Specification Tests for Temporal Heterogeneity in Spatial Panel Data Models with Fixed Effects*

Yuhong Xu and Zhenlin Yang†

[a] *School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903.*

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Abstract

We propose *adjusted quasi score* (AQS) tests for testing the existence of temporal heterogeneity in slope and spatial parameters in spatial panel data (SPD) models, allowing for the presence of individual-specific and/or time-specific fixed effects (or in general intercept heterogeneity). The SPD model with *spatial lag* is treated in detail by first considering the model with individual fixed effects only, and then extending it to the model with both individual and time fixed effects. Two types of AQS tests (*naïve and robust*) are proposed, and their asymptotic properties are presented. These tests are then fully extended to SPD models with both *spatial lag* and *spatial error*. Monte Carlo results show that the robust tests have much superior finite and large sample properties than the naive tests. Thus, the proposed robust tests provide reliable tools for identifying possible existence of temporal heterogeneity in regression and spatial coefficients. Empirical applications of the proposed tests are given.

Key Words: Spatial panels; Fixed effects; Time-Varying Covariate Effects; Time-Varying Spatial Effects; Change Points.

JEL Classification: C10, C13, C21, C23, C15

1. Introduction

Being able to control *unobserved heterogeneity* may be one of the most important features of a panel data (PD) model. Heterogeneity may occur on intercept, slope and error variance. In a spatial PD model (SPD), it may also occur on spatial parameters (Anselin, 1988). Heterogeneity in variance is often referred to as *heteroskedasticity*. Heterogeneity may occur in spatial and/or temporal dimension. When unobserved heterogeneity occurs on the intercept, it gives rise to individual-specific effects and/or time-specific effects, which may appear in the model *additively* or *interactively*. Change point or structural break may be considered as a special case of unobserved heterogeneity.

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†Corresponding Author: 90 Stamford Road, Singapore 178903. Phone: +65-6828-0852; Fax: +65-6828-0833. E-mail addresses: zlyang@smu.edu.sg.

Temporal heterogeneity is a common feature in an SPD model. It is an important issue but relatively unexplored in the spatial panel literature. Temporal heterogeneity may occur as a result of a credit crunch or debt, an oil price shock, a tax policy change, a fad or fashion in society, a discovery of a new medicine, and an enaction of new governmental program (Bai, 2010). Many economic processes, for example, housing decisions, technology adoption, unemployment, welfare participation, price decisions, crime rates, trade flows, etc., exhibit time heterogeneity patterns. Values observed at one location depend on the values of neighboring observations at nearby locations. Therefore, one may be interested in the question whether this dependence stays the same over time.

There is a sizable literature on temporal heterogeneity in regular panel data models, mostly on change points or structural breaks, see, Bai (2010), Liao (2008), Feng et al. (2009), to name a few. In spatial models, previous literature has focused more on the spatial heterogeneity (e.g., Aquaro et al., 2015; LeSage et al., 2016, 2017). The literature on temporal heterogeneity in spatial panel data models is rather thin. We are only aware of the following two works, Sengupta (2017) who proposes tests for a structural break in a spatial panel model without fixed effects, and Li (2018) who studies fixed effects SPD models with structural changes. SPD models with temporal heterogeneity also appear in finance literature, see, e.g., Blasques et al. (2016) and Catania and Billé (2017), but under a different setting where the time dimension is much larger than the spatial dimension.

In this paper, we consider the fixed effects SPD models with temporal heterogeneity in regression and spatial coefficients. We focus on the testing problems. The presence of temporal heterogeneity renders the usual fixed effects estimation method through transformation (Lee and Yu, 2010; Baltagi and Yang, 2013; Yang et al. 2016) inapplicable in handling the individual-specific fixed effects. A general method, the *adjusted quasi score* (AQS) method, is introduced for constructing tests for temporal homogeneity/heterogeneity on regression coefficients and spatial correlation coefficients in SPD models, allowing for presence of spatial-temporal heterogeneity in the intercepts (or fixed effects). The SPD model with spatial lag dependence is first treated in detail by first considering the model with individual-specific fixed effects only, and then extended to the model with both individual and time specific fixed effects. Two types of AQS tests (*naïve and robust*) are proposed, and their asymptotic properties are presented. These tests are then fully extended to the SPD models with both spatial lag (SL) and spatial error (SE) dependence. Monte Carlo results show that the robust tests have much superior finite and large sample properties than the naive tests. Thus, the proposed robust tests provide reliable tools for practitioners. Two empirical applications of the proposed tests are presented, and a detailed guidance is given to aid applied researchers in their empirical studies.

The rest of the paper is organized as follows. Section 2 presents AQS tests for the panel SL model with one-way and two-way fixed effects, where a general method for constructing non-normality robust AQS tests is outlined. Section 3 generalizes these tests to the SPD models with both SL and SE dependence. Section 4 presents Monte Carlo results. Section

5 presents two empirical applications to give a detailed illustration on how the proposed methods are implemented. Section 6 discuss possible extensions and concludes the paper.

2. Tests for Temporal Heterogeneity in Panel SL Model

In this section, we introduce the general AQS method for constructing the specification tests and a method for the practical implementations of these tests, using the simplest panel SL model with one-way FE (i.e., unobserved *spatial heterogeneity* in the intercept). Then, we extend these tests to a panel SL model with two-way FE (i.e., the unobserved *spatiotemporal heterogeneity* in intercepts). Asymptotic properties of the proposed tests are presented. Some key quantities for calculating the test statistics, the Hessian and expected Hessian matrices, and the variance-covariance matrix of the AQS function, are given in Appendix B, and proofs of theorems are sketched in Appendix C.

2.1. Panel SL model with one-way FE

Consider the following panel SL model with individual-specific FE, or one-way FE:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + V_{nt}, \tag{2.1}$$

where Y_{nt} is an $n \times 1$ vector of observations on the dependent variable for $t = 1, 2, \ldots, T$; X_{nt} is an $n \times k$ matrix containing the values of exogenous regressors and possibly their spatial lags, W_n is an $n \times n$ spatial weight matrix; V_{nt} is an $n \times 1$ vector of independent and identically distributed (iid) disturbances with mean zero and variance σ^2; λ_t is the *spatial lag parameter* and β_t is a $k \times 1$ vector of regression coefficients for the tth period; and c_n denotes the individual-specific fixed effects or the spatial heterogeneity in intercept.

Null hypotheses. We are primarily interested in tests for temporal homogeneity (TH) in regression and spatial coefficients, i.e., the tests of the null hypothesis:

$$H_0^{\mathtt{TH}} : \beta_1 = \cdots = \beta_T = \beta \text{ and } \lambda_1 = \cdots = \lambda_T = \lambda, \tag{2.2}$$

allowing for the presence of unobserved cross-sectional heterogeneity in intercept, i.e., the individual specific fixed effects c_n. If $H_0^{\mathtt{TH}}$ is rejected, one may wish to find the 'cause' of such a rejection instead of fitting the general heterogeneous model (2.1). Natural tests to proceed would be the tests of TH in regression coefficients only (RH), $H_0^{\mathtt{RH}} : \beta_1 = \cdots = \beta_T = \beta$, and the tests of TH in spatial coefficients only (SH): $H_0^{\mathtt{SH}} : \lambda_1 = \cdots = \lambda_T = \lambda$. If $H_0^{\mathtt{RH}}$ is not rejected, then one may infer that the cause of rejection of $H_0^{\mathtt{TH}}$ is the existence of temporal heterogeneity in spatial coefficients; if $H_0^{\mathtt{SH}}$ is not rejected, then the cause of rejection of $H_0^{\mathtt{TH}}$ may be the existence temporal heterogeneity in regression coefficients. In both cases, one would fit a simpler model of heterogeneous spatial coefficients only, or of heterogeneous regression coefficients only. If both $H_0^{\mathtt{RH}}$ and $H_0^{\mathtt{SH}}$ are rejected, one may need to fit the general model (2.1). However, rejection of both $H_0^{\mathtt{RH}}$ and $H_0^{\mathtt{SH}}$ may be due to the existence of *change points* (CPs) in β-coefficients and λ-coefficients, giving rise to a case of particular interest: change point detection in the spirit of Bai (2010) and Li (2018):

$$H_0^{\texttt{CP}}: \beta_1 = \cdots = \beta_{b_0} \neq \beta_{b_0+1} = \cdots = \beta_T \text{ and } \lambda_1 = \cdots = \lambda_{\ell_0} \neq \lambda_{\ell_0+1} = \cdots = \lambda_T, \quad (2.3)$$

where $1 < b_0, \ell_0 < T$, and b_0 and ℓ_0 can be the same or different. If $H_0^{\texttt{CP}}$ is not rejected, one may fit a much simpler model with one CP in β_t at $t = b_0$ and one CP for λ_t at $t = \ell_0$. These discussions can be extended to have more one CP in β_t and λ_t. All of these hypotheses can be put in a general framework and tests can be constructed in a general manner.[1]

Adjusted (quasi) score functions. As λ_t and β_t are allowed to change with t, the usual fixed-effects estimation methods, such as first differencing or orthogonal transformation, cannot be applied. We propose an *adjusted score* (AS) or *adjusted quasi score* (AQS) method for estimating the structural parameters in the model, which proceeds by first eliminating c_n through direct maximization of the loglikelihood function, given the structural parameters, and then adjusting the resulted concentrated (quasi) score function to give a set of estimating functions that are unbiased or asymptotically unbiased so as to achieve asymptotically unbiased estimation. The resulted set of AS or AQS functions then lead to a set of score-type of tests, referred to as the **AQS tests** in this paper, for identifying temporal heterogeneity in regression coefficients and spatial parameters.

We develop score-type tests as they require only the estimation of the null model. However, the construction of the score-type of tests requires the full quasi score (QS) function, derived from the quasi Gaussian loglikelihood, **as if** $\{V_{nt}\}$ are iid $N(0, \sigma^2 I_n)$:

$$\ell_{\texttt{SL1}}(\boldsymbol{\theta}, c_n) = -\frac{nT}{2}\ln(2\pi\sigma^2) + \sum_{t=1}^{T}\ln|A_n(\lambda_t)| - \frac{1}{2\sigma^2}\sum_{t=1}^{T} V'_{nt}(\lambda_t, \beta_t, c_n)V_{nt}(\lambda_t, \beta_t, c_n), \quad (2.4)$$

where $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \sigma^2)'$, $\boldsymbol{\beta} = (\beta'_1, \ldots, \beta'_T)'$ and $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_T)'$; $A_n(\lambda_t) = I_n - \lambda_t W_n$, I_n is an $n \times n$ identity matrix, and $V_{nt}(\beta_t, \lambda_t, c_n) = A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - c_n$, $t = 1, \ldots, T$.

First, given $\boldsymbol{\theta}$, $\ell_{\texttt{SL1}}(\boldsymbol{\theta}, c_n)$ is partially maximized at: $\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \frac{1}{T}\sum_{t=1}^{T}[A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t]$, which gives the concentrated loglikelihood function of $\boldsymbol{\theta}$ upon substitution:

$$\ell^c_{\texttt{SL1}}(\boldsymbol{\theta}) = -\frac{nT}{2}\ln(2\pi\sigma^2) + \sum_{t=1}^{T}\ln|A_n(\lambda_t)| - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})'\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \quad (2.5)$$

where $\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - \tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda})$. Then, differentiate $\ell^c_{\texttt{SL1}}(\boldsymbol{\theta})$ to get the concentrated score (CS) or concentrated quasi score (CQS) function of $\boldsymbol{\theta}$:

$$S^c_{\texttt{SL1}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X'_{nt}\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \quad t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_n Y_{nt})'\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}) - \text{tr}[G_n(\lambda_t)], \quad t = 1, \ldots, T, \\ -\frac{nT}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}\widetilde{V}'_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}), \end{cases} \quad (2.6)$$

where $G_n(\lambda_t) = W_n A_n^{-1}(\lambda_t), t = 1, \ldots, T$.

[1] Various conditional tests of, e.g., RH given SH, SH given RH, CP on β_t only given SH, and CP on λ_t only given RH, are also of interest, of which, the test of RH given SH is an extension of the well known Chow's (1960) test for a linear regression and Anselin's (1988, Sec. 9.2.2) test for a spatial error model.

Let $\theta_0 = (\beta_0', \lambda_0', \sigma_0^2)'$ be the true value of the general parameter vector $\theta = (\beta', \lambda', \sigma^2)'$. We view that Model (2.1) holds only under the true θ_0. The usual expectation and variance operators correspond to θ_0. At the true θ_0, we have $\tilde{c}_n(\beta_0, \lambda_0) = \overline{V}_n + c_n$ and thus $\widetilde{V}_{nt} \equiv \widetilde{V}_{nt}(\beta_0, \lambda_0) = V_{nt} - \overline{V}_n$, where $\overline{V}_n = \frac{1}{T}\sum_{t=1}^T V_{nt}$. Furthermore, $W_n Y_{nt} = G_n(\lambda_{t0})(X_{nt}\beta_{t0} + c_n + V_{nt})$. With these, it is easy to show that,

$$\mathrm{E}[S^c_{\mathtt{SL1}}(\theta_0)] = \left\{0'_{Tk,1},\ -\tfrac{1}{T}\mathrm{tr}[G_n(\lambda_{t0})],\ t = 1, \dots T,\ -\tfrac{n}{2\sigma_0^2}\right\}',$$

where $0_{m,r}$ denotes an $m \times r$ matrix of zeros. Clearly, $\frac{1}{nT}\mathrm{E}[S^c_{\mathtt{SL1}}(\theta_0)] \nrightarrow 0$, unless $T \to \infty$. A necessary condition for consistent estimation is violated. Therefore, the direct approach does not yield consistent estimators unless T goes to large. Even if T goes large with n, there will be an asymptotic bias of order $O(\frac{1}{T^2})$ for the estimation of $\{\lambda_t\}$, and an asymptotic bias of order $O(\frac{1}{T})$ for the estimation of σ^2.

To have a inference method that is consistent and asymptotically unbiased, CS or CQS function given in (2.6) should be adjusted by subtracting the above bias vector from it, leading to the AS or AQS function as

$$S^\star_{\mathtt{SL1}}(\theta) = \begin{cases} \frac{1}{\sigma^2} X_{nt}' \widetilde{V}_{nt}(\beta, \lambda),\ t = 1, \dots, T, \\ \frac{1}{\sigma^2}(W_n Y_{nt})' \widetilde{V}_{nt}(\beta, \lambda) - \frac{T-1}{T}\mathrm{tr}[G_n(\lambda_t)],\ t = 1, \dots, T, \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}_{nt}'(\beta, \lambda)\widetilde{V}_{nt}(\beta, \lambda). \end{cases} \tag{2.7}$$

It is easy to show that $\mathrm{E}[S^\star_{\mathtt{SL1}}(\theta_0)] = 0$, and that $\frac{1}{nT}S^\star_{\mathtt{SL1}}(\theta_0) \xrightarrow{p} 0$ as $n \to \infty$ alone, or both n and T go infinity. Thus, this AQS function gives a set of unbiased estimating functions, and paves the way for developing asymptotically valid score-type tests.[2]

Construction of AQS tests. Denote by $\tilde{\theta}_{\mathtt{SL1}}$ the constrained estimator of θ under H_0.[3] Let $J_{\mathtt{SL1}}(\theta) = -\frac{\partial}{\partial\theta'}S^\star_{\mathtt{SL1}}(\theta)$, $I_{\mathtt{SL1}}(\theta_0) = \mathtt{E}[J_{\mathtt{SL1}}(\theta_0)]$ and $\Sigma_{\mathtt{SL1}}(\theta_0) = \mathrm{Var}[S^\star_{\mathtt{SL1}}(\theta_0)]$, with their expressions given in Appendix B.1. The *usual* score test, treating $S^\star_{\mathtt{SL1}}(\theta)$ as a genuine score vector so that the information matrix equality (IME) holds, takes the form:

$$T_{\mathtt{SL1}} = S^\star_{\mathtt{SL1}}(\tilde{\theta}_{\mathtt{SL1}})' J^{-1}_{\mathtt{SL1}}(\tilde{\theta}_{\mathtt{SL1}}) S^\star_{\mathtt{SL1}}(\tilde{\theta}_{\mathtt{SL1}}), \tag{2.8}$$

where $J_{\mathtt{SL1}}(\tilde{\theta}_{\mathtt{SL1}})$ can be replaced by $I_{\mathtt{SL1}}(\tilde{\theta}_{\mathtt{SL1}})$ or $\Sigma_{\mathtt{SL1}}(\tilde{\theta}_{\mathtt{SL1}})$. However, $S^\star_{\mathtt{SL1}}(\theta)$ is not a genuine score function even if the errors are normal, as it comes from the original score function after some adjustments. In this case, the IME or its generalized version (Cameron and Trivedy, 2005; Wooldridge, 2010) does not hold. Hence, the test statistic $T_{\mathtt{SL1}}$ constructed in this *usual* way may not be valid even if the errors are normal, *unless* under 'specific' situations where $I_{\mathtt{SL1}}(\theta_0)$ and $\Sigma_{\mathtt{SL1}}(\theta_0)$ are asymptotically equivalent, i.e., the IME holds asymptotically. See the discussions below Theorem 2.1 for details.

[2] Solving the estimating equation, $S^\star_{\mathtt{SL1}}(\theta) = 0$, gives the unconstrained AQS estimator of θ. Simplifying this AQS function under the null gives AQS function of the null model, and the constrained estimates of the null model parameters. See the end of section for a general method for estimating the null models.

[3] For testing $H_0^{\mathtt{TH}}$ in (2.2), for example, $\tilde{\beta}_{\mathtt{SL1}} = 1_T \otimes \tilde{\beta}_{\mathtt{SL1}}$, $\tilde{\lambda}_{\mathtt{SL1}} = 1_T \otimes \tilde{\lambda}_{\mathtt{SL1}}$, and $\tilde{\theta}_{\mathtt{SL1}} = (\tilde{\beta}'_{\mathtt{SL1}}, \tilde{\lambda}'_{\mathtt{SL1}}, \tilde{\sigma}^2_{\mathtt{SL1}})'$, where $\tilde{\beta}_{\mathtt{SL1}}$ and $\tilde{\lambda}_{\mathtt{SL1}}$ are the estimators of the common β and λ, and 1_T is a $T \times 1$ vector of ones.

To address this issue, denoting $k_q = \dim(\theta) = (k+1)T+1$, we put our testing problem in a general framework with null hypothesis being written as

$$H_0 : C\theta_0 = 0, \tag{2.9}$$

where C is a $k_p \times k_q$ matrix generating k_p linear contrasts in the parameter vector θ.

For example, for testing $H_0^{\texttt{TH}}$ in (2.2), the number of constraints $k_p = (k+1)(T-1)$, and the linear contrast matrix $C = [\texttt{blkdiag}\{C_T^k,\ C_T^1\},\ 0_{k_p,1}]$, where $\texttt{blkdiag}\{\cdots\}$ forms a block diagonal matrix, and C_τ^m is an $m(\tau-1)\times m\tau$ matrix defined as

$$C_\tau^m = [(1_{\tau-1}\otimes I_m),\ -(I_{\tau-1}\otimes I_m)], \tag{2.10}$$

where $\otimes$ is the Kronecker product; for testing $H_0^{\texttt{RH}}$, $C = [C_T^k, 0_{k_p,T}, 0_{k_p,1}]$ and $k_p = (T-1)k$; for testing $H_0^{\texttt{SH}}$, $C = [0_{k_p,kT}, C_T^1, 0_{k_p,1}]$ and $k_p = T-1$; and for testing $H_0^{\texttt{CP}}$ in (2.3), $C = [\texttt{blkdiag}\{C_{b_0}^k, C_{T-b_0}^k, C_{\ell_0}^1, C_{T-\ell_0}^1\}, 0_{k_p}]$ and $k_p = (T-2)(k+1)$. The C matrices for tests of CP on β-coefficients only or tests of CP on λ-coefficients only can be formulated easily. The CP-test can be carried out repeatedly until the 'true' change points are detected. In all these and other interesting cases, k_p and C can be easily written out.

The score-type test is constructed based on the AQS function $S_{\texttt{SL1}}^\star(\tilde\theta_{\texttt{SL1}})$, and its asymptotic variance-covariance (VC) matrix. Denote by $N_0 = n(T-1)$ the *effective sample size* to differentiate from the overall sample size $N = nT$. Under mild regularity conditions, such as the $\sqrt{N_0}$-consistency of $\tilde\theta_{\texttt{SL1}}$ under the null, we have by Taylor expansion:

$$\tfrac{1}{\sqrt{N_0}}S_{\texttt{SL1}}^\star(\tilde\theta_{\texttt{SL1}}) = \tfrac{1}{\sqrt{N_0}}S_{\texttt{SL1}}^\star(\theta_0) + \tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)\sqrt{N_0}(\tilde\theta_{\texttt{SL1}}-\theta_0) + o_p(1), \text{ and}$$

$$[\tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S_{\texttt{SL1}}^\star(\tilde\theta_{\texttt{SL1}}) = [\tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S_{\texttt{SL1}}^\star(\theta_0) + \sqrt{N_0}(\tilde\theta_{\texttt{SL1}}-\theta_0) + o_p(1).$$

As $C\theta_0 = 0$ under H_0, we have $C\tilde\theta_{\texttt{SL1}} = 0$ (see Wooldridge 2010, p.424). It follows that

$$C[\tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S_{\texttt{SL1}}^\star(\tilde\theta_{\texttt{SL1}}) = C[\tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S_{\texttt{SL1}}^\star(\theta_0) + o_p(1), \tag{2.11}$$

leading to the asymptotic VC matrix of $C[\frac{1}{N}I_{\texttt{SL1}}(\theta_0)]^{-1}\frac{1}{\sqrt{N}}S_{\texttt{SL1}}^\star(\tilde\theta_{\texttt{SL1}})$ as

$$\Xi_{\texttt{SL1}}(\theta_0) = C[\tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)]^{-1}[\tfrac{1}{N_0}\Sigma_{\texttt{SL1}}(\theta_0)][\tfrac{1}{N_0}I_{\texttt{SL1}}(\theta_0)]^{-1}C'. \tag{2.12}$$

This gives an asymptotically valid and nonnormality robust AQS test:

$$T_{\texttt{SL1}}^{(r)} = \widetilde S_{\texttt{SL1}}^{\star\prime}\widetilde I_{\texttt{SL1}}^{-1}C'\big(C\widetilde I_{\texttt{SL1}}^{-1}\widetilde\Sigma_{\texttt{SL1}}\widetilde I_{\texttt{SL1}}^{-1}C'\big)^{-1}C\widetilde I_{\texttt{SL1}}^{-1}\widetilde S_{\texttt{SL1}}^\star, \tag{2.13}$$

where $\widetilde S_{\texttt{SL1}}^\star = S_{\texttt{SL1}}^\star(\tilde\theta_{\texttt{SL1}})$, $\widetilde I_{\texttt{SL1}} = I_{\texttt{SL1}}(\tilde\theta_{\texttt{SL1}})$, and $\widetilde\Sigma_{\texttt{SL1}} = \Sigma_{\texttt{SL1}}(\tilde\theta_{\texttt{SL1}})$.

