.50	1.009	1.008	0.994	0.998	0.997	0.996	0.510	0.509	0.993	0.499	0.498	0.996
Lognormal Error, n=250												
.50	1.011	1.010	0.986	0.999	0.998	0.987	0.511	0.511	0.982	0.498	0.498	0.983
.25	1.012	1.013	0.985	1.000	1.001	0.987	0.513	0.513	0.986	0.501	0.501	0.988
.00	1.010	1.009	0.983	0.998	0.998	0.985	0.511	0.511	0.984	0.499	0.499	0.987
-.25	1.010	1.009	0.982	0.999	0.997	0.985	0.512	0.510	0.984	0.500	0.498	0.987
-.50	1.007	1.007	0.985	0.996	0.997	0.987	0.509	0.508	0.983	0.498	0.497	0.986

Table 4-r. Empirical Means of the Non-Spatial Estimators, 2FE-SPD Model with SLD

$\beta = (1 \ 1)', \sigma = 1, \text{REG1}, T = 3$

λ	$\hat{\beta}_{1N}$	$\hat{\beta}_{2N}$	$\hat{\sigma}_N^2$	$\hat{\beta}_{1N}^{bc}$	$\hat{\beta}_{2N}^{bc}$	$\hat{\sigma}_N^{2,bc}$	$\hat{\beta}_{1N}$	$\hat{\beta}_{2N}$	$\hat{\sigma}_N^2$	$\hat{\beta}_{1N}^{bc}$	$\hat{\beta}_{2N}^{bc}$	$\hat{\sigma}_N^{2,bc}$
(a) Group Interaction, iid Bootstrap												
Normal Error, n=50												
.50	.999	.997	.982	.999	.996	.990	.998	.993	.980	.998	.992	.989
.25	.995	.999	.982	.997	.999	.992	.993	.998	.982	.993	.998	.992
.00	.993	.992	.981	.996	.993	.992	.998	.995	.980	.999	.996	.990
-.25	.991	.994	.978	.995	.996	.990	.996	.997	.979	.996	.997	.990
-.50	.993	.994	.975	.998	.997	.988	.997	.995	.981	.997	.995	.992
Normal Mixture, n=50												
.50	.997	1.000	.978	.998	.999	.987	.993	.998	.974	.993	.997	.983
.25	.996	.998	.969	.997	.998	.979	.999	.997	.972	1.000	.997	.981
.00	.991	.999	.974	.993	1.000	.985	.995	.996	.972	.995	.996	.983
-.25	.993	.997	.973	.997	.999	.985	.994	.994	.971	.995	.994	.981
-.50	.995	.992	.973	.999	.995	.986	.997	.999	.976	.997	.999	.986
Lognormal Error, n=50												
.50	.997	.997	.945	.997	.995	.953	.996	.999	.945	.996	.998	.954
.25	.999	.995	.942	1.000	.995	.951	.999	.995	.939	1.000	.995	.949
.00	.994	.991	.942	.997	.992	.952	.998	.997	.939	.999	.997	.949
-.25	.996	.996	.938	.999	.998	.950	.998	.995	.943	.998	.995	.953
-.50	.995	.997	.937	.999	1.000	.950	.996	.994	.939	.996	.994	.949
Normal Error, n=100												
.50	1.001	1.000	.992	1.001	1.000	.996	1.001	1.000	.992	1.000	.998	.996
.25	.999	.997	.990	.999	.997	.995	.996	.996	.991	.996	.995	.996
.00	.996	.999	.989	.997	1.000	.995	.999	.999	.991	.998	.999	.996
-.25	.995	.999	.989	.997	1.001	.995	1.002	.998	.990	1.002	.998	.995
-.50	.996	.994	.989	.999	.997	.995	1.000	.998	.990	1.000	.998	.995
Normal Mixture, n=100												
.50	.998	.999	.987	.998	.999	.991	.997	1.001	.986	.996	.999	.991
.25	.997	.998	.985	.997	.999	.989	1.001	.996	.989	1.000	.995	.994
.00	.999	.997	.984	1.000	.998	.989	1.001	.999	.990	1.001	.998	.995
-.25	.996	.997	.985	.998	.998	.991	1.000	.999	.985	1.000	.998	.991
-.50	.998	.996	.987	1.001	.998	.994	1.000	.998	.986	1.001	.998	.991
Lognormal Error, n=100												
.50	.999	.999	.976	.999	.998	.980	1.000	.998	.968	.999	.996	.972
.25	.999	.998	.966	1.000	.999	.970	.999	.998	.965	.998	.997	.969
.00	.999	.998	.964	1.000	1.000	.969	.999	.998	.969	.999	.998	.974
-.25	.996	.998	.964	.998	1.000	.970	.998	1.001	.967	.998	1.001	.972
-.50	.997	.996	.968	.999	.998	.974	1.000	.997	.964	1.000	.997	.969
Normal Error, n=250												
.50	1.000	1.002	.996	.999	1.001	.998	.999	1.001	.997	.999	1.001	.998
.25	.999	1.000	.997	.998	.999	.999	1.000	.999	.996	.999	.999	.998
.00	.999	.999	.996	.999	.998	.999	.999	.999	.996	.998	.998	.998
-.25	.997	.999	.996	.997	.999	.998	1.000	.999	.996	1.000	.999	.998
-.50	.998	1.000	.996	.998	1.000	.998	.999	.999	.996	.999	.999	.998
Normal Mixture, n=250												
.50	1.000	1.001	.996	.999	1.000	.998	.999	1.002	.994	.999	1.001	.996
.25	1.000	1.001	.995	.999	1.000	.997	1.000	1.001	.993	1.000	1.000	.995
.00	1.000	.999	.995	1.000	.999	.997	.999	.998	.994	.999	.998	.996
-.25	.999	.999	.993	.999	.999	.995	.999	.997	.995	.999	.997	.997
-.50	.998	1.000	.994	.998	1.000	.997	1.000	1.001	.996	1.000	1.001	.998
Lognormal Error, n=250												
.50	1.001	1.002	.986	1.000	1.001	.988	1.000	.998	.989	.999	.998	.991
.25	.999	.999	.987	.999	.999	.989	.999	.998	.983	.999	.997	.985
.00	1.000	.998	.984	1.000	.998	.986	.999	.999	.985	.999	.999	.987
-.25	1.000	1.000	.984	.999	1.000	.987	.998	.999	.983	.998	.999	.985
-.50	1.000	1.000	.986	1.000	1.000	.988	1.000	.998	.986	1.000	.998	.988

