A Supplement to Improved Likelihood Inferences for Weibull Regression Model

by Yan Shen and Zhenlin Yang

More simulation experiments were carried out to investigate the effect of different censoring percentages on the performance of the original MLE and the proposed methods. The experiments are designed for Type-I censored data, which are a special type of randomly censored data but with a fixed censoring time. Thus the censoring percentage can be controlled in simulation.

We consider the same Weibull regression model with an intercept as in the manuscript:

$$\log T = a_1 + a_2 X_2 + a_3 X_3 + Z/\beta.$$

For all the Monte Carlo experiments, $a' = (a_1, a_2, a_3)$ is set at $\{5, 1, 1\}$ or $\{5, 0, 1\}$, β takes values $\{0.5, 0.8, 1, 2, 5\}$, and n takes values $\{12, 20, 50\}$. The two covariates are generated independently, according to $\{x_{i2}\} \stackrel{iid}{\sim} N(0,1)/\sqrt{2}$ and $\{x_{i3}\} \stackrel{iid}{\sim} N(0,1)/\sqrt{2}$. The censoring percentage (cp) is set as $\{10\%, 20\%, 30\%\}$ respectively, and the corresponding censoring times for each β are obtained by generating random numbers from the model and taking the upper $10\%, 20\%, 30\%$ quantiles, which are summarized in the Table 1.

Table 1. The fixed censoring times

β	$a' = (5,1,1)$			$a' = (5,0,1)$		
	10%	20%	30%	10%	20%	30%
0.5	1076.8	450.44	229.38	925.40	415.68	221.36
0.8	665.94	343.97	208.91	537.79	304.71	196.82
1.0	584.54	321.41	205.87	457.43	278.26	190.39
2.0	495.54	302.45	210.70	361.29	247.49	187.07
5.0	494.06	314.82	227.07	344.29	248.95	196.67

1. Performance of the 2nd-order bias corrected MLEs

Tables 2-4 summarize the empirical mean, root-mean-square-error (rmse) and standard error (se) of the original and 2nd-order bias-corrected MLEs for the censoring percentages of $10\%, 20\%, 30\%$ respectively.

The results in the three tables show that, (i) for the shaper parameter β, the 2ndorder bias-corrected MLE $\hat{\beta}_n^{\mathrm{bc2}}$ is able to greatly reduce the bias of the original MLE $\hat{\beta}_n$ regardless of the censoring percentage, and also has smaller rmse's and se's; (ii) although the improvements of $\hat{a}_{in}^{\mathrm{bc2}}$ over $\hat{a}_{in}$, $i = 1, 2, 3$, are not so significant as that of $\hat{\beta}_n^{\mathrm{bc2}}$, $\hat{a}_{in}^{\mathrm{bc2}}$ is still generally better than $\hat{a}_{in}$ with respect to the bias. Hence the 2nd-order bias-corrected MLE $\hat{\theta}_n^{\mathrm{bc2}}$ is much superior to the original MLE $\hat{\theta}_n$ in terms of parameter estimation.

There are some other interesting observations. For example, when the censoring percentage increases, the bias of $\hat{\beta}_n$ also increases, but the other estimators, $\hat{\beta}_n^{\mathrm{bc2}}$, $\hat{a}_{in}$, $\hat{a}_{in}^{\mathrm{bc2}}$, behave in an irregular way. Moreover, the estimators $\hat{a}_{in}$ and $\hat{a}_{in}^{\mathrm{bc2}}$ with $\beta < 1$ are more sensitive to the change of the censoring percentage than the estimators with $\beta > 1$. The different performances for $\beta < 1$ and $\beta > 1$ also can be found in the following simulation results.

2. Performance of the significance tests

To compare the performances of the two t-ratios t_n and t_n^{bc2}, we set the covaraite coefficients to $a' = (a_1, a_2, a_3) = (5, 0, 1)$ and test $H_0 : a_2 = 0$. Table 5 reports the empirical significance levels of t_n and t_n^{bc2} for the censoring percentages of $10\%, 20\%, 30\%$. The variance estimate used in t_n^{bc2} is the inverse of the observed information matrix, $J_n^{-1}(\hat{\theta}_n^{\mathrm{bc2}})$, which is the same setting as in the manuscript.

Based on the results in the tables, the main observation is that the test t_n^{bc2} could offer huge reduction in significance level distortions given by t_n, and have the empirical levels getting close to their nominal levels faster than t_n. At the same time, two exceptional cases with $n = 12, \beta = 0.5$ and $n = 12, \beta = 1$ are found in Table 5 for the censoring percentage 30%. Their empirical significance levels indicate that when censoring percentage is high, t_n has a better performance than t_n^{bc2} for the scenario of small sample size and $\beta < 1$. We can find that for $\beta < 1$ and $n = 12$, the empirical significance levels decrease rather fast with the increase of the censoring percentage, which again implies the sensitiveness of the estimators for $\beta < 1$.

In general, the t_n^{bc2} based on the 2nd-order bias-corrected MLE outperforms the asymptotic test t_n greatly. If the effective data is few (i.g. small sample size and high censoring percentage) and $\beta < 1$, t_n is a better choice.

3. Confidence intervals for the shape parameter

Tables 6 summarizes the simulations results for the confidence interval for the shape parameter β. The 2nd-order variance estimate $\widehat{\mathrm{V}}_2(\hat{\theta}_n^{\mathrm{bc2}})$ is used for the construction of confidence interval.

The results show that, (*i*) for censoring percentage 10% , $1-\gamma = 0.90, 0.95$ and censoring percentage 20% , $1-\gamma = 0.90$, $\mathrm{CI}_2(\beta)$ with $\widehat{\mathrm{V}}_2(\hat{\theta}_n^{\mathrm{bc2}})$ has the coverage probabilities much closer to the nominal levels compared to $\mathrm{CI}_1(\beta)$; (*ii*) for almost all censored situations with $1-\gamma = 0.99$, $\mathrm{CI}_1(\beta)$ has better performances in terms of both coverage probability and interval length; (*iii*) for other censored cases, $\mathrm{CI}_2(\beta)$ has the advantage in constructing confidence intervals for $\beta > 1$ while $\mathrm{CI}_1(\beta)$ is preferred when $\beta < 1$. In overall, the two CIs are comparable to each other.