Remark 2.1. *Although the AQS test given in* (2.13) *is developed based on the panel SL model with 1FE, the general principles behind apply to all models considered in this paper. It also applies to more complicated spatial models as well as many non-spatial models.*

Asymptotic properties. In studying the asymptotic properties of the proposed tests, we focus on the tests of temporal homogeneity to ease the exposition. Therefore, some of the regularity conditions, i.e., Assumptions 2 and 4, correspond to the null model under $H_0^{\texttt{TH}}$ in (2.2) only. However, these assumptions can be easily relaxed to cater a non-homogeneous null model. Denote $X_{nt}^{\circ} = X_{nt} - \bar{X}_n$, where $\bar{X}_n = \frac{1}{T}\sum_{t=1}^{T} X_{nt}$.

Assumption 1. *The disturbances $\{v_{it}\}$ are iid across i and t with mean zero, variance σ_0^2, and $E\,|v_{it}|^{4+\epsilon_0} < \infty$ for some $\epsilon_0 > 0$.*

Assumption 2. *Under H_0, the parameter space Λ of the common λ is compact, and the true value λ_0 is in the interior of Λ. The matrix $A_n(\lambda)$ is invertible for all $\lambda \in \Lambda$.*

Assumption 3. *The elements of X_{nt} are non-stochastic, and are bounded uniformly in n and t, such that $\lim_{N_0\to\infty}\frac{1}{N_0}\sum_{t=1}^{T} X_{nt}^{\circ\prime}X_{nt}^{\circ}$ exists and nonsingular. The elements of c_n are uniformly bounded.*

Assumption 4. *W_n has zero diagonal elements, and is uniformly bounded in both row and column sums in absolute value. $A_n^{-1}(\lambda)$ is also uniformly bounded in both row and column sums in absolute value for λ in a neighborhood of λ_0.*

Theorem 2.1. *Under Assumptions 1-4, if further, (i) $\tilde{\theta}_{\texttt{SL1}}$ is $\sqrt{N_0}$-consistent for θ_0 under $H_0^{\texttt{TH}}$, and (ii) $I_{\texttt{SL1}}(\theta)$ and $\Xi_{\texttt{SL1}}(\theta)$ are positive definite for θ in a neighborhood of θ_0 when N_0 is large enough, then we have, under $H_0^{\texttt{TH}}$, $T_{\texttt{SL1}}^{(r)} \xrightarrow{D} \chi^2_{k_p}$, as $n \to \infty$.*

Note that in case of testing for temporal homogeneity, $k_p = (T-1)(k+1)$, and that in case of testing for a 'single change' of points, $k_p = (T-2)(k+1)$. It can easily be seen that $T_{\texttt{SL1}}$ is in general not an asymptotic pivotal quantity due to the violation of IME. However, if $I_{\texttt{SL1}}(\theta_0) \asymp \Sigma_{\texttt{SL1}}(\theta_0)$, where $\asymp$ denotes asymptotic equivalence, then $\widetilde{I}_{\texttt{SL1}}^{-1}C'\big(C\widetilde{I}_{\texttt{SL1}}^{-1}\widetilde{\Sigma}_{\texttt{SL1}}\widetilde{I}_{\texttt{SL1}}^{-1}C'\big)^{-1}C\widetilde{I}_{\texttt{SL1}}^{-1} \asymp \widetilde{I}_{\texttt{SL1}}^{-1}$ (see Wooldridge, 2010, p. 424), and hence $T_{\texttt{SL1}}$ becomes valid. This is in fact true when T is also large as seen from the expressions given in Appendix B.1, but this case needs an extra care as in Remark 2.2 below.

Remark 2.2. *When $T \to \infty$ as $n \to \infty$, the degrees of freedom (d.f) of the chi-square statistic increase with n. In this case, one may apply the arguments for 'double asymptotics' (see, e.g., Rempala and Wesolowski, 2016) to show that $(T_{\texttt{SL1}}^{(r)} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$ as $n/\sqrt{T} \to \infty$. This sample size requirement (n goes large faster than $\sqrt{T}$) is rather weak as it is typical in spatial panels that n is at least as large as T.*

Estimation of null models. The construction of the AQS tests requires estimation of various null models, which could be the homogeneous model as specified by $H_0^{\texttt{TH}}$ in (2.2), the model with homogeneity in β's only, the model with homogeneity in λ's only, or the model with change points as specified by $H_0^{\texttt{CP}}$ in (2.3), etc. Each null model can estimated by solving the simplified AQS equations by simplifying $S_{\texttt{SL1}}^{\star}(\theta)$ according to the null hypothesis, which is clearly inconvenient to the applied researchers. To facilitate **practical applications** of our methods, a general *Lagrange Multiplier* (LM) method is introduced. Let $l_{\texttt{SL1}}(\theta)$ be the objective function to be maximized subject to $C\theta_0 = 0$, with $S_{\texttt{SL1}}^{\star}(\theta)$ given in (2.7) being its partial derivatives. Define the Lagrangian

$$\mathscr{L}_{\texttt{SL1}}(\theta) = l_{\texttt{SL1}}(\theta) - \phi'(C\theta),$$

where ϕ is a $k_p \times 1$ vector of Lagrange multipliers. Taking partial derivatives and equating to 0, we have k_q equations $\frac{\partial \mathscr{L}_{\mathtt{SL1}}}{\partial \boldsymbol{\theta}} = S^{*}_{\mathtt{SL1}}(\boldsymbol{\theta}) - C'\phi = 0_{k_q,1}$. Together with the k_p constraints $C\boldsymbol{\theta} = 0$, we have $k_q + k_p$ equations for the $k_q + k_p$ unknowns $\boldsymbol{\theta}$ and ϕ, leading to

$$\begin{pmatrix} \tilde{\boldsymbol{\theta}}_{\mathtt{SL1}} \\ \tilde{\phi}_{\mathtt{SL1}} \end{pmatrix} = \arg \left\{ \begin{array}{l} S^{\star}_{\mathtt{SL1}}(\boldsymbol{\theta}) - C'\phi = 0_{k_q,1} \\ C\boldsymbol{\theta} = 0_{k_p,1} \end{array} \right\}. \tag{2.14}$$

To further aid the applications, we make the Matlab codes available upon request, or online at `http://www.mysmu.edu/faculty/zlyang/`.

Finally, from the expressions of $I_{\mathtt{SL1}}(\boldsymbol{\theta}_0)$ and $\Sigma_{\mathtt{SL1}}(\boldsymbol{\theta}_0)$ given in Appendix B.1, we see that they both contain c_n, which is estimated by plugging the null estimates $\tilde{\boldsymbol{\beta}}_{\mathtt{SL1}}$ and $\tilde{\boldsymbol{\lambda}}_{\mathtt{SL1}}$ into $\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda})$. Furthermore, in case of nonnormality, the VC matrix $\Sigma_{\mathtt{SL1}}(\boldsymbol{\theta}_0)$ contains two additional parameters, the skewness γ and excess kurtosis κ of the idiosyncratic errors $V_{n,it}$, and their estimates are obtained by applying Lemma 4.1(a) of Yang et al. (2016). See Sec. 5 for a detailed discussion on issues related to practical implementations.

However, as the hypothesis $H_0^{\mathtt{HT}}$ given in (2.2) and the corresponding homogeneous model plays an important role in studying the asymptotic properties of the test and in Monte Carlo simulation, an outline is given on how $S^{\star}_{\mathtt{SL1}}(\boldsymbol{\theta})$ is simplified and how it leads to constrained AQS estimators with the desired asymptotic properties. Let $\theta = (\beta', \lambda, \sigma^2)'$. The constrained estimate of c_n given (β, λ) becomes $\tilde{c}^{\circ}_n(\beta, \lambda) = A_n(\lambda)\bar{Y}_n - \bar{X}_n\beta$ where $\bar{Y}_n$ and $\bar{X}_n$ are the averages of $\{Y_{nt}\}$ and $\{X_{nt}\}$, respectively. Along the same line leading to (2.7), one can easily show that AQS function for the homogeneous model takes the form:

$$S^{\circ}_{\mathtt{SL1}}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t-1}^{T} X^{\circ\prime}_{nt}\widetilde{V}^{\circ}_{nt}(\beta, \lambda), \\ \frac{1}{\sigma^2}\sum_{t-1}^{T}(W_nY^{\circ}_{nt})'\widetilde{V}^{\circ}_{nt}(\beta, \lambda) - (T-1)\mathrm{tr}[G_n(\lambda)], \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}\widetilde{V}^{\circ\prime}_{nt}(\beta, \lambda)\widetilde{V}^{\circ}_{nt}(\beta, \lambda), \end{cases} \tag{2.15}$$

$\widetilde{V}^{\circ}_{nt}(\beta, \lambda) = A_n(\lambda)Y_{nt} - X_{nt}\beta - \tilde{c}^{\circ}_n(\beta, \lambda) = A_n(\lambda)Y^{\circ}_{nt} - X^{\circ}_{nt}\beta$, where $Y^{\circ}_{nt} = Y_{nt} - \bar{Y}_n$ and $X^{\circ}_{nt} = X_{nt} - \bar{X}_n$. Solving the estimating equations, $S^{\circ}_{\mathtt{SL1}}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\mathtt{SL1}}$ of θ. The AQS estimation provides an alternative to the QML estimation based on transformation of Lee and Yu (2010). The two can be shown to be asymptotically equivalent, and therefore $\tilde{\theta}_{\mathtt{SL1}}$ is $\sqrt{n(T-1)}$-consistent for θ.

2.2. Panel SL model with two-way FE

While the unit-specific fixed effects are important to the spatial panel data models, the time-specific effects often cannot be neglected. In this section, we extend our tests to panel SL model with two-way FE (2FE). The model takes the following form:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + \alpha_t 1_n + V_{nt}, \tag{2.16}$$

where $\{\alpha_t\}$ are the unobserved time-specific effects or the unobserved temporal heterogeneity in the intercept, and 1_n is an $n \times 1$ vector of ones. As the spatial parameters and regression coefficients change only with time. One can apply transformation method to eliminate the time-specific effects as is widely applied in the literature, see, e.g., Lee and Yu (2010), Baltagi and Yang (2013) and Yang et al. (2016). Define $J_n = I_n - \frac{1}{n}1_n1_n'$. Assume W_n is row-normalized (i.e., row sums are one). Then, $J_nW_n = J_nW_nJ_n$. Let $(F_{n,n-1}, \frac{1}{\sqrt{n}}1_n)$ be the orthonormal eigenvector matrix of J_n, where $F_{n,n-1}$ is the $n \times (n-1)$ sub-matrix corresponding to the eigenvalues of one. By *Spectral Theorem*, $J_n = F_{n,n-1}F_{n,n-1}'$. It follows that $F_{n,n-1}'W_n = F_{n,n-1}'W_nF_{n,n-1}F_{n,n-1}'$. Premultiplying $F_{n,n-1}'$ on both sides of (2.16), we have the following transformed model:

$$Y_{nt}^* = \lambda_t W_n^* Y_{nt}^* + X_{nt}^*\beta_t + c_n^* + V_{nt}^*,\ t = 1, \ldots, T, \tag{2.17}$$

where $Y_{nt}^* = F_{n,n-1}'Y_{nt}$, and so are X_{nt}^*, c_n^* and V_{nt}^* defined; and $W_n^* = F_{n,n-1}'W_nF_{n,n-1}$. After the transformation, the effective sample size is $(n-1)T$. Model (2.17) takes an identical form as Model (2.1). Furthermore, $V_{nt}^* \sim (0, \sigma_0^2 I_{n-1})$, which is normal if V_{nt}^* is, and is independent of V_{ns}^*, $s \neq t$.[4] Hence, the steps leading to the score-type tests and the consistent estimation of the null model are similar to those for the SL one-way FE model.

Define $A_n^*(\lambda_t) = I_{n-1} - \lambda_t W_n^*$, $t = 1, \ldots, T$. The quasi Gaussian loglikelihood function of $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \sigma^2)'$ and c_n^* of Model (2.17) is

$$\begin{aligned} \ell_{\texttt{SL2}}(\boldsymbol{\theta}, c_n^*) = & -\tfrac{(n-1)T}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A_n^*(\lambda_t)| \\ & -\tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T V_{nt}^{*\prime}(\lambda_t, \beta_t, c_n^*)V_{nt}^*(\lambda_t, \beta_t, c_n^*), \end{aligned} \tag{2.18}$$

where $V_{nt}^*(\beta_t, \lambda_t, c_n^*) = A_n^*(\lambda_t)Y_{nt}^* - X_{nt}^*\beta_t - c_n^*$. Given $\boldsymbol{\theta}$, $\ell_{\texttt{SL2}}(\boldsymbol{\theta}, c_n^*)$ is maximized at:

$$\tilde{c}_n^*(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \tfrac{1}{T}\textstyle\sum_{t=1}^T [A_n^*(\lambda_t)Y_{nt}^* - X_{nt}^*\beta_t], \tag{2.19}$$

which gives the concentrated loglikelihood function of $\boldsymbol{\theta}$ upon substitution:

$$\ell_{\texttt{SL2}}^c(\boldsymbol{\theta}) = -\tfrac{(n-1)T}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A_n^*(\lambda_t)| - \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T \widetilde{V}_{nt}^{*\prime}(\boldsymbol{\beta}, \boldsymbol{\lambda})'\widetilde{V}_{nt}^*(\boldsymbol{\beta}, \boldsymbol{\lambda}), \tag{2.20}$$

where $\widetilde{V}_{nt}^*(\boldsymbol{\beta}, \boldsymbol{\lambda}) = A_n^*(\lambda_t)Y_{nt}^* - X_{nt}^*\beta_t - \tilde{c}_n^*(\boldsymbol{\beta}, \boldsymbol{\lambda})$. Now, define $G_n^*(\lambda_t) = W_n^*A_n^{*-1}(\lambda_t)$. Differentiating $\ell_{\texttt{SL2}}^c(\boldsymbol{\theta})$ gives the CS or CQS function of $\boldsymbol{\theta}$ of Model (2.17):

$$S_{\texttt{SL2}}^c(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X_{nt}^{*\prime}\widetilde{V}_{nt}^*(\boldsymbol{\beta}, \boldsymbol{\lambda}),\ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_n^*Y_{nt}^*)'\widetilde{V}_{nt}^*(\boldsymbol{\beta}, \boldsymbol{\lambda}) - \mathrm{tr}[G_n^*(\lambda_t)],\ \ t = 1, \ldots, T, \\ -\frac{(n-1)T}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}_{nt}^{*\prime}(\boldsymbol{\beta}, \boldsymbol{\lambda})\widetilde{V}_{nt}^*(\boldsymbol{\beta}, \boldsymbol{\lambda}). \end{cases} \tag{2.21}$$

Takes the expectation of the above score, we have,

$$\mathrm{E}[S_{\texttt{SL2}}^c(\boldsymbol{\theta}_0)] = \big\{0_{Tk}',\ -\tfrac{1}{T}\mathrm{tr}[G_n^*(\lambda_{t0})],\ t = 1, \ldots T,\ -\tfrac{n-1}{2\sigma_0^2}\big\}',$$

which again shows that model estimation based on maximizing the quasi loglikelihood would not lead to consistent estimates of the model parameters. The CQS function given

[4] The time-specific effects can also be eliminated by pre-multiplying J_n on both sides of (2.16). However, the resulted disturbances J_nV_{nt} would not be linearly independent over the cross-section dimension.

in (2.21) should be adjusted by recentering, giving the AQS function of Model (2.17):

$$S^{\star}_{\text{SL2}}(\theta) = \begin{cases} \frac{1}{\sigma^2} X^{*\prime}_{nt} \widetilde{V}^{*}_{nt}(\beta, \lambda), \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2} (W^{*}_{n} Y^{*}_{nt})' \widetilde{V}^{*}_{nt}(\beta, \lambda) - \frac{T-1}{T} \text{tr}[G^{*}_{n}(\lambda_t)], \ t = 1, \ldots, T, \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{t=1}^{T} \widetilde{V}^{*\prime}_{nt}(\beta, \lambda) \widetilde{V}^{*}_{nt}(\beta, \lambda). \end{cases} \tag{2.22}$$

It is easy to show that $\text{E}[S^{\star}_{\text{SL2}}(\theta)] = 0$, and that $\frac{1}{nT} S^{\star}_{\text{SL2}}(\theta_0) \xrightarrow{p} 0$ as $n \to \infty$ alone, or both n and T go infinity. Thus, this AQS function gives a set of unbiased estimating functions, and paves the way for developing asymptotic valid score-type tests. Again, simplifying this AQS function under various null hypotheses gives the AQS functions of the null models and the constrained estimates. See the end of the Section for a general formulation.

Now, the tests concerning $\{\beta_t\}$ and $\{\lambda_t\}$ allow the existence of both unobserved cross-sectional and time-specific heterogeneity in the intercept, i.e., the existence of both individual specific fixed effects and the time specific fixed effects. As the transformed 2FE panel SL model takes an identical form as 1FE panel SL model, the tests developed for 1FE panel SL model extends directly to give tests for the 2FE panel SL model. Let $\tilde{\theta}_{\text{SL2}}$ be the null estimate of θ. Let $I_{\text{SL2}}(\theta_0)$ and $\Sigma_{\text{SL2}}(\theta_0)$ be, respectively, the expected negative Hessian and the VC matrix of $S^{\star}_{\text{SL2}}(\theta_0)$, given in Appendix B.2. The AQS test, robust against nonnormality and taking into account of the estimation of fixed effects, is:

$$T^{(r)}_{\text{SL2}} = \widetilde{S}^{\star\prime}_{\text{SL2}} \widetilde{I}^{-1}_{\text{SL2}} C' \left(C \widetilde{I}^{-1}_{\text{SL2}} \widetilde{\Sigma}_{\text{SL2}} \widetilde{I}^{-1}_{\text{SL2}} C' \right)^{-1} C \widetilde{I}^{-1}_{\text{SL2}} \widetilde{S}^{\star}_{\text{SL2}}, \tag{2.23}$$

where $\widetilde{S}^{\star}_{\text{SL2}} = S^{\star}_{\text{SL2}}(\tilde{\theta}_{\text{SL2}})$, $\widetilde{I}_{\text{SL2}} = I_{\text{SL2}}(\tilde{\theta}_{\text{SL2}})$, and $\widetilde{\Sigma}_{\text{SL2}} = \Sigma_{\text{SL2}}(\tilde{\theta}_{\text{SL2}})$. As in the case of 1FE-SL model, when $I_{\text{SL2}}(\theta_0) \asymp \Sigma_{\text{SL2}}(\theta_0)$, $\widetilde{I}^{-1}_{\text{SL2}} C' \left(C \widetilde{I}^{-1}_{\text{SL2}} \widetilde{\Sigma}_{\text{SL2}} \widetilde{I}^{-1}_{\text{SL2}} C' \right)^{-1} C \widetilde{I}^{-1}_{\text{SL2}} \asymp \widetilde{I}^{-1}_{\text{SL2}}$, and hence $T^{(r)}_{\text{SL2}}$ reduces to the *naïve* test: $T_{\text{SL2}} = \widetilde{S}^{\star\prime}_{\text{SL2}} \widetilde{J}^{-1}_{\text{SL2}} \widetilde{S}^{\star}_{\text{SL2}}$, where $\widetilde{J}_{\text{SL2}} = -\frac{\partial}{\partial \theta} S^{\star}_{\text{SL2}}(\tilde{\theta}_{\text{SL2}})$.

Asymptotic properties of these tests can be studied along the same line as the tests for 1FE panel SL model. Again we focus on the test of H^{TH}_0 for ease of exposition. The effective sample size becomes $N_0 = (n-1)(T-1)$ due to the 'estimation' of both individual- and time-specific FEs. Let $\Xi_{\text{SL2}}(\theta)$ and $X^{*\circ}_{nt}$ be defined as $\Xi_{\text{SL1}}(\theta)$ and X°_{nt} in Sec. 2.1.

Assumption $3'$: The elements of X_{nt} are nonstochastic, and are bounded uniformly in n and t, such that $\lim_{N_0 \to \infty} \frac{1}{N_0} \sum_{t=1}^{T} X^{*\circ\prime}_{nt} X^{*\circ}_{nt}$ exists and is nonsingular.