Table 5a. Empirical Sizes: Two-Sided Tests of Spatial Dependence in SARAR Model
 Group Interaction, REG2, $T = 3$, $\beta = (1, 1)'$, $\sigma = 1$, Wild Bootstrap

n	Test	10%			5%			1%			10%			5%			1%		
		Normal Errors			Normal Mixture			Lognormal Errors			Normal Errors			Normal Mixture			Lognormal Errors		
$H_0 : \lambda = \rho = 0$																			
50	\mathcal{W}_{11}	.1974	.1288	.0546	.1918	.1232	.0450	.1616	.1062	.0456									
	\mathcal{W}_{22}	.1896	.1196	.0516	.1846	.1222	.0470	.1584	.1008	.0408									
	\mathcal{W}_{33}	.1520	.0906	.0388	.1428	.0874	.0302	.1318	.0778	.0300									
100	\mathcal{W}_{11}	.1732	.1048	.0348	.1652	.0964	.0384	.1416	.0860	.0286									
	\mathcal{W}_{22}	.1754	.1116	.0366	.1684	.1070	.0388	.1416	.0858	.0284									
	\mathcal{W}_{33}	.1290	.0764	.0224	.1228	.0734	.0266	.1192	.0676	.0208									
250	\mathcal{W}_{11}	.1406	.0808	.0208	.1364	.0736	.0198	.1104	.0620	.0162									
	\mathcal{W}_{22}	.1390	.0788	.0234	.1350	.0758	.0206	.1170	.0712	.0196									
	\mathcal{W}_{33}	.1148	.0618	.0174	.1102	.0576	.0154	.1026	.0564	.0170									
500	\mathcal{W}_{11}	.1334	.0740	.0176	.1168	.0682	.0142	.1128	.0630	.0136									
	\mathcal{W}_{22}	.1358	.0752	.0178	.1270	.0674	.0176	.1338	.0730	.0196									
	\mathcal{W}_{33}	.1088	.0548	.0128	.1000	.0528	.0118	.1096	.0552	.0118									
$H_0 : \lambda = 0, (\text{true } \rho = 0)$																			
50	\mathcal{W}_{11}	.1660	.1024	.0392	.1436	.0920	.0320	.1450	.0920	.0360									
	\mathcal{W}_{22}	.1622	.1044	.0382	.1578	.0968	.0378	.1590	.0970	.0410									
	\mathcal{W}_{33}	.1354	.0842	.0294	.1260	.0758	.0246	.1284	.0798	.0286									
100	\mathcal{W}_{11}	.1362	.0798	.0256	.1352	.0812	.0268	.1302	.0734	.0230									
	\mathcal{W}_{22}	.1532	.0908	.0282	.1494	.0906	.0294	.1332	.0758	.0230									
	\mathcal{W}_{33}	.1174	.0668	.0212	.1162	.0686	.0202	.1186	.0670	.0178									
250	\mathcal{W}_{11}	.1232	.0732	.0174	.1228	.0690	.0158	.1134	.0576	.0154									
	\mathcal{W}_{22}	.1266	.0726	.0170	.1238	.0682	.0160	.1174	.0616	.0154									
	\mathcal{W}_{33}	.1126	.0630	.0132	.1100	.0594	.0118	.1052	.0542	.0126									
500	\mathcal{W}_{11}	.1108	.0578	.0142	.1094	.0556	.0116	.1116	.0616	.0138									
	\mathcal{W}_{22}	.1198	.0588	.0148	.1120	.0576	.0128	.1198	.0662	.0160									
	\mathcal{W}_{33}	.1050	.0530	.0122	.1030	.0524	.0098	.1070	.0572	.0130									
$H_0 : \rho = 0, (\text{true } \lambda = 0)$																			
50	\mathcal{W}_{11}	.1730	.1054	.0392	.1714	.1070	.0382	.1498	.0902	.0328									
	\mathcal{W}_{22}	.1366	.0850	.0326	.1418	.0822	.0312	.1202	.0692	.0192									
	\mathcal{W}_{33}	.1268	.0794	.0280	.1214	.0710	.0262	.1056	.0598	.0170									
100	\mathcal{W}_{11}	.1604	.0980	.0268	.1478	.0856	.0250	.1292	.0710	.0198									
	\mathcal{W}_{22}	.1302	.0758	.0252	.1274	.0732	.0260	.1142	.0672	.0220									
	\mathcal{W}_{33}	.1124	.0630	.0198	.1056	.0612	.0196	.0952	.0568	.0164									
250	\mathcal{W}_{11}	.1358	.0742	.0192	.1304	.0724	.0192	.1030	.0506	.0122									
	\mathcal{W}_{22}	.1216	.0694	.0166	.1226	.0670	.0176	.1036	.0552	.0168									
	\mathcal{W}_{33}	.1074	.0570	.0132	.1054	.0556	.0126	.0880	.0456	.0132									
500	\mathcal{W}_{11}	.1306	.0704	.0158	.1126	.0600	.0140	.0976	.0514	.0124									
	\mathcal{W}_{22}	.1208	.0682	.0170	.1110	.0590	.0150	.1154	.0616	.0146									
	\mathcal{W}_{33}	.1030	.0528	.0114	.0928	.0466	.0106	.0966	.0478	.0116									