4. Confidence intervals for percentiles with certain covariates

We also consider the confidence intervals for 50% percentile with given covariates $x_{\mathrm{reg}} = (1, 0, 0)'$, that is $y_p = 5 + z_p/\beta$ or $T_p = \exp(y_p)$ with the probability $p = 0.5$. The results are given in Tables 7.

It is show that, (*i*) for the censoring percentages 10%, 20%, the confidence interval based on the improved *t*-ratio, $\mathrm{CI}_2(y_p)$, has a overwhelming superior performance compared to the regular $\mathrm{CI}_1(y_p)$, with the coverage probabilities much closer to the nominal levels; (*ii*) the things differ when the censoring percentage is 30%, as $\mathrm{CI}_1(y_p)$ outperforms $\mathrm{CI}_2(y_p)$ in terms of coverage probability for $\beta < 1$. From the simulation results, we find again that the change of censoring percentage has different effects on the performance of the inferences based on either the original or the bias-corrected MLEs.

Conclusion

Based on all the simulation results for Type-I censoring, it can be concluded that the 2nd-order bias-corrected MLE and the improved inferences are generally better than the original MLE and the asymptotic inferences.

Of great interest is the the different effects of censoring percentage on the performances of the two types of inferences. It is found that when censoring percentage is high, the improved inferences still have a rather satisfactory performance for $\beta > 1$, but for $\beta < 1$, the asymptotic inferences based on the original MLE would become better.

Table 2. Empirical mean [rmse](se) of the estimators of all parameters, Type-I censored data with different censoring percentages: $n = 12$

β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
0.5	10%	0.6340 [.2445](.2045)	0.5272 [.1745](.1724)	5.0	10%	4.9778 [.7792](.7789)	4.9664 [.8341](.8335)
	20%	0.6381 [.2597](.2200)	0.5222 [.1852](.1839)		20%	5.0123 [.7668](.7668)	4.9913 [.7902](.7902)
	30%	0.6556 [.2987](.2549)	0.5225 [.2071](.2059)		30%	5.0498 [.8639](.8625)	5.0014 [.8460](.8461)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.4170 [1.607](1.552)	1.3047 [1.819](1.794)	1.0	10%	1.4116 [1.294](1.227)	1.3420 [1.450](1.409)
	20%	1.4671 [1.426](1.347)	1.4056 [1.572](1.519)		20%	1.4493 [1.139](1.046)	1.4272 [1.180](1.100)
	30%	1.5465 [1.354](1.239)	1.5131 [1.387](1.289)		30%	1.5571 [1.374](1.257)	1.5067 [1.408](1.314)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
0.8	10%	1.0241 [.4026](.3345)	0.8501 [.2841](.2797)	5.0	10%	4.9774 [.4076](.4070)	4.9916 [.4065](.4064)
	20%	1.0320 [.4367](.3700)	0.8387 [.3085](.3061)		20%	4.9959 [.4594](.4594)	4.9866 [.4536](.4535)
	30%	1.0483 [.4832](.4145)	0.8329 [.3331](.3315)		30%	5.0352 [.5298](.5287)	5.0161 [.5153](.5151)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.1440 [.6673](.6516)	1.1332 [.6853](.6723)	1.0	10%	1.1705 [.6980](.6769)	1.1568 [.7105](.6930)
	20%	1.2006 [.8037](.7783)	1.1740 [.8158](.7970)		20%	1.2145 [.7990](.7698)	1.2072 [.8281](.8018)
	30%	1.2477 [.8772](.8415)	1.2329 [.8972](.8665)		30%	1.2731 [.9274](.8864)	1.2541 [.9410](.9061)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
1.0	10%	1.2849 [.5036](.4153)	1.0642 [.3528](.3469)	5.0	10%	4.9672 [.3378](.3362)	4.9795 [.3391](.3385)
	20%	1.2790 [.5117](.4290)	1.0477 [.3589](.3558)		20%	5.0042 [.3824](.3824)	5.0043 [.3767](.3767)
	30%	1.3124 [.5936](.5048)	1.0396 [.4032](.4013)		30%	5.0286 [.4457](.4448)	5.0120 [.4362](.4360)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0799 [.5268](.5207)	1.0830 [.5493](.5430)	1.0	10%	1.1174 [.5829](.5710)	1.1229 [.6037](.5911)
	20%	1.1502 [.6637](.6465)	1.1389 [.6778](.6634)		20%	1.1256 [.6169](.6040)	1.1191 [.6275](.6162)
	30%	1.1651 [.7259](.7069)	1.1519 [.7495](.7340)		30%	1.1690 [.7277](.7079)	1.1621 [.7584](.7409)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
2.0	10%	2.6171 [1.072](.8762)	2.1627 [.7481](.7302)	5.0	10%	4.9819 [.1827](.1818)	4.9871 [.1870](.1866)
	20%	2.6957 [1.239](1.025)	2.1616 [.8330](.8173)		20%	4.9904 [.2126](.2124)	4.9935 [.2204](.2203)
	30%	2.7153 [1.289](1.073)	2.1261 [.8339](.8244)		30%	5.0051 [.2380](.2380)	4.9989 [.2386](.2387)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0297 [.3158](.3144)	1.0267 [.3272](.3261)	1.0	10%	1.0404 [.3376](.3352)	1.0379 [.3439](.3418)
	20%	1.0430 [.3295](.3267)	1.0422 [.3593](.3568)		20%	1.0412 [.3396](.3371)	1.0334 [.3454](.3438)
	30%	1.0696 [.3850](.3786)	1.0595 [.3963](.3919)		30%	1.0706 [.4151](.4091)	1.0612 [.4289](.4246)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
5.0	10%	6.7070 [2.877](2.316)	5.4711 [1.943](1.886)	5.0	10%	4.9882 [.0763](.0754)	4.9910 [.0785](.0779)
	20%	6.8373 [3.113](2.513)	5.4668 [2.057](2.004)		20%	4.9909 [.0854](.0849)	4.9918 [.0870](.0866)
	30%	7.1766 [3.653](2.934)	5.4838 [2.232](2.179)		30%	4.9920 [.1030](.1027)	4.9926 [.1045](.1042)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0106 [.1243](.1239)	1.0108 [.1292](.1288)	1.0	10%	1.0118 [.1258](.1253)	1.0128 [.1314](.1308)
	20%	1.0156 [.1525](.1517)	1.0152 [.1607](.1600)		20%	1.0124 [.1383](.1378)	1.0109 [.1432](.1428)
	30%	1.0144 [.1630](.1624)	1.0141 [.1709](.1703)		30%	1.0171 [.1655](.1647)	1.0185 [.1720](.1710)

