Supplementary Material to “A General Method for Third-Order Bias and Variance Corrections on a Nonlinear Estimator”

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Summary

This supplementary material contains extended Monte Carlo results including those reported in a paper entitled “A General Method for Third-Order Bias and Variance Corrections on a Nonlinear Estimator”. More detailed analyses relating to the general observations reported in the paper are also given.

1 Introduction

Extensive Monte Carlo experiments are carried out to investigate (i) the finite sample performance of the QMLE $\hat{\lambda}_n$ and the bias-corrected QMLEs $\hat{\lambda}_n^{\mathrm{bc2}}$ and $\hat{\lambda}_n^{\mathrm{bc3}}$ of the spatial lag parameter λ, (ii) the finite sample performance of the corrected standard errors (se) of the original QMLE as well the bias-corrected QMLE, and (iii) the impact of the bias and se corrections on the subsequent inferences for λ. Also, through the process of our investigation on the higher-order properties of the SAR model, we found some anomalies in the documented Monte Carlo results in Bao and Ullah (2007a) and in Lee (2004a). Their Monte Carlo experiments are rerun and the amended results reported. A comparison is thus made with the analytical approach of Bao and Ullah (2007a).

2 Monte Carlo Experiment I

To demonstrate the effectiveness of the proposed bias-correction procedure, and to investigate the impact of bias and se corrections on the subsequent interferences, a comprehensive set of Monte Carlo experiments is carried out based on the following general SAR model:

$$Y_n = \lambda W_n Y_n + \beta_0 1_n + X_{n1}\beta_1 + X_{n2}\beta_2 + u_n,$$

where 1_n is an n-vector of ones. For all the Monte Carlo experiments, $\beta' = \{\beta_0, \beta_1, \beta_2\}$ is set at $\{5, 1, 1\}$ or $\{.5, .1, .1\}$, σ at 1 or 2, λ takes values $\{.5, .25, 0, -.25, -.5\}$, and n takes values

$\{50, 100, 200, 500\}$.[1] Several ways of generating W_n, (X_{n1}, X_{n2}), and u_n are considered. First, the values $\{x_{1i}\}$ or $\{x_{1,ir}\}$ of X_{n1}, and the values $\{x_{2i}\}$ or $\{x_{2,ir}\}$ of X_{n2} are,

`MRSAR-A`: $\{x_{1i}\} \overset{iid}{\sim} N(0,1)/\sqrt{2}$, and $\{x_{2i}\} \overset{iid}{\sim} N(0,1)/\sqrt{2}$, or
`MRSAR-B`: $\{x_{1,ir}\} = (2z_r + z_{ir})/\sqrt{7}$, and $\{x_{2,ir}\} = (v_r + v_{ir})/\sqrt{7}$,

where in `MRSAR-B`, $\{z_r, z_{ir}, v_r, v_{ir}\} \overset{iid}{\sim} N(0,1)$, across all i and r. Apparently, `MRSAR-A` gives iid X values, and `MRSAR-B` gives non-iid X values, or different group means under group interaction, see Lee (2004a) and below for details. Both schemes give signal-to-noise ratios 1 or 1/2 when $\sigma = 1$ or 2.

Spatial layouts. Three general spatial layouts are considered in the Monte Carlo experiments. The first is based on Rook contiguity, the second is based on Queen contiguity and the third is based on the notion of group interactions. The methods used in generating these three spatial layouts are similar to those used in Yang (2010).

The detail for generating the W_n matrix under rook contiguity is as follows: (i) index the n spatial units by $\{1, 2, \cdots, n\}$, randomly permute these indices and then allocate them into a lattice of $k \times m (\geq n)$ squares, (ii) let $W_{ij} = 1$ if the index j is in a square which is on immediate left, or right, or above, or below the square which contains the index i, otherwise $W_{ij} = 0, i, j = 1, \cdots, n$, to form an $n \times n$ matrix, and (iii) divide each element of this matrix by its row sum to give W_n. Similarly, one generates the W_n matrix under Queen contiguity with additional neighbors sharing a common vertex with the unit of interest.

To generate the W_n matrix according to the group interaction scheme, suppose we have k groups of sizes $m_1, m_2, \cdots, m_k$. Define $W_n = \text{diag}\{W_j/(m_j - 1), j = 1, \cdots, k\}$, a matrix formed by placing the submatrices W_j along the diagonal direction, where W_j is an $m_j \times m_j$ matrix with ones on the off-diagonal positions and zeros on the diagonal positions. The group sizes $\{m_j\}$ can be the same or different, and independent or dependent on n, allowing for a full range of spatial scenarios considered in Lee (2004a). The details are as follows: (i) calculate the number of groups according to $k = K(n)$, and the approximate average group size $m = n/k$, (ii) generate the group sizes $(m_1, m_2, \cdots, m_k)$ according to a discrete distribution centered at m, and (iii) adjust the group sizes so that $\sum_{j=1}^{k} m_j = n$.[2]

In our Monte Carlo experiments, we use $K(n) = \text{Round}(n^{\epsilon})$ with $\epsilon = 0.35, 0.50$, and 0.75,

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[1] As in Lee (2007a), the maximization of $\ell_n^c(\lambda)$ is performed globally without imposing a restricted lower bound on λ. This is important when the true λ value is negative and big, because QMLE is downward biased and a restricted lower bound, -0.9999 say, would result in the searching process to hit the lower bound quite often, thus failing to reach the true maximum point. This would in turn give a wrong impression that the QMLE can be upward biased and the bias-correction may not work in certain cases. This is believed to be the reason for the incoherent Monte Carlo results of Bao and Ullah (2007a). See Anselin (1988, p. 78-79) for a theoretical discussion on the parameter space of λ in relation to the eigenvalues of W_n.

[2] Clearly, this design covers the scenario considered in Case (1991). Typical forms of $K(n)$ include $K(n) = n/m$ where m is a prespecified constant independent of n and $K(n) = \text{Round}(n^{\epsilon})$. Lee (2007b) shows that the group size variation plays an important role in the identification and estimation of econometric

representing respectively the situations where (a) there are few groups of many spatial units in each, (b) the number of groups and the sizes of the groups are of the same magnitude, and (c) there are many groups of few elements in each. Clearly, $h_n = O(n^{1-\epsilon})$. The group sizes are drawn from a discrete uniform distribution from $0.5m$ to $1.5m$.

Error distributions. To generate $u_n = \sigma e_n$, three distributions are considered: `dgp1:` the elements $\{e_i\}$ of e_n are iid standard normal, `dgp2:` $\{e_i\}$ are iid standardized normal mixture, and `dgp3:` $\{e_i\}$ are iid standardized log-normal. Specifically, for `dgp2`,

$$e_i = ((1-\xi_i)Z_i + \xi_i \tau Z_i)/(1-p+p*\sigma^2)^{0.5},$$

where $\xi \sim \text{Bernoulli}(p)$, and $Z_i \sim N(0,1)$ independent of ξ. The parameter p represents the proportion of mixing the two normal populations. In our experiments, we choose $p = 0.1$, meaning that 90% of the random variates are from standard normal and the remaining 10% are from another normal population with standard deviation τ. We choose $\tau = 4$ to simulate the situation where there are gross errors in the data. For `dgp3`,

$$e_i = [\exp(Z_i) - \exp(0.5)]/[\exp(2) - \exp(1)]^{0.5},$$

which gives an error distribution that is both skewed and leptokurtic. The normal mixture gives an error distribution that is still symmetric like normal but leptokurtic. Other non-normal distributions, such as normal-gamma mixture and chi-square, are also considered and the results (available from the author upon request) exhibit a similar pattern.

Finite sample performance of the spatial estimators. We report the Monte Carlo mean, rmse and sd of $\hat{\lambda}_n$, $\hat{\lambda}_n^{\text{bc2}}$ and $\hat{\lambda}_n^{\text{bc3}}$ under various combinations of the values for (n, λ, σ), the error distributions, and the spatial layouts. We also report the averages (over Monte Carlo samples) of the 1st-, 2nd- and 3rd-order ses: $\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}$, $\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}$, $\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}}$ and $\widehat{V}_3(\hat{\lambda}_n^{\text{bc3}})^{\frac{1}{2}}$, defined by Corollaries 3.3 and 3.4 and calculated based on the proposed bootstrap method. Each set of results is based on 10,000 Monte Carlo samples, and $B = 999 + floor(n^{0.75})$ bootstrap samples for each Monte Carlo sample. Table 1 summarizes the results (reported in the paper) with $\beta = \{5, 1, 1\}'$ and $\sigma = 1$, where the regressors' values are generated according to `MRSAR-A` for Queen contiguity spatial layout, and `MRSAR-B` for group interaction spatial layout. Table 1-1 contains additional results under the same setup as the group interaction scheme but with different group size configurations. Table 1-2 replicates the results of Table 1 under $\beta = \{.5, .1, .1\}'$, and Table 1-3 replicates the results of Table 1 under $\sigma = 2$. For the results in both Tables 1b and 1c, 1,000 Monte Carlo runs and $399 + floor(n^{0.5})$ bootstrap samples for each Monte Carlo run are used.

Some general observations are in order:

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models with group interactions, contextual factors and fixed effects. Yang (2010) shows that it also plays an important role in the robustness of the LM test of spatial error components.

(i) the bias-corrected QMLEs $\hat{\lambda}_n^{\rm bc2}$ and $\hat{\lambda}_n^{\rm bc3}$ are in general nearly unbiased and clearly outperform the original QMLE $\hat{\lambda}_n$;

(ii) $\hat{\lambda}_n$ is always downward biased and the biasness can be very serious depending on the spatial layout, the sample size and the error standard deviation;

(iii) $\hat{\lambda}_n^{\rm bc3}$ improves over $\hat{\lambda}_n^{\rm bc2}$, but using $\hat{\lambda}_n^{\rm bc2}$ seems to be sufficient under most of the situations as far as bias-correction is concerned;

(iv) spatial layouts can have a huge impact on the finite sample performance of $\hat{\lambda}_n$ – the stronger the spatial dependence the worse $\hat{\lambda}_n$ performs;

(v) the values of σ and the slope parameters also have a big impact – the bigger the σ is, or the smaller the $|\beta_1|$ and $|\beta_2|$ are, the bigger are the biases, rmses and sds of $\hat{\lambda}_n$;

(vi) the value of λ and the way the regressors being generated affect the finite sample performance of $\hat{\lambda}_n$ – as λ decreases, the bias of $\hat{\lambda}_n$ decreases under iid regressors but increases under non-iid regressors, whereas the [rmse](se) of $\hat{\lambda}_n$ always increases as λ decreases, with a sharper amount for the case of non-iid regressors;

(vii) the error distribution does not affect much on the general performance of the three estimators, showing the robustness of the proposed approach.

(viii) The empirical sd of $\hat{\lambda}_n$ can be slightly different from that of $\hat{\lambda}_n^{\rm bc3}$ when sample size is small, suggesting that the variances of $\hat{\lambda}_n$ and $\hat{\lambda}_n^{\rm bc3}$ may differ on higher-order term (see panels (a)-(c), Table 1). The results in the last four columns of Table 1 show that $\widehat{V}_3(\hat{\lambda}_n^{\rm bc3})$ provides the best approximation to the variance of $\hat{\lambda}_n^{\rm bc3}$. The empirical sds of $\hat{\lambda}_n^{\rm bc2}$ and $\hat{\lambda}_n^{\rm bc3}$ agree closely, suggesting that the finite sample variances of $\hat{\lambda}_n^{\rm bc2}$ and $\hat{\lambda}_n^{\rm bc3}$ are about the same. These are consistent with the result of Corollary 3.4.

In summary, the proposed bias-correction procedure works excellently in general, it is simple and widely applicable, and thus should be recommended for the practitioners.

The performance of t-ratios. The finite sample behavior of the t-ratios t_{ij} for testing $H_0 : \lambda = 0$, defined in (23) are investigated. Partial Monte Carlo results in terms of means, sds, and tail probabilities are reported in Table 2. From the results, the following conclusions can be drawn: (i) the asymptotic t-ratio t_{11} can perform quite badly with severe distortions on mean and sizes; (ii) use of second-order bias-corrected estimator only (t_{21}) immediately improves; (iii) use of both second-order bias-corrected estimator and second-order variance gives further improvements; and (iv) use of the third-order bias-corrected estimator and the third-order variance estimate gives the best results.

3 Monte Carlo Experiment II

The Monte Carlo experiment conducted by Bao and Ullah (2007a) is replicated and extended for two purposes: (i) to compare our bootstrap-based bias correction with their analytical bias correction, and (ii) to investigate the cause of incoherent Monte Carlo results of Bao and Ullah (2007a) for the case of $\lambda = -0.9$ and $J = 10$, where J is the number of neighbors each spatial unit has when the 'circular-world' design is used.

The empirical means and rmses for the four estimators: the MLE $\hat{\lambda}_n$, the second-order bias-corrected MLE $\hat{\lambda}_n^{\mathrm{bc2}}$, and the analytically bias-corrected MLE $\hat{\lambda}_n^{\mathrm{BU}}$ of Bao and Ullah (2007a), are summarized in Table 3. The results show that the second-order bias correction works excellently in general: (i) in case of normal errors, the bootstrap method and analytical method produce almost identical results, confirming the validity of the proposed bootstrap method, and (ii) in case of non-normal errors, bootstrap method produces slightly better results, showing that the proposed method is more robust against the error distribution than the analytical method that is based on the normality assumption. The results also show a very coherent behavior of the MLE and the bias-corrected MLEs in that the MLE is almost unbiased when J is small and is downward biased when J is big. In cases where the MLE is biased, the bias-corrected estimators always be able to correct the bias. However, the Monte Carlo results of Bao and Ullah show a different picture for the cases where J is large and λ is large and negative: the MLE is upward biased and the bias-correction does not work. A possible explanation for this can be found in Footnote 9.