Note: \mathcal{W}_{jj} are defined in (3.12) for joint tests and (3.13) for one-directional tests.

Table 5a-r. Empirical Sizes: Two-Sided Tests of Spatial Dependence in SARAR Model
Group Interaction, REG2, $T = 3, \beta = (1, 1)', \sigma = 1$, iid Bootstrap

n	Test	10%			5%			1%			10%			5%			1%		
		Normal Errors			Normal Mixture			Lognormal Errors			10%			5%			1%		
$H_0 : \lambda = \rho = 0$																			
50	\mathcal{W}_{11}	.1738	.1114	.0392	.1616	.1038	.0426	.1390	.0896	.0374									
	\mathcal{W}_{22}	.1556	.0978	.0332	.1454	.0906	.0368	.1326	.0824	.0346									
	\mathcal{W}_{33}	.1390	.0824	.0252	.1322	.0778	.0296	.1180	.0730	.0302									
100	\mathcal{W}_{11}	.1420	.0776	.0244	.1314	.0726	.0238	.1098	.0628	.0178									
	\mathcal{W}_{22}	.1268	.0668	.0214	.1156	.0662	.0212	.1172	.0704	.0200									
	\mathcal{W}_{33}	.1164	.0618	.0174	.1046	.0580	.0166	.0948	.0528	.0152									
250	\mathcal{W}_{11}	.1266	.0654	.0174	.1196	.0638	.0126	.1088	.0580	.0154									
	\mathcal{W}_{22}	.1242	.0666	.0150	.1114	.0604	.0132	.1086	.0566	.0146									
	\mathcal{W}_{33}	.1126	.0606	.0118	.1004	.0534	.0106	.0962	.0492	.0124									
500	\mathcal{W}_{11}	.1230	.0656	.0158	.1176	.0628	.0128	.1026	.0514	.0124									
	\mathcal{W}_{22}	.1182	.0656	.0162	.1126	.0600	.0130	.1070	.0532	.0144									
	\mathcal{W}_{33}	.1104	.0584	.0152	.1014	.0524	.0112	.0922	.0472	.0114									
$H_0 : \lambda = 0, (\text{true } \rho = 0)$																			
50	\mathcal{W}_{11}	.1510	.0942	.0354	.1392	.0858	.0324	.1432	.0888	.0370									
	\mathcal{W}_{22}	.1568	.1004	.0378	.1498	.0910	.0338	.1380	.0884	.0374									
	\mathcal{W}_{33}	.1296	.0804	.0258	.1246	.0742	.0266	.1246	.0744	.0308									
100	\mathcal{W}_{11}	.1242	.0748	.0188	.1222	.0692	.0174	.1136	.0620	.0168									
	\mathcal{W}_{22}	.1224	.0734	.0194	.1198	.0686	.0188	.1250	.0726	.0208									
	\mathcal{W}_{33}	.1096	.0622	.0144	.1066	.0578	.0146	.0992	.0552	.0142									
250	\mathcal{W}_{11}	.1224	.0648	.0156	.1152	.0630	.0138	.1144	.0608	.0172									
	\mathcal{W}_{22}	.1228	.0654	.0160	.1180	.0634	.0142	.1206	.0648	.0186									
	\mathcal{W}_{33}	.1118	.0580	.0134	.1056	.0548	.0118	.1022	.0540	.0142									
500	\mathcal{W}_{11}	.1114	.0612	.0134	.1102	.0568	.0132	.1046	.0544	.0128									
	\mathcal{W}_{22}	.1134	.0618	.0144	.1142	.0588	.0132	.1100	.0610	.0152									