Table 3. Empirical mean [rmse](se) of the estimators of all parameters, Type-I censored data with different censoring percentages: $n = 20$

β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
0.5	10%	0.5713 [.1469](.1285)	0.5147 [.1173](.1164)	5.0	10%	4.9577 [.5260](.5243)	4.9827 [.5296](.5293)
	20%	0.5715 [.1551](.1376)	0.5117 [.1246](.1240)		20%	4.9706 [.5582](.5575)	4.9741 [.5548](.5542)
	30%	0.5763 [.1721](.1543)	0.5111 [.1382](.1378)		30%	5.0059 [.6457](.6458)	4.9849 [.6381](.6380)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.2003 [.8171](.7922)	1.1712 [.8480](.8306)	1.0	10%	1.2122 [.7721](.7424)	1.1995 [.7884](.7628)
	20%	1.2987 [.9831](.9366)	1.2475 [.9973](.9661)		20%	1.2748 [.8820](.8382)	1.2448 [.8758](.8409)
	30%	1.2577 [.9791](.9446)	1.2132 [1.031](1.009)		30%	1.2380 [.8095](.7737)	1.2168 [.8124](.7830)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
0.8	10%	0.9170 [.2343](.2030)	0.8257 [.1851](.1833)	5.0	10%	4.9662 [.3266](.3248)	4.9857 [.3229](.3226)
	20%	0.9179 [.2492](.2196)	0.8206 [.1991](.1980)		20%	4.9795 [.3435](.3430)	4.9890 [.3379](.3378)
	30%	0.9211 [.2739](.2457)	0.8139 [.2188](.2184)		30%	5.0073 [.3982](.3981)	5.0015 [.3875](.3875)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0691 [.4977](.4929)	1.0641 [.4990](.4949)	1.0	10%	1.0680 [.5024](.4978)	1.0580 [.5025](.4991)
	20%	1.0879 [.5455](.5384)	1.0672 [.5397](.5355)		20%	1.0847 [.5277](.5209)	1.0744 [.5273](.5221)
	30%	1.1261 [.6196](.6066)	1.1016 [.6158](.6074)		30%	1.1136 [.5997](.5889)	1.0881 [.5935](.5869)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
1.0	10%	1.1442 [.2875](.2487)	1.0298 [.2267](.2248)	5.0	10%	4.9687 [.2630](.2612)	4.9853 [.2601](.2597)
	20%	1.1438 [.3021](.2657)	1.0218 [.2395](.2386)		20%	4.9846 [.2832](.2828)	4.9921 [.2789](.2788)
	30%	1.1672 [.3564](.3148)	1.0290 [.2773](.2757)		30%	4.9955 [.3299](.3299)	4.9918 [.3227](.3226)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0416 [.4215](.4194)	1.0347 [.4233](.4219)	1.0	10%	1.0423 [.4129](.4107)	1.0330 [.4143](.4130)
	20%	1.0584 [.4504](.4466)	1.0405 [.4473](.4454)		20%	1.0518 [.4235](.4203)	1.0390 [.4227](.4209)
	30%	1.0827 [.5193](.5127)	1.0602 [.5146](.5110)		30%	1.0807 [.5156](.5093)	1.0578 [.5102](.5069)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
2.0	10%	2.2982 [.5884](.5073)	2.0653 [.4607](.4561)	5.0	10%	4.9851 [.1359](.1351)	4.9944 [.1353](.1352)
	20%	2.3301 [.6497](.5596)	2.0757 [.5047](.4990)		20%	4.9870 [.1467](.1461)	4.9930 [.1460](.1458)
	30%	2.3750 [.7612](.6624)	2.0758 [.5781](.5732)		30%	4.9916 [.1686](.1684)	4.9935 [.1663](.1662)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0175 [.2176](.2169)	1.0176 [.2196](.2190)	1.0	10%	1.0141 [.2137](.2132)	1.0142 [.2168](.2164)
	20%	1.0223 [.2403](.2393)	1.0200 [.2427](.2418)		20%	1.0279 [.2493](.2477)	1.0211 [.2515](.2507)
	30%	1.0376 [.2918](.2894)	1.0294 [.2925](.2910)		30%	1.0371 [.2864](.2840)	1.0284 [.2874](.2860)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
5.0	10%	5.8061 [1.573](1.351)	5.2069 [1.228](1.211)	5.0	10%	4.9911 [.0562](.0555)	4.9955 [.0558](.0556)
	20%	5.9639 [1.800](1.520)	5.2533 [1.349](1.325)		20%	4.9920 [.0633](.0628)	4.9959 [.0633](.0631)
	30%	6.0464 [1.963](1.661)	5.2771 [1.453](1.426)		30%	4.9931 [.0708](.0705)	4.9969 [.0707](.0707)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
1.0	10%	1.0063 [.0908](.0906)	1.0064 [.0919](.0916)	1.0	10%	1.0047 [.0891](.0890)	1.0043 [.0903](.0902)
	20%	1.0106 [.1070](.1064)	1.0109 [.1087](.1082)		20%	1.0084 [.1055](.1052)	1.0084 [.1067](.1064)
	30%	1.0109 [.1184](.1179)	1.0109 [.1209](.1204)		30%	1.0118 [.1123](.1116)	1.0130 [.1142](.1134)