4 Monte Carlo Experiment III

The Monte Carlo experiments of Lee (2004) are rerun and extended again for two purposes: i) to further compare MLEs (estimators under normal errors) and second-order bias-corrected MLEs under a model without intercept, and ii) to amend the anomalies in his Monte Carlo results. Under the identical set-up but with more Monte Carlo replications (10,000 vs 400) for each case, we found that with the number of districts $R = (30, 60, 120)$ and the number of members in each district $m = (2, 3, 5, 1020, 50, 100)$, the MLEs all perform very well in all cases, showing a big contrast to the results of Lee (2004a). However, when the values for R and m are switched, the results obtained are more in line with those of Lee (2004a) in terms of bias but not in terms of sd.

Table 4a corresponds to Tables I & II in Lee (2004a). Tables 4b summarize the results of the Monte Carlo experiments when the values for R and m are switched, i.e., $R = (3, 5, 10, 20, 50, 100)$ and $m = (30, 60, 120)$. The results in Tables 4b are now more in line with Lee's results in terms of bias but not in terms of sd of $\hat{\lambda}_n$. The results further show that for a given R value, increasing the m value does not necessarily reduce the bias of $\hat{\lambda}_n$.

However, for a given m, increasing R clearly improves the performance of the estimators with both bias and sd significantly reduced.

From the theoretical point of view, our results make more sense as increasing m enlarges the degree of spatial dependence, making the estimation of the spatial parameter harder (or at least not easier). In contrast, increasing R clearly reduces the 'degree' of spatial dependence, making the estimation of the spatial parameter much easier.

Table 1. Empirical Mean[rmse](sd) of Estimators of λ, and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^{\mathrm{c}}$
`(a) Queen Contiguity, Normal Errors, MRSAR-A`								
.50	50	.411 [.195](.174)	.492 [.175](.175)	.497 [.175](.175)	.159	.171	.179	.179
	100	.459 [.123](.116)	.498 [.117](.117)	.500 [.117](.117)	.113	.116	.120	.120
	200	.480 [.078](.076)	.499 [.075](.075)	.499 [.075](.075)	.073	.074	.076	.075
	500	.493 [.049](.048)	.501 [.048](.048)	.501 [.048](.048)	.048	.048	.049	.049
.25	50	.163 [.222](.204)	.242 [.209](.209)	.246 [.210](.210)	.190	.203	.202	.207
	100	.212 [.146](.140)	.248 [.142](.142)	.250 [.143](.143)	.136	.139	.139	.141
	200	.231 [.094](.092)	.250 [.093](.093)	.250 [.093](.093)	.090	.092	.092	.092
	500	.242 [.060](.060)	.250 [.060](.060)	.250 [.060](.060)	.060	.060	.060	.060
.00	50	-.078 [.229](.216)	-.006 [.224](.224)	-.003 [.226](.226)	.210	.222	.215	.223
	100	-.034 [.157](.153)	-.002 [.156](.156)	-.001 [.157](.157)	.151	.154	.151	.155
	200	-.018 [.106](.104)	-.000 [.105](.105)	.000 [.105](.105)	.103	.104	.104	.104
	500	-.008 [.068](.067)	-.000 [.068](.068)	-.000 [.068](.068)	.068	.068	.068	.068
-.25	50	-.317 [.233](.223)	-.255 [.236](.236)	-.254 [.237](.237)	.221	.232	.220	.231
	100	-.279 [.164](.161)	-.253 [.166](.166)	-.253 [.166](.166)	.158	.161	.156	.161
	200	-.266 [.112](.111)	-.252 [.112](.112)	-.251 [.112](.112)	.110	.112	.110	.112
	500	-.256 [.073](.072)	-.250 [.073](.073)	-.250 [.073](.073)	.072	.073	.072	.072
-.50	50	-.552 [.228](.222)	-.504 [.236](.236)	-.504 [.237](.237)	.223	.232	.217	.230
	100	-.519 [.162](.161)	-.501 [.166](.166)	-.501 [.166](.166)	.159	.161	.155	.160
	200	-.514 [.113](.113)	-.502 [.114](.114)	-.502 [.114](.114)	.113	.114	.112	.114
	500	-.505 [.073](.073)	-.500 [.073](.073)	-.500 [.073](.073)	.074	.074	.073	.074
`(b) Queen Contiguity, Normal Mixture Errors, MRSAR-A`								
.50	50	.420 [.182](.164)	.494 [.165](.165)	.498 [.165](.165)	.149	.160	.167	.167
	100	.462 [.120](.114)	.499 [.114](.114)	.500 [.114](.114)	.108	.111	.115	.115
	200	.482 [.076](.074)	.500 [.074](.074)	.500 [.074](.074)	.071	.072	.074	.074
	500	.492 [.049](.048)	.500 [.048](.048)	.500 [.048](.048)	.048	.048	.048	.048
.25	50	.169 [.207](.190)	.241 [.195](.195)	.244 [.195](.195)	.179	.190	.191	.195
	100	.213 [.140](.135)	.248 [.136](.136)	.249 [.137](.137)	.130	.133	.134	.136
	200	.230 [.092](.090)	.249 [.090](.090)	.249 [.090](.090)	.088	.089	.090	.090
	500	.242 [.060](.060)	.250 [.060](.060)	.250 [.060](.060)	.059	.059	.060	.060
.00	50	-.070 [.217](.206)	-.004 [.213](.213)	-.002 [.214](.214)	.197	.207	.204	.211
	100	-.032 [.150](.147)	-.002 [.150](.150)	-.001 [.150](.150)	.145	.148	.146	.149
	200	-.018 [.104](.103)	-.001 [.103](.103)	-.001 [.103](.103)	.100	.101	.101	.102
	500	-.008 [.068](.067)	-.001 [.067](.067)	-.001 [.067](.067)	.067	.067	.067	.067
-.25	50	-.314 [.223](.213)	-.258 [.224](.224)	-.257 [.225](.225)	.208	.216	.209	.219
	100	-.275 [.155](.153)	-.251 [.157](.157)	-.250 [.157](.157)	.152	.154	.151	.155
	200	-.263 [.111](.110)	-.249 [.112](.112)	-.249 [.112](.112)	.108	.109	.108	.109
	500	-.257 [.072](.072)	-.251 [.072](.072)	-.251 [.072](.072)	.071	.072	.071	.072
-.50	50	-.550 [.218](.212)	-.506 [.224](.224)	-.505 [.225](.225)	.210	.216	.207	.218
	100	-.520 [.155](.153)	-.503 [.158](.158)	-.503 [.158](.158)	.152	.154	.150	.155
	200	-.513 [.112](.111)	-.502 [.113](.113)	-.502 [.113](.113)	.111	.111	.110	.112
	500	-.505 [.074](.073)	-.500 [.074](.074)	-.500 [.074](.074)	.073	.073	.073	.073

Note: $\overline{se}_1 = \mathrm{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \mathrm{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^{\mathrm{c}} = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n^{\mathrm{bc3}})^{\frac{1}{2}})$.

Table 1 (cont'd). Empirical Mean[rmse](sd) of Estimators of λ, and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^{\mathrm{c}}$
		`(c) Queen Contiguity, Lognormal Errors, MRSAR-A`						
.50	50	.426 [.163](.146)	.491 [.146](.146)	.493 [.146](.146)	.138	.144	.154	.154
	100	.465 [.110](.105)	.498 [.105](.105)	.498 [.105](.105)	.102	.103	.106	.106
	200	.482 [.072](.069)	.499 [.069](.069)	.499 [.069](.069)	.067	.067	.069	.069
	500	.491 [.047](.046)	.499 [.046](.046)	.499 [.046](.046)	.046	.046	.046	.046
.25	50	.179 [.185](.171)	.241 [.174](.174)	.244 [.174](.174)	.163	.170	.176	.179
	100	.216 [.128](.124)	.247 [.126](.125)	.248 [.126](.125)	.123	.123	.124	.126
	200	.232 [.087](.085)	.249 [.085](.085)	.249 [.085](.085)	.084	.083	.085	.085
	500	.242 [.058](.057)	.249 [.057](.057)	.249 [.057](.057)	.057	.057	.057	.057
.00	50	-.067 [.198](.186)	-.011 [.192](.191)	-.008 [.192](.192)	.180	.186	.190	.196
	100	-.029 [.139](.136)	-.003 [.138](.138)	-.002 [.138](.138)	.135	.136	.136	.138
	200	-.017 [.099](.097)	-.002 [.098](.098)	-.001 [.098](.098)	.095	.095	.096	.096
	500	-.007 [.065](.064)	-.000 [.065](.065)	.000 [.065](.065)	.065	.064	.065	.065
-.25	50	-.307 [.199](.191)	-.258 [.198](.198)	-.256 [.199](.199)	.189	.194	.197	.205
	100	-.272 [.142](.140)	-.252 [.144](.144)	-.251 [.144](.144)	.141	.142	.141	.145
	200	-.264 [.105](.104)	-.251 [.105](.105)	-.250 [.105](.105)	.102	.102	.102	.103
	500	-.256 [.070](.070)	-.250 [.070](.070)	-.250 [.070](.070)	.069	.069	.069	.069
-.50	50	-.548 [.196](.190)	-.509 [.200](.199)	-.507 [.200](.200)	.191	.195	.196	.206
	100	-.514 [.145](.144)	-.500 [.148](.148)	-.499 [.148](.148)	.141	.141	.141	.145
	200	-.511 [.106](.106)	-.501 [.107](.107)	-.501 [.107](.107)	.105	.105	.105	.106
	500	-.505 [.070](.070)	-.501 [.070](.070)	-.500 [.070](.070)	.071	.070	.070	.071
		`(d) Group Interaction with` $k = n^{0.5}$`, Normal Errors, MRSAR-B`						
.50	50	.426 [.145](.124)	.495 [.122](.122)	.499 [.122](.122)	.111	.123	.128	.121
	100	.449 [.112](.099)	.498 [.097](.097)	.500 [.097](.097)	.092	.097	.102	.098
	200	.474 [.073](.068)	.499 [.067](.067)	.500 [.067](.067)	.065	.067	.069	.068
	500	.491 [.042](.041)	.500 [.041](.041)	.500 [.041](.041)	.040	.041	.041	.041
.25	50	.142 [.209](.179)	.239 [.178](.178)	.244 [.179](.178)	.159	.175	.180	.174
	100	.177 [.160](.143)	.248 [.141](.141)	.250 [.141](.141)	.133	.140	.146	.142
	200	.212 [.108](.102)	.249 [.101](.101)	.249 [.101](.101)	.096	.098	.101	.100
	500	.236 [.062](.060)	.250 [.061](.061)	.250 [.061](.061)	.060	.060	.060	.060
.00	50	-.134 [.261](.224)	-.013 [.226](.226)	-.007 [.227](.227)	.202	.220	.223	.219
	100	-.094 [.206](.183)	-.005 [.182](.182)	-.003 [.182](.182)	.172	.180	.186	.183
	200	-.047 [.137](.128)	.001 [.128](.128)	.001 [.128](.128)	.125	.128	.131	.130
	500	-.019 [.081](.079)	-.000 [.079](.079)	-.000 [.079](.079)	.078	.079	.079	.080
-.25	50	-.404 [.303](.260)	-.265 [.266](.266)	-.259 [.267](.267)	.240	.258	.259	.259
	100	-.358 [.243](.218)	-.253 [.219](.219)	-.251 [.219](.219)	.207	.215	.221	.219
	200	-.308 [.170](.160)	-.251 [.160](.160)	-.250 [.160](.160)	.153	.156	.159	.159
	500	-.274 [.101](.098)	-.252 [.098](.098)	-.252 [.098](.098)	.097	.098	.098	.098
-.50	50	-.670 [.337](.291)	-.516 [.302](.301)	-.511 [.303](.302)	.273	.290	.288	.293
	100	-.620 [.281](.254)	-.501 [.257](.257)	-.499 [.257](.257)	.238	.246	.252	.252
	200	-.568 [.198](.186)	-.502 [.187](.187)	-.501 [.187](.187)	.179	.183	.186	.186
	500	-.527 [.118](.115)	-.500 [.116](.116)	-.500 [.116](.116)	.115	.116	.116	.117

Note: $\overline{se}_1 = \mathrm{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \mathrm{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^{\mathrm{c}} = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n^{\mathrm{bc3}})^{\frac{1}{2}})$.