Theorem 2.2. *Under Assumptions 1-2,* $3'$*, and 4, if further, (i)* $\tilde{\theta}_{\text{SL2}}$ *is* $\sqrt{N_0}$*-consistent for* θ_0 *under* H^{TH}_0*, and (ii)* $I_{\text{SL2}}(\theta)$ *and* $\Xi_{\text{SL2}}(\theta)$ *are positive definite for* θ *in a neighborhood of* θ_0 *when* N_0 *is large enough, then we have, under* H^{TH}_0*,* $T^{(r)}_{\text{SL2}} \xrightarrow{D} \chi^2_{k_p}$*, as* $n \to \infty$*.*

Note that while the effective sample size for the 2FE-SL model is smaller than that of the 1FE-SL model, the d.f. associated with the test statistics remain the same. As discussed below Theorem 2.1, T_{SL2} is not an asymptotic pivotal quantity unless T is also large. As in Remark 2.2, if T grows with n, $(T^{(r)}_{\text{SL2}} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0, 1)$, as $n/\sqrt{T} \to \infty$.

Estimation of null models. The general constrained root-finding method, the LM procedure, presented at the end of Sec. 2.1 for the panel SL model with 1FE directly applies to the panel SL model with 2FE to give constrained estimates of various null models. This greatly facilitates the practical applications. Again, the homogeneous model specified by H_0^{TH} in (2.2) and its AQS estimation play important roles in studying the asymptotic properties and performing Monte Carlo simulations, and therefore an outline is given on the estimation procedures based on the simplified AQS function. The constrained estimate of c_n^*, given (β, λ), becomes $\tilde{c}_n^{*\circ}(\beta, \lambda) = A_n^*(\lambda)\bar{Y}_n^* - \bar{X}_n^*\beta$, where $\bar{Y}_n^*$ and $\bar{X}_n^*$ are the averages of $\{Y_{nt}^*\}$ and $\{X_{nt}^*\}$, respectively. Along the same line leading to (2.15), we have the AS or AQS function for the homogeneous panel SL model with 2FE:

$$S_{\text{SL2}}^{\circ}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t-1}^{T} X_{nt}^{*\circ\prime}\widetilde{V}_{nt}^{*\circ}(\beta, \lambda), \\ \frac{1}{\sigma^2}\sum_{t-1}^{T}(W_n^*Y_{nt}^{*\circ})'\widetilde{V}_{nt}^{*\circ}(\beta, \lambda) - (T-1)\text{tr}[G_n^*(\lambda)], \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}\widetilde{V}_{nt}^{*\circ\prime}(\beta, \lambda)\widetilde{V}_{nt}^{*\circ}(\beta, \lambda), \end{cases} \tag{2.24}$$

where $\widetilde{V}_{nt}^{*\circ}(\beta, \lambda) = A_n^*(\lambda)Y_{nt}^* - X_{nt}^*\beta - \tilde{c}_n^{*\circ}(\beta, \lambda) = A_n(\lambda)Y_{nt}^{*\circ} - X_{nt}^{*\circ}\beta$, $Y_{nt}^{*\circ} = Y_{nt}^* - \bar{Y}_n^*$ and $X_{nt}^{*\circ} = X_{nt}^* - \bar{X}_n^*$. Solving the estimating equations, $S_{\text{SL2}}^{\circ}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\text{SL2}}$ of θ. Again, it can be shown to be asymptotically equivalent to the transformation-based QML estimator of Lee and Yu (2010). Thus, $\tilde{\theta}_{\text{SL2}}$ is $\sqrt{(n-1)(T-1)}$-consistent for θ. The estimation of c_n and γ and κ contained in $I_{\text{SL2}}(\boldsymbol{\theta}_0)$ and $\Sigma_{\text{SL2}}(\boldsymbol{\theta}_0)$ proceeds similarly.

3. Test for Temporal Heterogeneity in Panel SLE Model

The tests introduced in the earlier section can be easily extended to a more general SPD model where the the disturbances are also subject to spatial interactions, giving an SPD model with both spatial lag and error (SLE) dependence. Again, we first present results for the one-way FE model, and then the results for the two-way FE model.

3.1. Panel SLE model with one-way FE

The SLE model with one-way fixed effects has the form:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + U_{nt}, \quad U_{nt} = \rho_t M_n U_{nt} + V_{nt}, \tag{3.1}$$

where M_n is another spatial weight matrix capturing the spatial interactions among the disturbances, which can be the same as W_n, and $\{\rho_t\}$ are the spatial error parameters, possibly changing with time. Again, we are primarily interested in the test for temporal homogeneity, which now corresponds to a test of the following null hypothesis:

$$H_0^{\text{TH}}: \beta_1 = \cdots = \beta_T = \beta, \;\; \lambda_1 = \cdots = \lambda_T = \lambda, \;\; \text{and} \;\; \rho_1 = \cdots = \rho_T = \rho. \tag{3.2}$$

If this test is rejected, one would be interested in testing various hypotheses discussed in Sec. 2.1, including H_0^{CP} in (2.3) extended to include the ρ-component, to find out the cause of the rejection. An **interesting test** for the panel SLE model would be the conditional test: $H_0^{\text{THC}}: \beta_1 = \cdots = \beta_T = \beta$, and $\lambda_1 = \cdots = \lambda_T = \lambda$, given $\rho_1 = \cdots = \rho_T = \rho$. In this case, the alternative (full) model is a submodel of (3.1) with the disturbance following a homogeneous SAR process: $U_{nt} = \rho M_n U_{nt} + V_{nt}$. We present the most general case here, and give necessary details related to this submodel at the end of Sec. 3.2.

Following the same set of notation as in the earlier section, and further denoting $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_T)'$, $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\lambda}', \boldsymbol{\rho}', \sigma^2)'$, and $B_n(\rho_t) = I_n - \rho_t M_n, t = 1, \ldots, T$, we have the (quasi) Gaussian loglikelihood for $(\boldsymbol{\theta}, c_n)$:

$$\begin{aligned}\ell_{\texttt{SLE1}}(\boldsymbol{\theta}, c_n) = &-\tfrac{nT}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A_n(\lambda_t)| + \sum_{t=1}^T \ln|B_n(\rho_t)| \\ &- \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T V_{nt}'(\beta_t, \lambda_t, \rho_t, c_n) V_{nt}(\beta_t, \lambda_t, \rho_t, c_n),\end{aligned} \tag{3.3}$$

where $V_{nt}(\beta_t, \lambda_t, \rho_t, c_n) = B_n(\rho_t)[A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - c_n],\ t = 1, \ldots, T.$

Similarly to the developments in the previous section, we first eliminate c_n through a direct maximization of the loglikelihood function, given the other model parameters $\boldsymbol{\theta}$, and then adjust the resulted CS or CQS function to eliminate the asymptotic bias or inconsistency. Given $\boldsymbol{\theta}$, $\ell_{\texttt{SLE1}}(\boldsymbol{\theta}, c_n)$ is maximized at

$$\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = \big[\textstyle\sum_{t=1}^T B_n'(\rho_t)B_n(\rho_t)\big]^{-1}\sum_{t=1}^T \big[B_n'(\rho_t)B_n(\rho_t)\big(A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t\big)\big], \tag{3.4}$$

leading to the concentrated (quasi) Gaussian loglikelihood function of $\boldsymbol{\theta}$ upon substitution:

$$\begin{aligned}\ell^c_{\texttt{SLE1}}(\boldsymbol{\theta}) = &-\tfrac{nT}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A_n(\lambda_t)| + \sum_{t=1}^T \ln|B_n(\rho_t)| \\ &- \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}),\end{aligned} \tag{3.5}$$

where $\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = V_{nt}(\beta_t, \lambda_t, \rho_t, \tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})) = B_n(\rho_t)[A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t - \tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})]$.

To facilitate the subsequent derivations, denote $U^\circ_{nt}(\beta_t, \lambda_t) = A_n(\lambda_t)Y_{nt} - X_{nt}\beta_t$, $D_n(\rho_t) = B_n'(\rho_t)B_n(\rho_t)$ and $\mathbb{D}_n(\boldsymbol{\rho}) = \sum_{t=1}^T D_n(\rho_t)$. Then,

$$\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = B_n(\rho_t)U^\circ_{nt}(\beta_t, \lambda_t) - B_n(\rho_t)\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}),$$

$\tilde{c}_n(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = \mathbb{D}_n^{-1}(\boldsymbol{\rho})\sum_{t=1}^T D_n(\rho_t)U^\circ_{nt}(\beta_t, \lambda_t)$, and the key term in (3.5):

$$\begin{aligned}\textstyle\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) = &\textstyle\sum_{t=1}^T U^{\circ\prime}_{nt}(\beta_t, \lambda_t)D_n(\rho_t)U^\circ_{nt}(\beta_t, \lambda_t) \\ &- \big(\textstyle\sum_{t=1}^T D_n(\rho_t)U^\circ_{nt}(\beta_t, \lambda_t)\big)'\mathbb{D}_n^{-1}(\boldsymbol{\rho})\big(\sum_{t=1}^T D_n(\rho_t)U^\circ_{nt}(\beta_t, \lambda_t)\big).\end{aligned}$$

Differentiating $\ell^c_{\texttt{SLE1}}(\boldsymbol{\theta})$ gives the CS or CQS function of $\boldsymbol{\theta}$:

$$S^c_{\texttt{SLE1}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X_{nt}'B_n'(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_nY_{nt})'B_n'(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \text{tr}[G_n(\lambda_t)], \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}\widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})H_n(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}) - \text{tr}[H_n(\rho_t)], \ \ t = 1, \ldots, T, \\ -\frac{nT}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}_{nt}'(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\rho}), \end{cases} \tag{3.6}$$

where $H_n(\rho_t) = M_nB_n^{-1}(\rho_t),\ \ t = 1, \ldots, T$.

At the true $\boldsymbol{\theta}_0$, we have, $\tilde{c}_n(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = c_n + \mathbb{D}_n^{-1}\sum_{s=1}^T B_{ns}'V_{ns}$ and hence $\widetilde{V}_{nt} \equiv \widetilde{V}_{nt}(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = V_{nt} - B_{nt}\mathbb{D}_n^{-1}\sum_{s=1}^T B_{ns}'V_{ns}$, and $W_nY_{nt} = G_{nt}(X_{nt}\beta_0 + c_n + B_{nt}^{-1}V_{nt})$, where $B_{nt} = B_n(\rho_{t0})$, $G_{nt} = G_n(\lambda_{t0})$, and $\mathbb{D}_n = \mathbb{D}_n(\boldsymbol{\rho}_0)$. It is easy to show that,

$$\mathrm{E}[S^c_{\texttt{SLE1}}(\boldsymbol{\theta}_0)] = \begin{cases} 0_{Tk,1}, \\ -\mathrm{tr}[\mathbb{D}_n^{-1}(\boldsymbol{\rho}_0)B'_n(\rho_{t0})B_n(\rho_{t0})G_n(\lambda_{t0})],\ t = 1, \ldots T, \\ -\mathrm{tr}[B_n(\rho_{t0})\mathbb{D}_n^{-1}(\boldsymbol{\rho}_0)B'_n(\rho_{t0})H_n(\rho_{t0})],\ t = 1, \ldots T, \\ -\frac{n}{2\sigma_0^2}. \end{cases}$$

Therefore, the AS or AQS function of $\boldsymbol{\theta}$ for Model (3.1) takes the form:

$$S^\star_{\texttt{SLE1}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X'_{nt}B'_n(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}),\ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}(W_nY_{nt})'B'_n(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}) - \mathrm{tr}[R_{nt}(\boldsymbol{\rho})G_n(\lambda_t)],\ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2}\widetilde{V}'_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho})H_n(\rho_t)\widetilde{V}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}) - \mathrm{tr}[S_{nt}(\boldsymbol{\rho})H_n(\rho_t)],\ \ t = 1, \ldots, T, \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}'_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho})\widetilde{V}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}), \end{cases} \tag{3.7}$$

where $R_{nt}(\boldsymbol{\rho}) = I_n - \mathbb{D}_n^{-1}(\boldsymbol{\rho})B'_n(\rho_t)B_n(\rho_t)$ and $S_{nt}(\boldsymbol{\rho}) = I_n - B_n(\rho_t)\mathbb{D}_n^{-1}(\boldsymbol{\rho})B'_n(\rho_t)$.

It is easy to show that $\mathrm{E}[S^\star_{\texttt{SLE1}}(\boldsymbol{\theta})] = 0$, and that $\frac{1}{nT}S^\star_{\texttt{SLE1}}(\boldsymbol{\theta}_0) \xrightarrow{p} 0$ as $n \to \infty$ alone, or both n and T go infinity. Thus, this AQS function gives a set of unbiased estimating functions, and paves the way for developing asymptotic valid score-type tests.

Construction of AQS tests. Denote the constrained estimator (under H_0) of $\boldsymbol{\theta}$ by $\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}}$.[5] To test various hypotheses concerning temporal homogeneity/heterogeneity, one is tempt to use the *naïve* test, $T_{\texttt{SLE1}} = S^\star_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})'J^{-1}_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})S^\star_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})$, treating $S^\star_{\texttt{SLE1}}(\boldsymbol{\theta})$ as a genuine score function, where $J_{\texttt{SLE1}}(\boldsymbol{\theta}_0) = -\frac{\partial}{\partial\boldsymbol{\theta}'}S^\star_{\texttt{SLE1}}(\boldsymbol{\theta}_0)$, which can be replaced by $I_{\texttt{SLE1}}(\boldsymbol{\theta}_0) = \mathrm{E}[J_{\texttt{SLE1}}(\boldsymbol{\theta}_0)]$, or $\Sigma_{\texttt{SLE1}}(\boldsymbol{\theta}_0) = \mathrm{Var}[S^\star_{\texttt{SLE1}}(\boldsymbol{\theta}_0)]$ (see Appendix B.3 for their expressions). Again, $S^\star_{\texttt{SLE1}}(\boldsymbol{\theta})$ is not a genuine score function. Hence, the test constructed in the usual way may not be a valid test statistic, even if the errors are normal.

To give a general robust test, we again, as in the previous section, put our testing problem in a general framework with null hypothesis being written as H_0: $C\boldsymbol{\theta}_0 = 0$, with some modifications on C to include the ρ parameters. The dimensions of C are again denoted as $k_p \times k_q$ with k_p linear contrasts on the parameter vector $\boldsymbol{\theta}$ of dimension $k_q = (k+2)T+1$. For $H_0^{\texttt{TH}}$ in (3.2), we have $k_p = (T-1)(k+2)$ and $C = [\texttt{blkdiag}\{C^k_T, C^1_T, C^1_T\}, 0_{k_p,1}]$, where C^κ_τ is defined in (2.10). For tests of $\texttt{CP}$ in β_t, λ_t and ρ_t at time points b_0, ℓ_0 and r_0, respectively, $k_p = (T-2)(k+2)$ and $C = [\texttt{blkdiag}\{C^k_{b_0}, C^k_{T-b_0}, C^1_{\ell_0}, C^1_{T-\ell_0}, C^1_{r_0}, C^1_{T-r_0}\}, 0_{k_p,1}]$.

Similarly, the score-type test is based on the AQS function $S^\star_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})$ evaluated at the null estimate $\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}}$ of $\boldsymbol{\theta}$, and the asymptotic VC matrix of $S^\star_{\texttt{SLE1}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}})$. Now, the effective sample size is back to $N_0 = n(T-1)$ as for the 1FE panel SL model. Following the fundamental developments in Sec 2.1, we have, under mild regularity conditions such as the $\sqrt{N_0}$-consistency of $\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}}$, an asymptotically valid and nonnormality robust AQS

[5] In case of testing $H_0^{\texttt{TH}}$ given in (3.2), the constrained estimators of $\boldsymbol{\beta}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ are, respectively, $\tilde{\boldsymbol{\beta}}_{\texttt{SLE1}} = 1_T \otimes \tilde{\beta}_{\texttt{SLE1}}$, $\tilde{\boldsymbol{\lambda}}_{\texttt{SLE1}} = 1_T \otimes \tilde{\lambda}_{\texttt{SLE1}}$, and $\tilde{\boldsymbol{\rho}}_{\texttt{SLE1}} = 1_T \otimes \tilde{\rho}_{\texttt{SLE1}}$, where $\tilde{\beta}_{\texttt{SLE1}}$, $\tilde{\lambda}_{\texttt{SLE1}}$ and $\tilde{\rho}_{\texttt{SLE1}}$ are the estimators of the common β, λ and ρ, leading to the constrained estimator of $\boldsymbol{\theta}$ as $\tilde{\boldsymbol{\theta}}_{\texttt{SLE1}} = (\tilde{\boldsymbol{\beta}}'_{\texttt{SLE1}}, \tilde{\boldsymbol{\lambda}}'_{\texttt{SLE1}}, \tilde{\boldsymbol{\rho}}'_{\texttt{SLE1}}, \tilde{\sigma}^2_{\texttt{SLE1}})'$.

test:

$$T^{(r)}_{\mathtt{SLE1}} = \widetilde{S}^{\star\prime}_{\mathtt{SLE1}}\widetilde{I}^{-1}_{\mathtt{SLE1}}C'\big(C\widetilde{I}^{-1}_{\mathtt{SLE1}}\widetilde{\Sigma}_{\mathtt{SLE1}}\widetilde{I}^{-1}_{\mathtt{SLE1}}C'\big)^{-1}C\widetilde{I}^{-1}_{\mathtt{SLE1}}\widetilde{S}^{\star}_{\mathtt{SLE1}}, \tag{3.8}$$

where $\widetilde{S}^{\star}_{\mathtt{SLE1}} = S^{\star}_{\mathtt{SLE1}}(\tilde{\theta}_{\mathtt{SLE1}})$, $\widetilde{I}_{\mathtt{SLE1}} = I_{\mathtt{SLE1}}(\tilde{\theta}_{\mathtt{SLE1}})$, and $\widetilde{\Sigma}_{\mathtt{SLE1}} = \Sigma_{\mathtt{SLE1}}(\tilde{\theta}_{\mathtt{SLE1}})$.

Asymptotic properties of the proposed tests are established based on Assumptions 1-4 in Sec. 2, and the following additional conditions on M_n and $B_n(\rho)$.

Assumption 5. *Under H_0, the parameter space $\mathbb{P}$ of the common ρ is compact. The true value ρ_0 is in the interior of $\mathbb{P}$. The matrix $B_n(\rho)$ is invertible for all $\rho \in \mathbb{P}$. M_n has zero diagonal elements, and are uniformly bounded in both row and column sums in absolute value. $B_n^{-1}(\rho)$ is uniformly bounded in both row and column sums in absolute value for ρ in a neighborhood of ρ_0.*

Furthermore, the existence and consistency of the constrained estimator $\tilde{\beta}_{\mathtt{SLE1}}$ depends on the existence and nonsingularity of $\lim_{n\to\infty}\frac{1}{nT}\sum_{t=1}^{T}X^{\circ\prime}_{nt}B_n'B_nX^{\circ}_{nt}$, which follows from Assumption 2 and the positive definiteness of $B_n'B_n$. Denoting $\Xi_{\mathtt{SLE1}}(\theta) = CI^{-1}_{\mathtt{SLE1}}(\theta)\Sigma_{\mathtt{SLE1}}(\theta)I^{-1}_{\mathtt{SLE1}}(\theta)C'$, we have the following theorem.

Theorem 3.1. *Under Assumptions 1-5, if further, (i) $\tilde{\theta}_{\mathtt{SEL1}}$ is $\sqrt{N}$-consistent for θ_0 under $H_0^{\mathtt{TH}}$, and (ii) $I_{\mathtt{SLE1}}(\theta)$ and $\Xi_{\mathtt{SLE1}}(\theta)$ are positive definite for θ in a neighborhood of θ_0 when N_0 is large enough, then we have, under $H_0^{\mathtt{TH}}$, $T^{(r)}_{\mathtt{SLE1}} \xrightarrow{D} \chi^2_{k_p}$, as $n \to \infty$.*

Note that the d.f. associated with the test statistics is $k_p = (T-1)(k+2)$ for testing for temporal homogeneity, and $k_p = (T-2)(k+2)$ for testing for a 'single change'. Similarly, if T increases with n it can be shown that $T_{\mathtt{SLE1}}$ is not an asymptotic pivotal quantity, and that $(T^{(r)}_{\mathtt{SLE1}} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$, as $n/\sqrt{T} \to \infty$.

Estimation of null models. The general LM procedure presented in Sec. 2.1 can be applied to estimate various null (1FE-SLE) models based on $S^{\star}_{\mathtt{SLE1}}(\theta)$ and a properly specified linear contrast matrix C. To estimate the homogeneous model for asymptotic analyses and Monte Carlo simulation, let $\theta = (\beta', \lambda, \rho, \sigma^2)'$. Under $H_0^{\mathtt{TH}}$, the constrained estimate of c_n given (β, λ) becomes $\tilde{c}^{\circ}_n(\beta,\lambda) = A_n(\lambda)\bar{Y}_n - \bar{X}_n\beta$, and the error vector becomes $\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho) = B_n(\rho)[A_n(\lambda)Y^{\circ}_{nt} - X^{\circ}_{nt}\beta]$, where $Y^{\circ}_{nt} = Y_{nt} - \bar{Y}_n$, $X^{\circ}_{nt} = X_{nt} - \bar{X}_n$, and $\bar{Y}_n = \frac{1}{T}\sum_{t=1}^{T}Y_{nt}$ and $\bar{X}_n = \frac{1}{T}\sum_{t=1}^{T}X_{nt}$. The AQS function at $H_0^{\mathtt{TH}}$ takes the form:

$$S^{\circ}_{\mathtt{SLE1}}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t=1}^{T}X^{\circ}_{nt}B_n'(\rho)\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho), \\ \frac{1}{\sigma^2}\sum_{t=1}^{T}(W_nY^{\circ}_{nt})'B_n'(\rho)\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[G_n(\lambda)], \\ \frac{1}{\sigma^2}\sum_{t=1}^{T}\widetilde{V}^{\circ\prime}_{nt}(\beta,\lambda,\rho)H_n(\rho)\widetilde{V}^{\circ}_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[H_n(\rho)], \\ -\frac{n(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}\widetilde{V}^{\circ\prime}_{nt}(\beta,\lambda)\widetilde{V}^{\circ}_{nt}(\beta,\lambda). \end{cases} \tag{3.9}$$

Solving the estimating equations, $S^{\circ}_{\mathtt{SLE1}}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\mathtt{SLE1}}$ of θ, which is shown to be asymptotically equivalent to the transformation-based QML estimator of Lee and Yu (2010), and thus is $\sqrt{n(T-1)}$-consistent. To estimate c_n, γ and κ, refer to the discussions at the end of Section 2.1.