Table 4. Empirical mean [rmse](se) of the estimators of all parameters, Type-I censored data with different censoring percentages: $n = 50$

β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
	10%	0.5250 [.0762](.0720)	0.5036 [.0701](.0700)		10%	4.9659 [.3860](.3845)	4.9839 [.3838](.3835)
0.5	20%	0.5230 [.0770](.0735)	0.5009 [.0706](.0705)	5.0	20%	4.9896 [.3317](.3316)	4.9995 [.3284](.3284)
	30%	0.5237 [.0834](.0800)	0.5003 [.0766](.0766)		30%	5.0019 [.3647](.3648)	5.0024 [.3595](.3596)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
	10%	1.0326 [.4393](.4381)	1.0221 [.4357](.4352)		10%	1.0450 [.4552](.4530)	1.0340 [.4519](.4506)
1.0	20%	1.0459 [.4737](.4715)	1.0296 [.4685](.4675)	1.0	20%	1.0605 [.4866](.4829)	1.0445 [.4811](.4790)
	30%	1.0627 [.5184](.5146)	1.0439 [.5106](.5087)		30%	1.0632 [.5246](.5208)	1.0453 [.5162](.5143)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
	10%	0.8414 [.1170](.1094)	0.8064 [.1053](.1051)		10%	4.9755 [.2998](.2988)	4.9872 [.2990](.2987)
0.8	20%	0.8420 [.1247](.1174)	0.8058 [.1126](.1125)	5.0	20%	4.9884 [.2143](.2140)	4.9954 [.2126](.2125)
	30%	0.8425 [.1349](.1280)	0.8034 [.1224](.1224)		30%	4.9942 [.2330](.2329)	4.9966 [.2305](.2305)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
	10%	1.0172 [.3030](.3025)	1.0090 [.3013](.3012)		10%	1.0206 [.3059](.3053)	1.0129 [.3043](.3040)
1.0	20%	1.0227 [.3248](.3240)	1.0114 [.3221](.3220)	1.0	20%	1.0234 [.3198](.3190)	1.0115 [.3174](.3172)
	30%	1.0279 [.3500](.3489)	1.0139 [.3469](.3466)		30%	1.0332 [.3433](.3417)	1.0199 [.3404](.3398)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
	10%	1.0499 [.1434](.1345)	1.0055 [.1288](.1287)		10%	4.9719 [.2774](.2760)	4.9816 [.2766](.2760)
1.0	20%	1.0516 [.1533](.1443)	1.0055 [.1382](.1381)	5.0	20%	4.9917 [.1695](.1693)	4.9975 [.1682](.1682)
	30%	1.0550 [.1683](.1590)	1.0058 [.1520](.1519)		30%	4.9937 [.1876](.1875)	4.9954 [.1856](.1856)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
	10%	1.0130 [.2334](.2330)	1.0068 [.2330](.2329)		10%	1.0129 [.2311](.2307)	1.0065 [.2303](.2303)
1.0	20%	1.0184 [.2628](.2621)	1.0093 [.2615](.2614)	1.0	20%	1.0220 [.2619](.2610)	1.0119 [.2602](.2600)
	30%	1.0207 [.2969](.2962)	1.0092 [.2951](.2950)		30%	1.0249 [.2836](.2825)	1.0121 [.2812](.2809)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
	10%	2.0939 [.3040](.2892)	2.0037 [.2764](.2764)		10%	4.9789 [.2404](.2395)	4.9846 [.2402](.2397)
2.0	20%	2.1112 [.3147](.2944)	2.0149 [.2816](.2812)	5.0	20%	4.9928 [.0889](.0886)	4.9975 [.0885](.0884)
	30%	2.1112 [.3376](.3188)	2.0067 [.3027](.3026)		30%	4.9940 [.1002](.1000)	4.9972 [.0998](.0997)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
	10%	1.0066 [.1261](.1259)	1.0038 [.1259](.1259)		10%	1.0074 [.1227](.1225)	1.0047 [.1225](.1224)
1.0	20%	1.0083 [.1391](.1388)	1.0043 [.1388](.1388)	1.0	20%	1.0088 [.1362](.1359)	1.0052 [.1362](.1361)
	30%	1.0097 [.1508](.1505)	1.0044 [.1508](.1507)		30%	1.0095 [.1552](.1549)	1.0046 [.1549](.1549)
β	cp	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	a_1	cp	$\hat{a}_{1n}$	$\hat{a}_{1n}^{\text{bc2}}$
	10%	5.2333 [.8098](.7755)	5.0001 [.7398](.7398)		10%	4.9848 [.2286](.2281)	4.9874 [.2285](.2282)
5.0	20%	5.2913 [.8462](.7945)	5.0302 [.7546](.7541)	5.0	20%	4.9969 [.0376](.0375)	4.9996 [.0375](.0375)
	30%	5.3171 [.8922](.8340)	5.0297 [.7871](.7866)		30%	4.9971 [.0428](.0428)	4.9997 [.0429](.0429)
a_2	cp	$\hat{a}_{2n}$	$\hat{a}_{2n}^{\text{bc2}}$	a_3	cp	$\hat{a}_{3n}$	$\hat{a}_{3n}^{\text{bc2}}$
	10%	1.0026 [.0505](.0504)	1.0020 [.0506](.0506)		10%	1.0016 [.0505](.0505)	1.0011 [.0507](.0507)
1.0	20%	1.0042 [.0598](.0596)	1.0034 [.0600](.0599)	1.0	20%	1.0037 [.0577](.0576)	1.0030 [.0578](.0578)
	30%	1.0052 [.0700](.0698)	1.0042 [.0704](.0702)		30%	1.0038 [.0659](.0658)	1.0031 [.0662](.0661)

