5 Implications for Option Pricing

Probably the most important application of the SV model is the pricing of options. Under a set of assumptions, Hull and White (1987) show that the value of a European call option on stocks based on a general specification of stochastic volatility is the Black-Scholes price integrated over the distribution of the mean volatility. Using the characteristic function approach, Heston (1993) derives a closed form solution for a European call option based on a square-root specification of volatility. For most other SV models, including our newly proposed N-SV model, option prices have no closed form solution and hence have to be approximated. A flexible way for approximating option prices is via Monte Carlo simulations. For example, Hull and White (1987) design an efficient procedure of carrying out the Monte Carlo simulation to calculate a European call option on stocks.

To examine the economic importance of our N-SV models on option pricing, we price options using both the LN and N-SV models provided the true model is the estimated N-SV model. To price options, we follow Mahieu and Schotman (1998).

Let C be the value of a European call option on a currency with maturity τ (measured in number of days), strike price X, current volatility σ_0^2, current exchange rate S_0, and the difference between the domestic and the foreign interest rates $r_d - r_f$. Under the same set of assumptions in Hull and White (1987), it can be shown that

$$C = e^{-\tau r_d} \int_0^\infty \mathrm{BS}(w_\tau) pdf(w_\tau | h_0) dw_\tau, \tag{20}$$

where w_τ^2 is given by

$$w_\tau^2 = \int_0^\tau g(h_s, \delta) ds, \tag{21}$$

and $\mathrm{BS}(w_\tau)$ is the Black-Scholes price for a currency option

$$\mathrm{BS}(w_\tau) = F_0 \mathrm{N}(d_1) - X\mathrm{N}(d_2), \tag{22}$$

in which $F_0 = S_0 e^{(r_d - r_f)\tau}$ is the forward exchange rate applying to time τ, d_1 and d_2 are given, respectively, by

$$d_1 = \frac{\ln(F_0/X) + w_\tau^2}{w_\tau}, \tag{23}$$

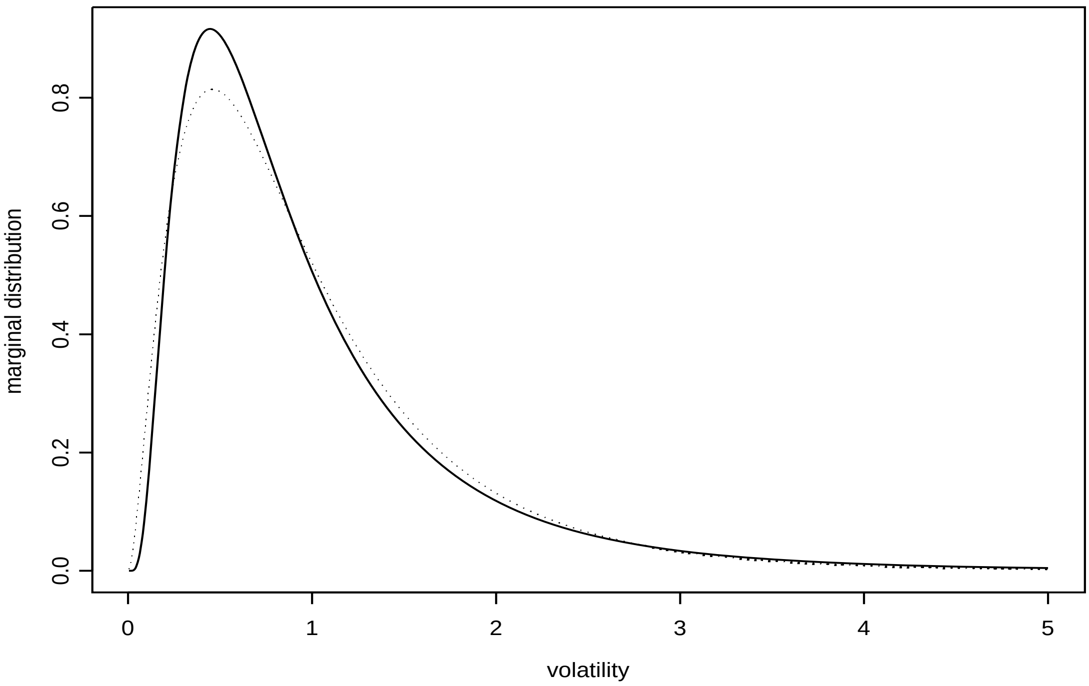

Figure 3: Marginal densities of dollar/pound exchange rate volatility implies from the LN SV model and the N-SV model. The solid line is for the LN SV model; the point line is for the N-SV model.

$$d_2 = d_1 - w_\tau. \tag{24}$$

In discrete time we have to approximate w_τ^2. In this paper we follow the suggestion of Amin and Ng (1993):

$$w_\tau^2 \approx \sum_{t=1}^{n} g(h_i, \delta), \tag{25}$$

where n is the number of discrete time periods until maturity of the option. In this paper, we choose the unit discrete time period to be one trading day and hence n $(= \tau)$ is the number of trading days before the maturity.