3.2. Panel SLE model with two-way FE

The panel SLE model with two-way fixed effects has the form:

$$Y_{nt} = \lambda_t W_n Y_{nt} + X_{nt}\beta_t + c_n + \alpha_t 1_n + U_{nt}, \quad U_{nt} = \rho_t M_n U_{nt} + V_{nt}, \tag{3.10}$$

which extends Model (2.16) by adding the spatial error dependence term. Applying the same orthonormal transformation as that for Model (2.16), i.e., premultiplying $F'_{n,n-1}$ on both sides of (3.10), and using $J_n W_n = J_n W_n J_n$, $J_n M_n = J_n M_n J_n$ and $J_n = F_{n,n-1}F'_{n,n-1}$, we have the following transformed model:

$$Y^*_{nt} = \lambda_t W^*_n Y^*_{nt} + X^*_{nt}\beta_t + c^*_n + U^*_{nt}, \quad U^*_{nt} = \rho_t M^*_n U^*_{nt} + V^*_{nt}, \tag{3.11}$$

where Y^*_{nt}, X^*_{nt}, c^*_n, W^*_n and V^*_{nt} are defined as in Model (2.17), and $M^*_n = F'_{n,n-1} M_n F_{n,n-1}$. After the transformation, the effective sample size becomes $N_0 = (n-1)(T-1)$ as for the 2FE panel SL model. As Model (3.11) takes an identical form as Model (3.1) and the elements of V^*_{nt} are iid normal if the original errors are normal, the steps leading to the score-type test and the steps leading to consistent estimation of the null models are similar. We first present the results for the general model, and then give the necessary details for the submodel with constant ρ at the end of this section and in Appendix B.5.

Define $A^*_n(\rho_t) = I_{n-1} - \lambda_t W^*_n$ and $B^*_n(\rho_t) = I_{n-1} - \rho_t M^*_n$, $t = 1, \ldots, T$. Similar to the previous section, we eliminate c^*_n through a direct maximization of the loglikelihood function to give the concentrated loglikelihood function of $\boldsymbol{\theta}$:

$$\begin{aligned}\ell^c_{\texttt{SLE2}}(\boldsymbol{\theta}) = &-\tfrac{nT}{2}\ln(2\pi\sigma^2) + \textstyle\sum_{t=1}^T \ln|A^*_n(\lambda_t)| + \sum_{t=1}^T \ln|B^*_n(\rho_t)| \\ the &- \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho})\widetilde{V}^*_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho})\end{aligned} \tag{3.12}$$

where $\widetilde{V}^*_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}) = B^*_n(\rho_t)U^{\circ *}_{nt}(\beta_t,\lambda_t) - B^*_n(\rho_t)\mathbb{D}^{*-1}_n(\boldsymbol{\rho})\sum_{s=1}^T D^*_n(\rho_s)U^{\circ *}_{ns}(\beta_s,\lambda_s)$, $\mathbb{D}^*_n(\boldsymbol{\rho}) = \sum_{t=1}^T D^*_n(\rho_t)$, $D^*_n(\rho_t) = B^{*\prime}_n(\rho_t)B^*_n(\rho_t)$, and $U^{\circ *}_{nt}(\beta_t,\lambda_t) = A^*_n(\lambda_t)Y^*_{nt} - X^*_{nt}\beta_t$. As in the previous subsection, we can obtain the AS or AQS function of $\boldsymbol{\theta}$ for Model (3.10) as

$$S^\star_{\texttt{SLE2}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2}X^{*\prime}_{nt}B^{*\prime}_n(\rho_t)\widetilde{V}^*_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}), \quad t = 1,\ldots,T, \\ \frac{1}{\sigma^2}(W^*_nY^*_{nt})'B^{*\prime}_n(\rho_t)\widetilde{V}^*_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}) - \mathrm{tr}[R^*_{nt}(\boldsymbol{\rho})G^*_n(\lambda_t)], \quad t = 1,\ldots,T, \\ \frac{1}{\sigma^2}\widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho})H^*_n(\rho_t)\widetilde{V}^*_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}) - \mathrm{tr}[S^*_{nt}(\boldsymbol{\rho})H^*_n(\rho_t)], \quad t = 1,\ldots,T, \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho})\widetilde{V}^*_{nt}(\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{\rho}), \end{cases} \tag{3.13}$$

where $R^*_{nt}(\boldsymbol{\rho}) = I_{n-1} - \mathbb{D}^{*-1}_n(\boldsymbol{\rho})D^*_{nt}(\rho_t)$, and $S^*_{nt}(\boldsymbol{\rho}) = I_{n-1} - B^*_{nt}(\rho_t)\mathbb{D}^{*-1}_n(\boldsymbol{\rho})B^{*\prime}_{nt}(\rho_t)$.

Denote the null estimator of $\boldsymbol{\theta}$ by $\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}$. Let $J_{\texttt{SLE2}}(\boldsymbol{\theta}) = -\frac{\partial}{\partial\boldsymbol{\theta}'}S^\star_{\texttt{SLE2}}(\boldsymbol{\theta})$, $I_{\texttt{SLE2}}(\boldsymbol{\theta}_0) = \mathrm{E}[J_{\texttt{SLE2}}(\boldsymbol{\theta}_0)]$ and $\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0) = \mathrm{Var}[S^\star_{\texttt{SLE2}}(\boldsymbol{\theta}_0)]$ with their expressions given in Appendix B.4. The robust AQS test, taking into account of estimation of fixed effects, has the forms:

$$T^{(r)}_{\texttt{SLE2}} = \widetilde{S}^{\star\prime}_{\texttt{SLE2}}\widetilde{I}^{-1}_{\texttt{SLE2}}C'\big(C\widetilde{I}^{-1}_{\texttt{SLE2}}\widetilde{\Sigma}_{\texttt{SLE2}}\widetilde{I}^{-1}_{\texttt{SLE2}}C'\big)^{-1}C\widetilde{I}^{-1}_{\texttt{SLE2}}\widetilde{S}^\star_{\texttt{SLE2}}, \tag{3.14}$$

where $\widetilde{S}^\star_{\texttt{SLE2}} = S^\star_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}})$, $\widetilde{I}_{\texttt{SLE2}} = I_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}})$, $\widetilde{\Sigma}_{\texttt{SLE2}} = \Sigma_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}})$, and the linear contrast matrix C has the same form as that for the 1FE panel SLE model. Similarly, when $I_{\texttt{SLE2}}(\theta_0) \asymp \Sigma_{\texttt{SLE2}}(\theta_0)$, $T^{(r)}_{\texttt{SLE2}}$ reduces to the *naïve* test: $T_{\texttt{SLE2}} = \widetilde{S}^{\star\prime}_{\texttt{SLE2}} J^{-1}_{\texttt{SLE2}}(\tilde{\theta}_{\texttt{SLE2}})\widetilde{S}^{\star}_{\texttt{SLE2}}$.

Let $\Xi_{\texttt{SLE2}}(\theta)$ be defined similarly as $\Xi_{\texttt{SLE1}}(\theta)$ for the 1FE panel SLE model.

Theorem 3.2. *Under Assumptions 1-2, 3′, and 4-5, if (i)* $\tilde{\theta}_{\texttt{SLE2}}$ *is* $\sqrt{N}$*-consistent for* θ_0 *under* $H_0^{\texttt{TH}}$*, and (ii)* $I_{\texttt{SLE2}}(\theta)$ *and* $\Xi_{\texttt{SLE2}}(\theta)$ *are positive definite for* θ *in a neighborhood of* θ_0 *when* N_0 *is large enough, then we have, under* $H_0^{\texttt{TH}}$, $T^{(r)}_{\texttt{SLE2}} \xrightarrow{D} \chi^2_{k_p}$, *as* $n \to \infty$.

The d.f. k_p associated with these tests remain the same as that in Theorem 3.1. Similarly, it can be shown that $T_{\texttt{SLE2}}$ is not an asymptotic pivotal quantity, and that $(T^{(r)}_{\texttt{SLE2}} - k_p)/\sqrt{2k_p} \xrightarrow{D} N(0,1)$, as $n/\sqrt{T} \to \infty$.

Estimation of the null model. Again, the general LM procedure can be adapted to estimated a null (panel SLE-2FE) model based on the AQS function $S^{\star}_{\texttt{SLE2}}(\theta)$ and a properly specified linear contrast matrix C. To estimate the null model specified by $H_0^{\texttt{TH}}$, the constrained estimate of c_n given (β, λ) becomes $\tilde{c}^{\circ *}_n(\beta,\lambda) = A^*_n(\lambda)\bar{Y}^*_n - \bar{X}^*_n\beta$ where $\bar{Y}^*_n$ and $\bar{X}^*_n$ are the averages of $\{Y^*_{nt}\}$ and $\{X^*_{nt}\}$, respectively. Along the same line leading to (3.13), one can easily show that AQS function of Model (3.11) at $H_0^{\texttt{TH}}$ takes the form:

$$S^{\circ *}_{\texttt{SLE2}}(\theta) = \begin{cases} \frac{1}{\sigma^2}\sum_{t=1}^T X^{\circ *\prime}_{nt} B^{*\prime}_n(\rho)\widetilde{V}^{\circ *}_{nt}(\beta,\lambda,\rho), \\ \frac{1}{\sigma^2}\sum_{t=1}^T (W^*_n Y^{\circ *}_{nt})' B^{*\prime}_n(\rho)\widetilde{V}^{\circ *}_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[G^*_n(\lambda)], \\ \frac{1}{\sigma^2}\sum_{t=1}^T \widetilde{V}^{\circ *\prime}_{nt}(\beta,\lambda,\rho) H^*_n(\rho)\widetilde{V}^{\circ *}_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[H^*_n(\lambda)], \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}^{\circ *\prime}_{nt}(\beta,\lambda,\rho)\widetilde{V}^{\circ *}_{nt}(\beta,\lambda,\rho), \end{cases} \tag{3.15}$$

$\widetilde{V}^{\circ *}_{nt}(\beta,\lambda,\rho) = B^*_n(\rho)[A^*_n(\lambda)Y^*_{nt} - X^*_{nt}\beta - \tilde{c}^{\circ *}_n(\beta,\lambda)] = B^*_n(\rho)[A^*_n(\lambda)Y^{\circ *}_{nt} - X^{\circ *}_{nt}\beta]$, where $Y^{\circ *}_{nt} = Y^*_{nt} - \bar{Y}^*_n$ and $X^{\circ *}_{nt} = X^*_{nt} - \bar{X}^*_n$. Solving the estimating equations, $S^{\circ *}_{\texttt{SLE2}}(\theta) = 0$, gives the null estimator $\tilde{\theta}_{\texttt{SLE2}}$ of $\theta = (\beta', \lambda, \rho, \sigma^2)'$, which is shown to be asymptotically equivalent to the transformation-based estimator of Lee and Yu (2010). Thus, $\tilde{\theta}_{\texttt{SLE2}}$ is $\sqrt{(n-1)(T-1)}$-consistent for θ. Estimation of c_n, γ and κ proceeds similarly.

A special submodel is the **panel SLE model homogeneous ρ-coefficients**. With two-way FE, the AQS function of $\theta = (\beta', \lambda', \rho, \sigma^2)'$ is obtained by simplifying $S^{\star}_{\texttt{SLE2}}(\theta)$:

$$S^{\star 0}_{\texttt{SLE2}}(\theta) = \begin{cases} \frac{1}{\sigma^2} X^{*\prime}_{nt} B^{*\prime}_n(\rho)\widetilde{V}^*_{nt}(\beta,\lambda,\rho), \quad t = 1,\ldots,T, \\ \frac{1}{\sigma^2}(W^*_n Y^*_{nt})' B^{*\prime}_n(\rho)\widetilde{V}^*_{nt}(\beta,\lambda,\rho) - \frac{T-1}{T}\mathrm{tr}[G^*_n(\lambda_t)], \quad t = 1,\ldots,T, \\ \frac{1}{\sigma^2}\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\beta,\lambda,\rho) H^*_n(\rho)\widetilde{V}^*_{nt}(\beta,\lambda,\rho) - (T-1)\mathrm{tr}[H^*_n(\rho)], \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^T \widetilde{V}^{*\prime}_{nt}(\beta,\lambda,\rho)\widetilde{V}^*_{nt}(\beta,\lambda,\rho). \end{cases} \tag{3.16}$$

This provides a channel for carrying out various conditional tests, given the temporal homogeneity in ρ. Necessary details for constructing these tests are provided in Appendix B.5., and these can easily be simplified to give AQS tests for the 1FE model.

Finally, a very special submodel, the SPD model with spatial errors (SE), deserves some discussions as it parallels with the panel SL models popular in practical applications. The

AQS function of $\boldsymbol{\theta} = (\boldsymbol{\beta}', \boldsymbol{\rho}', \sigma^2)'$ of the **panel SE model with 2FE** takes the form:

$$S^{\star}_{\mathtt{SE2}}(\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma^2} X^{*\prime}_{nt} B^{*\prime}_n(\rho_t) \widetilde{V}^{*}_{nt}(\boldsymbol{\beta}, \boldsymbol{\rho}), \ \ t = 1, \ldots, T, \\ \frac{1}{\sigma^2} \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\rho}) H^{*}_n(\rho_t) \widetilde{V}^{*}_{nt}(\boldsymbol{\beta}, \boldsymbol{\rho}) - \mathrm{tr}[S^{*}_{nt}(\boldsymbol{\rho}) H^{*}_n(\rho_t)], \ \ t = 1, \ldots, T, \\ -\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{t=1}^{T} \widetilde{V}^{*\prime}_{nt}(\boldsymbol{\beta}, \boldsymbol{\rho}) \widetilde{V}^{*}_{nt}(\boldsymbol{\beta}, \boldsymbol{\rho}), \end{cases} \tag{3.17}$$

where $\widetilde{V}^{*}_{nt}(\boldsymbol{\beta}, \boldsymbol{\rho}) = B^{*}_n(\rho_t) U^{\circ *}_{nt}(\beta_t) - B^{*}_n(\rho_t) \mathbb{D}^{*-1}_n(\boldsymbol{\rho}) \sum_{s=1}^{T} D^{*}_n(\rho_s) U^{\circ *}_{ns}(\beta_s)$, and $U^{\circ *}_{nt}(\beta_t) = Y^{*}_{nt} - X^{*}_{nt}\beta_t$. This can be used to perform tests concerning $\{\beta_t\}$ and $\{\rho_t\}$ in the panel SE model with 2FE. The necessary detail for constructing these tests are given in Appendix B.6, which can easily be simplified to give the AQS tests for panel SE model with 1FE.

4. Monte Carlo Study

Extensive Monte Carlo experiments are conducted to investigate the finite sample performance of the proposed tests, based on the following four data generation processes (DGPs), the SDP models with, respectively, 1FE-SL, 2FE-SL, 1FE-SLE and 2FE-SLE:

$$\begin{aligned}
&\mathtt{DGP1}: \ Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + V_{nt}, \ t = 1, 2, \ldots, T, \\
&\mathtt{DGP2}: \ Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + \alpha_{t0} 1_n + V_{nt}, \ t = 1, 2, \ldots, T. \\
&\mathtt{DGP3}: \ Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + U_{nt}, \\
&\qquad\qquad U_{nt} = \rho_{t0} M_n U_{nt} + V_{nt}, \ t = 1, 2, \ldots, T. \\
&\mathtt{DGP4}: \ Y_{nt} = \lambda_{t0} W_n Y_{nt} + X_{1nt}\beta_{1t0} + X_{2nt}\beta_{2t0} + c_{n0} + \alpha_{t0} 1_n + U_{nt}, \\
&\qquad\qquad U_{nt} = \rho_{t0} M_n U_{nt} + V_{nt}, \ t = 1, 2, \ldots, T.
\end{aligned}$$

We concentrate on the tests of temporal homogeneity. In all the Monte Carlo experiments for simulating the empirical sizes of the tests, $\beta_t = (\beta_{1t}, \beta_{2t})' = (1, 1)'$, $\lambda_t \in \{0.5, 0, -0.5\}$, and $\rho_t \in \{0.5, 0, -0.5\}$ for all $t = 1, \ldots, T$, $\sigma_0^2 = 1$, $n \in \{50, 100, 200, 500\}$, and $T = \{3, 6\}$. Each set of Monte Carlo results is based on 10,000 Monte Carlo samples for the two SL models, and 5,000 for the two SLE models.

The **weight matrices** are generated based on three different methods: (*i*) `Rook Contiguity`, (*ii*) `Queen Contiguity`, and (*iii*) `Group Interaction`, with details given in Yang (2015a). In spatial layouts (*i*)-(*ii*), the degree of spatial interactions (number of neighbors each unit has) is fixed, while in (*iii*) it may grow with the sample size. This is attained by allowing the number of groups, G, in the sample of spatial units to be directly related to the sample size n, e.g., $G = n^{0.5}$. Hence, the average group size, $m = n/G$, gives a measure of the degree of spatial dependence among the n spatial units. The actual sizes of the groups are generated from a discrete uniform distribution from $.5m$ to $1.5m$.

The two **exogenous regressors** are generated according to `REG1:` $X_{jnt} \overset{iid}{\sim} N(0, I_n)$ for $j = 1, 2$ and $t = 1, \ldots, T$; and `REG2:` the *i*th value of the *j*th regressor in the *g*th group is such that $X_{jt,ig} \overset{iid}{\sim} (2z_g + z_{ig})/\sqrt{10}$, where $(z_g, z_{ig}) \overset{iid}{\sim} N(0, 1)$ when group interaction scheme is followed; $\{X_{jt,ig}\}$ are thus independent across j and t, but not across i.

The **errors**, $v_{it} = \sigma_0 e_{it}$, are generated according to `err1:` $\{e_{it}\}$ are iid standard normal; `err2:` $\{e_{it}\}$ are iid normal mixture with 10% of values from $N(0,4)$ and the remaining from $N(0,1)$, standardized to have mean 0 and variance 1; and `err3:` $\{e_{it}\}$ iid log-normal (i.e., $\log e_{it} \overset{iid}{\sim} N(0,1)$) standardized to have mean 0 and variance 1.

Partial Monte Carlo results are reported in Tables 1 & 2 for the panel SL models, and Tables 3 & 4 for the panel SLE models. The results in Tables 1 & 2 show the following.

(*i*) The proposed robust test performs very well in general with empirical coverage probabilities all very close to their nominal levels, except that in cases of heavy spatial dependence (`Group Interaction`) and not-so-large n, it can be slightly undersized. As sample size increases, the empirical sizes quickly converge to their nominal levels.

(*ii*) In contrast, the naïve test can perform quite badly, with empirical sizes being as high as 35% for tests of 10% nominal level, when the erorrs are fairly non-normal (e.g., log-normal). It is interesting to note that the size distortions for the naïve tests also drop as sample size increase.

(*iii*) A larger T seems lead to a worsened performance for the naïve tests under `Queen Contiguity` but not under `Group Interaction`.

(*iv*) The finite sample performance of the tests for 1FE panel SL model do not seem to differ much from those for 2FE panel SL model.

From the results for the panel SLE model, reported (in Tables 3 & 4) and unreported (available from the authors upon request), similar patterns are observed for the finite sample performance of the proposed tests. In summary, the proposed robust tests are reliable and easy to apply, and hence are recommended for the applied researchers. The Monte Carlo experiments for the power of the tests, and the size and power of the other tests, e.g., tests for change points, are also carried out, and the results (available from the authors upon request) show similar patterns.

5. Empirical Applications

The specification tests of temporal homogeneity in spatial panel data models proposed in this paper are demonstrated in empirical settings using two well known data sets: *Public Capital Productivity* (Munnell, 1990) and *Cigarette Demand* (Baltagi and Levin, 1992). We endeavor to provide a detailed guidance to aid applied researchers in their empirical studies. First, a general discussion is given on the issues of spatial interaction and spatiotemporal heterogeneity commonly existed in economic studies.

5.1. Spatial interaction and spatiotemporal heterogeneity.

A wide range of empirical studies, such as urban economics, international trade, public finance, industrial organization, real estate analyses and regional economics, deal with spatial interaction. Values observed at one location depend on the values of neighboring observations at nearby locations due to budget spillovers, difference in tax rates, copycatting, network effects, et. However, this dependence may not stay the same over time. There are two major reasons for specifying, estimating, and testing for the time-varying spatial effects in the regression models. One is the growing interest in using theoretical economics that include time-varying spatial effects to analyze economic phenomenon such as externalities, group patterns and some other economic processes, for example, housing decisions, unemployment, price decisions, crime rates, trade flows, etc., which exhibit time heterogeneity patterns. The effects of relevant variables, including interactions among agents, on economic activities are changing over time. This may be due to the change of government policy, an unexpected accident, the change of the benefit from the interactions. The second driver is the need from geographic research and environmental study, where researchers usually face a large set of geocoded data when analyzing the relationships between different variables. Under this situation, due to the spatial interaction and the fact that everything in nature is changing over time, time-varying spatial autoregression model is more outstanding than many other econometric models. Adding the time-varying spatial effects in the regression model may be necessary.

One empirical problem we discuss in this paper is the U.S. *cigarette demand* in state level from 1963-1992 (Baltagi and Levin, 1992). The tax policy on cigarette differs by states, and this leads to substantial cross-state sales. Due to the government interventions (in 1965, 1967, 1971) and the reports about the health hazards of smoking (in 1983), the effects of the spatial lag, spatial error and the variables (price per pack of cigarettes, population, per capita disposable income, and etc) on the US cigarette demand might be subject to the temporal heterogeneity. The other empirical problem we discuss is the U.S. *public capital productivity* in state level from 1970-1986 (Munnell, 1990). The private production of each state may subject to spillover effects of infrastructure improvement from other states. Temporal homogeneity may be in question due to the change in policies and the change of economic environment such as 1973 oil crisis and the 1979 energy crisis. These two data sets have been extensively used in Baltagi (2013) for the illustrations of various standard panel data techniques.