Table 5. Empirical significance levels: two-sided tests of $H_0 : a_2 = 0$, Type-I censored data with different censoring percentages

β	cp	Test	10%	5%	1%	10%	5%	1%	10%	5%	1%
			$n = 12$			$n = 20$			$n = 50$		
0.5	10%	(1)	0.1518	0.0876	0.0234	0.1430	0.0816	0.0249	0.1208	0.0657	0.0165
		(2)	0.0992	0.0508	0.0113	0.1032	0.0571	0.0137	0.1019	0.0523	0.0119
	20%	(1)	0.1382	0.0740	0.0174	0.1359	0.0716	0.0200	0.1184	0.0620	0.0147
		(2)	0.0876	0.0408	0.0095	0.0945	0.0475	0.0109	0.0994	0.0497	0.0099
	30%	(1)	0.1139	0.0563	0.0125	0.1245	0.0629	0.0139	0.1157	0.0604	0.0138
		(2)	0.0725	0.0321	0.0061	0.0907	0.0432	0.0069	0.0948	0.0481	0.0089
0.8	10%	(1)	0.1605	0.0933	0.0262	0.1447	0.0830	0.0245	0.1242	0.0669	0.0180
		(2)	0.1025	0.0523	0.0112	0.1038	0.0534	0.0123	0.1026	0.0534	0.0125
	20%	(1)	0.1408	0.0767	0.0216	0.1352	0.0764	0.0220	0.1208	0.0651	0.0165
		(2)	0.0942	0.0478	0.0133	0.0986	0.0508	0.0126	0.1025	0.0516	0.0119
	30%	(1)	0.1135	0.0564	0.0145	0.1280	0.0703	0.0171	0.1180	0.0632	0.0151
		(2)	0.0741	0.0331	0.0065	0.0933	0.0458	0.0091	0.0983	0.0510	0.0106
1.0	10%	(1)	0.1592	0.0964	0.0318	0.1528	0.0932	0.029	0.1194	0.0659	0.0162
		(2)	0.1039	0.0590	0.0161	0.1145	0.0620	0.0171	0.1007	0.0511	0.0116
	20%	(1)	0.1547	0.0887	0.0247	0.1442	0.0840	0.0250	0.1133	0.0625	0.0151
		(2)	0.1038	0.0525	0.0145	0.1098	0.0562	0.0146	0.0947	0.0498	0.0112
	30%	(1)	0.1372	0.0723	0.0189	0.1331	0.0733	0.0188	0.1127	0.0594	0.0130
		(2)	0.0868	0.0415	0.0097	0.1018	0.0480	0.0096	0.0949	0.0470	0.0099
2.0	10%	(1)	0.1684	0.1045	0.0361	0.1478	0.0856	0.0274	0.1195	0.0655	0.0149
		(2)	0.1115	0.0613	0.0168	0.1087	0.0596	0.0164	0.1016	0.0529	0.0110
	20%	(1)	0.1622	0.0985	0.0362	0.1472	0.0819	0.0268	0.1242	0.0674	0.0142
		(2)	0.1095	0.0584	0.0186	0.1058	0.0556	0.0156	0.1051	0.0527	0.0103
	30%	(1)	0.1466	0.0892	0.0326	0.1465	0.0833	0.0270	0.1223	0.0627	0.0169
		(2)	0.0949	0.0526	0.0151	0.1076	0.0580	0.0155	0.1012	0.0502	0.0114
5.0	10%	(1)	0.1877	0.1232	0.0519	0.1498	0.0883	0.0302	0.1248	0.0712	0.0196
		(2)	0.1298	0.0759	0.0268	0.1099	0.0620	0.0179	0.1076	0.0568	0.0138
	20%	(1)	0.2007	0.1372	0.0648	0.1535	0.0937	0.0317	0.1246	0.0705	0.0195
		(2)	0.1413	0.0880	0.0332	0.1146	0.0635	0.0188	0.1055	0.0572	0.0147
	30%	(1)	0.2056	0.1418	0.0669	0.1609	0.0951	0.0357	0.1255	0.0711	0.0195
		(2)	0.1412	0.0869	0.0353	0.1159	0.0653	0.0219	0.1038	0.0564	0.0140

Table 6. Empirical coverage probability (average length) of confidence intervals for β, Type-I censored data with different censoring percentages: $\hat{\beta}_n$ with variance $J_n^{-1}(\hat{\theta}_n)$, $\hat{\beta}_n^{\text{bc2}}$ with variance $\widehat{\text{V}}_2(\hat{\theta}_n^{\text{bc2}})$