	\mathcal{W}_{33}	.1026	.0552	.0114	.1032	.0504	.0118	.0970	.0510	.0122									
$H_0 : \rho = 0 \text{ (true } \lambda = 0\text{)}$																			
50	\mathcal{W}_{11}	.1530	.0882	.0256	.1494	.0906	.0316	.1248	.0744	.0266									
	\mathcal{W}_{22}	.1104	.0582	.0136	.1098	.0622	.0192	.0994	.0572	.0212									
	\mathcal{W}_{33}	.1154	.0634	.0176	.1160	.0648	.0214	.1030	.0600	.0212									
100	\mathcal{W}_{11}	.1400	.0810	.0202	.1226	.0638	.0170	.1124	.0590	.0112									
	\mathcal{W}_{22}	.1120	.0584	.0150	.0946	.0514	.0128	.0920	.0496	.0138									
	\mathcal{W}_{33}	.1108	.0572	.0156	.0934	.0516	.0124	.0858	.0462	.0134									
250	\mathcal{W}_{11}	.1184	.0636	.0122	.1164	.0614	.0120	.0988	.0514	.0104									
	\mathcal{W}_{22}	.1098	.0552	.0104	.1004	.0504	.0130	.0910	.0460	.0120									
	\mathcal{W}_{33}	.1082	.0530	.0102	.0962	.0480	.0120	.0836	.0426	.0108									
500	\mathcal{W}_{11}	.1234	.0684	.0132	.1148	.0608	.0106	.0922	.0442	.0112									
	\mathcal{W}_{22}	.1132	.0562	.0150	.1044	.0516	.0104	.0856	.0454	.0114									
	\mathcal{W}_{33}	.1094	.0534	.0150	.1010	.0490	.0100	.0810	.0420	.0102									

Note: \mathcal{W}_{jj} are defined in (3.12) for joint tests and (3.13) for one-directional tests.

Table 5b. Empirical Sizes: Two-Sided Tests of $H_0 : \lambda = 0$ in SLD ModelGroup Interaction, REG2, $T = 3, \beta = (1, 1)', \sigma = 1$. \mathcal{T}_{jj} are defined in (3.14)

n	Test	10%	5%	1%	10%	5%	1%	10%	5%	1%
		Normal Errors			Normal Mixture			Lognormal Errors		
		.1422	.0850	.0232	.1254	.0676	.0190	.1068	.0552	.0140
50	\mathcal{T}_{11}	.1348	.0808	.0212	.1154	.0586	.0162	.1042	.0586	.0134
	\mathcal{T}_{22}	.1120	.0616	.0146	.0992	.0472	.0126	.0918	.0484	.0102
	\mathcal{T}_{33}									
100	\mathcal{T}_{11}	.1224	.0622	.0174	.1186	.0660	.0136	.1070	.0590	.0116
	\mathcal{T}_{22}	.1142	.0604	.0128	.1214	.0654	.0158	.1108	.0600	.0130
	\mathcal{T}_{33}	.1004	.0478	.0102	.1046	.0518	.0118	.0958	.0502	.0084
250	\mathcal{T}_{11}	.1148	.0584	.0176	.1042	.0540	.0112	.1006	.0512	.0142
	\mathcal{T}_{22}	.1130	.0622	.0172	.1128	.0604	.0128	.1140	.0572	.0150
	\mathcal{T}_{33}	.1006	.0526	.0130	.0946	.0506	.0086	.0996	.0466	.0124
500	\mathcal{T}_{11}	.1126	.0560	.0106	.1082	.0528	.0122	.0970	.0472	.0082
	\mathcal{T}_{22}	.1154	.0646	.0140	.1066	.0564	.0118	.1064	.0554	.0106
	\mathcal{T}_{33}	.1010	.0554	.0110	.0972	.0484	.0104	.0960	.0474	.0080