Many other empirical studies have documented the existence of spatial interaction or spatial spillover effects, and these naturally raise the question whether these spillover effects as well as the economic variables effects remain constant over time due to policy change. Care (1991) studied spatial patterns in household demand. Case et al. (1993) showed that the U.S. states' budget expenditure depends on the spending of similar states. Policies have changed over the years, and one might be interested in testing if the *spatial patterns* and *budget spillovers* remain the same over time. Acemoglu et al. (2012) studied the inter-sectoral input-output linkages in the U.S. Baltagi et al. (2016) studied intra-sectoral spillovers in total factor productivity (TFP) across Chinese producers in the chemical industry using a panel data on 12,552 firms over 2004-2006, by modeling spatial spillovers in TFP through contextual effects of observable variables and the spatial dependence of the disturbances. Test of stability/homogeneity of the covariate effects as well as spatial effects may be interesting, perhaps based on extended data.

Therefore, it is highly desirable to have a general procedure to identify the possible existence of temporal heterogeneity in spatial panel data models to aid the applied researchers in their empirical studies. The AQS test we propose may serve the purpose.

We provide a detailed instruction, through two empirical applications, of how to construct AQS-tests for testing certain null hypothesis in an SPD model allowing spatiotemporal heterogeneity in the intercept (fixed effects), i.e., the model specified by (2.1), (2.16), (3.1), or (3.10), based on the AQS function defined by (2.7), (2.22), (3.7), or (3.13). Given a null hypothesis, the linear contrast matrix C is defined, the null model is estimated by solving the LM-equations (defined as in (2.14) for the panel SL model with 1FE), and the corresponding test statistic defined by (2.13), (2.23), (3.8), or (3.14) is computed.

5.2. Public capital productivity

Munnell (1990) investigated the productivity of public capital in private production based on data for 48 U.S. states observed over 17 years (1970-1986). Baltagi and Pinnoi (1995) considered a Cobb-Douglas production function of the form:

$$\ln(\texttt{gsp}) = \beta_1 \ln(\texttt{pcap}) + \beta_2 \ln(\texttt{pc}) + \beta_3 \ln(\texttt{emp}) + \beta_4 \texttt{unemp} + \epsilon,$$

with state-specific fixed effects, where '`gsp`' is the gross social product of a given state, '`pcap`', '`pc`' and '`emp`' are the inputs of private capital, public capital, and labor respectively. In order to capture business cycle effects, an additional variable '`unemp`' is also added which indicates the state unemployment rate. The model now is extended by adding the time-specific fixed effects and the spatial effects. The latter is for capturing the possible spill over effects of public capital. The spatial weight matrix (W_n) is specified using a contiguity form where (i, j)th element is indicated as 1 if state i and j share a common border, otherwise 0. The final W_n is row normalized. The data file `Product.csv` and the spatial weights matrix `weight_Product.csv`, and the associated matlab files can be found in the website: `http://www.mysmu.edu/faculty/zlyang/`.

It is well known that 1970-86 is the period that U.S. had experienced several social and economic shocks such as the baby booms in the early 1970s, the oil crises in 1973 and 1979, and economic recession between 1980-82. It is therefore questionable that the above production relationship would remain stable over time. We demonstrate how our AQS test can answer this question, and how it may it may help detecting change points.

To test $H_0^{\texttt{TH}}$, the temporal homogeneity, assign $k = 4$. Based on full data, $T = 17$, $k_p = (k+1)(T-1) = 80$ and $C = [\texttt{blkdiag}\{C_T^k, C_T^1\}, 0_{k_p,1}]$ for the SL models; and $(k+2)(T-1) = 96$ and $C = [\texttt{blkdiag}\{C_T^k, C_T^1, C_T^1\}, 0_{k_p,1}]$ for the SLE models, where C_T^m is defined in (2.10) for $m = 1, k$. To test $H_0^{\texttt{TH}}$ based on first four periods, $T = 4$,

$k_p = (k+1)(T-1) = 15$ for the SL models, and $(k+2)(T-1) = 18$ for the SLE models. The C matrices remain in the same forms. Note that k_p is also the degrees of freedom (df) of the chi-squared test statistics, based on which the asymptotic critical values and p-values are found.

Table below summarize the values of the test statistics and their p-values, for the *naïve* tests and the nonnormality robust AQS tests for temporal homogeneity based on both the full dataset and a subset of data, fitted using the four models: 1FE-SL, 2FE-SL, 1FE-SLE and 2FE-SLE. From the table we see that all tests based on full data (t_1–t_{17}) give a clean rejection of the temporal homogeneity hypothesis $H_0^{\mathtt{TH}}$.

Tests for Temporal Homogeneity: Public Capital Productivity

Data	$T_{\mathtt{SL1}}$	$T_{\mathtt{SL1}}^{(r)}$	$T_{\mathtt{SL2}}$	$T_{\mathtt{SL2}}^{(r)}$	$T_{\mathtt{SLE1}}^{(r)}$	$T_{\mathtt{SLE1}}^{(r)}$	$T_{\mathtt{SLE2}}$	$T_{\mathtt{SLE2}}^{(r)}$
t_1–t_{17}	1621	321	3189	328	1971	289	1556	326
	.000	.000	.000	.000	.000	.000	.000	.000
t_1–t_5	215.60	68.14	22.34	18.22	47.18	38.43	33.08	19.57
	.000	.000	.322	.573	.003	.031	.102	.721
t_1–t_4	10.24	9.37	9.59	8.69	11.78	10.61	7.07	11.43
	.804	.857	.845	.893	.858	.910	.990	.875

Note: p-values are in every second row.

As discussed in Section 2.1, a rejection of $H_0^{\mathtt{TH}}$ may be due to the existence of change points instead of full heterogeneity. Thus, we break down the panel into sub-periods to test whether $H_0^{\mathtt{TH}}$ holds for a smaller panel. Indeed, based on the first four periods (t_1–t_4), all tests do not reject $H_0^{\mathtt{TH}}$, indicating that the panel consisting of the first four periods is fairly homogeneous. Furthermore, based on t_1–t_5, the tests $T_{\mathtt{SL1}}^{(r)}$ and $T_{\mathtt{SLE1}}^{(r)}$ reject $H_0^{\mathtt{TH}}$ but $T_{\mathtt{SL2}}^{(r)}$ and $T_{\mathtt{SLE2}}^{(r)}$ do not, suggesting that if temporal heterogeneity in intercepts is not controlled for, the first change point is t_5 or 1974, the year after the first oil crisis. However, $T_{\mathtt{SL2}}^{(r)}$ and $T_{\mathtt{SLE2}}^{(r)}$ do not reject $H_0^{\mathtt{TH}}$ **up to** first six periods, meaning that after controlling both spatial and temporal heterogeneity in intercepts, the panel is homogeneous in first six periods but changes in structure from 7th period onwards.[6] Applying the pair of test $T_{\mathtt{SL2}}^{(r)}$ and $T_{\mathtt{SLE2}}^{(r)}$ to test $H_0^{\mathtt{TH}}$ based on other sub-periods from 1976 onwards, all tests reject $H_0^{\mathtt{TH}}$ at 10% level, except the tests based on the following tow sub-periods: t_7–t_8 and t_{12}–t_{13}. These suggest that there exist multiple change points in this panel, and hence the standard applications of homogeneous penal methods are not valid.[7]

Based on the above results, we recommend the pairs of tests $T_{\mathtt{SL2}}^{(r)}$ and $T_{\mathtt{SLE2}}^{(r)}$ for practical applications as they control both spatial and temporal heterogeneity in intercepts (two-way fixed effects). We can further carry out the tests for detecting change points. However,the tests for temporal homogeneity based on sub-panels have revealed quite a clear picture, we therefore do not pursue `CP` tests in this application.

[6]The p-values for these two tests are .513 and .633 based on t_1–t_6, and .000 and .000 based on t_1–t_7, suggesting that the structure has changed since year 7 (or 1977) onwards.

[7]The relatively much bigger values of the usual or naïve tests show that they are rather unreliable, in line with the Monte Carlo results.

5.3. Cigarette demand.

Second application of the proposed tests uses another well known data set, *the Cigarettes Demand for the United States* (Baltagi and Levin, 1992). It contains a panel of 46 states over 30 time periods (1963-1992). The data file `cigarette.csv`, spatial weight matrix `weight_cigarette.csv`, and the associated matlab codes can be found in the website: `http://www.mysmu.edu/faculty/zlyang/`. Our analysis is based on the response variable Y = Cigarette sales in packs per capita; and the covariates X_1 = Price per pack of cigarettes; X_2 = Population above the age of 16; X_3 = Per capita disposable income; and X_4 = Minimum price in adjoining states per pack of cigarettes. Earlier studies include Hamilton (1972), McGuiness and Cowling (1975), Baltagi and Levin (1986, 1992), Baltagi et al. (2000), and Yang et al. (2006), all under homogeneity assumption and in log-log form except in Yang et al. (2006) who estimated the Box-Cox functional form. The spatial weight matrix is specified using a contiguity form where (i, j)th element is 1 if state i and j share a common border, otherwise 0, and then row normalized.

Tests for temporal homogeneity/heterogeneity is of particular interest in cigarette demand, due to government's policy interventions (in 1965, 1967, 1971) in attempting reducing the consumptions of cigarettes, and the reports from medial journals as well as Surgeon General warning (in 1983) about the health hazards of smoking (see Baltagi and Levin, 1986). The table below summarize the values of the test statistics and their p-values, for tests of homogeneity based on the full panel or sub-panels and using the log-log form.

Tests for Temporal Homogeneity: Cigarette Demand

	$T^{(r)}_{\texttt{SL1}}$	$T^{(r)}_{\texttt{SL2}}$	$T^{(r)}_{\texttt{SLE1}}$	$T^{(r)}_{\texttt{SLE2}}$		$T^{(r)}_{\texttt{SL1}}$	$T^{(r)}_{\texttt{SL2}}$	$T^{(r)}_{\texttt{SLE1}}$	$T^{(r)}_{\texttt{SLE2}}$
t_1–t_{30}	443	517	507	587	t_1–t_{10}	122	118	116	126
	.000	.000	.000	.000		.000	.000	.000	.000
t_{11}–t_{20}	99	90	104	112	t_{21}–t_{30}	135	114	121	106
	.000	.000	.000	.000		.000	.000	.000	.000
t_1–t_3	13.13	9.38	9.68	8.75	t_4–t_5	6.72	6.23	7.86	8.10
	.217	.497	.644	.724		.242	.285	.248	.230
t_1–t_5	43.0	30.7	45.0	40.8	t_5–t_8	21.7	19.2	21.2	17.4
	.002	.060	.006	.018		.116	.204	.271	.495

Note: p-values are in every second row.

From the results we see that all tests based on the full data, and the first, second and last ten years data clearly reject $H_0^{\texttt{TH}}$, the hypothesis of temporal homogeneity in regression and spatial coefficients. Therefore, the `Cigarette Demand` panel is temporally heterogeneous. Further breaking down the panel and repeatedly applying the set of robust tests, we see that only the sub-panels 1963-65, 1966-67, and 1967-70 are fairly stable, suggesting that panel structures have changed after 1965, 1967, and 1970, in line with the policy interventions in 1965, 1967 and 1971. From the results, we also see that controlling the temporal heterogeneity in intercepts seems increase the stability of the overall model structure as seen from the larger p-values associated with $T^{(r)}_{\texttt{SL2}}$ and $T^{(r)}_{\texttt{SLE2}}$.

Furthermore, applying $T^{(r)}_{\texttt{SL2}}$ to test $H^{\texttt{CP}}_0$ based on data from t_1–t_5 with $b_0 = \ell_0 = 3$ gives a p-value of 0.632 compared with 0.06 from the test of $H^{\texttt{TH}}_0$given in the table above. This confirms that 1965 is a point after which the structure has changed. Similarly, based on data from t_4–t_9, the p-value is 0.231 for testing $H^{\texttt{CP}}_0$ using $T^{(r)}_{\texttt{SL2}}$ with $b_0 = \ell_0 = 3$, suggesting that 1978 is another change point. The `CP` tests with multiple change points can be carried out as well based on the general LM procedure we propose.

However, the matlab function `fsolve` that our LM-procedure depends upon may not alway perform well. This seems to be an interesting computation problem, and is beyond the scope of this paper. In any situation, one can always repeatedly apply our robust tests for testing temporal homogeneity as they are based up the optimization functions such as `fminbnd` and `fmincon`, which are numerically much more stable stable than `fsolve`.

In summary, our tests show that there exit multiple change points in the the `Cigarette Demand` panel, and hence in real applications, one should base their analyses either on a shorter panel so that a homogeneous SPD model can be used, or a relatively longer panel and the corresponding SPD model with 'specified' change points.

6. Conclusion and Discussion

We introduce *adjusted quasi score tests* for temporal homogeneity/heterogeneity in regression and spatial coefficients in spatial panel data models allowing the existence of spatial and temporal heterogeneity in the intercepts of the model. The proposed tests are robust against nonnormality, they are simple and reliable as shown by the Monte Carlo results, and can be repeatedly applied to identity a 'parsimonious model' instead of the model with full temporal heterogeneity. That is, once the null hypothesis of homogeneity is rejected (as in the two empirical applications), one may proceed with further tests of hypotheses with known change points suggested by the data (as in Cigarette Demand application). Thus, the proposed tests provide useful tools for the applied researchers.

The tests can be extended by (i) adding higher-order spatial terms and spatial Durbin terms in the model, (ii) treating individual- and time-specific effects as random effects, or correlated random effects, (iii) allowing spatial-temporal heterogeneity in error variance (i.e., heteroskedasticity), (iv) allowing interactive fixed effects, and (v) by allowing dynamic effects in the model. These extensions are interesting but clearly beyond the scope of the current paper, which will be in our future research agenda.

Appendix A: Some Basic Lemmas

Lemma A.1 (Kelejian and Prucha, 1999; Lee, 2002): *Let* $\{A_n\}$ *and* $\{B_n\}$ *be two sequences of* $n \times n$ *matrices that are uniformly bounded in both row and column sums. Let* C_n *be a sequence of conformable matrices whose elements are uniformly bounded. Then*

(i) the sequence $\{A_n B_n\}$ *are uniformly bounded in both row and column sums,*

(ii) the elements of A_n *are uniformly bounded and* $tr(A_n) = O(n)$*, and*

(iii) the elements of $A_n C_n$ *and* $C_n A_n$ *are uniformly bounded.*

Lemma A.2 (Yang, 2015b, Lemma A.1, extended). *For* $t = 1, 2$*, let* A_{nt} *be* $n \times n$ *matrices and* c_{nt} *be* $n \times 1$ *vectors. Let* ε_n *be an* $n \times 1$ *random vector of iid elements with mean zero, variance* σ^2*, and finite 3rd and 4th cumulants* μ_3 *and* μ_4*. Let* a_{nt} *be the vector of diagonal elements of* A_{nt}*. Define* $Q_{nt} = c'_{nt}\varepsilon_n + \varepsilon'_n A_{nt}\varepsilon_n$*,* $t = 1, 2$*. Then, for* $t, s = 1, 2$*,*

$$\begin{aligned}&\mathrm{Cov}(Q_{nt}, Q_{ns}) \equiv f(A_{nt}, c_{nt}; A_{ns}, c_{ns})\\ =\ &\sigma^4 \mathrm{tr}[(A'_{nt} + A_{nt})A_{ns})] + \mu_3 a'_{nt}c_{ns} + \mu_3 c'_{nt}a_{ns} + \mu_4 a'_{nt}a_{ns} + \sigma^2 c'_{nt}c_{ns}.\end{aligned}$$

Lemma A.3 (CLT for Linear-Quadratic Forms, Kelejian and Prucha, 2001). *Let* A_n, a_n, c_n *and* ε_n *be as in Lemma A.2. Assume (i)* A_n *is bounded uniformly in row and column sums, (ii)* $n^{-1}\sum_{i=1}^n |c_{n,i}^{2+\eta_1}| < \infty, \eta_1 > 0$*, and (iii)* $E|\varepsilon_{n,i}^{4+\eta_2}| < \infty$*,* $\eta_2 > 0$*. Then,*

$$\frac{\varepsilon'_n A_n \varepsilon_n + c'_n\varepsilon_n - \sigma^2 \mathrm{tr}(A_n)}{\{\sigma^4 \mathrm{tr}(A'_n A_n + A_n^2) + \mu_4 a'_n a_n + \sigma^2 c'_n c_n + 2\mu_3 a'_n c_n\}^{\frac{1}{2}}} \xrightarrow{D} N(0, 1).$$

Appendix B: Hessian, Expected Hessian and VC Matrices

Notation. For $t, s = 1, \ldots, T$, `blkdiag`$\{A_t\}$ forms a block-diagonal matrix by placing A_t diagonally, $\{A_t\}$ forms a matrix by stacking A_t horizontally, and $\{B_{ts}\}$ forms a matrix by the component matrices B_{ts}. The expected negative Hessian $I_\varpi(\boldsymbol{\theta}_0)$ and the VC matrix $\Sigma_\varpi(\boldsymbol{\theta}_0)$ of the AQS function, ϖ=`SL1`, `SL2`, `SLE1`, `SLE2`, are both partitioned according to the slope parameters $\boldsymbol{\beta}$, the spatial lag parameters $\boldsymbol{\lambda}$, spatial error parameters $\boldsymbol{\rho}$ (if existing in the model), and the error variance σ^2, with the sub-matrices denoted by, e.g., $I_{\boldsymbol{\beta\beta}}$, $I_{\boldsymbol{\beta\lambda}}$, $\Sigma_{\boldsymbol{\beta\beta}}$, $\Sigma_{\boldsymbol{\beta\lambda}}$. Furthermore, `diag`$(\cdot)$ forms a diagonal matrix and `diagv`$(\cdot)$ a column vector, based on the diagonal elements of a square matrix.

Parametric quantities, e.g., $A_n(\lambda_{t0})$ and $B_n(\rho_{t0})$, evaluated at the true parameters are denoted as A_{nt} and B_{nt}. For a matrix A_n, denote $A_n^s = A_n + A'_n$. The bold $\mathbf{0}$ represents generically a vector or a matrix of zeros, to distinguish from the scalar 0.

Let $\mathbb{V}_N = (V'_{n1}, \ldots, V'_{nT})'$ be the vector of original errors with elements $\{v_{it}\}$ being iid of mean 0, variance σ^2, skewness γ and excess kurtosis κ. We present here results sufficient for the implementation of the tests introduced in the paper. More details can be found in a `Supplementary Appendix` available at: `http://www.mysmu.edu/faculty/zlyang/`.

B.1. Panel SL model with one-way FE. The negative Hessian matrix $J_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ is given in the `Supplementary Appendix`. Its expectation $I_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ has the components:

$$I_{\beta\beta} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}X'_{nt}X_{nt}\} - \{\tfrac{1}{T\sigma_0^2}X'_{nt}X_{ns}\},\ I_{\lambda\beta} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}\eta'_{nt}X_{nt}\} - \{\tfrac{1}{T\sigma_0^2}\eta'_{nt}X_{ns}\},$$
$$I_{\lambda\lambda} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}\eta'_{nt}\eta_{nt} + \tfrac{T-1}{T}\text{tr}(G^s_{nt}G_{nt})\} - \{\tfrac{1}{T\sigma_0^2}\eta'_{nt}\eta_{ns}\},\quad I_{\sigma^2\lambda} = \{\tfrac{T-1}{T\sigma_0^2}\text{tr}(G_{nt})\},$$
$$I_{\sigma^2\beta} = \mathbf{0},\ I_{\sigma^2\sigma^2} = \tfrac{n(T-1)}{2\sigma_0^4},\ \text{ where } \eta_{nt} = G_{nt}(X_{nt}\beta_{t0} + c_n) \text{ and } G^s_{nt} = G_{nt} + G'_{nt}.$$

The VC matrix $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ is obtained by applying Lemma A.2 with ε replaced by $\mathbb{V}_N$, c_{nt} by Π_{1t} and Π_{2t}, and A_{nt} by Φ_t and Ψ:

$$\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0) = \begin{pmatrix} \{f(\mathbf{0},\Pi_{1t};\mathbf{0},\Pi_{1s})\}, & \{f(\mathbf{0},\Pi_{1t};\Phi_s,\Pi_{2s})\}, & \{f(\mathbf{0},\Pi_{1t};\Psi,\mathbf{0})\} \\ \sim, & \{f(\Phi_t,\Pi_{2t};\Phi_s,\Pi_{2s})\}, & \{f(\Phi_t,\Pi_{2t};\Psi,\mathbf{0})\} \\ \sim, & \sim, & f(\Psi,\mathbf{0};\Psi,\mathbf{0}) \end{pmatrix},$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}X_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}\eta_{nt}$, $\Phi_t = \frac{1}{\sigma_0^2}Z_{Nt}G'_{nt}Z^{\circ\prime}_{Nt}$, and $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^T Z^\circ_{Nt}Z^{\circ\prime}_{Nt}$; $Z^\circ_{Nt} = Z_{Nt} - \bar{Z}_N$, $Z_{Nt} = z_t \otimes I_n$, $\bar{Z}_N = \frac{1}{T}(l_T \otimes I_n)$, and z_t be a $T \times 1$ vector of element 1 in the tth position and 0 elsewhere.