β_0	cp	$1-\gamma=0.90$		$1-\gamma=0.95$		$1-\gamma=0.99$	
		$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$
$n=12$							
0.5	10%	0.8501 (0.5249)	0.9033 (0.5375)	0.9259 (0.6255)	0.9457 (0.6405)	0.9897 (0.8220)	0.9831 (0.8417)
	20%	0.8724 (0.5699)	0.8943 (0.5680)	0.9424 (0.6791)	0.9411 (0.6768)	0.9927 (0.8925)	0.9807 (0.8894)
	30%	0.8913 (0.6430)	0.8986 (0.6175)	0.9570 (0.7661)	0.9460 (0.7358)	0.9962 (1.0069)	0.9877 (0.9670)
0.8	10%	0.8563 (0.8499)	0.9031 (0.8707)	0.9293 (1.0127)	0.9501 (1.0375)	0.9911 (1.3309)	0.9857 (1.3635)
	20%	0.8577 (0.9078)	0.8998 (0.9179)	0.9306 (1.0817)	0.9446 (1.0937)	0.9908 (1.4215)	0.9816 (1.4374)
	30%	0.8841 (1.0587)	0.9094 (1.0320)	0.9514 (1.2616)	0.9535 (1.2297)	0.9956 (1.6580)	0.9894 (1.6161)
1.0	10%	0.8504 (1.0616)	0.9067 (1.0917)	0.9289 (1.2649)	0.9509 (1.3009)	0.9904 (1.6624)	0.9848 (1.7096)
	20%	0.8592 (1.1443)	0.9000 (1.1596)	0.9298 (1.3636)	0.9445 (1.3818)	0.9914 (1.7920)	0.9821 (1.8160)
	30%	0.8896 (1.3049)	0.9162 (1.2744)	0.9554 (1.5549)	0.9560 (1.5186)	0.9957 (2.0434)	0.9914 (1.9957)
2.0	10%	0.8331 (2.1289)	0.9204 (2.2137)	0.9179 (2.5367)	0.9590 (2.6378)	0.9901 (3.3338)	0.9874 (3.4667)
	20%	0.8371 (2.3318)	0.9154 (2.4162)	0.9242 (2.7785)	0.9549 (2.8790)	0.9926 (3.6516)	0.9884 (3.7837)
	30%	0.8449 (2.6784)	0.9207 (2.7201)	0.9268 (3.1915)	0.9613 (3.2411)	0.9932 (4.1944)	0.9907 (4.2596)
5.0	10%	0.8192 (5.4360)	0.9176 (5.7231)	0.9041 (6.4774)	0.9572 (6.8195)	0.9872 (8.5128)	0.9900 (8.9623)
	20%	0.8163 (6.1445)	0.9200 (6.4614)	0.9063 (7.3217)	0.9603 (7.6992)	0.9893 (9.6223)	0.9882 (10.119)
	30%	0.8152 (7.5104)	0.9383 (7.7271)	0.9181 (8.9492)	0.9712 (9.2074)	0.9920 (11.761)	0.9939 (12.101)
$n=20$							
0.5	10%	0.8649 (0.3672)	0.8926 (0.3665)	0.9317 (0.4376)	0.9412 (0.4367)	0.9886 (0.5751)	0.9825 (0.5740)
	20%	0.8780 (0.3979)	0.8923 (0.3915)	0.9409 (0.4741)	0.9374 (0.4665)	0.9903 (0.6231)	0.9796 (0.6131)
	30%	0.8844 (0.4390)	0.8806 (0.4207)	0.9437 (0.5231)	0.9289 (0.5013)	0.9904 (0.6875)	0.9751 (0.6589)
0.8	10%	0.8699 (0.5874)	0.9028 (0.5877)	0.9343 (0.7000)	0.9477 (0.7003)	0.9885 (0.9199)	0.9860 (0.9204)
	20%	0.8821 (0.6342)	0.8896 (0.6270)	0.9407 (0.7557)	0.9401 (0.7472)	0.9889 (0.9932)	0.9825 (0.9820)
	30%	0.8820 (0.6972)	0.8824 (0.6768)	0.9444 (0.8307)	0.9340 (0.8065)	0.9894 (1.0918)	0.9781 (1.0599)
1.0	10%	0.8720 (0.7252)	0.9019 (0.7277)	0.9366 (0.8641)	0.9471 (0.8671)	0.9895 (1.1356)	0.9866 (1.1396)
	20%	0.8773 (0.7857)	0.8959 (0.7804)	0.9390 (0.9362)	0.9412 (0.9298)	0.9906 (1.2304)	0.9820 (1.2220)
	30%	0.8823 (0.8685)	0.8902 (0.8509)	0.9398 (1.0349)	0.9343 (1.0139)	0.9898 (1.3601)	0.9806 (1.3325)
2.0	10%	0.8632 (1.4361)	0.9041 (1.4525)	0.9346 (1.7112)	0.9492 (1.7307)	0.9891 (2.2489)	0.9857 (2.2746)
	20%	0.8617 (1.5572)	0.8976 (1.5649)	0.9314 (1.8555)	0.9447 (1.8647)	0.9883 (2.4385)	0.9858 (2.4506)
	30%	0.8659 (1.6962)	0.9035 (1.6985)	0.9354 (2.0212)	0.9492 (2.0239)	0.9898 (2.6563)	0.9861 (2.6598)
5.0	10%	0.8469 (3.6353)	0.9009 (3.6892)	0.9233 (4.3318)	0.9485 (4.3960)	0.9873 (5.6929)	0.9868 (5.7773)
	20%	0.8469 (3.9881)	0.9047 (4.0669)	0.9202 (4.7521)	0.9520 (4.8460)	0.9867 (6.2453)	0.9893 (6.3687)
	30%	0.8517 (4.4099)	0.9164 (4.5259)	0.9286 (5.2547)	0.9571 (5.3930)	0.9886 (6.9058)	0.9893 (7.0876)
$n=50$							
0.5	10%	0.8877 (0.2135)	0.8921 (0.2118)	0.9474 (0.2538)	0.9410 (0.2517)	0.9901 (0.3325)	0.9868 (0.3298)
	20%	0.8948 (0.2312)	0.8920 (0.2279)	0.9479 (0.2755)	0.9415 (0.2716)	0.9886 (0.3621)	0.9820 (0.3570)
	30%	0.8971 (0.2546)	0.8844 (0.2482)	0.9457 (0.3034)	0.9354 (0.2957)	0.9887 (0.3988)	0.9806 (0.3887)
0.8	10%	0.8807 (0.3363)	0.8885 (0.3333)	0.9384 (0.4001)	0.9429 (0.3965)	0.9882 (0.5249)	0.9838 (0.5201)
	20%	0.8976 (0.3619)	0.8983 (0.3574)	0.9487 (0.4313)	0.9472 (0.4259)	0.9912 (0.5668)	0.9856 (0.5597)
	30%	0.8922 (0.3975)	0.8906 (0.3895)	0.9480 (0.4736)	0.9405 (0.4641)	0.9905 (0.6224)	0.9842 (0.6099)
1.0	10%	0.8874 (0.4182)	0.8909 (0.4127)	0.9419 (0.4977)	0.9448 (0.4911)	0.9882 (0.6531)	0.9867 (0.6445)
	20%	0.8888 (0.4493)	0.8882 (0.4426)	0.9445 (0.5354)	0.9407 (0.5273)	0.9897 (0.7036)	0.9849 (0.6931)
	30%	0.8923 (0.4911)	0.8895 (0.4811)	0.9459 (0.5852)	0.9403 (0.5732)	0.9896 (0.7691)	0.9837 (0.7534)
2.0	10%	0.8856 (0.8174)	0.8905 (0.8041)	0.9403 (0.9734)	0.9415 (0.9575)	0.9890 (1.2782)	0.9845 (1.2574)
	20%	0.8906 (0.8724)	0.8987 (0.8585)	0.9462 (1.0396)	0.9475 (1.0230)	0.9889 (1.3662)	0.9852 (1.3444)
	30%	0.8923 (0.9443)	0.8925 (0.9286)	0.9479 (1.1252)	0.9450 (1.1065)	0.9889 (1.4787)	0.9855 (1.4542)
5.0	10%	0.8852 (2.0274)	0.8990 (2.0003)	0.9450 (2.4152)	0.9483 (2.3828)	0.9887 (3.1731)	0.9867 (3.1306)
	20%	0.8753 (2.1581)	0.8870 (2.1306)	0.9386 (2.5715)	0.9422 (2.5388)	0.9891 (3.3796)	0.9851 (3.3365)
	30%	0.8747 (2.3028)	0.8917 (2.2761)	0.9398 (2.7439)	0.9407 (2.7121)	0.9887 (3.6061)	0.9866 (3.5643)