Table 1 (cont'd). Empirical Mean[rmse](sd) of Estimators of λ, and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^{\mathrm{c}}$
		(e) Group Interaction with $k = n^{0.5}$, Normal Mixture Errors, MRSAR-B						
.50	50	.427 [.144](.124)	.489 [.121](.120)	.493 [.120](.120)	.104	.114	.120	.115
	100	.449 [.111](.098)	.495 [.096](.096)	.497 [.096](.096)	.088	.093	.098	.095
	200	.474 [.073](.068)	.498 [.068](.068)	.499 [.068](.068)	.064	.065	.067	.067
	500	.490 [.042](.041)	.500 [.041](.041)	.500 [.041](.041)	.040	.040	.041	.041
.25	50	.147 [.203](.175)	.234 [.172](.171)	.239 [.172](.171)	.149	.163	.171	.166
	100	.179 [.157](.140)	.245 [.138](.137)	.248 [.138](.138)	.128	.133	.140	.137
	200	.214 [.104](.097)	.249 [.097](.097)	.249 [.097](.097)	.093	.095	.099	.098
	500	.236 [.062](.060)	.250 [.060](.060)	.250 [.060](.060)	.059	.059	.060	.060
.00	50	-.126 [.250](.215)	-.017 [.214](.213)	-.011 [.214](.214)	.191	.207	.215	.212
	100	-.089 [.203](.182)	-.006 [.180](.180)	-.004 [.181](.181)	.164	.171	.179	.176
	200	-.047 [.137](.129)	-.002 [.129](.128)	-.001 [.129](.129)	.122	.124	.128	.128
	500	-.020 [.082](.080)	-.002 [.080](.080)	-.002 [.080](.080)	.078	.078	.079	.079
-.25	50	-.393 [.294](.256)	-.267 [.259](.259)	-.261 [.260](.259)	.227	.244	.251	.252
	100	-.358 [.241](.216)	-.259 [.216](.216)	-.257 [.216](.216)	.199	.205	.213	.212
	200	-.306 [.165](.156)	-.251 [.155](.155)	-.250 [.155](.155)	.149	.151	.156	.156
	500	-.274 [.101](.098)	-.251 [.099](.099)	-.251 [.099](.099)	.096	.096	.097	.098
-.50	50	-.658 [.330](.289)	-.519 [.298](.297)	-.513 [.298](.298)	.260	.277	.282	.287
	100	-.618 [.270](.243)	-.507 [.245](.245)	-.505 [.245](.245)	.230	.237	.245	.246
	200	-.566 [.197](.185)	-.503 [.186](.186)	-.502 [.186](.186)	.175	.178	.183	.183
	500	-.528 [.120](.117)	-.501 [.117](.117)	-.501 [.117](.117)	.113	.114	.115	.116
		(f) Group Interaction with $k = n^{0.5}$, Lognormal Errors, MRSAR-B						
.50	50	.437 [.127](.110)	.488 [.107](.106)	.492 [.106](.106)	.094	.102	.117	.113
	100	.452 [.104](.092)	.492 [.090](.090)	.494 [.090](.090)	.083	.085	.092	.090
	200	.475 [.071](.067)	.497 [.066](.066)	.497 [.066](.066)	.060	.060	.065	.064
	500	.490 [.041](.039)	.499 [.040](.040)	.499 [.040](.040)	.038	.037	.039	.039
.25	50	.165 [.180](.158)	.236 [.156](.155)	.241 [.155](.155)	.135	.146	.166	.162
	100	.184 [.146](.131)	.242 [.128](.128)	.244 [.128](.128)	.118	.120	.130	.128
	200	.215 [.101](.094)	.247 [.094](.093)	.247 [.093](.093)	.088	.088	.094	.094
	500	.235 [.060](.059)	.248 [.059](.059)	.248 [.059](.059)	.056	.056	.058	.058
.00	50	-.109 [.231](.203)	-.020 [.202](.201)	-.014 [.201](.201)	.174	.187	.210	.207
	100	-.081 [.179](.160)	-.008 [.159](.159)	-.006 [.159](.158)	.151	.152	.164	.163
	200	-.046 [.131](.123)	-.005 [.122](.122)	-.004 [.122](.122)	.114	.114	.122	.122
	500	-.018 [.078](.076)	-.001 [.077](.077)	-.001 [.077](.076)	.074	.073	.077	.077
-.25	50	-.377 [.274](.243)	-.272 [.245](.244)	-.265 [.244](.244)	.210	.225	.253	.252
	100	-.346 [.216](.193)	-.261 [.194](.194)	-.259 [.194](.193)	.181	.181	.194	.195
	200	-.307 [.159](.148)	-.258 [.148](.148)	-.257 [.148](.148)	.140	.139	.148	.148
	500	-.273 [.099](.096)	-.252 [.097](.097)	-.252 [.097](.097)	.092	.091	.095	.095
-.50	50	-.639 [.312](.279)	-.524 [.286](.285)	-.516 [.285](.285)	.241	.258	.285	.288
	100	-.610 [.245](.219)	-.516 [.222](.221)	-.514 [.222](.221)	.209	.208	.220	.224
	200	-.565 [.186](.174)	-.508 [.174](.174)	-.507 [.174](.174)	.165	.164	.173	.174
	500	-.527 [.117](.114)	-.503 [.114](.114)	-.502 [.114](.114)	.110	.109	.113	.114

Note: $\overline{se}_1 = \mathrm{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \mathrm{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^{\mathrm{c}} = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n^{\mathrm{bc3}})^{\frac{1}{2}})$.

Table 1-1. Additional Results in Connection to Table 1

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
		`(g) Group Interaction with` $k = n^{0.35}$`, Normal Errors, MRSAR-B`						
.50	50	.421 [.149](.126)	.491 [.129](.128)	.494 [.129](.129)	.113	.124	.126	.125
	100	.435 [.127](.109)	.496 [.112](.112)	.498 [.113](.113)	.102	.109	.111	.112
	200	.467 [.086](.080)	.499 [.080](.080)	.500 [.080](.080)	.076	.078	.081	.080
	500	.486 [.051](.049)	.500 [.050](.050)	.500 [.050](.050)	.048	.049	.049	.050
.25	50	.136 [.213](.180)	.237 [.185](.185)	.241 [.186](.186)	.165	.181	.183	.182
	100	.152 [.189](.161)	.241 [.166](.166)	.243 [.166](.166)	.151	.161	.163	.165
	200	.200 [.130](.120)	.247 [.120](.120)	.248 [.120](.120)	.113	.116	.120	.119
	500	.229 [.076](.073)	.249 [.074](.074)	.249 [.074](.074)	.072	.073	.073	.074
.00	50	-.142 [.272](.232)	-.014 [.241](.241)	-.009 [.241](.241)	.213	.232	.233	.235
	100	-.125 [.246](.212)	-.008 [.219](.219)	-.006 [.219](.219)	.198	.210	.213	.216
	200	-.066 [.170](.156)	-.003 [.157](.157)	-.001 [.157](.157)	.149	.152	.158	.158
	500	-.025 [.100](.097)	.001 [.098](.098)	.001 [.098](.098)	.096	.097	.097	.098
-.25	50	-.423 [.328](.279)	-.271 [.291](.290)	-.265 [.291](.291)	.259	.279	.279	.284
	100	-.407 [.301](.256)	-.264 [.266](.266)	-.262 [.267](.267)	.245	.258	.261	.266
	200	-.332 [.211](.195)	-.255 [.195](.195)	-.253 [.195](.195)	.184	.189	.196	.195
	500	-.283 [.125](.120)	-.250 [.122](.122)	-.250 [.122](.122)	.119	.120	.121	.122
-.50	50	-.705 [.384](.325)	-.529 [.343](.342)	-.523 [.343](.343)	.303	.325	.323	.332
	100	-.688 [.360](.307)	-.520 [.320](.320)	-.518 [.321](.320)	.289	.303	.306	.313
	200	-.593 [.247](.229)	-.501 [.230](.230)	-.499 [.230](.230)	.219	.224	.232	.231
	500	-.539 [.150](.145)	-.501 [.146](.146)	-.501 [.146](.146)	.142	.143	.144	.146
		`(h) Group Interaction with` $k = n^{0.35}$`, Normal Mixture, MRSAR-B`						
.50	50	.423 [.149](.127)	.485 [.127](.126)	.490 [.127](.126)	.106	.115	.124	.122
	100	.434 [.128](.110)	.491 [.112](.112)	.493 [.112](.112)	.098	.103	.108	.109
	200	.467 [.086](.080)	.498 [.079](.079)	.498 [.079](.079)	.074	.075	.079	.079
	500	.487 [.050](.048)	.500 [.048](.048)	.500 [.048](.048)	.048	.048	.049	.049
.25	50	.140 [.214](.183)	.229 [.185](.183)	.235 [.184](.183)	.154	.166	.179	.178
	100	.155 [.187](.161)	.238 [.165](.165)	.241 [.165](.165)	.145	.152	.159	.161
	200	.200 [.128](.118)	.246 [.117](.117)	.247 [.117](.117)	.110	.111	.118	.118
	500	.229 [.076](.074)	.248 [.074](.074)	.248 [.074](.074)	.071	.072	.073	.074
.00	50	-.142 [.277](.238)	-.029 [.242](.240)	-.021 [.241](.240)	.200	.215	.231	.231
	100	-.124 [.243](.210)	-.015 [.215](.214)	-.012 [.214](.214)	.190	.198	.207	.211
	200	-.065 [.169](.156)	-.004 [.156](.156)	-.003 [.156](.156)	.145	.147	.156	.155
	500	-.026 [.101](.097)	-.001 [.098](.098)	-.001 [.098](.098)	.095	.095	.097	.098
-.25	50	-.416 [.324](.278)	-.280 [.284](.283)	-.271 [.283](.282)	.244	.260	.279	.282
	100	-.403 [.297](.254)	-.270 [.262](.262)	-.266 [.262](.261)	.234	.243	.254	.259
	200	-.328 [.211](.196)	-.254 [.196](.196)	-.252 [.196](.196)	.180	.182	.193	.192
	500	-.284 [.124](.119)	-.252 [.121](.121)	-.252 [.121](.121)	.118	.118	.120	.122
-.50	50	-.698 [.384](.329)	-.543 [.339](.336)	-.533 [.338](.336)	.284	.302	.321	.327
	100	-.680 [.353](.304)	-.525 [.314](.313)	-.520 [.313](.313)	.277	.287	.299	.306
	200	-.593 [.248](.230)	-.505 [.231](.231)	-.503 [.231](.231)	.214	.216	.229	.229
	500	-.538 [.148](.143)	-.500 [.145](.145)	-.500 [.145](.145)	.141	.141	.144	.145

Note: $\overline{se}_1 = \mathrm{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \mathrm{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \mathrm{mean}(\widehat{V}_3(\hat{\lambda}_n^{\mathrm{bc3}})^{\frac{1}{2}})$.

Table 1-1 (cont'd). Empirical Mean[rmse](sd), and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
		`(i) Group Interaction with` $k = n^{0.35}$`, Lognormal Errors, MRSAR-B`						
.50	50	.434 [.137](.120)	.484 [.118](.117)	.489 [.117](.116)	.093	.104	.145	.138
	100	.440 [.124](.108)	.488 [.108](.107)	.492 [.107](.107)	.090	.093	.108	.108
	200	.469 [.085](.080)	.496 [.079](.079)	.497 [.079](.078)	.069	.069	.079	.078
	500	.487 [.051](.050)	.499 [.050](.050)	.499 [.050](.050)	.046	.046	.049	.049
.25	50	.154 [.199](.174)	.225 [.172](.170)	.232 [.170](.169)	.136	.151	.208	.200
	100	.164 [.177](.155)	.234 [.156](.155)	.239 [.154](.154)	.133	.137	.158	.159
	200	.204 [.126](.117)	.245 [.116](.116)	.246 [.116](.116)	.102	.103	.117	.116
	500	.229 [.077](.074)	.247 [.075](.075)	.247 [.075](.075)	.069	.068	.073	.073
.00	50	-.117 [.252](.223)	-.025 [.221](.220)	-.017 [.220](.219)	.176	.197	.269	.261
	100	-.116 [.231](.200)	-.024 [.201](.200)	-.018 [.199](.198)	.174	.179	.207	.208
	200	-.064 [.171](.159)	-.011 [.157](.156)	-.010 [.156](.156)	.135	.136	.155	.154
	500	-.027 [.102](.098)	-.003 [.099](.099)	-.003 [.099](.099)	.092	.091	.096	.097
-.25	50	-.400 [.319](.282)	-.289 [.282](.279)	-.278 [.280](.278)	.217	.242	.334	.325
	100	-.392 [.289](.252)	-.280 [.255](.254)	-.272 [.252](.251)	.214	.220	.254	.257
	200	-.327 [.208](.194)	-.261 [.192](.192)	-.259 [.192](.191)	.168	.169	.192	.191
	500	-.283 [.126](.121)	-.254 [.122](.122)	-.253 [.122](.122)	.114	.113	.120	.121
-.50	50	-.676 [.376](.333)	-.548 [.334](.330)	-.536 [.330](.328)	.255	.283	.387	.380
	100	-.667 [.333](.288)	-.537 [.294](.292)	-.528 [.291](.290)	.252	.258	.298	.303
	200	-.588 [.243](.226)	-.511 [.224](.224)	-.508 [.224](.224)	.200	.201	.227	.226
	500	-.537 [.152](.147)	-.502 [.149](.149)	-.501 [.149](.149)	.136	.135	.143	.145
		`(j) Group Interaction with` $k = n^{0.65}$`, Normal Error, MRSAR-B`						
.50	50	.460 [.099](.091)	.498 [.089](.089)	.500 [.089](.089)	.086	.095	.094	.092
	100	.473 [.078](.074)	.501 [.072](.072)	.502 [.072](.072)	.069	.074	.075	.073
	200	.483 [.058](.055)	.499 [.054](.054)	.499 [.054](.054)	.053	.055	.055	.054
	500	.493 [.035](.035)	.500 [.035](.035)	.500 [.035](.035)	.034	.035	.035	.035
.25	50	.199 [.134](.123)	.249 [.123](.123)	.251 [.124](.124)	.118	.129	.126	.125
	100	.212 [.109](.102)	.250 [.101](.101)	.252 [.101](.101)	.098	.103	.104	.102
	200	.228 [.080](.077)	.250 [.077](.077)	.251 [.077](.077)	.076	.078	.078	.077
	500	.240 [.052](.051)	.250 [.051](.051)	.250 [.051](.051)	.050	.050	.051	.050
.00	50	-.061 [.159](.146)	-.003 [.149](.149)	-.002 [.149](.149)	.143	.154	.149	.150
	100	-.044 [.133](.125)	.001 [.125](.125)	.002 [.125](.125)	.121	.126	.126	.125
	200	-.028 [.102](.098)	.000 [.098](.098)	.001 [.098](.098)	.096	.098	.098	.098
	500	-.013 [.066](.065)	-.001 [.065](.065)	-.001 [.065](.065)	.064	.065	.065	.065
-.25	50	-.315 [.178](.166)	-.254 [.171](.171)	-.254 [.172](.172)	.160	.170	.161	.166
	100	-.302 [.150](.141)	-.254 [.142](.142)	-.253 [.143](.143)	.139	.144	.142	.143
	200	-.283 [.120](.116)	-.251 [.116](.116)	-.251 [.116](.116)	.113	.115	.115	.115
	500	-.264 [.078](.077)	-.249 [.077](.077)	-.249 [.077](.077)	.077	.078	.078	.078
-.50	50	-.564 [.182](.171)	-.504 [.179](.179)	-.504 [.180](.180)	.169	.177	.166	.173
	100	-.548 [.159](.151)	-.500 [.155](.155)	-.500 [.155](.155)	.150	.154	.150	.154
	200	-.534 [.131](.127)	-.501 [.128](.128)	-.500 [.128](.128)	.126	.128	.127	.128
	500	-.516 [.091](.090)	-.499 [.090](.090)	-.499 [.090](.090)	.089	.089	.089	.090

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\mathrm{bc3}})^{\frac{1}{2}})$.