The Monte Carlo algorithm for calculating the value of a European call option on a currency may be summarized as follows:

1. Obtain the initial value of h_0 based on the initial value of σ_0^2;
2. Draw independent standard normal variates ν_i for $1 \leq i \leq n$;
3. Generate h_i according to
$$h_i = \mu + \phi(h_{i-1} - \mu) + \sigma\nu_i, \text{ for } i = 1, ..., n;$$
4. Calculate w_τ^2 using equation (25);
5. Calculate BS(w_τ) using equation (22) and call it p_1;
6. Repeat Steps 3-5 using $\{-\nu_i\}$ and define the value of BS(w_τ) by p_2;
7. Calculate the average value of p_1 and p_2 and call it y;
8. Repeat Steps 2-7 for K times and hence we should have a sequence of y's;
9. Calculate the mean of y's and this is the estimate of the option price.

Our algorithm is closely related to the one suggested by Mahieu and Schotman (1998), but there are two differences. The first difference is we use an antithetic method in Step 6 to reduce the variance of simulation errors. Finally, we use a much larger value of K (10,000 as opposed to 500) to ensure that the approximation errors in calculating equation (20) are very small.

The algorithm is then applied to price a half-year call option based on the LN and N-SV models with the estimated parameter values in Table 2 imposed.[11] In both models, we choose $n = 126$, $S_0 = 1.5$, $r_d = 0$, $r_f = 0$, $K = 10,000$, $\sigma_0 = 0.006349$,[12] and S_0/X

[11]Since the parameter estimates reported in Table 2 are based on the scaled data, for the purpose of pricing options, we have to scale the data back by multiplying the mean equation by the sample standard error of raw data which equals 0.006321 for the dollar/pound exchange rate.

[12]This initial value of standard error is very close to the sample standard error of the dollar/pound exchange rate and corresponds to a square root of volatility of 160% per year.

takes each of the following values, $0.75, 0.8, 0.85, 0.9, 0.95, 1, 1.05, 1.1, 1.15, 1.2, 1.25$. Table 4 compares the option prices and percentage differences between the prices based on the two estimated SV models.

The main conclusion we draw from the table is that the LN SV model tends to overprice the options. In fact the N-SV option price is always no bigger than the LN option prices. This finding is not surprising because we have found that while both models have a similar value of long term variance the N-SV model tends to generate a smoother volatility series. Prices of all the out-of-money options based on the N-SV model are systematically lower than those based on the LN model and the deep-out-of-the-money options show the largest percentage of discrepancies. The differences in the percentage term are much smaller for in-the-money options and eventually disappear when the in-the-money option goes very deep. Since near out-of-money options where the strike price is within about 10% of the spot price are traded very frequently over the counter and on exchanges, our results have important practical implications.

Table 4

	LN SV	N- SV	Percentage
S_0/X	Option Price	Option Price	Difference
0.75	2.401e-5	1.172e-5	-104.86
0.8	1.511e-4	1.032e-4	-46.41
0.85	8.645e-4	7.231e-4	-19.55
0.9	0.00415	0.00386	-7.513
0.95	0.01548	0.01507	-2.721
1	0.04257	0.04213	-1.044
1.05	0.08701	0.08661	-0.462
1.1	0.1413	0.1410	-0.213
1.15	0.1971	0.1969	-0.102
1.2	0.2504	0.2503	-0.040
1.25	0.3001	0.3001	0.000

6 Conclusions and Extensions

In this paper a class of nonlinear SV models has been proposed. The new class facilitates comparing and testing all standard parametric SV models. Since these alternative para-

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metric SV models coexist in the literature, our approach is useful in the sense that it can provide evidence to support or against some of the classical specifications. The MCMC approach is developed to provide a likelihood-based inference for the analysis of proposed models. Simulation studies confirm that the proposed MCMC algorithm works well for the new models. Empirical applications are performed first using daily dollar/pound exchange rate series. Empirical results show that all the standard SV models are rejected and hence suggest evidence of nonlinear stochastic volatility. Furthermore, model diagnostics indicate that, without sacrificing the overall goodness-of-fit the nonlinear SV model improves the fit to the data when the market has little movement. Moreover, this nonlinearity has important implications for pricing currency options. In particular the LN models tend to overprice out-of-money options. The deeper the out-of-money options, the larger the percentage bias. For all the other four major exchange rate series considered, the only standard "classical" SV model which cannot be rejected is the LN model. As a result, daily exchange volatility is well described by the LN distribution as its marginal distribution, consistent with the results found in recent literature (Andersen et al. (2001)).

Although in this paper we focus on one-factor SV models which are, at the same time, free of jumps, there are some possible extensions to our work. One possibility is to use the suggested methodology to analyze stock data. However, since stock data often display a strong volatility feedback feature as well as a higher kurtosis than that could be generated from the mixture of distributions, one has to incorporate a leverage effect and a fat tail error distribution into the nonlinear SV model. Also, because equity data often have more than one volatility factors (Gallant and Tauchen (2001)), one needs to apply the Box-Cox transformation to all the factors. Other interesting extensions would be to incorporate jumps and long memory volatility into the model; see for example Duffie, Pan and Singleton (2000) and Breidt, Crato and De Lima (1998). Thirdly, although we implement our theory based on the discrete time SV models in the connection with MCMC, one can estimate the continuous time N-SV models using alternative estimation methods. Finally, it would be interesting to evaluate the out-of-sample forecasting performances of the nonlinear SV models relative to standard SV models.

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