Table 7. Empirical coverage probability (average length) of confidence intervals for $y_{0.5}$, Type-I censored data with different censoring percentages: $\hat{y}_{n,0.5}$ with variance $J_n^{-1}(\hat{\theta}_n)$, $\hat{y}_{n,0.5}^{\text{bc2}}$ with variance $\widehat{\text{V}}_2(\hat{\theta}_n^{\text{bc2}})$

β_0	cp	$1-\gamma=0.90$		$1-\gamma=0.95$		$1-\gamma=0.99$	
		$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$	$\hat{\beta}_n$	$\hat{\beta}_n^{\text{bc2}}$
$n=12$							
4.2670	10%	0.8527 (2.1291)	0.9252 (2.7500)	0.9131 (2.5370)	0.9618 (3.2768)	0.9695 (3.3341)	0.9882 (4.3065)
	20%	0.8633 (2.2494)	0.9450 (3.0216)	0.9255 (2.6804)	0.9728 (3.6004)	0.9767 (3.5226)	0.9915 (4.7317)
	30%	0.8872 (2.4396)	0.9585 (3.3913)	0.9463 (2.9070)	0.9837 (4.0410)	0.9880 (3.8204)	0.9969 (5.3108)
4.5419	10%	0.8446 (1.2982)	0.9213 (1.6849)	0.9048 (1.5469)	0.9590 (2.0077)	0.9641 (2.0330)	0.9875 (2.6386)
	20%	0.8623 (1.4281)	0.9383 (1.9354)	0.9251 (1.7017)	0.9725 (2.3062)	0.9762 (2.2364)	0.9919 (3.0309)
	30%	0.8934 (1.6122)	0.9533 (2.2531)	0.9457 (1.9211)	0.9803 (2.6848)	0.9871 (2.5247)	0.9955 (3.5284)
4.6335	10%	0.8387 (1.0598)	0.9131 (1.3780)	0.9002 (1.2629)	0.9518 (1.6420)	0.9607 (1.6597)	0.9851 (2.1579)
	20%	0.8535 (1.1770)	0.9285 (1.5572)	0.9181 (1.4024)	0.9657 (1.8556)	0.9746 (1.8431)	0.9906 (2.4386)
	30%	0.8729 (1.2659)	0.9422 (1.7831)	0.9314 (1.5084)	0.9746 (2.1247)	0.9816 (1.9823)	0.9945 (2.7923)
4.8167	10%	0.8207 (0.5414)	0.8918 (1.3328)	0.8839 (0.6451)	0.9345 (1.5881)	0.9493 (0.8478)	0.9765 (2.0871)
	20%	0.8309 (0.6048)	0.9053 (0.8630)	0.8941 (0.7206)	0.9432 (1.0284)	0.9579 (0.9471)	0.9789 (1.3515)
	30%	0.8488 (0.6732)	0.9223 (0.9431)	0.9085 (0.8021)	0.9562 (1.1237)	0.9674 (1.0542)	0.9861 (1.4768)
4.9267	10%	0.8029 (0.2127)	0.8826 (0.2858)	0.8686 (0.2535)	0.9316 (0.3405)	0.9412 (0.3331)	0.9717 (0.4475)
	20%	0.8066 (0.2345)	0.8843 (0.3234)	0.8692 (0.2794)	0.9296 (0.3854)	0.9414 (0.3672)	0.9725 (0.5065)
	30%	0.7911 (0.2681)	0.8754 (0.3930)	0.8590 (0.3195)	0.9218 (0.4683)	0.9326 (0.4199)	0.9673 (0.6155)
$n=20$							
4.2670	10%	0.8613 (1.6871)	0.9020 (1.9167)	0.9172 (2.0097)	0.9481 (2.2833)	0.9748 (2.6402)	0.9870 (2.9997)
	20%	0.8774 (1.7543)	0.9225 (2.0210)	0.9322 (2.0904)	0.9658 (2.4081)	0.9831 (2.7473)	0.9934 (3.1648)
	30%	0.8826 (1.8863)	0.9286 (2.2182)	0.9386 (2.2477)	0.9666 (2.6431)	0.9880 (2.9539)	0.9949 (3.4736)
4.5419	10%	0.8667 (1.0606)	0.9077 (1.2071)	0.9230 (1.2631)	0.9489 (1.4377)	0.9751 (1.6590)	0.9875 (1.8885)
	20%	0.8619 (1.0852)	0.9095 (1.2434)	0.9203 (1.2931)	0.9559 (1.4817)	0.9777 (1.6994)	0.9904 (1.9472)
	30%	0.8855 (1.2177)	0.9344 (1.4278)	0.9457 (1.4509)	0.9713 (1.7013)	0.9889 (1.9068)	0.9952 (2.2359)