Table 1-1 (cont'd). Empirical Mean[rmse](sd), and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\rm bc2}$	$\hat{\lambda}_n^{\rm bc3}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
		`(k) Group Interaction with` $k = n^{0.65}$`, Normal Mixture, MRSAR-B`						
.50	50	.463 [.094](.086)	.498 [.084](.084)	.500 [.084](.084)	.081	.088	.088	.086
	100	.472 [.077](.072)	.499 [.070](.070)	.500 [.070](.070)	.067	.071	.072	.070
	200	.484 [.057](.054)	.499 [.054](.054)	.500 [.054](.054)	.052	.053	.054	.053
	500	.493 [.035](.035)	.500 [.035](.035)	.500 [.035](.035)	.034	.034	.035	.035
.25	50	.200 [.128](.118)	.246 [.118](.118)	.247 [.118](.118)	.111	.120	.118	.117
	100	.213 [.106](.099)	.249 [.098](.098)	.250 [.098](.098)	.094	.098	.099	.098
	200	.227 [.080](.076)	.249 [.076](.076)	.249 [.076](.076)	.074	.076	.077	.076
	500	.240 [.050](.049)	.250 [.049](.049)	.250 [.049](.049)	.049	.050	.050	.050
.00	50	-.057 [.152](.141)	-.004 [.143](.143)	-.003 [.143](.143)	.134	.143	.139	.141
	100	-.044 [.128](.120)	-.002 [.120](.120)	-.001 [.120](.120)	.116	.121	.121	.120
	200	-.026 [.099](.095)	.001 [.095](.095)	.002 [.095](.095)	.093	.095	.096	.096
	500	-.013 [.066](.065)	-.001 [.065](.065)	-.000 [.065](.065)	.063	.064	.064	.064
-.25	50	-.311 [.168](.157)	-.256 [.161](.161)	-.255 [.161](.161)	.150	.158	.152	.156
	100	-.297 [.146](.138)	-.252 [.140](.140)	-.251 [.140](.140)	.134	.138	.136	.138
	200	-.281 [.117](.113)	-.251 [.114](.114)	-.251 [.114](.114)	.111	.112	.112	.113
	500	-.265 [.079](.078)	-.250 [.078](.078)	-.250 [.078](.078)	.076	.077	.077	.077
-.50	50	-.558 [.173](.163)	-.505 [.170](.170)	-.504 [.171](.170)	.161	.166	.158	.165
	100	-.549 [.155](.147)	-.505 [.152](.152)	-.505 [.152](.152)	.148	.150	.146	.150
	200	-.532 [.129](.125)	-.500 [.127](.127)	-.500 [.127](.127)	.126	.126	.125	.127
	500	-.516 [.091](.089)	-.500 [.090](.090)	-.500 [.090](.090)	.089	.089	.089	.089
		`(l) Group Interaction with` $k = n^{0.65}$`, Lognormal Errors, MRSAR-B`						
.50	50	.467 [.085](.078)	.496 [.076](.076)	.498 [.076](.076)	.073	.078	.081	.080
	100	.473 [.072](.067)	.497 [.065](.065)	.498 [.065](.065)	.064	.065	.066	.065
	200	.485 [.053](.050)	.499 [.050](.050)	.500 [.050](.050)	.049	.050	.051	.050
	500	.493 [.035](.034)	.500 [.034](.034)	.500 [.034](.034)	.033	.033	.034	.034
.25	50	.209 [.111](.104)	.247 [.103](.103)	.249 [.103](.103)	.100	.106	.108	.109
	100	.215 [.097](.091)	.247 [.090](.090)	.248 [.090](.090)	.088	.089	.091	.090
	200	.230 [.073](.070)	.250 [.070](.070)	.250 [.070](.070)	.070	.070	.071	.071
	500	.240 [.049](.049)	.250 [.048](.048)	.250 [.048](.048)	.048	.047	.048	.048
.00	50	-.050 [.135](.125)	-.007 [.126](.126)	-.005 [.126](.126)	.122	.128	.130	.132
	100	-.042 [.117](.109)	-.004 [.109](.109)	-.003 [.109](.109)	.108	.109	.110	.110
	200	-.027 [.092](.088)	-.003 [.088](.088)	-.003 [.088](.088)	.088	.088	.089	.089
	500	-.012 [.063](.062)	-.000 [.062](.062)	-.000 [.062](.062)	.061	.060	.061	.061
-.25	50	-.302 [.150](.141)	-.256 [.144](.144)	-.254 [.144](.144)	.139	.144	.144	.148
	100	-.293 [.133](.125)	-.253 [.127](.127)	-.252 [.127](.127)	.126	.126	.125	.127
	200	-.278 [.109](.106)	-.251 [.106](.106)	-.251 [.106](.106)	.106	.104	.105	.106
	500	-.265 [.076](.075)	-.251 [.075](.075)	-.251 [.075](.075)	.074	.073	.074	.074
-.50	50	-.552 [.161](.153)	-.508 [.159](.158)	-.506 [.159](.159)	.150	.154	.152	.158
	100	-.542 [.145](.138)	-.504 [.143](.143)	-.503 [.143](.143)	.141	.139	.136	.141
	200	-.527 [.124](.121)	-.500 [.123](.123)	-.500 [.123](.123)	.123	.120	.119	.121
	500	-.515 [.089](.088)	-.501 [.088](.088)	-.501 [.088](.088)	.086	.085	.086	.086

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\rm bc3})^{\frac{1}{2}})$.

Table 1-2. Replication of Table 1, under $\beta = \{.5, .1, .1\}'$

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\text{bc2}}$	$\hat{\lambda}_n^{\text{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
`(a) Queen, Normal Error, MRSAR-A`								
.50	50	.370 [.252](.215)	.486 [.215](.215)	.493 [.216](.216)	.191	.207	.223	.219
	100	.445 [.150](.140)	.500 [.139](.139)	.501 [.139](.139)	.130	.134	.142	.141
	200	.468 [.098](.093)	.497 [.093](.093)	.497 [.093](.093)	.091	.093	.096	.095
	500	.489 [.059](.058)	.500 [.058](.058)	.500 [.058](.058)	.057	.057	.058	.058
.25	50	.121 [.277](.246)	.236 [.253](.253)	.241 [.255](.255)	.229	.245	.249	.254
	100	.192 [.181](.171)	.246 [.173](.173)	.247 [.174](.174)	.160	.165	.167	.169
	200	.218 [.115](.111)	.247 [.112](.112)	.247 [.112](.112)	.113	.115	.116	.116
	500	.238 [.073](.071)	.249 [.071](.071)	.249 [.071](.071)	.071	.072	.072	.072
.00	50	-.133 [.296](.264)	-.027 [.282](.281)	-.025 [.284](.283)	.255	.270	.262	.276
	100	-.064 [.198](.188)	-.015 [.193](.193)	-.014 [.194](.193)	.181	.186	.183	.188
	200	-.030 [.135](.131)	-.003 [.133](.133)	-.003 [.133](.133)	.128	.130	.129	.130
	500	-.009 [.080](.080)	.001 [.080](.080)	.001 [.080](.080)	.081	.081	.081	.081
-.25	50	-.342 [.284](.269)	-.250 [.290](.290)	-.250 [.292](.292)	.267	.279	.263	.282
	100	-.280 [.197](.195)	-.238 [.203](.202)	-.237 [.203](.203)	.191	.195	.189	.196
	200	-.271 [.135](.133)	-.247 [.136](.136)	-.247 [.136](.136)	.136	.138	.136	.138
	500	-.261 [.088](.087)	-.252 [.088](.088)	-.252 [.088](.088)	.087	.087	.086	.087
-.50	50	-.581 [.272](.259)	-.510 [.284](.284)	-.511 [.286](.286)	.270	.277	.256	.279
	100	-.526 [.190](.188)	-.494 [.197](.197)	-.494 [.198](.198)	.195	.198	.190	.198
	200	-.510 [.138](.137)	-.492 [.141](.140)	-.492 [.141](.141)	.139	.140	.137	.140
	500	-.507 [.086](.086)	-.500 [.087](.087)	-.500 [.087](.087)	.088	.089	.088	.089
`(b) Queen, Normal Mixture, MRSAR-A`								
.50	50	.384 [.234](.203)	.497 [.203](.203)	.503 [.204](.204)	.181	.197	.212	.208
	100	.443 [.150](.138)	.496 [.138](.138)	.497 [.138](.138)	.127	.131	.138	.137
	200	.474 [.094](.090)	.502 [.089](.089)	.502 [.089](.089)	.089	.090	.094	.093
	500	.490 [.059](.058)	.501 [.058](.058)	.501 [.058](.058)	.056	.057	.058	.058
.25	50	.130 [.259](.229)	.243 [.235](.235)	.248 [.237](.237)	.220	.236	.239	.244
	100	.198 [.166](.158)	.252 [.160](.160)	.253 [.161](.160)	.155	.160	.163	.164
	200	.229 [.113](.111)	.257 [.112](.112)	.258 [.112](.112)	.110	.112	.113	.114
	500	.240 [.071](.070)	.252 [.071](.071)	.252 [.071](.071)	.070	.071	.071	.071
.00	50	-.096 [.268](.251)	.009 [.264](.264)	.012 [.267](.266)	.242	.256	.251	.262
	100	-.048 [.181](.175)	.001 [.180](.180)	.002 [.180](.180)	.175	.180	.178	.182
	200	-.022 [.128](.127)	.005 [.128](.128)	.005 [.128](.128)	.125	.127	.126	.128
	500	-.010 [.079](.078)	.001 [.079](.079)	.001 [.079](.079)	.080	.080	.080	.081
-.25	50	-.352 [.266](.246)	-.262 [.265](.265)	-.262 [.267](.267)	.258	.269	.254	.272
	100	-.294 [.185](.180)	-.253 [.186](.186)	-.253 [.187](.187)	.186	.190	.185	.191
	200	-.275 [.139](.137)	-.252 [.139](.139)	-.252 [.139](.139)	.134	.135	.133	.136
	500	-.260 [.087](.087)	-.251 [.087](.087)	-.251 [.087](.087)	.086	.086	.086	.086
-.50	50	-.578 [.266](.255)	-.507 [.278](.277)	-.508 [.279](.279)	.260	.268	.249	.270
	100	-.531 [.197](.195)	-.500 [.204](.204)	-.501 [.204](.204)	.190	.193	.185	.193
	200	-.518 [.136](.135)	-.500 [.138](.138)	-.500 [.138](.138)	.136	.138	.135	.138
	500	-.512 [.087](.086)	-.505 [.087](.087)	-.505 [.087](.087)	.088	.088	.087	.088

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\text{bc3}})^{\frac{1}{2}})$.

Table 1-2 (Cont'd). Empirical Mean[rmse](sd), and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\text{bc2}}$	$\hat{\lambda}_n^{\text{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^{c}$
			`(c) Queen, lognormal, MRSAR-A`					
.50	50	.383 [.214](.179)	.493 [.178](.177)	.499 [.178](.178)	.176	.188	.200	.197
	100	.448 [.135](.125)	.499 [.124](.124)	.500 [.124](.124)	.123	.123	.129	.128
	200	.475 [.089](.086)	.502 [.085](.085)	.502 [.085](.085)	.087	.086	.088	.088
	500	.487 [.055](.054)	.499 [.054](.054)	.499 [.054](.054)	.056	.055	.056	.055
.25	50	.137 [.234](.205)	.247 [.210](.210)	.251 [.211](.211)	.213	.224	.225	.230
	100	.197 [.159](.150)	.248 [.151](.151)	.249 [.151](.151)	.150	.151	.152	.154
	200	.221 [.111](.108)	.248 [.108](.108)	.249 [.108](.108)	.108	.107	.108	.108
	500	.239 [.068](.067)	.250 [.067](.067)	.250 [.067](.067)	.069	.068	.069	.069
.00	50	-.123 [.263](.233)	-.023 [.246](.245)	-.020 [.247](.246)	.237	.247	.238	.250
	100	-.040 [.166](.161)	.007 [.165](.165)	.008 [.165](.165)	.168	.170	.167	.171
	200	-.032 [.127](.123)	-.006 [.125](.125)	-.006 [.125](.125)	.122	.122	.120	.122
	500	-.010 [.080](.079)	.001 [.080](.080)	.001 [.080](.080)	.078	.078	.078	.078
-.25	50	-.352 [.262](.242)	-.265 [.260](.259)	-.264 [.261](.261)	.250	.259	.244	.260
	100	-.297 [.177](.170)	-.258 [.176](.176)	-.257 [.176](.176)	.180	.182	.176	.182
	200	-.274 [.130](.128)	-.251 [.130](.130)	-.251 [.130](.130)	.130	.130	.128	.130
	500	-.257 [.081](.081)	-.248 [.081](.081)	-.248 [.081](.081)	.084	.084	.083	.084
-.50	50	-.565 [.254](.245)	-.496 [.266](.266)	-.496 [.268](.268)	.252	.258	.240	.260
	100	-.530 [.174](.171)	-.500 [.178](.178)	-.500 [.178](.178)	.184	.186	.178	.186
	200	-.517 [.129](.128)	-.500 [.131](.131)	-.499 [.131](.131)	.133	.134	.131	.133
	500	-.508 [.084](.083)	-.501 [.084](.084)	-.501 [.084](.084)	.086	.086	.085	.086
			`(d) Group Interaction with` $k = n^{0.5}$`, Normal Errors, MRSAR-B`					
.50	50	.315 [.302](.238)	.488 [.198](.198)	.502 [.198](.198)	.171	.197	.234	.180
	100	.390 [.195](.161)	.499 [.138](.138)	.504 [.137](.137)	.131	.143	.170	.141
	200	.434 [.141](.125)	.502 [.112](.112)	.504 [.112](.112)	.104	.109	.128	.112
	500	.454 [.100](.089)	.498 [.082](.082)	.498 [.082](.082)	.080	.083	.093	.086
.25	50	.012 [.381](.297)	.247 [.257](.257)	.262 [.260](.260)	.237	.268	.314	.255
	100	.089 [.285](.235)	.242 [.208](.208)	.248 [.208](.208)	.189	.205	.240	.205
	200	.155 [.209](.186)	.255 [.168](.168)	.257 [.168](.168)	.153	.159	.185	.165
	500	.185 [.151](.136)	.249 [.127](.127)	.250 [.127](.127)	.118	.122	.136	.126
.00	50	-.286 [.464](.365)	-.003 [.333](.333)	.010 [.338](.338)	.295	.327	.374	.323
	100	-.201 [.364](.304)	-.010 [.277](.277)	-.004 [.278](.278)	.242	.258	.298	.263
	200	-.127 [.263](.230)	.001 [.209](.209)	.003 [.209](.209)	.200	.207	.239	.215
	500	-.086 [.188](.167)	-.001 [.156](.156)	-.001 [.156](.155)	.156	.160	.179	.166
-.25	50	-.602 [.564](.441)	-.288 [.428](.427)	-.280 [.435](.434)	.346	.374	.414	.382
	100	-.468 [.396](.330)	-.249 [.306](.306)	-.244 [.309](.309)	.286	.303	.345	.313
	200	-.396 [.330](.296)	-.241 [.271](.271)	-.240 [.272](.272)	.242	.249	.287	.261
	500	-.356 [.241](.216)	-.253 [.204](.204)	-.252 [.204](.204)	.192	.196	.219	.204
-.50	50	-.864 [.568](.436)	-.534 [.444](.442)	-.529 [.450](.449)	.386	.407	.437	.424
	100	-.757 [.465](.387)	-.517 [.375](.374)	-.514 [.378](.377)	.328	.342	.380	.358
	200	-.692 [.374](.321)	-.512 [.297](.297)	-.511 [.298](.298)	.287	.294	.337	.309
	500	-.608 [.260](.237)	-.489 [.223](.223)	-.488 [.224](.223)	.225	.229	.255	.239

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^{c} = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\text{bc3}})^{\frac{1}{2}})$.