Table 5c. Empirical Sizes: Two-Sided Tests of $H_0 : \rho = 0$ in SED ModelGroup Interaction, REG2, $T = 3, \beta = (1, 1)', \sigma = 1$. \mathcal{T}_{jj} are defined in (3.14)

n	Test	10%	5%	1%	10%	5%	1%	10%	5%	1%
		Normal Errors			Normal Mixture			Lognormal Errors		
		.1572	.0920	.0282	.1492	.0846	.0236	.1282	.0666	.0164
50	\mathcal{T}_{11}	.1386	.0758	.0234	.1242	.0734	.0220	.1030	.0572	.0152
	\mathcal{T}_{22}	.1146	.0620	.0172	.1152	.0640	.0176	.0928	.0518	.0142
	\mathcal{T}_{33}									
100	\mathcal{T}_{11}	.1420	.0798	.0224	.1324	.0738	.0142	.1170	.0598	.0126
	\mathcal{T}_{22}	.1274	.0736	.0202	.1248	.0700	.0160	.1010	.0550	.0140
	\mathcal{T}_{33}	.1116	.0594	.0154	.1054	.0540	.0112	.0840	.0444	.0116
250	\mathcal{T}_{11}	.1224	.0630	.0140	.1128	.0568	.0114	.1028	.0544	.0124
	\mathcal{T}_{22}	.1190	.0656	.0172	.1096	.0560	.0142	.1056	.0566	.0166
	\mathcal{T}_{33}	.1006	.0518	.0124	.0882	.0450	.0114	.0880	.0466	.0114
500	\mathcal{T}_{11}	.1124	.0578	.0120	.1126	.0526	.0098	.1004	.0518	.0116
	\mathcal{T}_{22}	.1136	.0624	.0142	.1202	.0604	.0148	.1164	.0610	.0178
	\mathcal{T}_{33}	.0952	.0492	.0098	.1004	.0482	.0108	.0982	.0476	.0126

Table 6. Empirical Sizes: Two-Sided Tests of $H_0 : \beta_1 = \beta_2$ in SARAR ModelGroup Interaction, REG2, $T = 3, \sigma = 1, \lambda = \rho = 0$

n	Test	10%	5%	1%	10%	5%	1%	10%	5%	1%
		Normal Errors			Normal Mixture			Lognormal Errors		
		.1608	.1020	.0386	.1630	.1046	.0386	.1604	.0978	.0344
50	\mathcal{T}_{22}	.1154	.0650	.0214	.1190	.0678	.0206	.1138	.0614	.0204
	\mathcal{T}_{11}	.1334	.0744	.0228	.1344	.0794	.0218	.1334	.0782	.0218
	\mathcal{T}_{22}	.1012	.0546	.0138	.1042	.0536	.0126	.1032	.0534	.0120
100	\mathcal{T}_{11}	.1240	.0642	.0166	.1210	.0680	.0204	.1196	.0670	.0184
	\mathcal{T}_{22}	.1066	.0524	.0120	.1060	.0564	.0152	.1018	.0580	.0114
	\mathcal{T}_{11}	.1092	.0548	.0116	.1100	.0564	.0140	.1154	.0616	.0200
250	\mathcal{T}_{22}	.0958	.0472	.0092	.0978	.0472	.0100	.1022	.0536	.0146
	\mathcal{T}_{11}	.1624	.1004	.0376	.1624	.1024	.0390	.1610	.0992	.0376
	\mathcal{T}_{22}	.1136	.0654	.0196	.1204	.0666	.0208	.1136	.0640	.0216
500	\mathcal{T}_{11}	.1282	.0742	.0196	.1394	.0810	.0208	.1420	.0808	.0250
	\mathcal{T}_{22}	.0968	.0496	.0114	.1068	.0540	.0090	.1060	.0564	.0118
	\mathcal{T}_{11}	.1254	.0688	.0190	.1224	.0642	.0140	.1146	.0622	.0180
100	\mathcal{T}_{22}	.1050	.0568	.0142	.1024	.0480	.0094	.0990	.0526	.0132
	\mathcal{T}_{11}	.1240	.0626	.0152	.1130	.0594	.0130	.1220	.0650	.0160
	\mathcal{T}_{22}	.1102	.0502	.0124	.0978	.0482	.0096	.1084	.0552	.0122

Note: $\beta = (1, 1)'$ for upper panel, and $(.5, .5)'$ for lower panel. \mathcal{T}_{jj} are defined in (3.15).