B.2. Panel SL model with two-way FE. The negative Hessian matrix $J_{\texttt{SL2}}(\boldsymbol{\theta}_0)$ is given in the `Supplementary Appendix`. Its expectation $I_{\texttt{SL2}}(\boldsymbol{\theta}_0)$ has the components:

$$I_{\beta\beta} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}X^{*\prime}_{nt}X^*_{nt}\} - \{\tfrac{1}{T\sigma_0^2}X^{*\prime}_{nt}X^*_{ns}\},\ I_{\lambda\beta} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}\eta^{*\prime}_{nt}X^*_{nt}\} - \{\tfrac{1}{T\sigma_0^2}\eta^{*\prime}_{nt}X^*_{ns}\},$$
$$I_{\lambda\lambda} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}\eta^{*\prime}_{nt}\eta^*_{nt} + \tfrac{T-1}{T}\text{tr}(G^{*s}_{nt}G^*_{nt})\} - \{\tfrac{1}{T\sigma_0^2}\eta^{*\prime}_{nt}\eta^*_{ns}\},\ I_{\sigma^2\lambda} = \{\tfrac{T-1}{T\sigma_0^2}\text{tr}(G^*_{nt})\},$$

$$I_{\sigma^2\beta} = \mathbf{0},\ I_{\sigma^2\sigma^2} = \tfrac{(n-1)(T-1)}{2\sigma_0^4},\quad \text{where } \eta^*_{nt} = G^*_{nt}(X^*_{nt}\beta_{t0} + c^*_n) \text{ and } G^{*s}_{nt} = G^*_{nt} + G^{*\prime}_{nt}.$$

$\Sigma_{\texttt{SL2}}(\boldsymbol{\theta}_0)$ has an identical form as $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ with the relevant quantities replaced by $\Pi_{1t} = \frac{1}{\sigma_0^2}Z^{\circ*}_{Nt}X^*_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z^\circ_{Nt}F_{n,n-1}\eta^*_{nt}$, $\Phi_t = \frac{1}{\sigma_0^2}Z^*_{Nt}G^{*\prime}_{nt}Z^{\circ*\prime}_{Nt}$, and $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^T Z^{\circ*}_{Nt}Z^{\circ*\prime}_{Nt}$, where $Z^*_{Nt} = Z_{Nt}F_{n,n-1}$ and $Z^{\circ*}_{Nt} = Z^\circ_{Nt}F_{n,n-1}$.

B.3. Panel SLE model with one-way FE. The negative Hessian matrix $J_{\texttt{SEL1}}(\boldsymbol{\theta}_0)$ is in the `Supplementary Appendix`. Its expectation $I_{\texttt{SEL1}}(\boldsymbol{\theta}_0)$ has the components:

$$I_{\beta\beta} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}X'_{nt}D_{nt}X_{nt}\} - \{\tfrac{1}{\sigma_0^2}X'_{nt}D_{nt}\mathbb{D}_n^{-1}D_{ns}X_{ns}\};$$
$$I_{\lambda\beta} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}\eta'_tD_{nt}X_{nt}\} - \{\tfrac{1}{\sigma_0^2}\eta'_tD_{nt}\mathbb{D}_n^{-1}D_{ns}X_{ns}\},\quad I_{\rho\beta} = 0_{Tk}$$
$$I_{\lambda\lambda} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}\eta'_{nt}D_{nt}\eta_{nt} + \text{tr}(S_{nt}\bar{G}^s_{nt}\bar{G}_{nt})\} - \{\tfrac{1}{\sigma_0^2}\eta'_{nt}D_{nt}\mathbb{D}_n^{-1}D_{ns}\eta_{ns}\},$$
$$I_{\lambda\rho} = \texttt{blkdiag}\{\text{tr}(\bar{G}'_{nt}S_{nt}H^s_{nt})\};\quad I_{\sigma^2\sigma^2} = -\tfrac{n(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^4}\textstyle\sum_{t=1}^T\text{tr}(S_{nt})$$
$$I_{\rho\lambda} = \texttt{blkdiag}\{\text{tr}(\bar{G}'_{nt}S_{nt}H^s_{nt}S_{nt})\} - \{\text{tr}(G'_{ns}D_{ns}\mathbb{D}_n^{-1}\dot{D}_{nt}\mathbb{D}_n^{-1})\}$$
$$I_{\rho\rho} = \texttt{blkdiag}\{\text{tr}(H^s_{nt}S_{nt}H_{nt} - B_{nt}\mathbb{D}_n^{-1}\dot{D}_{nt}B_{nt}^{-1}H_{nt})\} + \{\text{tr}(B_{nt}\mathbb{D}_n^{-1}\dot{D}_{ns}\mathbb{D}_n^{-1}B'_{nt}H_{nt})\}$$
$$I_{\sigma^2\beta} = \mathbf{0},\quad I_{\sigma^2\lambda} = \{\tfrac{1}{\sigma_0^2}\text{tr}(R_{nt}G_{nt})\},\quad I_{\sigma^2\rho} = \tfrac{1}{\sigma_0^2}\text{tr}(S_{nt}H_{nt}).$$

where $\dot{D}_{nt} = -\frac{d}{d\rho_{t0}}D_{nt} = M'_nB_{nt} + B'_{nt}M_n$, and $\bar{G}_{nt} = B_{nt}G_{nt}B_{nt}^{-1}$.

The VC matrix $\Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)$ is obtained by applying Lemma A.2 with ε replaced by $\mathbb{V}_N$, c_{nt} by Π_{1t}, or Π_{2t}, and A_{nt} by Φ_{1t}, Φ_{2t}, or Ψ:

$\Sigma_{\texttt{SLE1}}(\boldsymbol{\theta}_0) =$

$$\begin{pmatrix} \{f(\mathbf{0},\Pi_{1t};\mathbf{0},\Pi_{1s})\}, \{f(\mathbf{0},\Pi_{1t};\Phi_{1s},\Pi_{2s})\}, & \{f(\mathbf{0},\Pi_{1t};\Phi_{2s},\mathbf{0})\}, & \{f(\mathbf{0},\Pi_{1t};\Psi,\mathbf{0})\} \\ \sim, \qquad \{f(\Phi_{1t},\Pi_{2t};\Phi_{1s},\Pi_{2s})\}, & \{f(\Phi_{1t},\Pi_{2t};\Phi_{2s},\mathbf{0})\}, & \{f(\Phi_{1t},\Pi_{2t};\Psi,\mathbf{0})\} \\ \sim, \qquad \sim, & \{f(\Phi_{2t},\mathbf{0};\Phi_{2s},\mathbf{0})\}, & \{f(\Phi_{2t},\mathbf{0};\Psi,\mathbf{0})\} \\ \sim, \qquad \sim, & \sim, & f(\Psi,\mathbf{0};\Psi,\mathbf{0}) \end{pmatrix},$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2} Z^{\diamond}_{Nt} B_{nt} X_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2} Z^{\diamond}_{Nt} B_{nt} \eta_{nt}$, $\Phi_{1t} = \frac{1}{\sigma_0^2} Z_{Nt} B_{nt}^{-1\prime} G'_{nt} B'_{nt} Z^{\diamond\prime}_{Nt}$, $\Phi_{2t} = \frac{1}{\sigma_0^2} Z^{\diamond}_{Nt} H_{nt} Z^{\diamond\prime}_{Nt}$, $\Psi = \frac{1}{2\sigma_0^4}\sum_{t=1}^{T} Z^{\diamond}_{Nt} Z^{\diamond\prime}_{Nt}$, with $Z^{\diamond\prime}_{Nt} = [Z'_{Nt} - B_{nt}\mathbb{D}_n^{-1}(l'_T \otimes I_n)\mathbb{B}_N]$ and $\mathbb{B}_N = \texttt{blkdiag}(B_{n1},\ldots,B_{nT})$.

B.4. Panel SLE model with two-way FE. The negative Hessian matrix $J_{\texttt{SEL2}}(\boldsymbol{\theta}_0)$ is in the `Supplementary Appendix`. Its expectation $I_{\texttt{SEL2}}(\boldsymbol{\theta}_0)$ has the components:

$$I_{\boldsymbol{\beta\beta}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} D^*_{nt} X^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} X{*\prime}_{nt} D^*_{nt} \mathbb{D}_n^{*-1} D^*_{ns} X^*_{ns}\big\};$$

$$I_{\boldsymbol{\lambda\beta}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_t D^*_{nt} X^*_{nt}\big\} - \big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_t D^*_{nt} \mathbb{D}_n^{*-1} D^*_{ns} X^*_{ns}\big\};\quad I_{\boldsymbol{\rho\beta}} = 0;$$

$$I_{\boldsymbol{\lambda\lambda}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_{nt} D^*_{nt} \eta^*_{nt} + \mathrm{tr}(S^*_{nt}\bar{G}^{*s}_{nt}\bar{G}^*_{nt})\big\} - \big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_{nt} D^*_{nt} \mathbb{D}_n^{*-1} D^*_{ns} \eta^*_{ns}\big\};$$

$$I_{\boldsymbol{\lambda\rho}} = \texttt{blkdiag}\big\{\mathrm{tr}(\bar{G}^{*\prime}_{nt} S^*_{nt} H^{*s}_{nt})\big\};\quad I_{\sigma^2\sigma^2} = -\tfrac{(n-1)(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^4}\textstyle\sum_{t=1}^{T}\mathrm{tr}(S^*_{nt});$$

$$I_{\boldsymbol{\rho\lambda}} = \texttt{blkdiag}\big\{\mathrm{tr}(\bar{G}^{*\prime}_{nt} S^*_{nt} H^{*s}_{nt} S^*_{nt})\big\} - \big\{\mathrm{tr}(G^{*\prime}_{ns} D^*_{ns} \mathbb{D}_n^{*-1} \dot{D}^*_{nt} \mathbb{D}_n^{*-1})\big\};$$

$$I_{\boldsymbol{\rho\rho}} = \texttt{blkdiag}\big\{\mathrm{tr}(H^{*s}_{nt} S^*_{nt} H^*_{nt} - B^*_{nt}\mathbb{D}_n^{*-1}\dot{D}^*_{nt} B^{*-1}_{nt} H^*_{nt})\big\} + \big\{\mathrm{tr}(B^*_{nt}\mathbb{D}_n^{*-1}\dot{D}^*_{ns}\mathbb{D}_n^{*-1} B^{*\prime}_{nt} H^*_{nt})\big\};$$

$$I_{\sigma^2\boldsymbol{\beta}} = \mathbf{0};\quad I_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{1}{\sigma_0^2}\mathrm{tr}(R^*_{nt} G^*_{nt})\big\};\quad I_{\sigma^2\boldsymbol{\rho}} = \big\{\tfrac{1}{\sigma_0^2}\mathrm{tr}(S^*_{nt} H^*_{nt})\big\},$$

where $\dot{D}^*_{nt} = -\frac{d}{d\rho_{t0}} D^*_{nt} = M^{*\prime}_n B^*_{nt} + B^{*\prime}_{nt} M^*_n$, and $\bar{G}^*_{nt} = B^*_n G^*_{nt} B^{*-1}_{nt}$.

The VC matrix $\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$ takes an identical form as $\Sigma_{\texttt{SLE1}}(\boldsymbol{\theta}_0)$, but with $\Pi_{1t} = \frac{1}{\sigma_0^2} Z^{\diamond*}_{Nt} B^*_{nt} X^*_{nt}$, $\Pi_{2t} = \frac{1}{\sigma_0^2} Z^{\diamond*}_{Nt} B^*_{nt} \eta^*_{nt}$, $\Phi_{1t} = \frac{1}{\sigma_0^2} Z^*_{Nt} B^{*-1\prime}_{nt} G^{*\prime}_{nt} B^{*\prime}_{nt} Z^{\diamond*\prime}_{Nt}$, $\Phi_{2t} = \frac{1}{\sigma_0^2} Z^{\diamond*}_{Nt} H^*_{nt} Z^{\diamond*\prime}_{Nt}$, and $\Psi = \frac{1}{2\sigma_0^4}\sum_{t=1}^{T} Z^{\diamond*}_{Nt} Z^{\diamond*\prime}_{Nt}$, where $Z^*_{Nt} = Z_{Nt} F_{n,n-1}$ and $Z^{\diamond*}_{Nt} = Z^{\diamond}_{Nt} F_{n,n-1}$.

B.5. Panel SLE model with two-way FE and homogeneous ρ. The expected negative Hessian corresponding to the AQS function given in (3.16) has components:

$$\begin{aligned} I_{\boldsymbol{\beta\beta}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} X^{*\prime}_{nt} D^*_n X^*_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2} X^{*\prime}_{nt} D^*_n X^*_{ns}\big\},, \\ I_{\boldsymbol{\lambda\beta}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_{nt} D^*_n X^*_{nt}\big\} - \big\{\tfrac{1}{T\sigma_0^2} \eta^{*\prime}_{nt} D^*_n X^*_{ns}\big\},\quad I_{\rho\boldsymbol{\beta}} = \mathbf{0} \\ I_{\boldsymbol{\lambda\lambda}} &= \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2} \eta^{*\prime}_{nt} D^*_n \eta^*_{nt} + \tfrac{T-1}{T}\mathrm{tr}(\bar{G}^{*s}_{nt}\bar{G}^*_{nt})\big\} - \big\{\tfrac{1}{T\sigma_0^2} \eta^{*\prime}_{nt} D^*_n \eta^*_{ns}\big\}, \\ I_{\boldsymbol{\lambda}\rho} &= \big\{\tfrac{T-1}{T}\mathrm{tr}(\bar{G}^{*\prime}_{nt} H^{*s}_n)\big\},\quad I_{\rho\rho} = (T-1)\mathrm{tr}(H^{*s}_n H^*_n), \\ I_{\sigma^2\boldsymbol{\beta}} &= 0'_{tk},\quad I_{\sigma^2\boldsymbol{\lambda}} = \big\{\tfrac{T-1}{T\sigma_0^2}\mathrm{tr}(G^*_{nt})\big\},\quad I_{\sigma^2\rho} = \tfrac{T-1}{\sigma_0^2}\mathrm{tr}(H^*_n),\quad I_{\sigma^2\sigma^2} = \tfrac{n(T-1)}{2\sigma_0^4}. \end{aligned}$$

The VC matrix of the AQS function given in (3.16) is obtained by applying Lemma A.2 with ε_n replaced by $\mathbb{V}_N$, c_{nt} by $\Pi_{1t} = \frac{1}{\sigma_0^2} Z^{\circ*}_{Nt} B^*_n X^*_{nt}$, or $\Pi_{2t} = \frac{1}{\sigma_0^2} Z^{\circ*}_{Nt} B^*_n \eta^*_{nt}$, and A_{nt} by $\Phi_{1t} = \frac{1}{\sigma_0^2} Z^*_{Nt} B^{*-1\prime}_n G^{*\prime}_{nt} B^{*\prime}_n Z^{\circ*\prime}_{Nt}$, or $\Phi_2 = \frac{1}{\sigma_0^2}\sum_{t=1}^{T} Z^{\circ*}_{Nt} H^*_n Z^{\circ*\prime}_{Nt}$, or $\Psi = \frac{1}{2\sigma_0^4}\sum_{t=1}^{T} Z^{\circ*}_{Nt} Z^{\circ*\prime}_{Nt}$, where $Z^*_{Nt} = Z_{Nt} F_{n,n-1}$, and $Z^{\circ*}_{Nt} = Z^{\circ}_{Nt} F_{n,n-1}$:

$$\begin{pmatrix} \{f(\mathbf{0},\Pi_{1t};\mathbf{0},\Pi_{1s})\}, \{f(\mathbf{0},\Pi_{1t};\Phi_{1s},\Pi_{2s})\}, & \{f(\mathbf{0},\Pi_{1t};\Phi_2,\mathbf{0})\}, & \{f(\mathbf{0},\Pi_{1t};\Psi,\mathbf{0})\} \\ \sim, \qquad \{f(\Phi_{1t},\Pi_{2t};\Phi_{1s},\Pi_{2s})\}, & \{f(\Phi_{1t},\Pi_{2t};\Phi_2,\mathbf{0})\}, & \{f(\Phi_{1t},\Pi_{2t};\Psi,\mathbf{0})\} \\ \sim, \qquad \sim, & f(\Phi_2,\mathbf{0};\Phi_2,\mathbf{0}), & f(\Phi_2,\mathbf{0};\Psi,\mathbf{0}) \\ \sim, \qquad \sim, & \sim, & f(\Psi,\mathbf{0};\Psi,\mathbf{0}) \end{pmatrix}.$$

B.6. Panel SE model with two-way FE The expected negative Hessian matrix corresponding to the AQS function given in (3.17) has the components:

$$I_{\boldsymbol{\beta\beta}} = \texttt{blkdiag}\{\tfrac{1}{\sigma_0^2}X_{nt}^{*\prime}D_{nt}^*X_{nt}^*\} - \{\tfrac{1}{\sigma_0^2}X_{nt}^{*\prime}D_{nt}^*\mathbb{D}_n^{*-1}D_{ns}^*X_{ns}^*\}, \quad I_{\boldsymbol{\rho\beta}} = \mathbf{0};$$

$$I_{\boldsymbol{\rho\rho}} = \texttt{blkdiag}\{\mathrm{tr}(H_{nt}^{*s}S_{nt}^*H_{nt}^* - B_{nt}^*\mathbb{D}_n^{*-1}\dot{D}_{nt}B_{nt}^{*-1}H_{nt}^*)\} + \{\mathrm{tr}(B_{nt}^*\mathbb{D}_n^{*-1}\dot{D}_{ns}\mathbb{D}_n^{*-1}B_{nt}^{*\prime}H_{nt}^*)\};$$

$$I_{\sigma^2\boldsymbol{\beta}} = \mathbf{0}; \quad I_{\sigma^2\boldsymbol{\rho}} = \{\tfrac{1}{\sigma_0^2}\mathrm{tr}(S_{nt}^*H_{nt}^*)\}; \quad I_{\sigma^2\sigma^2} = -\tfrac{(n-1)(T-1)}{2\sigma_0^4} + \tfrac{1}{\sigma_0^4}\textstyle\sum_{t=1}^T \mathrm{tr}(S_{nt}^*).$$

Applying Lemma A.2 with ε_n being replaced by $\mathbb{V}_N$, c_{nt} by $\Pi_t = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond *}B_{nt}^*X_{nt}^*$, and A_{nt} by Φ_t or Ψ, we obtain the corresponding VC matrix of the AQS function (3.17):

$$\begin{pmatrix} \{f(\mathbf{0},\Pi_t;\mathbf{0},\Pi_s)\}, & \{f(\mathbf{0},\Pi_t;\Phi_s,\mathbf{0})\}, & \{f(\mathbf{0},\Pi_t;\Psi,\mathbf{0})\} \\ \sim, & \{f(\Phi_t,\mathbf{0};\Phi_s,\mathbf{0})\}, & \{f(\Phi_t,\mathbf{0};\Psi,\mathbf{0})\} \\ \sim, & \sim, & f(\Psi,\mathbf{0};\Psi,\mathbf{0}) \end{pmatrix},$$

where $\Phi_t = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond *}H_{nt}^*Z_{Nt}^{\diamond *\prime}$, $\Psi = \frac{1}{2\sigma^4}\sum_{t=1}^T Z_{Nt}^{\diamond *}Z_{Nt}^{\diamond *\prime}$, and $Z_{Nt}^{\diamond *} = Z_{Nt}^{\diamond}F_{n,n-1}$.

Appendix C: Proof of the Theorems

The four theorems share some similar features. We provide here only the proof of the most general Theorem 3.2. The detailed proofs of all theorems can be found in the `Supplementary Appendix`, available at: `http://www.mysmu.edu/faculty/zlyang/`.

Proof of Theorem 3.2. Consider the AQS function $S_{\texttt{SLE2}}^{\star}(\boldsymbol{\theta})$ given in (3.13). We need to show that $\frac{1}{\sqrt{N_0}}S_{\texttt{SLE2}}^{\star}(\boldsymbol{\theta}_0) \xrightarrow{D} N\big(0, \lim_{N_0\to\infty}\frac{1}{N_0}\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)\big)$, as $N_0 \to \infty$. We have

$$\widetilde{V}_{nt}^* \equiv \widetilde{V}_{nt}^*(\boldsymbol{\beta}_0, \boldsymbol{\lambda}_0, \boldsymbol{\rho}_0) = V_{nt}^* - B_{nt}^*\mathbb{D}_n^{*-1}\textstyle\sum_{s=1}^T B_{ns}^{*\prime}V_{ns}^* = F_{n,n-1}'Z_{Nt}^{\diamond\prime}\mathbb{V}_N, \text{ and}$$
$$W_n^*Y_{nt}^* = G_{nt}^*(X_{nt}^*\beta_{t0} + c_n^* + B_{nt}^{-*1}V_{nt}^*) = \eta_{nt}^* + G_{nt}^*B_{nt}^{*-1}F_{n,n-1}'Z_{Nt}'\mathbb{V}_N.$$

Hence, the AQS function at true $\boldsymbol{\theta}_0$ can be written as

$$S_{\texttt{SLE2}}^*(\boldsymbol{\theta}_0) = \begin{cases} \Pi_{1t}'\mathbb{V}_N, \quad t = 1,\ldots,T, \\ \Pi_{2t}'\mathbb{V}_N + \mathbb{V}_N'\Phi_{1t}\mathbb{V}_N - \mathrm{tr}(R_{nt}^*G_{nt}^*), \quad t = 1,\ldots,T, \\ \mathbb{V}_N'\Phi_{2t}\mathbb{V}_N - \mathrm{tr}(S_{nt}^*H_{nt}^*), \quad t = 1,\ldots,T, \\ \mathbb{V}_N'\Psi\mathbb{V}_N - \frac{(n-1)(T-1)}{2\sigma^2}, \end{cases} \tag{C.1}$$

where $\Pi_{1t} = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond *}B_{nt}^*X_{nt}^*$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond *}B_{nt}^*\eta_{nt}^*$, $\Phi_{1t} = \frac{1}{\sigma_0^2}Z_{Nt}^*B_{nt}^{*-1\prime}G_{nt}^{*\prime}B_{nt}^{*\prime}Z_{Nt}^{\diamond *\prime}$, $\Phi_{2t} = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond *}H_{nt}^*Z_{Nt}^{\diamond *\prime}$, and $\Psi = \frac{1}{2\sigma_0^4}\sum_{t=1}^T Z_{Nt}^{\diamond *}Z_{Nt}^{\diamond *\prime}$, with $Z_{Nt}^* = Z_{Nt}F_{n,n-1}$ and $Z_{Nt}^{\diamond *} = Z_{Nt}^{\diamond}F_{n,n-1}$;

$Z_{Nt} = z_t \otimes I_n$ and z_t is a $T \times 1$ vector with tth element being 1 and other elements being zero; and $Z^{\diamond\prime}_{Nt} = [Z'_{Nt} - B_{nt}\mathbb{D}_n^{-1}(l'_T \otimes I_n)\mathbb{B}_N]$ and $\mathbb{B}_N = \texttt{blkdiag}(B_{n1}, \ldots, B_{nT})$.