Table 1-2 (Cont'd). Empirical Mean[rmse](sd), and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\text{bc2}}$	$\hat{\lambda}_n^{\text{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
		`(e) Group Interaction with` $k = n^{0.5}$`, Normal Mixture, MRSAR-B`						
.50	50	.325 [.284](.224)	.493 [.184](.184)	.507 [.184](.184)	.163	.191	.226	.175
	100	.397 [.189](.158)	.502 [.137](.136)	.507 [.136](.136)	.127	.138	.164	.137
	200	.434 [.135](.117)	.502 [.105](.105)	.504 [.105](.105)	.103	.108	.126	.111
	500	.458 [.099](.090)	.501 [.083](.083)	.502 [.083](.083)	.079	.082	.092	.085
.25	50	.033 [.362](.290)	.258 [.255](.255)	.272 [.258](.257)	.226	.260	.302	.246
	100	.094 [.272](.223)	.243 [.198](.198)	.249 [.198](.198)	.184	.199	.233	.200
	200	.151 [.200](.174)	.250 [.157](.157)	.252 [.157](.157)	.151	.158	.183	.163
	500	.189 [.145](.131)	.252 [.123](.123)	.253 [.123](.123)	.117	.120	.135	.125
.00	50	-.274 [.451](.358)	-.001 [.328](.328)	.011 [.334](.334)	.289	.325	.368	.319
	100	-.199 [.355](.294)	-.012 [.267](.267)	-.007 [.269](.269)	.236	.254	.291	.258
	200	-.136 [.263](.226)	-.008 [.206](.205)	-.006 [.206](.206)	.198	.206	.238	.214
	500	-.090 [.196](.174)	-.006 [.163](.162)	-.006 [.162](.162)	.155	.159	.178	.166
-.25	50	-.557 [.503](.399)	-.254 [.385](.385)	-.246 [.393](.393)	.342	.377	.413	.379
	100	-.476 [.399](.329)	-.261 [.307](.307)	-.258 [.310](.310)	.284	.302	.339	.310
	200	-.399 [.302](.262)	-.247 [.241](.241)	-.246 [.242](.242)	.240	.249	.285	.259
	500	-.354 [.231](.206)	-.251 [.194](.194)	-.251 [.194](.194)	.191	.195	.218	.203
-.50	50	-.846 [.576](.461)	-.526 [.477](.476)	-.524 [.484](.484)	.389	.418	.442	.428
	100	-.743 [.427](.351)	-.508 [.341](.341)	-.507 [.346](.346)	.329	.346	.377	.358
	200	-.681 [.351](.301)	-.505 [.279](.279)	-.503 [.280](.280)	.283	.293	.333	.306
	500	-.615 [.265](.239)	-.495 [.224](.224)	-.495 [.224](.224)	.226	.231	.257	.241
		`(f) Group Interaction with` $k = n^{0.5}$`, lognormal, MRSAR-B`						
.50	50	.354 [.235](.184)	.511 [.155](.154)	.523 [.156](.154)	.153	.176	.207	.161
	100	.391 [.183](.147)	.495 [.127](.127)	.499 [.127](.127)	.124	.133	.156	.132
	200	.441 [.123](.108)	.506 [.097](.097)	.506 [.097](.097)	.099	.101	.118	.105
	500	.454 [.095](.083)	.497 [.077](.077)	.497 [.077](.077)	.078	.079	.089	.082
.25	50	.033 [.336](.257)	.251 [.228](.228)	.263 [.230](.230)	.221	.250	.288	.237
	100	.110 [.246](.202)	.255 [.179](.179)	.259 [.180](.179)	.176	.186	.216	.187
	200	.160 [.179](.155)	.256 [.140](.140)	.257 [.140](.140)	.146	.149	.173	.155
	500	.186 [.140](.125)	.250 [.116](.116)	.249 [.116](.116)	.115	.116	.130	.121
.00	50	-.269 [.417](.319)	-.004 [.291](.291)	.007 [.295](.295)	.284	.316	.354	.309
	100	-.179 [.309](.252)	-.001 [.229](.229)	.003 [.231](.231)	.227	.237	.270	.241
	200	-.128 [.243](.207)	-.003 [.189](.189)	-.003 [.189](.189)	.192	.195	.225	.203
	500	-.077 [.178](.161)	.005 [.151](.151)	.004 [.151](.151)	.151	.151	.170	.158
-.25	50	-.551 [.475](.368)	-.256 [.359](.359)	-.250 [.365](.365)	.339	.369	.399	.368
	100	-.445 [.343](.282)	-.241 [.264](.264)	-.239 [.267](.266)	.273	.281	.313	.288
	200	-.403 [.300](.257)	-.254 [.237](.237)	-.255 [.237](.237)	.235	.239	.272	.249
	500	-.352 [.225](.200)	-.252 [.187](.187)	-.253 [.187](.187)	.188	.189	.211	.198
-.50	50	-.816 [.510](.401)	-.503 [.407](.407)	-.503 [.414](.414)	.391	.417	.434	.421
	100	-.715 [.385](.320)	-.492 [.312](.312)	-.494 [.316](.316)	.320	.325	.349	.334
	200	-.662 [.323](.279)	-.492 [.260](.260)	-.493 [.261](.261)	.277	.280	.316	.291
	500	-.608 [.247](.222)	-.492 [.209](.209)	-.493 [.210](.210)	.223	.225	.249	.234

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\text{bc3}})^{\frac{1}{2}})$.

Table 1-3. Replication of Table 1, under $\sigma = 2$

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\text{bc2}}$	$\hat{\lambda}_n^{\text{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
`(a) Queen, Normal Error, MRSAR-A`								
.50	50	.382 [.234](.203)	.487 [.203](.203)	.493 [.204](.203)	.181	.196	.209	.207
	100	.450 [.143](.134)	.500 [.134](.134)	.501 [.133](.133)	.124	.128	.135	.134
	200	.472 [.089](.085)	.497 [.085](.084)	.498 [.085](.084)	.085	.087	.089	.089
	500	.490 [.054](.053)	.500 [.053](.053)	.500 [.053](.053)	.053	.053	.054	.054
.25	50	.135 [.256](.229)	.238 [.235](.235)	.243 [.236](.236)	.217	.232	.234	.240
	100	.196 [.173](.164)	.245 [.166](.166)	.246 [.166](.166)	.153	.157	.159	.161
	200	.220 [.109](.105)	.245 [.105](.105)	.246 [.105](.105)	.106	.108	.109	.109
	500	.240 [.067](.066)	.250 [.066](.066)	.250 [.066](.066)	.066	.066	.067	.067
.00	50	-.118 [.275](.249)	-.023 [.264](.263)	-.020 [.266](.266)	.242	.256	.248	.260
	100	-.057 [.188](.179)	-.013 [.184](.184)	-.012 [.184](.184)	.172	.176	.174	.178
	200	-.027 [.125](.122)	-.004 [.124](.123)	-.003 [.124](.124)	.120	.122	.121	.122
	500	-.009 [.074](.074)	.001 [.075](.075)	.001 [.075](.075)	.075	.075	.075	.076
-.25	50	-.331 [.268](.255)	-.249 [.273](.273)	-.248 [.275](.275)	.253	.264	.250	.266
	100	-.278 [.187](.185)	-.239 [.191](.191)	-.239 [.192](.192)	.181	.185	.180	.186
	200	-.271 [.129](.127)	-.250 [.130](.130)	-.250 [.130](.130)	.128	.130	.128	.130
	500	-.260 [.082](.082)	-.252 [.082](.082)	-.252 [.082](.082)	.080	.081	.080	.081
-.50	50	-.575 [.260](.249)	-.511 [.270](.270)	-.512 [.272](.272)	.256	.264	.245	.265
	100	-.523 [.182](.180)	-.494 [.188](.188)	-.494 [.188](.188)	.185	.188	.180	.188
	200	-.508 [.130](.130)	-.491 [.132](.132)	-.491 [.133](.132)	.131	.132	.130	.132
	500	-.505 [.080](.080)	-.499 [.081](.081)	-.499 [.081](.081)	.082	.082	.082	.083
`(b) Queen, Normal Mixture, MRSAR-A`								
.50	50	.397 [.215](.188)	.497 [.189](.189)	.502 [.190](.190)	.171	.185	.197	.195
	100	.449 [.141](.131)	.497 [.131](.131)	.498 [.131](.131)	.121	.124	.130	.130
	200	.477 [.088](.085)	.501 [.084](.084)	.502 [.084](.084)	.083	.084	.087	.087
	500	.491 [.054](.053)	.501 [.053](.053)	.501 [.053](.053)	.052	.052	.053	.053
.25	50	.145 [.243](.219)	.244 [.225](.225)	.248 [.226](.226)	.207	.221	.224	.228
	100	.205 [.156](.149)	.252 [.151](.151)	.254 [.151](.151)	.147	.152	.154	.155
	200	.231 [.108](.106)	.256 [.107](.107)	.256 [.107](.107)	.103	.105	.106	.106
	500	.241 [.065](.064)	.251 [.064](.064)	.251 [.064](.064)	.065	.066	.066	.066
.00	50	-.084 [.249](.235)	.008 [.246](.246)	.010 [.248](.247)	.228	.241	.235	.245
	100	-.044 [.170](.165)	-.000 [.169](.169)	.001 [.169](.169)	.166	.170	.168	.172
	200	-.021 [.120](.118)	.003 [.119](.119)	.003 [.120](.120)	.117	.119	.118	.120
	500	-.008 [.073](.073)	.001 [.073](.073)	.001 [.073](.073)	.074	.075	.074	.075
-.25	50	-.340 [.253](.236)	-.262 [.251](.251)	-.262 [.253](.253)	.242	.253	.240	.255
	100	-.289 [.175](.171)	-.253 [.176](.176)	-.252 [.176](.176)	.176	.179	.175	.180
	200	-.272 [.129](.128)	-.252 [.129](.129)	-.252 [.130](.130)	.126	.127	.125	.127
	500	-.259 [.081](.080)	-.251 [.081](.081)	-.251 [.081](.081)	.080	.080	.080	.080
-.50	50	-.570 [.251](.242)	-.508 [.261](.260)	-.508 [.262](.262)	.245	.252	.236	.254
	100	-.528 [.188](.186)	-.501 [.194](.194)	-.501 [.194](.194)	.179	.181	.175	.182
	200	-.516 [.129](.128)	-.500 [.131](.131)	-.500 [.131](.131)	.128	.129	.127	.130
	500	-.510 [.081](.081)	-.505 [.082](.082)	-.505 [.082](.082)	.081	.082	.081	.082

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\text{bc3}})^{\frac{1}{2}})$.