Table 6-r. Empirical Sizes: Two-Sided Tests of $H_0 : \beta_1 = \beta_2$ in SARAR Model
Group Interaction, REG2, $T = 3, \sigma = 1, \lambda = 0, \rho = 0$.

n	Test	10%			5%			1%			10%			5%			1%		
		Normal Errors			Normal Mixture			Lognormal Errors			10%			5%			1%		
<i>iid Bootstrap, RGE2, $k = n^{0.6}$, $\beta = (1, 1)'$</i>																			
50	1	.1424	.0806	.0232	.1314	.0748	.0218	.1294	.0722	.0232									
	2	.1126	.0582	.0154	.1078	.0548	.0152	.1014	.0566	.0142									
100	1	.1202	.0684	.0180	.1190	.0672	.0198	.1168	.0628	.0178									
	2	.1062	.0582	.0142	.1046	.0556	.0146	.1022	.0530	.0142									
$\beta = (.5, .5)'$																			
50	1	.1376	.0770	.0224	.1338	.0804	.0212	.1294	.0746	.0220									
	2	.1066	.0564	.0136	.1068	.0574	.0134	.1026	.0574	.0126									
100	1	.1184	.0648	.0166	.1164	.0666	.0158	.1224	.0656	.0168									
	2	.1022	.0540	.0132	.1046	.0534	.0118	.1062	.0536	.0114									
<i>Wild Bootstrap, RGE2, $k = n^{0.6}$, $\beta = (1, 1)'$</i>																			
50	1	.1398	.0756	.0224	.1326	.0774	.0252	.1298	.0796	.0212									
	2	.1256	.0680	.0180	.1240	.0694	.0214	.1220	.0704	.0214									
100	1	.1098	.0620	.0186	.1172	.0616	.0160	.1184	.0634	.0180									
	2	.1060	.0598	.0162	.1106	.0606	.0132	.1132	.0588	.0138									
$\beta = (.5, .5)'$																			
50	1	.1370	.0738	.0204	.1346	.0812	.0230	.1412	.0800	.0214									
	2	.1200	.0646	.0174	.1252	.0710	.0192	.1218	.0678	.0190									
100	1	.1236	.0704	.0184	.1198	.0636	.0176	.1170	.0628	.0154									
	2	.1210	.0684	.0190	.1118	.0578	.0142	.1106	.0582	.0120									
<i>iid Bootstrap, RGE1, $k = n^{0.5}$, $\beta = (1, 1)'$</i>																			
50	1	.1166	.0602	.0126	.1202	.0598	.0146	.1058	.0552	.0128									
	2	.1150	.0600	.0128	.1204	.0596	.0148	.1082	.0536	.0130									
100	1	.1076	.0596	.0138	.1108	.0600	.0142	.1122	.0570	.0152									
	2	.1050	.0610	.0134	.1120	.0584	.0138	.1108	.0548	.0150									
$\beta = (.5, .5)'$																			
50	1	.1132	.0612	.0128	.1216	.0644	.0152	.0984	.0542	.0144									
	2	.1148	.0598	.0136	.1192	.0646	.0162	.1016	.0556	.0142									
100	1	.1112	.0628	.0134	.1064	.0556	.0124	.1054	.0536	.0098									
	2	.1110	.0614	.0134	.1054	.0540	.0120	.1052	.0528	.0096									

Note: Test 1 = \mathcal{T}_{11} , and Test 2 = \mathcal{T}_{22} , which are defined in (3.15).

Table 6-rr. Empirical Sizes: Two-Sided Tests of $H_0 : \beta_1 = \beta_2$ in SARAR Model
Group Interaction, REG2, $T = 3, \sigma = 1, \lambda = .25, \rho = .25$.