First, as the elements of X_{nt} are non-stochastic and uniformly bounded (by Assumption 3), the row and column sums of B^*_{nt} are uniformly bounded in absolute values by Assumption 5 and Lemma A.1. It follows that the elements of Π_{1t} are uniformly bounded. By Assumption A.4 and Lemma A.1(i), G_{nt} is uniformly bounded in both row and column sums. By Lemma A.2 of Lee and Yu (2010),

$$(I_n - \lambda F'_{n,n-1}W_n F_{n,n-1})^{-1} = F'_{n,n-1}(I_n - \lambda W_n)^{-1}F_{n,n-1}. \tag{C.2}$$

We have $A^{*-1}_{nt} = F'_{n,n-1}A^{-1}_{nt}F_{n,n-1}$. Thus, G^*_{nt} is uniformly bounded in both row and column sums by Lemma A.1(iii), and the elements of $\eta^*_{nt} = G^*_{nt}(X^*_{nt}\beta_{t0} + c^*_n)$ are uniformly bounded by Assumption A3. It follows that the elements of Π_{2t} are uniformly bounded. Similarly, $B^{*-1}_{nt} = F'_{n,n-1}B^{-1}_{nt}F_{n,n-1}$, and therefore the elements of H^*_{nt} is uniformly bounded in both row and column sums. With these and the definitions of Z_{Nt} and $Z^{\diamond}_{Nt}$, it is easy to show that Φ_{1t}, Φ_{2t} and Ψ are uniformly bounded in both row and column sums. Thus, under Assumptions 1-5, the central limit theorem (CLT) of linear-quadratic (LQ) form of Kelejian and Prucha (2001) or its simplified version (under iid errors) given in Lemma A.3 can be applied to each element of $S^\star_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$ to establish its asymptotic normality. Then, an application of Cramér-Wold device under a finite T gives, $\frac{1}{\sqrt{N_0}}S^\star_{\texttt{SLE2}}(\boldsymbol{\theta}_0) \xrightarrow{D} N\big(0,\ \lim_{N_0\to\infty}\frac{1}{N_0}\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)\big)$, as $N_0 \to \infty$. Then, by (2.11) and (2.12),

$$C[\tfrac{1}{N_0}I_{\texttt{SLE2}}(\boldsymbol{\theta}_0)]^{-1}\tfrac{1}{\sqrt{N_0}}S^\star_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) \xrightarrow{D} N\big(0,\ \lim_{N_0\to\infty}\boldsymbol{\Xi}_{\texttt{SLE2}}(\boldsymbol{\theta}_0)\big).$$

It left to show that, as $N_0 \to \infty$,

(a) $\frac{1}{N_0}[I_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}) - I_{\texttt{SLE2}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$,

(b) $\frac{1}{N_0}[\Sigma_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) - \Sigma_{\texttt{SL1}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$.

Under the $\sqrt{N_0}$-consistency of $\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}$ and with the analytical expressions of $I_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$ and $\Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$ given in Appendix B.4, the proofs of these results are repeated applications of the mean value theorem (MVT) to each component of $\frac{1}{N_0}[I_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) - I_{\texttt{SLE2}}(\boldsymbol{\theta}_0)]$ and each component of $\frac{1}{N_0}[\Sigma_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) - \Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)]$.

To show (a), we pick a typical element of $I_{\texttt{SLE2}}(\boldsymbol{\theta}_0)$ given in Appendix B.4,

$$I_{\boldsymbol{\lambda\lambda}} = \texttt{blkdiag}\big\{\tfrac{1}{\sigma_0^2}\eta^{*\prime}_{nt}D^*_{nt}\eta^*_{nt} + \mathrm{tr}(S^*_{nt}\bar{G}^{*s}_{nt}\bar{G}^*_{nt})\big\} - \big\{\tfrac{1}{\sigma_0^2}\eta^{*\prime}_{nt}D^*_{nt}\mathbb{D}^{*-1}_n D^*_{ns}\eta^*_{ns}\big\}$$

to show that $\frac{1}{N_0}(\widetilde{I}_{\boldsymbol{\lambda\lambda}} - I_{\boldsymbol{\lambda\lambda}}) \xrightarrow{p} 0$. The proofs for the other components follow similarly. Recall: $\eta^*_{nt} = G^*_{nt}(X^*_{nt}\beta_{t0} + c^*_n)$, $\mathbb{D}^*_n(\boldsymbol{\rho}) = \sum_{t=1}^T D^*_n(\rho_t)$, $D^*_n(\rho_t) = B^{*\prime}_n(\rho_t)B^*_n(\rho_t)$, $B^*_n(\rho_t) = I_{n-1} - \rho_t M^*_n$, $S^*_{nt}(\boldsymbol{\rho}) = I_{n-1} - B^*_{nt}(\rho_t)\mathbb{D}^{*-1}_n(\boldsymbol{\rho})B^{*\prime}_{nt}(\rho_t)$, and $\bar{G}^*_{nt} = B^*_{nt}G^*_{nt}B^{*-1}_{nt}$.

By Assumptions 4 and 5 and Lemma A.1(i), it is straightforward to show the two matrices, $D^*_n(\rho_t)$ and $\bar{G}^*_{nt}(\lambda_t, \rho_t)$, are uniformly bounded in both row and column sums in a neighborhood of $(\lambda_{t0}, \rho_{t0})$ for each t, and so are their derivatives. Clearly with the properties of $D^*_n(\rho_t)$ and a finite T, $\mathbb{D}^*_n(\boldsymbol{\rho})$ is uniformly bounded in both row and column sums in a neighborhood of $\boldsymbol{\rho}_0$, and so are its derivatives.

By Assumption 5 and Lemma A.1(i), $D_n^{*-1}(\rho_t)$ is uniformly bounded in both row and column sums in a neighborhood of ρ_{t0} for each t, and so are its derivatives. By a matrix result that for two invertible matrices A_n and B_n, $(A_n+B_n)^{-1} = A_n^{-1} + \frac{1}{1+c}A_n^{-1}B_nA_n^{-1}$, where $c = \text{tr}(B_nA_n^{-1})$, we infer that for a finite T, $\mathbb{D}_n^*(\boldsymbol{\rho})$ is uniformly bounded in both row and column sums in a neighborhood of $\boldsymbol{\rho}_0$, and so are its derivatives. It follows that $S_{nt}^*(\boldsymbol{\rho})$ is uniformly bounded in both row and column sums in a neighborhood of $\boldsymbol{\rho}_0$, and so are its derivatives. Noting that $\tilde{I}_{\boldsymbol{\lambda\lambda}} = I_{\boldsymbol{\lambda\lambda}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}})$ and $I_{\boldsymbol{\lambda\lambda}} = I_{\boldsymbol{\lambda\lambda}}(\boldsymbol{\theta}_0)$, we have by MVT, for each component of $I_{\boldsymbol{\lambda\lambda}}(\boldsymbol{\theta})$ denoted as $I_{\boldsymbol{\lambda\lambda},ts}(\boldsymbol{\theta})$, $t, s = 1,\ldots,T$,

$$\tfrac{1}{N_0}I_{\boldsymbol{\lambda\lambda},ts}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) = \tfrac{1}{N_0}I_{\boldsymbol{\lambda\lambda},ts}(\boldsymbol{\theta}_0) + [\tfrac{1}{N_0}\tfrac{\partial}{\partial\boldsymbol{\theta}'}I_{\boldsymbol{\lambda\lambda},ts}(\bar{\boldsymbol{\theta}})](\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}} - \boldsymbol{\theta}_0),$$

where $\bar{\boldsymbol{\theta}}$ lies elementwise between $\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}$ and $\boldsymbol{\theta}_0$, with $\bar{\boldsymbol{\theta}}$ being $\sqrt{N_0}$-consistent as $\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}$ is. With the above argument and Lemma A.1(ii), we have $\frac{1}{N_0}\frac{\partial}{\partial\boldsymbol{\theta}'}I_{\boldsymbol{\lambda\lambda},ts}(\bar{\boldsymbol{\theta}}) = O_p(1)$. Therefore, $\frac{1}{N_0}[I_{\boldsymbol{\lambda\lambda},ts}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) - I_{\boldsymbol{\lambda\lambda},ts}(\boldsymbol{\theta}_0)] = o_p(1)$ for each (t,s), and $\frac{1}{N_0}[I_{\boldsymbol{\lambda\lambda}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) - I_{\boldsymbol{\lambda\lambda}}(\boldsymbol{\theta}_0)] = o_p(1)$. Note that the easily proved results such as $\frac{1}{N_0}(\tilde{c}_n\tilde{G}_{nt}\tilde{c}_n - c_nG_{nt}c_n) \xrightarrow{p} 0$, has been used. The proofs of the other components of $\frac{1}{N_0}[I_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SL1}}) - I_{\texttt{SL1}}(\boldsymbol{\theta}_0)] \xrightarrow{p} \mathbf{0}$ proceeds similarly.

To show (b), we again choose the most complicated term, $f(\Phi_{1t},\Pi_{2t};\Phi_{1s},\Pi_{2s})$ that corresponds to $\boldsymbol{\lambda}$, to show in details where the quantities involved are given at the end of Appendix B.4: $\Pi_{1t} = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond*}B_{nt}^*X_{nt}^*$, $\Pi_{2t} = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond*}B_{nt}^*\eta_{nt}^*$, $\Phi_{1t} = \frac{1}{\sigma_0^2}Z_{Nt}^*B_{nt}^{*-1\prime}G_{nt}^{*\prime}B_{nt}^{*\prime}Z_{Nt}^{\diamond*\prime}$, and $\Phi_{2t} = \frac{1}{\sigma_0^2}Z_{Nt}^{\diamond*}H_{nt}^*Z_{Nt}^{\diamond*\prime}$, where $Z_{Nt}^* = Z_{Nt}F_{n,n-1}$ and $Z_{Nt}^{\diamond*} = Z_{Nt}^{\diamond}F_{n,n-1}$.

Applying Lemma A.2 with A_{nt} replaced by Φ_{1t}, a_{nt} by $\phi_{1t} = \texttt{diagv}(\Phi_{1t})$, and c_{nt} by Π_{2t} (similarly for the quantities with subscript s), and noting that $\mu_3 = \gamma$ and $\mu_4 = \kappa$, we obtain the covariance between the λ_t- and λ_s-components of the AQS function:

$$f(\Phi_{1t},\Pi_{2t};\Phi_{1s},\Pi_{2s}) = \sigma_0^4\text{tr}[(\Phi_{1t}'+\Phi_{1t})\Phi_{1s}] + \gamma\phi_{1t}'\Pi_{2s} + \gamma\Pi_{2t}'\phi_{1s} + \kappa\phi_{1t}'\phi_{1s} + \sigma_0^2\Pi_{2t}'\Pi_{2s}.$$

Applying MVT and following the similar arguments as in (a), the convergence of the relevant terms can easily be proved, e.g., $\frac{1}{N_0}\{\text{tr}[(\widetilde{\Phi}_{1t}'+\widetilde{\Phi}_{1t})\widetilde{\Phi}_{1s}] - \text{tr}[(\Phi_{1t}'+\Phi_{1t})\Phi_{1s}]\} = o_p(1)$, $\frac{1}{N_0}[\phi_{1t}'\Pi_{2s} - \phi_{1t}'\Pi_{2s}] = o_p(1)$, etc. Furthermore, $\tilde{\sigma}_{\texttt{SLE2}}^2 - \sigma_0^2 = o_p(1)$, and hence $\tilde{\sigma}_{\texttt{SLE2}}^4 - \sigma_0^4 = o_p(1)$; for the estimates obtained from Lemma 4.1(a) of Yang et al. (2016), it is easy to show that $\tilde{\gamma} - \gamma \xrightarrow{p} 0$ and $\tilde{\kappa} - \kappa \xrightarrow{p} 0$. It follows that

$$[\tilde{f}(\widetilde{\Phi}_{1t},\widetilde{\Pi}_{2t};\widetilde{\Phi}_{1s},\widetilde{\Pi}_{2s}) - f(\Phi_{1t},\Pi_{2t};\Phi_{1s},\Pi_{2s})] = o_p(1).$$

Similarly, the convergence of the other elements of $\frac{1}{N_0}[\Sigma_{\texttt{SLE2}}(\tilde{\boldsymbol{\theta}}_{\texttt{SLE2}}) - \Sigma_{\texttt{SLE2}}(\boldsymbol{\theta}_0)]$ is proved.
■

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Table 1a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model

One-Way Fixed Effects, `Queen Contiguity`

		$T=3$						$T=6$					
λ	n	$T_{\mathtt{SL1}}$			$T_{\mathtt{SL1}}^{(r)}$			$T_{\mathtt{SL1}}$			$T_{\mathtt{SL1}}^{(r)}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.208	.135	.052	.096	.045	.007	.216	.138	.050	.095	.044	.008
	100	.150	.086	.024	.098	.046	.009	.161	.097	.028	.103	.050	.009
	200	.128	.068	.015	.103	.049	.008	.129	.069	.018	.099	.051	.010
	500	.107	.054	.010	.097	.046	.007	.110	.054	.011	.098	.049	.009
0	50	.204	.135	.053	.102	.048	.008	.214	.137	.050	.095	.046	.009
	100	.147	.086	.025	.099	.048	.008	.160	.096	.027	.105	.051	.009
	200	.127	.069	.015	.104	.049	.009	.127	.068	.018	.100	.049	.010
	500	.111	.056	.011	.100	.048	.008	.109	.056	.012	.099	.050	.010
-.5	50	.204	.133	.055	.102	.048	.008	.212	.136	.051	.097	.046	.009
	100	.147	.086	.025	.099	.049	.008	.160	.097	.027	.103	.050	.009
	200	.129	.068	.015	.103	.048	.009	.127	.070	.017	.100	.050	.010
	500	.108	.055	.012	.101	.048	.009	.110	.056	.012	.100	.049	.010
		Normal Mixture Error											
.5	50	.201	.129	.053	.096	.047	.006	.229	.154	.061	.121	.070	.023
	100	.149	.088	.027	.100	.048	.009	.163	.096	.029	.099	.050	.010
	200	.130	.073	.019	.105	.052	.011	.133	.073	.018	.103	.054	.010
	500	.112	.058	.012	.102	.051	.009	.118	.061	.012	.102	.051	.010
0	50	.197	.126	.052	.099	.047	.007	.229	.150	.061	.103	.053	.011
	100	.149	.087	.028	.102	.049	.010	.161	.094	.029	.099	.048	.010
	200	.129	.073	.019	.105	.052	.010	.132	.073	.018	.104	.054	.011
	500	.111	.059	.012	.103	.051	.010	.120	.061	.012	.102	.053	.009
-.5	50	.193	.129	.052	.097	.048	.008	.231	.151	.062	.103	.053	.012
	100	.150	.088	.028	.101	.050	.010	.162	.094	.030	.101	.050	.010
	200	.130	.073	.019	.104	.052	.011	.132	.073	.018	.103	.053	.011
	500	.113	.059	.013	.102	.051	.010	.118	.062	.013	.101	.052	.010
		Log-normal Error											
.5	50	.180	.119	.045	.089	.043	.008	.211	.145	.060	.100	.054	.017
	100	.149	.087	.027	.097	.047	.009	.164	.102	.032	.101	.057	.012
	200	.133	.071	.018	.097	.045	.009	.147	.087	.030	.101	.055	.014
	500	.127	.071	.018	.100	.051	.011	.142	.078	.030	.101	.050	.011
0	50	.180	.118	.046	.093	.044	.008	.193	.130	.056	.099	.054	.015
	100	.132	.078	.023	.094	.047	.009	.146	.086	.024	.100	.052	.010
	200	.109	.057	.013	.089	.042	.008	.114	.064	.017	.094	.051	.012
	500	.099	.052	.012	.010	.050	.010	.110	.058	.013	.102	.053	.011
-.5	50	.194	.128	.049	.097	.045	.008	.225	.154	.072	.106	.058	.016
	100	.142	.083	.024	.096	.047	.010	.191	.118	.042	.104	.057	.013
	200	.120	.067	.017	.095	.046	.009	.166	.102	.032	.102	.054	.012
	500	.118	.065	.016	.098	.050	.011	.151	.102	.032	.102	.050	.010

Table 1b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
One-Way Fixed Effects, `Group Interaction`

		$T=3$						$T=6$					
λ	n	$T_{\texttt{SL1}}$			$T_{\texttt{SL1}}^{(r)}$			$T_{\texttt{SL1}}$			$T_{\texttt{SL1}}^{(r)}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
							Normal Error						
.5	50	.222	.144	.057	.086	.034	.004	.219	.136	.048	.085	.039	.007
	100	.150	.089	.025	.088	.039	.006	.165	.094	.028	.089	.042	.007
	200	.124	.067	.018	.092	.042	.008	.128	.070	.016	.094	.045	.008
	500	.110	.059	.014	.097	.049	.011	.113	.057	.012	.095	.048	.009
0	50	.232	.157	.065	.087	.036	.005	.232	.151	.056	.084	.040	.007
	100	.155	.091	.027	.089	.040	.006	.173	.099	.030	.091	.044	.008
	200	.124	.068	.020	.090	.042	.008	.131	.071	.016	.095	.044	.008
	500	.110	.060	.015	.098	.049	.010	.114	.058	.013	.096	.048	.009
-.5	50	.238	.163	.071	.086	.038	.004	.239	.159	.063	.085	.038	.007
	100	.157	.092	.029	.088	.040	.005	.178	.102	.033	.089	.043	.008
	200	.126	.069	.020	.091	.043	.008	.133	.072	.016	.096	.043	.008
	500	.111	.061	.014	.098	.049	.010	.115	.059	.012	.096	.048	.009
							Normal Mixture Error						
.5	50	.230	.151	.056	.087	.033	.004	.215	.143	.051	.088	.046	.009
	100	.154	.088	.025	.087	.041	.006	.165	.094	.025	.087	.041	.009
	200	.131	.070	.017	.095	.043	.008	.133	.071	.018	.093	.043	.009
	500	.114	.061	.013	.100	.048	.009	.116	.059	.011	.096	.048	.008
0	50	.241	.163	.068	.088	.036	.005	.231	.155	.061	.088	.046	.008
	100	.157	.092	.029	.089	.041	.006	.170	.098	.029	.089	.041	.008
	200	.133	.070	.018	.095	.044	.008	.133	.072	.019	.094	.042	.009
	500	.114	.059	.014	.099	.048	.010	.133	.072	.019	.094	.042	.009
-.5	50	.259	.181	.081	.093	.043	.007	.270	.186	.083	.096	.050	.010
	100	.168	.103	.033	.096	.046	.007	.193	.118	.040	.093	.046	.010
	200	.136	.075	.020	.097	.045	.009	.142	.079	.023	.094	.045	.010
	500	.116	.060	.015	.098	.048	.009	.117	.059	.012	.097	.048	.008
							Log-normal Error						
.5	50	.218	.143	.054	.081	.035	.005	.206	.137	.050	.079	.040	.009
	100	.151	.088	.026	.084	.037	.005	.176	.107	.034	.091	.048	.012
	200	.130	.069	.018	.091	.043	.006	.142	.081	.022	.095	.051	.012
	500	.108	.057	.012	.094	.045	.008	.126	.066	.016	.101	.049	.010
0	50	.227	.151	.064	.084	.036	.006	.243	.166	.075	.087	.045	.010
	100	.152	.091	.029	.088	.040	.006	.185	.122	.046	.097	.049	.013
	200	.137	.077	.019	.096	.047	.008	.136	.078	.025	.097	.052	.011
	500	.107	.059	.014	.098	.048	.009	.115	.057	.014	.098	.048	.010
-.5	50	.263	.188	.086	.093	.043	.008	.350	.259	.139	.106	.057	.015
	100	.179	.114	.042	.101	.049	.010	.260	.186	.090	.105	.054	.014
	200	.161	.096	.029	.107	.056	.010	.185	.114	.043	.103	.052	.013
	500	.123	.067	.018	.100	.051	.010	.131	.072	.021	.101	.051	.010

Table 2a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
Two-Way Fixed Effects, `Queen Contiguity`