Table 1-3 (cont'd). Empirical Mean[rmse](sd), and Averaged Bootstrap SEs

λ	n	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\rm bc2}$	$\hat{\lambda}_n^{\rm bc3}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^{\rm c}$
		`(c) Queen, Lognormal Error, MRSAR-A`						
.50	50	.399 [.194](.166)	.491 [.166](.165)	.496 [.166](.166)	.163	.173	.183	.181
	100	.452 [.129](.119)	.497 [.119](.119)	.498 [.119](.119)	.116	.117	.121	.121
	200	.479 [.084](.082)	.502 [.081](.081)	.502 [.081](.081)	.080	.080	.082	.082
	500	.489 [.050](.048)	.498 [.048](.048)	.498 [.048](.048)	.051	.050	.051	.051
.25	50	.150 [.223](.199)	.242 [.203](.203)	.246 [.204](.204)	.197	.206	.207	.211
	100	.206 [.150](.143)	.250 [.145](.145)	.251 [.145](.145)	.141	.142	.143	.144
	200	.225 [.105](.102)	.248 [.103](.103)	.249 [.103](.103)	.100	.099	.100	.100
	500	.241 [.062](.061)	.250 [.062](.062)	.250 [.062](.062)	.064	.063	.063	.064
.00	50	-.105 [.242](.218)	-.022 [.228](.227)	-.019 [.229](.228)	.217	.226	.219	.228
	100	-.037 [.161](.156)	.003 [.160](.160)	.004 [.160](.160)	.158	.159	.157	.160
	200	-.030 [.120](.116)	-.008 [.118](.118)	-.007 [.118](.118)	.113	.113	.113	.114
	500	-.009 [.074](.073)	.000 [.073](.073)	.000 [.073](.073)	.072	.072	.072	.072
-.25	50	-.343 [.244](.225)	-.271 [.240](.239)	-.269 [.241](.241)	.229	.235	.224	.237
	100	-.295 [.169](.163)	-.261 [.168](.167)	-.261 [.168](.168)	.168	.170	.165	.170
	200	-.269 [.121](.120)	-.250 [.121](.121)	-.249 [.121](.121)	.121	.121	.120	.121
	500	-.257 [.076](.075)	-.249 [.076](.076)	-.249 [.076](.076)	.078	.077	.077	.078
-.50	50	-.554 [.236](.230)	-.496 [.246](.246)	-.495 [.247](.247)	.230	.235	.222	.237
	100	-.524 [.166](.164)	-.498 [.170](.170)	-.497 [.170](.170)	.171	.173	.167	.173
	200	-.516 [.121](.120)	-.500 [.123](.123)	-.500 [.123](.123)	.124	.125	.123	.125
	500	-.506 [.078](.078)	-.500 [.079](.079)	-.500 [.079](.079)	.079	.079	.079	.080
		`(d) Group Interaction with` $k = n^{0.5}$`, Normal Error, MRSAR-B`						
.50	50	.362 [.233](.188)	.486 [.171](.170)	.495 [.171](.171)	.146	.166	.186	.159
	100	.423 [.145](.122)	.498 [.115](.115)	.501 [.115](.115)	.112	.120	.132	.121
	200	.450 [.114](.102)	.501 [.096](.096)	.502 [.096](.096)	.092	.095	.106	.098
	500	.469 [.076](.070)	.497 [.068](.068)	.497 [.068](.068)	.065	.067	.071	.069
.25	50	.066 [.314](.254)	.238 [.237](.237)	.248 [.238](.238)	.206	.231	.255	.226
	100	.135 [.214](.181)	.241 [.173](.173)	.244 [.174](.173)	.161	.172	.187	.175
	200	.179 [.169](.154)	.254 [.145](.145)	.255 [.145](.145)	.134	.139	.154	.144
	500	.206 [.112](.103)	.247 [.100](.100)	.248 [.100](.100)	.096	.098	.105	.101
.00	50	-.214 [.366](.297)	-.004 [.284](.284)	.006 [.287](.287)	.257	.284	.308	.283
	100	-.141 [.275](.237)	-.007 [.230](.230)	-.004 [.231](.231)	.206	.218	.235	.223
	200	-.097 [.218](.195)	-.001 [.185](.185)	.001 [.185](.185)	.176	.181	.200	.188
	500	-.056 [.145](.134)	-.003 [.130](.130)	-.002 [.130](.130)	.127	.129	.137	.133
-.25	50	-.520 [.455](.366)	-.281 [.363](.362)	-.273 [.366](.365)	.306	.331	.352	.336
	100	-.406 [.305](.263)	-.251 [.257](.257)	-.247 [.258](.258)	.247	.260	.277	.267
	200	-.364 [.277](.253)	-.247 [.241](.241)	-.246 [.241](.241)	.214	.219	.241	.228
	500	-.316 [.179](.167)	-.250 [.163](.163)	-.249 [.163](.163)	.156	.159	.169	.164
-.50	50	-.783 [.467](.371)	-.527 [.382](.381)	-.521 [.385](.385)	.344	.365	.379	.375
	100	-.682 [.368](.320)	-.509 [.320](.320)	-.506 [.322](.322)	.284	.295	.311	.306
	200	-.643 [.302](.266)	-.508 [.256](.256)	-.506 [.256](.256)	.252	.257	.282	.269
	500	-.573 [.204](.191)	-.496 [.187](.187)	-.495 [.187](.187)	.185	.187	.199	.193

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat{\lambda}_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat{\lambda}_n)^{\frac{1}{2}})$ and $\overline{se}_3^{\rm c} = \text{mean}(\widehat{V}_3(\hat{\lambda}_n^{\rm bc3})^{\frac{1}{2}})$.

Table 1-3 (cont'd). Empirical Mean[rmse](sd), and Averaged Bootstrap SEs

λ	n	$\hat\lambda_n$	$\hat\lambda_n^{\text{bc2}}$	$\hat\lambda_n^{\text{bc3}}$	$\overline{se}_1$	$\overline{se}_2$	$\overline{se}_3$	$\overline{se}_3^c$
		`(e) Group Interaction with` $k = n^{0.5}$`, Normal Mixture, MRSAR-B`						
.50	50	.375 [.220](.181)	.490 [.164](.164)	.498 [.164](.164)	.137	.157	.175	.152
	100	.430 [.134](.115)	.500 [.109](.109)	.503 [.109](.109)	.107	.114	.125	.116
	200	.448 [.116](.103)	.498 [.097[(.097)	.499 [.097](.097)	.090	.094	.104	.097
	500	.475 [.073](.068)	.502 [.066](.066)	.502 [.066](.066)	.064	.065	.070	.067
.25	50	.094 [.276](.227)	.253 [.212](.212)	.262 [.214](.213)	.194	.219	.241	.215
	100	.142 [.208](.177)	.242 [.170](.170)	.245 [.170](.170)	.155	.166	.180	.169
	200	.174 [.165](.146)	.248 [.138](.138)	.249 [.138](.138)	.132	.137	.152	.142
	500	.213 [.109](.103)	.254 [.100](.100)	.254 [.100](.100)	.095	.097	.103	.100
.00	50	-.203 [.363](.301)	-.006 [.289](.289)	.003 [.291](.291)	.248	.276	.299	.277
	100	-.142 [.273](.233)	-.015 [.225](.225)	-.012 [.226](.225)	.201	.212	.228	.217
	200	-.104 [.215](.189)	-.009 [.180](.179)	-.008 [.180](.180)	.173	.178	.196	.185
	500	-.059 [.145](.133)	-.006 [.129](.129)	-.006 [.129](.129)	.126	.128	.136	.132
-.25	50	-.482 [.406](.333)	-.262 [.330](.330)	-.254 [.333](.333)	.294	.322	.341	.328
	100	-.408 [.307](.263)	-.262 [.259](.259)	-.259 [.260](.260)	.240	.252	.266	.259
	200	-.364 [.253](.226)	-.251 [.217](.217)	-.249 [.217](.217)	.210	.216	.237	.225
	500	-.317 [.171](.158)	-.252 [.155](.155)	-.251 [.155](.155)	.155	.158	.167	.163
-.50	50	-.771 [.486](.404)	-.531 [.419](.418)	-.526 [.422](.421)	.338	.364	.377	.374
	100	-.676 [.341](.292)	-.514 [.293](.292)	-.512 [.294](.294)	.278	.290	.302	.299
	200	-.639 [.294](.259)	-.506 [.250](.250)	-.505 [.250](.250)	.248	.254	.277	.265
	500	-.578 [.209](.194)	-.502 [.189](.189)	-.502 [.189](.189)	.185	.187	.199	.194
		`(f) Group Interaction with` $k = n^{0.5}$`, Lognormal Error, MRSAR-B`						
.50	50	.392 [.192](.159)	.492 [.146](.146)	.498 [.146](.146)	.127	.143	.158	.141
	100	.425 [.135](.111)	.489 [.105](.105)	.491 [.105](.105)	.102	.107	.117	.110
	200	.457 [.102](.093)	.504 [.088](.088)	.504 [.088](.088)	.085	.087	.096	.090
	500	.470 [.074](.067)	.496 [.065](.065)	.496 [.065](.065)	.062	.063	.067	.065
.25	50	.096 [.257](.206)	.238 [.196](.196)	.245 [.197](.197)	.184	.204	.224	.206
	100	.158 [.185](.160)	.248 [.154](.154)	.251 [.155](.155)	.145	.150	.162	.155
	200	.187 [.142](.127)	.255 [.121](.121)	.256 [.121](.121)	.126	.128	.141	.133
	500	.208 [.107](.098)	.247 [.096](.095)	.247 [.096](.095)	.092	.093	.099	.096
.00	50	-.193 [.330](.268)	-.020 [.260](.259)	-.012 [.261](.261)	.236	.259	.279	.263
	100	-.122 [.234](.200)	-.008 [.195](.195)	-.006 [.195](.195)	.186	.191	.204	.199
	200	-.101 [.206](.180)	-.012 [.171](.171)	-.012 [.171](.171)	.167	.169	.186	.176
	500	-.048 [.136](.128)	.002 [.125](.125)	.002 [.125](.125)	.121	.121	.129	.126
-.25	50	-.468 [.384](.316)	-.273 [.319](.318)	-.266 [.320](.320)	.282	.305	.323	.313
	100	-.391 [.269](.229)	-.260 [.228](.228)	-.258 [.228](.228)	.224	.227	.239	.238
	200	-.362 [.248](.221)	-.256 [.212](.212)	-.255 [.212](.212)	.203	.206	.225	.215
	500	-.313 [.170](.158)	-.252 [.154](.154)	-.252 [.154](.154)	.151	.152	.161	.157
-.50	50	-.727 [.411](.343)	-.510 [.351](.351)	-.505 [.354](.354)	.328	.350	.364	.362
	100	-.654 [.298](.255)	-.510 [.259](.259)	-.509 [.260](.260)	.261	.261	.271	.274
	200	-.627 [.273](.242)	-.504 [.234](.234)	-.504 [.234](.234)	.241	.243	.264	.254
	500	-.566 [.193](.182)	-.494 [.178](.178)	-.494 [.178](.178)	.180	.181	.192	.187

Note: $\overline{se}_1 = \text{mean}(\widehat{V}_1(\hat\lambda_n)^{\frac{1}{2}}), \overline{se}_2 = \text{mean}(\widehat{V}_2(\hat\lambda_n)^{\frac{1}{2}}), \overline{se}_3 = \text{mean}(\widehat{V}_3(\hat\lambda_n)^{\frac{1}{2}})$ and $\overline{se}_3^c = \text{mean}(\widehat{V}_3(\hat\lambda_n^{\text{bc3}})^{\frac{1}{2}})$.

Table 2a. Null Behavior of Wald Tests for $H_0 : \lambda = 0$: Group Interaction with $k = n^{0.35}$

					Empirical Tail Probabilities: L=left, R=right					
n	Test	dgp	Mean	SD	L-1%	L-2.5%	L-5%	R-5%	R-2.5%	R-1%
Nominal Values			0.0000	1.0000	0.0100	0.0250	0.0500	0.0500	0.0250	0.0100
50	t_{11}	1	-0.5904	1.0572	0.0470	0.0965	0.1553	0.0210	0.0113	0.0051
		2	-0.6080	1.0801	0.0554	0.0974	0.1585	0.0209	0.0109	0.0042
		3	-0.5607	1.1100	0.0622	0.1072	0.1590	0.0159	0.0069	0.0030
	t_{21}	1	0.0088	1.1571	0.0193	0.0404	0.0729	0.0796	0.0472	0.0265
		2	-0.0526	1.1665	0.0246	0.0505	0.0794	0.0712	0.0436	0.0244
		3	-0.0655	1.1712	0.0339	0.0601	0.0926	0.0610	0.0340	0.0151
	t_{22}	1	-0.0085	1.0573	0.0143	0.0315	0.0591	0.0574	0.0323	0.0156
		2	-0.0644	1.0830	0.0201	0.0417	0.0705	0.0563	0.0323	0.0152
		3	-0.0739	1.0780	0.0280	0.0487	0.0809	0.0425	0.0208	0.0077
	t_{33}	1	0.0106	1.0583	0.0141	0.0309	0.0564	0.0588	0.0343	0.0157
		2	-0.0254	1.0350	0.0153	0.0322	0.0580	0.0512	0.0295	0.0141
		3	-0.0258	0.9535	0.0185	0.0332	0.0533	0.0327	0.0145	0.0055
100	t_{11}	1	-0.5341	1.0220	0.0383	0.0771	0.1376	0.0185	0.0084	0.0038
		2	-0.5089	1.0464	0.0387	0.0828	0.1385	0.0202	0.0101	0.0035
		3	-0.5296	1.0904	0.0518	0.0959	0.1508	0.0241	0.0111	0.0047
	t_{21}	1	0.0300	1.0906	0.0138	0.0315	0.0590	0.0687	0.0403	0.0203
		2	0.0339	1.1103	0.0169	0.0351	0.0657	0.0745	0.0423	0.0200
		3	-0.0398	1.1400	0.0205	0.0468	0.0812	0.0722	0.0407	0.0199
	t_{22}	1	0.0189	1.0320	0.0111	0.0274	0.0529	0.0574	0.0304	0.0137
		2	0.0216	1.0671	0.0153	0.0326	0.0612	0.0647	0.0339	0.0151
		3	-0.0479	1.1219	0.0209	0.0470	0.0791	0.0664	0.0365	0.0161
	t_{33}	1	0.0293	1.0090	0.0095	0.0245	0.0485	0.0534	0.0272	0.0119
		2	0.0401	1.0104	0.0105	0.0250	0.0466	0.0552	0.0270	0.0122
		3	-0.0091	0.9954	0.0099	0.0252	0.0505	0.0502	0.0257	0.0109
200	t_{11}	1	-0.3593	1.0045	0.0254	0.0539	0.0978	0.0236	0.0125	0.0059
		2	-0.3578	1.0367	0.0283	0.0581	0.1062	0.0295	0.0148	0.0063
		3	-0.3633	1.0686	0.0326	0.0628	0.1104	0.0328	0.0163	0.0053
	t_{21}	1	0.0483	1.0508	0.0114	0.0277	0.0523	0.0628	0.0346	0.0173
		2	0.0393	1.0823	0.0137	0.0302	0.0555	0.0690	0.0414	0.0199
		3	0.0054	1.1089	0.0159	0.0363	0.0626	0.0701	0.0433	0.0206
	t_{22}	1	0.0445	1.0266	0.0105	0.0255	0.0498	0.0578	0.0313	0.0146
		2	0.0346	1.0680	0.0130	0.0295	0.0550	0.0648	0.0378	0.0176
		3	0.0003	1.1114	0.0176	0.0372	0.0648	0.0703	0.0419	0.0199
	t_{33}	1	0.0408	0.9954	0.0095	0.0236	0.0462	0.0500	0.0262	0.0120
		2	0.0352	1.0140	0.0105	0.0250	0.0477	0.0575	0.0308	0.0121
		3	0.0161	1.0048	0.0104	0.0232	0.0460	0.0548	0.0292	0.0111

Note: (1) X_1 and X_2 are generated from `MRSAR-A` schme, $\sigma = 1$, and $\beta = (5, 1, 1)'$;
(2) dgp: 1=normal, 2=normal mixture($\tau = 4, p = .1$), 3=lognormal;
(3) t_{ij}: t-test with ith-order corrected estimator and jth-order corrected variance of it.