4.6335	10%	0.8538 (0.8450)	0.8939 (0.9548)	0.9110 (1.0063)	0.9414 (1.1371)	0.9725 (1.3214)	0.9846 (1.4935)
	20%	0.8681 (0.8925)	0.9097 (1.0205)	0.9280 (1.0634)	0.9546 (1.2160)	0.9794 (1.3976)	0.9898 (1.5982)
	30%	0.8794 (0.9504)	0.9240 (1.1079)	0.9342 (1.1325)	0.9638 (1.3202)	0.9852 (1.4883)	0.9922 (1.7350)
4.8167	10%	0.8519 (0.4238)	0.8898 (0.4772)	0.9116 (0.5043)	0.9381 (0.5680)	0.9685 (0.6618)	0.9812 (0.7454)
	20%	0.8612 (0.4508)	0.9006 (0.5124)	0.9186 (0.5372)	0.9447 (0.6106)	0.9732 (0.7060)	0.9856 (0.8024)
	30%	0.8670 (0.5008)	0.9093 (0.5797)	0.9285 (0.5968)	0.9522 (0.6908)	0.9795 (0.7843)	0.9871 (0.9078)
4.9267	10%	0.8508 (0.1738)	0.8885 (0.1958)	0.9121 (0.2065)	0.9386 (0.2327)	0.9701 (0.2704)	0.9810 (0.3048)
	20%	0.8461 (0.1868)	0.8842 (0.2133)	0.9055 (0.2226)	0.9307 (0.2542)	0.9630 (0.2925)	0.9772 (0.3340)
	30%	0.8410 (0.2151)	0.8792 (0.2499)	0.9018 (0.2564)	0.9309 (0.2978)	0.9655 (0.3369)	0.9747 (0.3914)
$n=50$							
4.2670	10%	0.8844 (1.0833)	0.9030 (1.1390)	0.9386 (1.2903)	0.9504 (1.3566)	0.9848 (1.6947)	0.9881 (1.7819)
	20%	0.8915 (1.1131)	0.9120 (1.1731)	0.9417 (1.3263)	0.9550 (1.3979)	0.9857 (1.7431)	0.9905 (1.8371)
	30%	0.8935 (1.1645)	0.9171 (1.2348)	0.9469 (1.3876)	0.9604 (1.4713)	0.9888 (1.8236)	0.9926 (1.9336)
4.5419	10%	0.8836 (0.6811)	0.9039 (0.7161)	0.9394 (0.8109)	0.9498 (0.8527)	0.9850 (1.0648)	0.9884 (1.1196)
	20%	0.8815 (0.7048)	0.9004 (0.7422)	0.9385 (0.8398)	0.9518 (0.8843)	0.9859 (1.1037)	0.9900 (1.1622)
	30%	0.8892 (0.7543)	0.9101 (0.7988)	0.9451 (0.8988)	0.9602 (0.9519)	0.9885 (1.1812)	0.9912 (1.2510)
4.6335	10%	0.8817 (0.5474)	0.8973 (0.5758)	0.9319 (0.6517)	0.9452 (0.6856)	0.9835 (0.8555)	0.9881 (0.9000)
	20%	0.8882 (0.5652)	0.9086 (0.5950)	0.9409 (0.6734)	0.9529 (0.7090)	0.9858 (0.8851)	0.9888 (0.9318)
	30%	0.8964 (0.6100)	0.9132 (0.6454)	0.9471 (0.7269)	0.9591 (0.7691)	0.9901 (0.9553)	0.9932 (1.0107)
4.8167	10%	0.8845 (0.2761)	0.9009 (0.2901)	0.9392 (0.3284)	0.9488 (0.3451)	0.9833 (0.4306)	0.9882 (0.4525)
	20%	0.8793 (0.2907)	0.8945 (0.3050)	0.9346 (0.3464)	0.9450 (0.3634)	0.9843 (0.4553)	0.9876 (0.4776)
	30%	0.8812 (0.3207)	0.8965 (0.3360)	0.9375 (0.3822)	0.9485 (0.4004)	0.9850 (0.5022)	0.9882 (0.5262)
4.9267	10%	0.8861 (0.1155)	0.9011 (0.1210)	0.9377 (0.1370)	0.9486 (0.1436)	0.9828 (0.1791)	0.9870 (0.1877)
	20%	0.8760 (0.1217)	0.8907 (0.1273)	0.9329 (0.1450)	0.9436 (0.1517)	0.9821 (0.1905)	0.9851 (0.1994)
	30%	0.8762 (0.1376)	0.8867 (0.1431)	0.9339 (0.1640)	0.9408 (0.1706)	0.9838 (0.2156)	0.9852 (0.2242)

Note: $\{4.2670, 4.5419, 4.6335, 4.8167, 4.9267\}$ corresponds to $y_{0.5}$ with $\{\beta = 0.5, 0.8, 1.0, 2.0, 5.0\}$.