		$T=3$						$T=6$					
λ	n	T_{SL2}			$T_{\mathrm{SL2}}^{(r)}$			T_{SL2}			$T_{\mathrm{SL2}}^{(r)}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.192	.123	.047	.093	.045	.007	.228	.148	.059	.100	.050	.010
	100	.140	.080	.023	.096	.048	.009	.157	.094	.029	.102	.050	.011
	200	.120	.064	.015	.098	.049	.009	.128	.068	.017	.101	.052	.009
	500	.103	.051	.013	.098	.048	.011	.105	.056	.012	.095	.049	.010
0	50	.194	.123	.048	.094	.046	.008	.224	.147	.059	.101	.049	.010
	100	.138	.082	.023	.095	.050	.009	.126	.069	.017	.099	.051	.009
	200	.115	.064	.016	.096	.049	.009	.157	.095	.027	.101	.049	.010
	500	.101	.052	.012	.098	.048	.009	.126	.069	.017	.099	.051	.009
-.5	50	.192	.123	.047	.093	.045	.009	.225	.148	.058	.100	.049	.009
	100	.138	.081	.023	.096	.049	.008	.157	.092	.027	.101	.048	.010
	200	.116	.063	.015	.096	.049	.009	.125	.069	.016	.102	.050	.009
	500	.105	.055	.011	.096	.048	.009	.108	.056	.013	.097	.051	.011
		Normal Mixture Error											
.5	50	.198	.131	.052	.100	.048	.008	.232	.155	.063	.106	.054	.013
	100	.140	.080	.025	.096	.047	.010	.165	.100	.030	.107	.055	.012
	200	.124	.067	.016	.101	.051	.009	.132	.071	.019	.104	.051	.013
	500	.110	.055	.013	.100	.050	.010	.106	.056	.012	.097	.051	.010
0	50	.199	.132	.052	.102	.048	.009	.234	.154	.064	.110	.055	.013
	100	.139	.080	.024	.097	.047	.009	.166	.100	.031	.109	.054	.011
	200	.124	.067	.017	.102	.051	.010	.129	.072	.019	.102	.051	.013
	500	.110	.055	.012	.102	.050	.010	.106	.055	.013	.096	.049	.010
-.5	50	.199	.130	.053	.101	.049	.009	.234	.157	.066	.112	.057	.013
	100	.143	.084	.025	.101	.048	.009	.164	.097	.031	.107	.053	.012
	200	.123	.069	.016	.103	.051	.010	.133	.073	.020	.105	.053	.012
	500	.109	.056	.012	.101	.050	.009	.107	.056	.014	.096	.048	.012
		Log-normal Error											
.5	50	.196	.131	.055	.100	.050	.009	.242	.171	.079	.107	.067	.018
	100	.139	.081	.027	.095	.050	.011	.171	.112	.041	.105	.055	.015
	200	.128	.070	.018	.106	.053	.010	.141	.081	.026	.104	.052	.013
	500	.109	.060	.014	.101	.052	.011	.123	.068	.019	.101	.051	.010
0	50	.196	.133	.059	.106	.055	.010	.239	.167	.081	.110	.055	.021
	100	.137	.078	.024	.095	.048	.010	.166	.110	.039	.107	.054	.018
	200	.126	.070	.018	.104	.052	.010	.133	.079	.025	.105	.049	.015
	500	.107	.056	.013	.100	.051	.010	.116	.061	.016	.102	.051	.013
-.5	50	.205	.141	.066	.112	.062	.011	.249	.177	.083	.108	.055	.026
	100	.154	.089	.028	.106	.052	.012	.172	.110	.042	.099	.048	.019
	200	.129	.074	.019	.107	.056	.012	.145	.088	.030	.098	.049	.020
	500	.110	.058	.014	.103	.052	.010	.122	.068	.018	.100	.049	.014

Table 2b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SL Model
Two-Way Fixed Effects, `Group Interaction`

		T = 3						T = 6					
λ	n	$T_{\mathtt{SL2}}$			$T^{(r)}_{\mathtt{SL2}}$			$T_{\mathtt{SL2}}$			$T^{(r)}_{\mathtt{SL2}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.226	.148	.059	.086	.038	.005	.223	.142	.052	.087	.040	.007
	100	.155	.090	.025	.090	.036	.006	.166	.095	.029	.089	.043	.007
	200	.124	.070	.018	.091	.044	.006	.131	.073	.016	.093	.045	.008
	500	.112	.060	.015	.097	.050	.010	.114	.057	.013	.096	.047	.010
0	50	.240	.159	.068	.088	.039	.005	.237	.154	.059	.086	.040	.007
	100	.159	.094	.025	.090	.037	.006	.174	.102	.031	.088	.042	.007
	200	.127	.072	.018	.091	.044	.007	.133	.074	.016	.094	.046	.008
	500	.112	.060	.014	.097	.050	.010	.116	.059	.013	.097	.046	.010
-.5	50	.244	.167	.075	.088	.039	.005	.249	.164	.065	.086	.040	.007
	100	.163	.096	.028	.089	.038	.006	.179	.104	.033	.085	.043	.007
	200	.127	.073	.019	.092	.045	.007	.134	.076	.017	.094	.045	.008
	500	.113	.059	.014	.098	.049	.010	.117	.059	.013	.097	.046	.010
		Normal Mixture Error											
.5	50	.232	.150	.058	.080	.034	.005	.222	.144	.055	.082	.041	.008
	100	.159	.090	.024	.088	.039	.006	.164	.095	.027	.083	.041	.008
	200	.130	.072	.018	.095	.045	.007	.133	.071	.017	.089	.043	.010
	500	.114	.059	.014	.097	.048	.009	.118	.060	.012	.098	.047	.009
0	50	.245	.167	.069	.085	.038	.006	.247	.165	.071	.083	.039	.007
	100	.164	.098	.027	.089	.040	.006	.175	.103	.032	.080	.038	.007
	200	.131	.072	.018	.094	.043	.008	.132	.072	.019	.089	.041	.009
	500	.115	.059	.014	.096	.048	.009	.119	.060	.012	.096	.047	.009
-.5	50	.269	.185	.085	.097	.047	.009	.298	.209	.100	.101	.052	.012
	100	.177	.110	.035	.099	.046	.007	.205	.127	.045	.094	.046	.008
	200	.138	.077	.020	.096	.045	.008	.145	.082	.023	.095	.045	.010
	500	.115	.059	.014	.096	.047	.009	.122	.063	.013	.099	.049	.009
		Log-normal Error											
.5	50	.217	.143	.057	.078	.036	.005	.215	.142	.055	.076	.036	.008
	100	.152	.088	.025	.079	.034	.005	.176	.111	.036	.082	.041	.009
	200	.132	.073	.018	.089	.044	.006	.141	.080	.023	.088	.046	.010
	500	.113	.057	.013	.094	.047	.008	.119	.062	.014	.096	.048	.009
0	50	.240	.165	.073	.085	.040	.006	.246	.174	.079	.085	.038	.008
	100	.164	.099	.034	.086	.041	.006	.191	.129	.051	.091	.040	.008
	200	.135	.076	.020	.092	.043	.007	.143	.083	.027	.095	.044	.009
	500	.111	.057	.014	.092	.045	.008	.113	.060	.013	.097	.045	.010
-.5	50	.287	.207	.104	.112	.060	.013	.347	.269	.151	.119	.068	.022
	100	.201	.131	.054	.109	.057	.012	.270	.195	.099	.119	.065	.019
	200	.156	.095	.028	.105	.054	.010	.191	.122	.049	.105	.056	.014
	500	.120	.067	.017	.098	.050	.009	.141	.081	.021	.103	.052	.010

Table 3a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
One-Way Fixed Effects, `Queen Contiguity`, $\lambda = 0.5$.

		$T=3$						$T=6$					
ρ	n	$T_{\texttt{SLE1}}$			$T^{(r)}_{\texttt{SLE1}}$			$T_{\texttt{SLE1}}$			$T^{(r)}_{\texttt{SLE1}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
							Normal Error						
.5	50	.199	.142	.075	.082	.039	.005	.161	.099	.036	.090	.042	.011
	100	.123	.068	.025	.094	.043	.009	.097	.050	.012	.092	.043	.006
	200	.084	.044	.009	.099	.046	.007	.079	.038	.008	.102	.049	.011
	500	.070	.034	.006	.104	.049	.009	.064	.030	.005	.102	.054	.009
0	50	.223	.164	.093	.090	.042	.006	.171	.104	.041	.093	.047	.010
	100	.132	.076	.029	.095	.046	.012	.105	.058	.014	.097	.047	.007
	200	.087	.046	.011	.103	.050	.010	.082	.039	.008	.104	.050	.011
	500	.069	.036	.006	.102	.050	.011	.063	.028	.005	.103	.054	.010
-.5	50	.232	.174	.098	.093	.042	.006	.181	.120	.048	.096	.047	.010
	100	.134	.083	.033	.097	.045	.011	.118	.064	.014	.098	.048	.008
	200	.097	.047	.013	.105	.050	.012	.079	.039	.008	.102	.052	.011
	500	.070	.035	.006	.102	.052	.009	.061	.028	.005	.102	.049	.011
							Normal Mixture Error						
.5	50	.196	.139	.072	.081	.037	.004	.168	.106	.044	.092	.047	.008
	100	.121	.070	.025	.087	.040	.008	.107	.057	.017	.096	.053	.012
	200	.084	.043	.011	.092	.046	.006	.082	.044	.010	.101	.052	.013
	500	.071	.035	.008	.099	.052	.012	.070	.036	.009	.097	.046	.014
0	50	.212	.151	.080	.087	.042	.005	.167	.110	.044	.089	.045	.010
	100	.131	.076	.028	.089	.041	.009	.105	.054	.015	.097	.046	.011
	200	.085	.046	.011	.095	.046	.008	.078	.039	.009	.100	.047	.012
	500	.071	.036	.007	.097	.050	.010	.064	.032	.006	.104	.054	.012
-.5	50	.226	.164	.090	.093	.040	.006	.197	.131	.057	.104	.056	.013
	100	.140	.083	.030	.094	.043	.009	.126	.073	.023	.104	.055	.013
	200	.094	.050	.013	.102	.051	.010	.086	.048	.013	.103	.055	.014
	500	.073	.038	.009	.101	.051	.012	.074	.034	.005	.102	.055	.011
							Log-normal Error						
.5	50	.150	.102	.046	.083	.038	.006	.169	.108	.044	.092	.048	.010
	100	.115	.075	.035	.091	.044	.010	.106	.058	.015	.098	.051	.010
	200	.109	.067	.027	.095	.046	.009	.073	.036	.008	.090	.046	.010
	500	.089	.050	.016	.100	.049	.011	.064	.032	.006	.104	.052	.012
0	50	.217	.160	.090	.082	.041	.009	.179	.118	.045	.092	.048	.011
	100	.126	.077	.031	.087	.042	.009	.108	.062	.017	.100	.055	.008
	200	.101	.055	.015	.103	.048	.010	.074	.035	.007	.095	.044	.008
	500	.071	.035	.008	.096	.048	.010	.059	.031	.006	.099	.050	.011
-.5	50	.192	.138	.069	.090	.045	.006	.202	.136	.054	.098	.050	.011
	100	.137	.087	.038	.092	.048	.010	.128	.074	.019	.108	.057	.010
	200	.094	.045	.014	.101	.048	.011	.081	.041	.008	.099	.049	.009
	500	.078	.040	.010	.102	.051	.010	.064	.030	.005	.105	.050	.012

Table 3b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
One-Way Fixed Effects, `Queen Contiguity`, $\lambda = -0.5$.

		$T=3$						$T=6$					
ρ	n	$T_{\texttt{SLE1}}$			$T^{(r)}_{\texttt{SLE1}}$			$T_{\texttt{SLE1}}$			$T^{(r)}_{\texttt{SLE1}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
Normal Error													
.5	50	.190	.131	.058	.088	.037	.007	.167	.102	.036	.088	.042	.010
	100	.116	.068	.022	.093	.044	.009	.098	.050	.013	.091	.044	.007
	200	.079	.042	.009	.094	.046	.007	.078	.040	.010	.100	.050	.012
	500	.071	.033	.007	.101	.050	.009	.060	.029	.005	.102	.053	.009
0	50	.209	.149	.073	.091	.040	.006	.169	.104	.040	.094	.043	.010
	100	.125	.073	.027	.099	.050	.011	.102	.056	.013	.093	.047	.006
	200	.084	.043	.010	.098	.048	.010	.079	.040	.008	.104	.051	.010
	500	.072	.033	.007	.103	.050	.011	.059	.029	.005	.096	.054	.010
-.5	50	.225	.162	.085	.095	.040	.006	.172	.111	.044	.094	.043	.010
	100	.131	.081	.031	.101	.050	.011	.109	.059	.013	.099	.047	.008
	200	.089	.044	.013	.105	.049	.011	.082	.039	.009	.104	.052	.010
	500	.069	.032	.007	.100	.049	.010	.057	.030	.005	.096	.049	.012
Normal Mixture Error													
.5	50	.187	.129	.061	.079	.034	.004	.176	.111	.043	.092	.047	.008
	100	.111	.068	.022	.086	.042	.008	.105	.054	.016	.097	.051	.013
	200	.083	.044	.009	.091	.047	.006	.085	.046	.010	.102	.056	.012
	500	.072	.033	.008	.102	.049	.011	.074	.036	.008	.099	.053	.010
0	50	.200	.140	.071	.086	.039	.006	.166	.105	.041	.090	.047	.010
	100	.126	.074	.027	.092	.042	.009	.103	.056	.016	.095	.049	.011
	200	.079	.045	.009	.095	.047	.008	.076	.041	.010	.098	.050	.012
	500	.071	.035	.008	.100	.049	.010	.064	.031	.007	.101	.050	.012
-.5	50	.218	.156	.080	.088	.041	.007	.191	.124	.052	.100	.054	.013
	100	.136	.079	.031	.096	.045	.008	.119	.068	.021	.105	.055	.013
	200	.087	.048	.013	.098	.048	.009	.088	.048	.014	.106	.057	.014
	500	.073	.037	.009	.103	.053	.011	.075	.034	.007	.104	.053	.011
Log-normal Error													
.5	50	.175	.125	.063	.084	.036	.009	.174	.110	.043	.092	.046	.010
	100	.138	.087	.038	.089	.042	.010	.099	.055	.016	.098	.050	.011
	200	.096	.048	.014	.096	.045	.008	.075	.037	.008	.098	.046	.011
	500	.075	.038	.009	.101	.052	.011	.066	.028	.006	.100	.053	.013
0	50	.207	.145	.081	.086	.042	.011	.173	.111	.044	.093	.046	.010
	100	.122	.078	.029	.090	.044	.009	.105	.056	.013	.096	.048	.009
	200	.091	.047	.010	.095	.047	.008	.076	.037	.007	.099	.046	.009
	500	.071	.035	.008	.099	.049	.011	.057	.027	.006	.101	.047	.011
-.5	50	.201	.138	.072	.093	.043	.008	.191	.125	.051	.097	.049	.012
	100	.141	.092	.039	.096	.048	.010	.118	.067	.017	.104	.053	.010
	200	.089	.045	.012	.104	.050	.009	.084	.041	.008	.104	.051	.010
	500	.072	.034	.007	.104	.049	.010	.062	.029	.006	.103	.046	.012

Table 4a. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
Two-Way Fixed Effects, `Queen Contiguity`, $\lambda = 0.5$.

		$T=3$						$T=6$					
ρ	n	T_{SLE2}			$T^{(r)}_{\text{SLE2}}$			T_{SLE2}			$T^{(r)}_{\text{SLE2}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
							Normal Error						
.5	50	.235	.181	.105	.083	.038	.006	.310	.226	.115	.087	.044	.008
	100	.212	.151	.086	.093	.045	.008	.190	.111	.036	.090	.041	.007
	200	.182	.121	.054	.098	.044	.006	.139	.079	.021	.101	.049	.011
	500	.134	.073	.022	.100	.048	.010	.121	.064	.014	.102	.055	.009
0	50	.272	.208	.117	.088	.043	.007	.314	.224	.111	.094	.045	.010
	100	.217	.143	.070	.094	.046	.011	.197	.116	.036	.093	.043	.008
	200	.161	.097	.032	.100	.051	.008	.142	.083	.022	.103	.050	.011
	500	.125	.065	.017	.105	.049	.011	.119	.064	.014	.102	.053	.010
-.5	50	.302	.233	.136	.094	.042	.005	.321	.239	.114	.092	.045	.009
	100	.209	.142	.062	.095	.046	.011	.205	.128	.042	.096	.047	.009
	200	.153	.090	.029	.102	.050	.009	.151	.081	.023	.098	.052	.010
	500	.119	.064	.015	.102	.054	.009	.115	.061	.014	.103	.051	.010
							Normal Mixture Error						
.5	50	.221	.159	.090	.083	.037	.004	.315	.242	.127	.090	.044	.008
	100	.212	.154	.085	.085	.044	.008	.201	.128	.050	.097	.053	.010
	200	.183	.122	.059	.092	.046	.008	.150	.090	.029	.101	.052	.009
	500	.137	.082	.028	.100	.053	.012	.139	.079	.022	.100	.053	.010
0	50	.269	.201	.114	.089	.043	.005	.315	.235	.124	.092	.052	.012
	100	.212	.149	.075	.089	.045	.009	.189	.118	.043	.096	.047	.010
	200	.158	.098	.033	.096	.048	.008	.143	.078	.025	.099	.050	.013
	500	.121	.070	.016	.099	.050	.010	.120	.063	.016	.102	.053	.012
-.5	50	.285	.225	.137	.093	.046	.008	.380	.286	.164	.103	.056	.011
	100	.229	.161	.083	.100	.047	.010	.229	.152	.061	.108	.060	.012
	200	.166	.102	.036	.101	.053	.009	.176	.106	.034	.104	.058	.012
	500	.132	.070	.018	.106	.054	.012	.136	.075	.020	.097	.050	.010
							Log-normal Error						
.5	50	.239	.181	.105	.085	.039	.006	.314	.232	.123	.091	.043	.008
	100	.222	.154	.086	.090	.043	.007	.196	.117	.041	.095	.047	.009
	200	.185	.126	.056	.096	.047	.008	.138	.079	.020	.097	.047	.009
	500	.138	.074	.024	.102	.049	.011	.123	.064	.016	.105	.052	.010
0	50	.246	.188	.108	.085	.042	.010	.319	.235	.115	.095	.047	.011
	100	.204	.141	.074	.090	.045	.007	.194	.115	.040	.095	.051	.008
	200	.180	.114	.047	.095	.047	.009	.142	.076	.021	.095	.048	.009
	500	.129	.075	.022	.097	.048	.010	.115	.060	.014	.100	.050	.011
-.5	50	.300	.235	.146	.093	.044	.008	.344	.246	.126	.097	.050	.011
	100	.214	.145	.064	.094	.045	.010	.208	.133	.050	.101	.055	.010
	200	.156	.092	.028	.099	.046	.008	.154	.086	.023	.101	.049	.011
	500	.123	.066	.015	.104	.051	.010	.121	.061	.014	.102	.050	.010

Table 4b. Empirical Sizes of Tests for Temporal Homogeneity in Panel SLE Model
Two-Way Fixed Effects, `Queen Contiguity`, $\lambda = -0.5$.

		$T=3$						$T=6$					
ρ	n	T_{SLE2}			$T^{(r)}_{\text{SLE2}}$			T_{SLE2}			$T^{(r)}_{\text{SLE2}}$		
		.10	.05	.01	.10	.05	.01	.10	.05	.01	.10	.05	.01
		Normal Error											
.5	50	.235	.173	.105	.086	.039	.007	.313	.225	.117	.089	.044	.009
	100	.216	.158	.086	.093	.046	.009	.189	.113	.037	.088	.044	.006
	200	.180	.117	.054	.093	.047	.007	.143	.079	.023	.100	.049	.012
	500	.134	.076	.021	.103	.048	.010	.118	.062	.014	.100	.053	.010
0	50	.271	.206	.116	.089	.040	.007	.315	.226	.109	.093	.044	.009
	100	.220	.149	.072	.098	.048	.011	.197	.115	.038	.092	.047	.008
	200	.160	.096	.032	.100	.051	.009	.146	.085	.024	.104	.052	.011
	500	.127	.062	.017	.103	.049	.011	.111	.059	.015	.094	.050	.010
-.5	50	.301	.233	.130	.095	.038	.007	.325	.232	.112	.092	.044	.009
	100	.214	.146	.065	.101	.048	.011	.206	.127	.039	.096	.046	.008
	200	.158	.092	.029	.103	.050	.011	.152	.087	.022	.102	.053	.010
	500	.117	.065	.014	.100	.050	.010	.111	.057	.013	.096	.048	.011
		Normal Mixture Error											
.5	50	.220	.161	.088	.080	.035	.005	.316	.243	.129	.093	.047	.009
	100	.213	.153	.085	.088	.043	.009	.204	.129	.048	.103	.051	.012
	200	.182	.121	.059	.096	.047	.006	.153	.089	.032	.106	.058	.013
	500	.139	.083	.030	.104	.049	.010	.137	.080	.022	.101	.051	.010
0	50	.256	.194	.113	.084	.043	.006	.321	.242	.124	.093	.049	.011
	100	.214	.151	.079	.091	.046	.008	.189	.121	.042	.098	.046	.011
	200	.155	.100	.033	.097	.048	.009	.146	.079	.028	.095	.051	.013
	500	.124	.068	.018	.099	.049	.011	.118	.064	.017	.102	.053	.012
-.5	50	.279	.219	.138	.089	.043	.007	.378	.288	.162	.111	.059	.016
	100	.232	.157	.082	.097	.049	.010	.234	.151	.058	.110	.057	.013
	200	.166	.103	.035	.102	.050	.010	.170	.104	.035	.106	.052	.014
	500	.128	.072	.019	.103	.054	.011	.134	.078	.018	.098	.047	.010
		Log-normal Error											
.5	50	.230	.178	.105	.086	.039	.008	.317	.232	.125	.089	.043	.009
	100	.218	.156	.087	.093	.045	.008	.197	.116	.042	.093	.049	.009
	200	.184	.122	.055	.093	.044	.008	.143	.080	.022	.100	.047	.010
	500	.139	.077	.024	.101	.052	.010	.119	.063	.015	.102	.053	.011
0	50	.242	.184	.107	.087	.043	.011	.315	.230	.113	.095	.046	.010
	100	.202	.142	.074	.091	.043	.010	.196	.115	.039	.093	.047	.008
	200	.176	.114	.046	.098	.046	.010	.141	.082	.023	.099	.049	.010
	500	.128	.074	.024	.098	.050	.011	.110	.055	.013	.102	.050	.010
-.5	50	.298	.230	.138	.095	.042	.008	.332	.245	.127	.097	.050	.010
	100	.220	.146	.067	.100	.045	.011	.212	.129	.048	.100	.052	.010
	200	.156	.092	.029	.100	.046	.009	.154	.089	.022	.105	.051	.011
	500	.123	.065	.015	.104	.051	.009	.115	.060	.015	.100	.048	.011