Table 2b. Null Behavior of Wald Tests for $H_0 : \lambda = 0$: Group Interaction with $k = n^{0.5}$

					Empirical Tail Probabilities: L=left, R=right					
n	stat	dgp	Mean	SD	L-1%	L-2.5%	L-5%	R-5%	R-2.5%	R-1%
Nominal Values			0.0000	1.0000	0.0100	0.0250	0.0500	0.0500	0.0250	0.0100
50	t_{11}	1	-0.5396	1.0523	0.0430	0.0833	0.1395	0.0220	0.0118	0.0055
		2	-0.5609	1.0624	0.0468	0.0875	0.1460	0.0199	0.0096	0.0054
		3	-0.5135	1.0627	0.0427	0.0841	0.1398	0.0227	0.0111	0.0049
	t_{21}	1	0.0607	1.1225	0.0167	0.0342	0.0627	0.0767	0.0467	0.0245
		2	0.0036	1.1299	0.0218	0.0410	0.0675	0.0706	0.0413	0.0209
		3	-0.0106	1.1193	0.0201	0.0393	0.0696	0.0660	0.0386	0.0186
	t_{22}	1	0.0412	1.0279	0.0120	0.0269	0.0517	0.0569	0.0317	0.0139
		2	-0.0115	1.0438	0.0171	0.0327	0.0572	0.0530	0.0277	0.0117
		3	-0.0205	1.0448	0.0162	0.0333	0.0589	0.0530	0.0281	0.0121
	t_{33}	1	0.0700	1.0439	0.0126	0.0261	0.0497	0.0639	0.0363	0.0172
		2	0.0227	1.0307	0.0137	0.0284	0.0504	0.0547	0.0293	0.0135
		3	0.0183	0.9889	0.0121	0.0267	0.0456	0.0498	0.0256	0.0126
100	t_{11}	1	-0.3930	1.0200	0.0292	0.0619	0.1088	0.0219	0.0126	0.0049
		2	-0.3850	1.0367	0.0288	0.0632	0.1112	0.0266	0.0131	0.0059
		3	-0.3872	1.0523	0.0332	0.0677	0.1129	0.0271	0.0134	0.0055
	t_{21}	1	0.0542	1.0577	0.0121	0.0289	0.0543	0.0625	0.0351	0.0171
		2	0.0470	1.0737	0.0128	0.0292	0.0576	0.0710	0.0405	0.0185
		3	0.0103	1.0824	0.0167	0.0355	0.0643	0.0638	0.0364	0.0162
	t_{22}	1	0.0472	1.0117	0.0100	0.0245	0.0496	0.0533	0.0274	0.0129
		2	0.0391	1.0373	0.0108	0.0268	0.0524	0.0629	0.0335	0.0148
		3	0.0053	1.0654	0.0161	0.0347	0.0620	0.0597	0.0332	0.0146
	t_{33}	1	0.0570	1.0058	0.0097	0.0235	0.0483	0.0530	0.0273	0.0126
		2	0.0520	1.0107	0.0090	0.0231	0.0456	0.0601	0.0312	0.0130
		3	0.0240	0.9940	0.0105	0.0243	0.0474	0.0520	0.0260	0.0108
200	t_{11}	1	-0.3265	1.0085	0.0213	0.0499	0.0939	0.0265	0.0124	0.0050
		2	-0.3182	1.0250	0.0239	0.0524	0.0979	0.0288	0.0133	0.0055
		3	-0.3165	1.0360	0.0237	0.0552	0.0972	0.0322	0.0173	0.0072
	t_{21}	1	0.0418	1.0376	0.0094	0.0251	0.0492	0.0640	0.0343	0.0141
		2	0.0433	1.0531	0.0121	0.0280	0.0521	0.0663	0.0353	0.0149
		3	0.0217	1.0610	0.0125	0.0287	0.0575	0.0649	0.0377	0.0182
	t_{22}	1	0.0377	1.0101	0.0087	0.0220	0.0463	0.0575	0.0296	0.0119
		2	0.0386	1.0330	0.0116	0.0268	0.0503	0.0602	0.0316	0.0134
		3	0.0179	1.0592	0.0129	0.0290	0.0575	0.0634	0.0358	0.0175
	t_{33}	1	0.0396	0.9997	0.0085	0.0215	0.0446	0.0557	0.0278	0.0114
		2	0.0435	1.0076	0.0102	0.0232	0.0467	0.0557	0.0287	0.0113
		3	0.0310	0.9997	0.0081	0.0193	0.0466	0.0551	0.0301	0.0132

Note: (1) X_1 and X_2 are generated from `MRSAR-A` schme, $\sigma = 1$, and $\beta = (5, 1, 1)'$;

(2) dgp: 1=normal, 2=normal mixture($\tau = 4, p = .1$), 3=lognormal;

(3) t_{ij}: t-test with ith-order corrected estimator and jth-order corrected variance of it.

Table 2c. Null Behavior of Wald Tests for $H_0 : \lambda = 0$: Group Interaction with $k = n^{0.65}$

					Empirical Tail Probabilities: L=left, R=right					
n	stat	dgp	Mean	SD	L-1%	L-2.5%	L-5%	R-5%	R-2.5%	R-1%
Nominal Values			0.0000	1.0000	0.0100	0.0250	0.0500	0.0500	0.0250	0.0100
50	t_{11}	1	-0.3796	1.0296	0.0310	0.0613	0.1033	0.0257	0.0122	0.0046
		2	-0.3877	1.0428	0.0340	0.0648	0.1086	0.0255	0.0139	0.0053
		3	-0.3680	1.0194	0.0226	0.0532	0.0990	0.0297	0.0169	0.0088
	t_{21}	1	0.0285	1.0676	0.0164	0.0326	0.0582	0.0650	0.0349	0.0168
		2	0.0012	1.0778	0.0173	0.0373	0.0638	0.0623	0.0334	0.0163
		3	-0.0183	1.0485	0.0119	0.0275	0.0555	0.0594	0.0336	0.0176
	t_{22}	1	0.0157	0.9872	0.0125	0.0265	0.0491	0.0476	0.0240	0.0094
		2	-0.0101	1.0100	0.0148	0.0314	0.0565	0.0476	0.0238	0.0098
		3	-0.0246	0.9986	0.0103	0.0241	0.0476	0.0491	0.0273	0.0139
	t_{33}	1	0.0268	1.0229	0.0145	0.0292	0.0532	0.0563	0.0282	0.0119
		2	0.0050	1.0298	0.0154	0.0332	0.0570	0.0527	0.0271	0.0128
		3	-0.0002	0.9930	0.0090	0.0217	0.0454	0.0516	0.0289	0.0148
100	t_{11}	1	-0.3143	1.0287	0.0261	0.0556	0.0968	0.0287	0.0145	0.0059
		2	-0.3188	1.0151	0.0244	0.0524	0.0958	0.0273	0.0133	0.0057
		3	-0.3026	0.9885	0.0163	0.0381	0.0752	0.0302	0.0174	0.0096
	t_{21}	1	0.0260	1.0592	0.0131	0.0315	0.0594	0.0624	0.0333	0.0150
		2	0.0124	1.0458	0.0132	0.0305	0.0568	0.0581	0.0306	0.0142
		3	0.0035	1.0153	0.0083	0.0226	0.0449	0.0551	0.0326	0.0177
	t_{22}	1	0.0165	1.0215	0.0121	0.0294	0.0555	0.0526	0.0274	0.0113
		2	0.0026	1.0212	0.0133	0.0295	0.0550	0.0512	0.0258	0.0111
		3	-0.0048	1.0137	0.0089	0.0243	0.0466	0.0540	0.0304	0.0167
	t_{33}	1	0.0252	1.0299	0.0118	0.0294	0.0562	0.0562	0.0288	0.0123
		2	0.0141	1.0194	0.0120	0.0280	0.0524	0.0533	0.0275	0.0115
		3	0.0155	0.9961	0.0073	0.0198	0.0404	0.0545	0.0312	0.0165
200	t_{11}	1	-0.2484	1.0103	0.0194	0.0425	0.0826	0.0315	0.0156	0.0065
		2	-0.2542	1.0038	0.0195	0.0462	0.0834	0.0284	0.0131	0.0052
		3	-0.2411	0.9991	0.0140	0.0374	0.0728	0.0348	0.0181	0.0083
	t_{21}	1	0.0422	1.0266	0.0103	0.0242	0.0483	0.0612	0.0313	0.0142
		2	0.0322	1.0215	0.0110	0.0277	0.0523	0.0566	0.0292	0.0123
		3	0.0292	1.0166	0.0078	0.0212	0.0439	0.0592	0.0351	0.0158
	t_{22}	1	0.0380	1.0033	0.0092	0.0226	0.0454	0.0561	0.0270	0.0120
		2	0.0272	1.0065	0.0110	0.0276	0.0507	0.0530	0.0266	0.0103
		3	0.0256	1.0233	0.0091	0.0230	0.0453	0.0588	0.0351	0.0157
	t_{33}	1	0.0423	1.0048	0.0090	0.0224	0.0454	0.0568	0.0278	0.0124
		2	0.0328	1.0002	0.0097	0.0261	0.0490	0.0528	0.0263	0.0100
		3	0.0353	1.0045	0.0076	0.0193	0.0418	0.0576	0.0339	0.0151

Note: (1) X_1 and X_2 are generated from `MRSAR-A` schme, $\sigma = 1$, and $\beta = (5, 1, 1)'$;

(2) dgp: 1=normal, 2=normal mixture($\tau = 4, p = .1$), 3=lognormal;

(3) t_{ij}: t-test with ith-order corrected estimator and jth-order corrected variance of it.

Table 3. Empirical Means [sds] of the MLE and Second-Order Bias-Corrected MLEs of λ: Comparison with Analytical Approach of Bao and Ullah (2007a), Pure SAR Model

J	λ	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{BU}}$	$\hat{\lambda}_n$	$\hat{\lambda}_n^{\mathrm{bc2}}$	$\hat{\lambda}_n^{\mathrm{BU}}$
			$n = 30,\ u_n \sim N(0, I_n)$			$n = 100,\ u_n \sim N(0, I_n)$	
2	0.9	0.881 [.057]	0.899 [.054]	0.899 [.054]	0.894 [.027]	0.900 [.026]	0.900 [.026]
	0.4	0.380 [.158]	0.397 [.164]	0.398 [.164]	0.395 [.086]	0.401 [.087]	0.401 [.087]
	0.2	0.189 [.176]	0.198 [.185]	0.198 [.184]	0.199 [.096]	0.202 [.098]	0.202 [.098]
	0.0	0.001 [.181]	0.000 [.190]	0.001 [.190]	-0.001 [.099]	-0.001 [.100]	-0.001 [.100]
	-0.2	-0.194 [.174]	-0.204 [.182]	-0.204 [.182]	-0.196 [.095]	-0.199 [.097]	-0.199 [.097]
	-0.4	-0.384 [.160]	-0.402 [.166]	-0.401 [.166]	-0.394 [.086]	-0.400 [.087]	-0.400 [.087]
	-0.9	-0.882 [.057]	-0.900 [.054]	-0.900 [.054]	-0.895 [.027]	-0.900 [.026]	-0.900 [.026]
6	0.9	0.853 [.109]	0.898 [.097]	0.897 [.097]	0.887 [.041]	0.899 [.038]	0.899 [.038]
	0.4	0.322 [.267]	0.397 [.271]	0.397 [.270]	0.376 [.135]	0.400 [.135]	0.400 [.135]
	0.2	0.126 [.300]	0.198 [.312]	0.198 [.312]	0.178 [.159]	0.201 [.161]	0.201 [.160]
	0.0	-0.072 [.324]	-0.008 [.344]	-0.007 [.343]	-0.025 [.174]	-0.004 [.177]	-0.004 [.177]
	-0.2	-0.259 [.341]	-0.207 [.366]	-0.206 [.366]	-0.216 [.188]	-0.198 [.192]	-0.198 [.192]
	-0.4	-0.442 [.349]	-0.403 [.378]	-0.402 [.378]	-0.414 [.197]	-0.401 [.202]	-0.401 [.202]
	-0.9	-0.898 [.346]	-0.900 [.377]	-0.899 [.377]	-0.900 [.193]	-0.900 [.198]	-0.900 [.198]
10	0.9	0.826 [.165]	0.899 [.139]	0.897 [.139]	0.880 [.055]	0.899 [.051]	0.899 [.051]
	0.4	0.255 [.369]	0.388 [.374]	0.387 [.373]	0.353 [.178]	0.395 [.176]	0.395 [.176]
	0.2	0.054 [.411]	0.186 [.431]	0.185 [.430]	0.154 [.208]	0.197 [.208]	0.197 [.208]
	0.0	-0.143 [.441]	-0.018 [.474]	-0.018 [.473]	-0.046 [.235]	-0.004 [.238]	-0.004 [.238]
	-0.2	-0.327 [.457]	-0.215 [.501]	-0.214 [.500]	-0.239 [.249]	-0.199 [.255]	-0.200 [.255]
	-0.4	-0.497 [.475]	-0.401 [.530]	-0.401 [.529]	-0.437 [.266]	-0.402 [.274]	-0.402 [.274]
	-0.9	-0.942 [.484]	-0.901 [.556]	-0.900 [.555]	-0.917 [.283]	-0.901 [.296]	-0.901 [.296]
			$n = 100,\ u_n \sim LN(0, I_n)$			$n = 200,\ u_n \sim LN(0, I_n)$	
6	0.4	0.386 [.121]	0.402 [.122]	0.409 [.120]	0.393 [.085]	0.401 [.085]	0.405 [.084]
	0.2	0.187 [.143]	0.202 [.145]	0.211 [.144]	0.192 [.100]	0.201 [.101]	0.204 [.101]
	0.0	-0.008 [.159]	0.004 [.162]	0.013 [.161]	-0.006 [.113]	0.001 [.115]	0.005 [.114]
	-0.2	-0.207 [.173]	-0.199 [.177]	-0.189 [.176]	-0.206 [.122]	-0.201 [.123]	-0.197 [.123]
	-0.4	-0.405 [.182]	-0.401 [.187]	-0.391 [.187]	-0.403 [.130]	-0.400 [.132]	-0.396 [.132]
10	0.4	0.370 [.159]	0.404 [.159]	0.412 [.156]	0.386 [.106]	0.403 [.106]	0.407 [.105]
	0.2	0.172 [.189]	0.206 [.190]	0.216 [.188]	0.183 [.132]	0.201 [.132]	0.206 [.131]
	0.0	-0.026 [.216]	0.005 [.220]	0.017 [.218]	-0.016 [.147]	0.001 [.149]	0.006 [.148]
	-0.2	-0.224 [.233]	-0.197 [.241]	-0.184 [.238]	-0.214 [.163]	-0.200 [.165]	-0.194 [.165]
	-0.4	-0.420 [.249]	-0.400 [.259]	-0.385 [.257]	-0.411 [.174]	-0.399 [.177]	-0.393 [.177]
14	0.4	0.357 [.193]	0.412 [.192]	0.417 [.187]	0.376 [.131]	0.402 [.130]	0.406 [.129]
	0.2	0.155 [.235]	0.210 [.237]	0.219 [.233]	0.177 [.158]	0.203 [.158]	0.209 [.157]
	0.0	-0.039 [.258]	0.013 [.264]	0.025 [.260]	-0.023 [.179]	0.003 [.180]	0.010 [.179]
	-0.2	-0.234 [.284]	-0.187 [.294]	-0.172 [.290]	-0.223 [.199]	-0.199 [.202]	-0.192 [.201]
	-0.4	-0.433 [.305]	-0.393 [.318]	-0.377 [.315]	-0.422 [.213]	-0.400 [.217]	-0.392 [.217]