n	Test	10%			5%			1%			10%			5%			1%		
		Normal Errors			Normal Mixture			Lognormal Errors			10%			5%			1%		
<i>iid Bootstrap, RGE1, $k = n^{0.5}, \beta = (1, 1)'$</i>																			
50	1	.1112	.0604	.0124	.1124	.0554	.0116	.1120	.0600	.0118									
	2	.1144	.0602	.0136	.1154	.0586	.0110	.1136	.0614	.0132									
100	1	.1136	.0570	.0120	.1056	.0578	.0108	.1018	.0548	.0112									
	2	.1136	.0578	.0118	.1072	.0578	.0120	.1018	.0558	.0122									
$\beta = (.5, .5)'$																			
50	1	.1114	.0606	.0126	.1206	.0626	.0160	.1188	.0602	.0162									
	2	.1148	.0598	.0138	.1220	.0644	.0160	.1218	.0628	.0166									
100	1	.1072	.0530	.0094	.1006	.0480	.0096	.1030	.0508	.0134									
	2	.1086	.0564	.0108	.1030	.0498	.0098	.1030	.0526	.0130									
<i>iid Bootstrap, RGE2, $k = n^{0.5}, \beta = (1, 1)'$</i>																			
50	1	.1484	.0882	.0310	.1428	.0842	.0286	.1402	.0820	.0260									
	2	.1138	.0644	.0194	.1114	.0600	.0186	.1088	.0570	.0158									
100	1	.1290	.0752	.0256	.1228	.0734	.0214	.1256	.0702	.0222									
	2	.1022	.0554	.0154	.1006	.0528	.0124	.0980	.0516	.0144									
$\beta = (.5, .5)'$																			
50	1	.1496	.0900	.0308	.1510	.0836	.0274	.1492	.0894	.0306									
	2	.1146	.0636	.0212	.1124	.0572	.0158	.1120	.0626	.0180									
100	1	.1306	.0742	.0208	.1292	.0728	.0230	.1394	.0790	.0238									
	2	.1012	.0508	.0108	.0992	.0504	.0138	.1084	.0574	.0144									
<i>Wild Bootstrap, RGE2, $k = n^{0.5}, \beta = (1, 1)'$</i>																			
50	1	.1484	.0888	.0312	.1388	.0798	.0276	.1456	.0880	.0306									
	2	.1268	.0730	.0244	.1108	.0608	.0200	.1244	.0718	.0186									
100	1	.1330	.0740	.0210	.1356	.0758	.0244	.1284	.0716	.0248									
	2	.1074	.0648	.0170	.1166	.0644	.0192	.1120	.0624	.0186									
$\beta = (.5, .5)'$																			
50	1	.1502	.0898	.0316	.1406	.0836	.0264	.1436	.0806	.0280									
	2	.1270	.0722	.0250	.1120	.0602	.0188	.1106	.0616	.0154									
100	1	.1272	.0754	.0212	.1330	.0730	.0226	.1406	.0784	.0224									
	2	.1076	.0606	.0150	.1116	.0568	.0172	.1206	.0644	.0176									

Note: Test 1 = \mathcal{T}_{11} , and Test 2 = \mathcal{T}_{22} , which are defined in (3.15).

Table 7: QMLEs and Bias-Corrected QMLEs for the SPD-FE Models Based on the Full Data (Years 1970-86).

	SARAR		SAR		SED		Durbin-SAR		Durbin-SED	
	$\hat{\theta}_{ML}$	$\hat{\theta}_{ML}^{bc2}$								
SLD effect (λ)	0.0270	0.0294	0.2100	0.2129			0.4124	0.4205		
s.e	(0.0384)	(0.0388)	(0.0284)	(0.0287)			(0.0433)	(0.0440)		
t-ratio	0.7037	0.7563	7.3923	7.4118			9.5186	9.5559		
SED effect (ρ)	0.4068	0.4095			0.4374	0.4403			0.4101	0.4168
s.e	(0.0536)	(0.0552)			(0.0425)	(0.0429)			0.0436	(0.0443)
t-ratio	7.5937	7.4208			10.2813	10.2547			9.4120	9.4037
lg(pcap)	-0.0145	-0.0145	-0.0352	-0.0352	-0.0122	-0.0121	-0.0090	-0.0088	-0.0184	-0.0183
s.e	(0.0258)	(0.0254)	(0.0258)	(0.0256)	(0.0257)	(0.0251)	(0.0433)	(0.0258)	0.0268	(0.0263)
t-ratio	-0.5599	-0.5697	-1.3637	-1.3771	-0.4749	-0.4817	-0.3420	-0.3430	-0.6867	-0.6960
lg(pcp)	0.1553	0.1553	0.1585	0.1583	0.1548	0.1547	0.1591	0.1591	0.1662	0.1662
s.e	(0.0265)	(0.0264)	(0.0265)	(0.0258)	(0.0264)	(0.0264)	(0.0266)	(0.0264)	0.0272	(0.0268)
t-ratio	5.8638	5.8822	5.9803	6.1402	5.8581	5.8662	5.9888	6.0288	6.1140	6.1957
lg(emp)	0.7555	0.7553	0.6824	0.6812	0.7584	0.7583	0.7514	0.7515	0.7539	0.7539
s.e	(0.0294)	(0.0298)	(0.0298)	(0.0302)	(0.0290)	(0.0294)	(0.0299)	(0.0301)	0.0294	(0.0297)
t-ratio	25.7262	25.3540	22.8939	22.5454	26.1169	25.7695	25.1208	24.9504	25.6309	25.4225
unemp	-0.0012	-0.0012	-0.0015	-0.0015	-0.0012	-0.0012	-0.0006	-0.0006	-0.0009	-0.0009
s.e	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0005)	0.0005	(0.0005)
t-ratio	-2.3652	-2.3935	-3.1327	-3.1614	-2.3511	-2.3807	-1.1295	-1.1574	-1.7158	-1.7599
$W \cdot \lg(pcap)$							-0.0567	-0.0558	-0.0750	-0.0745
s.e							(0.0481)	(0.0475)	0.0575	(0.0569)
t-ratio							-1.1809	-1.1734	-1.3044	-1.3110
$W \cdot \lg(pcp)$							0.0066	0.0046	0.0901	0.0897
s.e							(0.0476)	(0.0483)	0.0594	(0.0608)
t-ratio							0.1391	0.0950	1.5161	1.4757
$W \cdot \lg(emp)$							-0.3159	-0.3228	-0.0130	-0.0141
s.e							(0.0544)	(0.0556)	0.0507	(0.0510)
t-ratio							-5.8105	-5.7999	-0.2559	-0.2758
$W \cdot unemp$							-0.0013	-0.0013	-0.0017	-0.0017
s.e							(0.0008)	(0.0008)	0.0010	(0.0010)
t-ratio							-1.5365	-1.5334	-1.7525	-1.7440