Note: $\hat{\lambda}_n^{\mathrm{BU}}$: 2nd-Order Analytically Bias-Corrected MLE of Bao and Ullah (2007a).

Table 4a. MC Means and sds (2nd row) of $\hat{\lambda}_n$ and $\hat{\lambda}_n^{\mathrm{bc2}}$ (2nd Column): **Replication I of Lee (2004a)**

R	θ	$m=3$		$m=5$		$m=10$		$m=20$		$m=50$		$m=100$	
`SAR:`		$Y_n = \lambda W_n + u_n,\ u_n \sim N(0, \sigma^2 I_n),\ \lambda = 0.5$ and $\sigma = 1$											
30	λ	0.493	0.500	0.490	0.500	0.489	0.500	0.488	0.500	0.488	0.500	0.487	0.500
		.068	0.068	.067	.066	.069	.068	.068	.067	.069	.068	.069	.067
	σ	0.992	0.989	0.995	0.993	0.998	0.997	0.998	0.998	0.999	0.999	1.000	1.000
		.079	0.079	.059	.059	.042	.041	.029	.029	.018	.018	.013	.013
60	λ	0.496	0.500	0.496	0.501	0.494	0.500	0.494	0.500	0.493	0.500	0.493	0.500
		.047	0.047	.046	.046	.047	.046	.047	.046	.047	.047	.047	.047
	σ	0.997	0.996	0.999	0.997	0.999	0.998	0.999	0.999	1.000	1.000	1.000	1.000
		.055	0.055	.042	.042	.029	.029	.020	.020	.013	.013	.009	.009
120	λ	0.498	0.499	0.498	0.500	0.497	0.500	0.497	0.500	0.496	0.499	0.497	0.500
		.033	0.033	.033	.033	.033	.033	.032	.032	.033	.032	.033	.033
	σ	0.998	0.997	0.999	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		.040	0.040	.030	.030	.021	.021	.015	.015	.009	.009	.007	.007
`MRSAR-1:`		$Y_n = \lambda W_n + X_n\beta + u_n$, where $u_n \sim N(0, \sigma^2 I_n),\ X_n \sim N(0, I_n),\ \lambda = .5, \beta = 1$, and $\sigma = 1$											
30	λ	0.494	0.499	0.492	0.499	0.492	0.500	0.492	0.500	0.491	0.500	0.489	0.500
		.055	.055	.060	.059	.057	.056	.053	.052	.056	.055	.060	.059
	β	0.999	0.995	1.000	0.999	1.000	0.999	1.000	0.999	1.000	1.000	1.000	1.000
		.117	.117	.085	.085	.056	.056	.043	.043	.026	.026	.019	.019
	σ	0.987	0.985	0.991	0.990	0.995	0.995	0.998	0.998	0.999	0.999	1.000	0.999
		.077	.077	.059	.059	.041	.041	.029	.029	.018	.018	.013	.013
60	λ	0.497	0.500	0.497	0.500	0.496	0.500	0.496	0.500	0.495	0.499	0.496	0.500
		.039	.039	.039	.039	.039	.039	.038	.038	.038	.037	.038	.037
	β	1.000	0.999	1.000	0.999	0.999	0.999	1.000	0.999	1.000	1.000	1.000	1.000
		.082	.082	.062	.062	.041	.041	.029	.029	.018	.018	.013	.013
	σ	0.993	0.992	0.996	0.995	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000
		.055	.055	.042	.042	.029	.029	.020	.020	.013	.013	.009	.009
120	λ	0.499	0.500	0.498	0.500	0.498	0.500	0.498	0.500	0.498	0.500	0.498	0.500
		.027	.027	.027	.027	.026	.026	.026	.026	.026	.026	.027	.027
	β	1.000	0.999	1.000	0.999	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		.053	.053	0.041	.041	.028	.028	.020	.020	.013	.013	.009	.009
	σ	0.997	0.996	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
		.039	.039	.030	.030	.021	.021	.015	.015	.009	.009	.007	.007
`MRSAR-2:`		As `MRSAR-1` but with $X_n = \{(z_r + z_{ir})/\sqrt{2}\}$, where $z_r{}'s$ and $z_{ir}{}'s$ are iid $N(0, 1)$											
30	λ	0.493	0.499	0.491	0.499	0.494	0.500	0.493	0.500	0.496	0.500	0.498	0.500
		.059	.058	.059	.058	.047	.046	.045	.045	.033	.033	.025	.025
	β	1.002	0.995	1.001	0.999	1.002	0.998	1.002	0.999	1.001	0.999	1.001	1.000
		.146	.145	.109	.108	.069	.069	.055	.055	.034	.034	.025	.025
	σ	0.987	0.985	0.991	0.990	0.995	0.995	0.998	0.998	0.999	0.999	1.000	1.000
		.078	.077	.059	.059	.041	.041	.029	.029	.018	.018	.013	.013
60	λ	0.497	0.500	0.497	0.500	0.497	0.501	0.498	0.500	0.498	0.500	0.499	0.500
		.041	.041	.038	.038	.035	.034	.030	.029	.022	.022	.019	.019
	β	1.000	0.998	1.001	0.999	1.000	0.999	1.000	0.999	1.000	0.999	1.000	1.000
		.103	.103	.073	.073	.052	.052	.037	.037	.024	.024	.018	.018
	σ	0.993	0.992	0.996	0.995	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000
		.055	.055	.042	.042	.029	.029	.020	.020	.013	.013	.009	.009
120	λ	0.498	0.500	0.498	0.500	0.498	0.500	0.499	0.500	0.499	0.500	0.499	0.500
		.028	.028	.028	.028	.025	.024	.021	.021	.017	.017	.014	.014
	β	1.000	0.999	1.000	0.999	1.000	0.999	1.001	1.000	1.000	1.000	1.000	1.000
		.066	.066	.054	.054	.036	.036	.026	.026	.017	.017	.012	.012
	σ	0.997	0.996	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
		.039	.039	.030	.030	.021	.021	.015	.015	.009	.009	.007	.007

Table 4b. MC Means and sds (2nd row) of $\hat{\lambda}_n$ and $\hat{\lambda}_n^{\mathrm{bc2}}$ (2nd Column): **Replication II of Lee (2004a)**

m	θ	$R = 3$		$R = 5$		$R = 10$		$R = 20$		$R = 50$		$R = 100$	
`SAR`: $Y_n = \lambda W_n + u_n,\ u_n \sim N(0, \sigma^2 I_n),\ \lambda = 0.5$ and $\sigma = 1$													
30	λ	0.325	0.482	0.407	0.492	0.460	0.499	0.481	0.500	0.493	0.500	0.497	0.500
		.446	.358	.247	.215	.136	.127	.087	.084	.053	.052	.036	.035
	σ	0.991	0.989	0.995	0.993	0.998	0.997	0.998	0.998	0.999	0.999	1.000	1.000
		.075	.075	.058	.058	.041	.041	.029	.029	.018	.018	.013	.013
60	λ	0.314	0.479	0.409	0.495	0.462	0.501	0.480	0.499	0.492	0.499	0.497	0.500
		.480	.376	.245	.210	.135	.126	.087	.084	.052	.051	.036	.036
	σ	0.996	0.996	0.998	0.998	0.999	0.998	0.999	0.999	1.000	1.000	1.000	1.000
		.053	.052	.041	.041	.029	.029	.020	.020	.013	.013	.009	.009
120	λ	0.309	0.478	0.410	0.497	0.461	0.501	0.480	0.499	0.492	0.500	0.496	0.500
		.527	.377	.256	.205	.139	.124	.090	.084	.052	.051	.037	.036
	σ	0.998	0.997	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		.038	.037	.029	.029	.020	.020	.014	.014	.009	.009	.007	.007
`MRSAR-1`: $Y_n = \lambda W_n + X_n\beta + u_n$, where $u_n \sim N(0, \sigma^2 I_n)$, $X_n \sim N(0, I_n)$, $\lambda = .5, \beta = 1$, and $\sigma = 1$													
30	λ	0.348	0.476	0.435	0.494	0.473	0.500	0.488	0.500	0.495	0.500	0.498	0.500
		.386	.320	.183	.165	.104	.099	.064	.063	.042	.042	.029	.029
	β	0.995	0.996	1.000	0.998	1.000	0.999	1.000	0.999	1.000	1.000	1.000	1.000
		.112	.112	.085	.085	.056	.056	.043	.043	.026	.026	.019	.019
	σ	0.986	0.985	0.991	0.990	0.995	0.995	0.998	0.998	0.999	0.999	1.000	0.999
		.075	.075	.058	.057	.041	.041	.029	.029	.018	.018	.013	.013
60	λ	0.360	0.480	0.434	0.496	0.474	0.500	0.486	0.499	0.494	0.499	0.498	0.500
		.378	.313	.193	.173	.101	.096	.068	.066	.043	.042	.030	.030
	β	0.999	0.998	1.000	0.999	0.999	0.999	1.000	0.999	1.000	1.000	1.000	1.000
		.079	.079	.061	.061	.041	.041	.029	.029	.018	.018	.013	.013
	σ	0.992	0.992	0.996	0.995	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000
		.053	.053	.041	.041	.029	.029	.020	.020	.013	.013	.009	.009
120	λ	0.395	0.483	0.451	0.496	0.469	0.497	0.486	0.499	0.495	0.501	0.497	0.500
		.274	.235	.144	.132	.106	.101	.066	.065	.041	.041	.029	.029
	β	1.000	0.999	1.000	0.999	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		.052	.052	.041	.041	.028	.028	.020	.020	.013	.013	.009	.009
	σ	0.997	0.996	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
		.037	.037	.029	.029	.021	.021	.015	.015	.009	. 009	.007	.007
`MRSAR-2`: As `MRSAR-1` but with $X_n = \{(z_r + z_{ir})/\sqrt{2}\}$, where $z_r{}'s$ and $z_{ir}{}'s$ are iid $N(0, 1)$													
30	λ	0.440	0.492	0.446	0.495	0.486	0.500	0.493	0.500	0.497	0.500	0.498	0.500
		.165	.154	.153	.140	.066	.065	.048	.048	.031	.031	.022	.022
	β	1.014	0.988	1.004	0.995	1.003	0.997	1.002	0.999	1.000	0.999	1.001	1.000
		.151	.153	.114	.114	.072	.073	.055	.055	.034	.034	.025	.025
	σ	0.987	0.986	0.991	0.990	0.995	0.995	0.998	0.998	0.999	0.999	1.000	1.000
		.075	.074	.058	.057	.041	.041	.029	.029	.018	.018	.013	.013
60	λ	0.468	0.496	0.476	0.499	0.493	0.502	0.495	0.500	0.498	0.500	0.499	0.500
		.103	.099	.090	.087	.051	.051	.039	.039	.024	.024	.017	.017
	β	1.008	0.995	1.005	0.996	1.002	0.998	1.001	0.999	1.000	0.999	1.000	1.000
		.097	.098	.080	.080	.053	.054	.038	.038	.024	.024	.017	.017
	σ	0.993	0.992	0.996	0.995	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000
		.053	.053	.041	.041	.029	.029	.020	.020	.013	.013	.009	.009
120	λ	0.483	0.499	0.479	0.498	0.496	0.500	0.496	0.500	0.499	0.500	0.500	0.500
		.071	.070	.080	.078	.035	.035	.033	.033	.018	.018	.013	.013
	β	1.005	0.997	1.002	0.998	1.001	0.999	1.001	1.000	1.000	1.000	1.000	1.000
		.070	.071	.054	.055	.038	.039	.027	.027	.017	.017	.012	.012
	σ	0.997	0.997	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
		.037	.037	.029	.029	.020	.020	.015	.015	.009	.009	.007	.007