Table 8: QMLEs and Bias-Corrected QMLEs for the SPD-FE Models Based on a Subset Data (Years 1982-84).

	SARAR		SAR		SED		Durbin-SAR		Durbin-SED	
	$\hat{\theta}_{ML}^{bc2}$									
SLD effect (λ)	0.0552	0.0524	0.3074	0.3231			0.4963	0.5635		
s.e	(0.1218)	(0.1326)	(0.0763)	(0.0804)			(0.1117)	(0.1225)		
<i>t</i> -ratio	4.4529	3.9448	4.0296	4.0207			4.4443	4.6018		
SED effect (ρ)	0.5516	0.5940			0.6160	0.6371			0.5230	0.5788
s.e	(0.1360)	(0.1545)			(0.0979)	(0.1028)			(0.1104)	(0.1274)
<i>t</i> -ratio	4.0558	3.8449			6.2920	6.1987			4.7379	4.5433
$lg(pcap)$	-0.2469	-0.2361	-0.2839	-0.2799	-0.2322	-0.2262	-0.1069	-0.1108	-0.1168	-0.1161
s.e	(0.1046)	(0.1133)	(0.0853)	(0.0883)	(0.1065)	(0.1135)	(0.1177)	(0.1230)	(0.1139)	(0.1176)
<i>t</i> -ratio	-2.3605	-2.0841	-3.3297	-3.1705	-2.1801	-1.9933	-0.9088	-0.9007	-1.0261	-0.9876
$lg(pcp)$	0.5663	0.5368	0.5132	0.4902	0.5522	0.5344	0.3309	0.3325	0.4619	0.4768
s.e	(0.2343)	(0.2478)	(0.2078)	(0.2164)	(0.2289)	(0.2415)	(0.2439)	(0.2522)	(0.2328)	(0.2392)
<i>t</i> -ratio	2.4170	2.1668	2.4694	2.2659	2.4118	2.2126	1.3570	1.3184	1.9837	1.9932
$lg(emp)$	1.1873	1.1880	1.1149	1.1113	1.1796	1.1795	1.1393	1.1385	1.1046	1.0963
s.e	(0.0848)	(0.0862)	(0.0877)	(0.0890)	(0.0826)	(0.0828)	(0.0863)	(0.0886)	(0.0911)	(0.0942)
<i>t</i> -ratio	13.9952	13.7878	12.7139	12.4853	14.2798	14.2521	13.1989	12.8462	12.1188	11.6417
$unemp$	-0.0009	-0.0008	-0.0014	-0.0014	-0.0008	-0.0008	-0.0010	-0.0010	-0.0015	-0.0015
s.e	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)
<i>t</i> -ratio	-1.0818	-1.0203	-1.7243	-1.6917	-1.0505	-1.0297	-1.3149	-1.2655	-1.7725	-1.8093
$W \cdot lg(pcap)$							-0.0698	-0.0302	-0.1609	-0.1305
s.e							(0.1751)	(0.1816)	(0.2069)	(0.2134)
<i>t</i> -ratio							-0.3984	-0.1663	-0.7779	-0.6117
$W \cdot lg(pcp)$							0.3929	0.3195	0.9698	0.9718
s.e							(0.3661)	(0.3858)	(0.4193)	(0.4364)
<i>t</i> -ratio							1.0732	0.8280	2.3128	2.2267
$W \cdot lg(emp)$							-0.6881	-0.7761	-0.2377	-0.2664
s.e							(0.1959)	(0.2076)	(0.1862)	(0.1937)
<i>t</i> -ratio							-3.5131	-3.7384	-1.2768	-1.3755
$W \cdot unemp$							-0.0023	-0.0022	-0.0034	-0.0033
s.e							(0.0015)	(0.0015)	(0.0018)	(0.0018)
<i>t</i> -ratio							-1.5803	-1.4514	-1.9087	-1.8428

Additional References

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