Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

A Robust LM Test for Spatial Error Components[1]

Zhenlin Yang

School of Economics, Singapore Management University

email: zlyang@smu.edu.sg

August, 2009

Abstract

This paper presents a modified LM test of spatial error components, which is shown to be robust against distributional misspecifications and spatial layouts. The proposed test differs from the LM test of Anselin (2001) by a term in the denominators of the test statistics. This term disappears when either the errors are normal, or the variance of the diagonal elements of the product of spatial weights matrix and its transpose is zero or approaches to zero as sample size goes large. When neither is true, as is often the case in practice, the effect of this term can be significant even when sample size is large. As a result, there can be severe size distortions of the Anselin's LM test, a phenomenon revealed by the Monte Carlo results of Anselin and Moreno (2003) and further confirmed by the Monte Carlo results presented in this paper. Our Monte Carlo results also show that the proposed test performs well in general.

Key Words: Distributional misspecifications; Robustness, Spatial layouts; Spatial error components; LM tests.

JEL Classification: C23, C5

[1]I would like to thank Ingmar Prucha, Steven Farber, and the conference participants of II World Conference of the Spatial Econometrics Association, Nov. 17-19, 2008, New York City, for the helpful comments. I am grateful to the support from a research grant (Grant number: C244/MSS8E010) from Singapore Management University.

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

1 Introduction.

The spatial error components model proposed by Kelejian and Robinson (1995) provides a useful alternative to the traditional spatial models with a spatial autoregressive (SAR) or a spatial moving average (SMA) error process, in particular in the situation where the range of spatial autocorrelation is constrained to close neighbors, e.g., spatial spillovers in the productivity of infrastructure investments (Kelejian and Robinson, 1997; Anselin and Moreno, 2003). Anselin (2001) derived an LM test for spatial error components based on the assumptions that the errors are normally distributed. Anselin and Moreno (2003) conducted Monte Carlo experiments to assess the finite sample behavior of Anselin's test and to compare it with other tests such as the GMM test of Kelejian and Robinson (1995) and Moran's (1950) I test, and found that none seems to perform satisfactorily in general. While Anselin and Moreno (2003) recognized that the LM test for spatial error components of Anselin (2001) is sensitive to distributional misspecifications and the spatial layouts, it is generally unclear on the exact cause of it and how this normal-theory based test performs under alternative distributions for the errors and under different spatial layouts.

In this paper, we present a modified LM test of spatial error components, which is shown to be robust against distributional misspecifications and spatial layouts. We show that the proposed test differs from the LM test of Anselin (2001) by a term in the denominator of the test statistic. This term disappears when either the errors are normal, or the variance of the diagonal elements of the product of spatial weights matrix and its transpose is zero or approaches to zero as sample size goes large. When neither is true, as is often the case in practice, we show that (i) if the elements of the weights matrix are fixed, this term poses a large sample effect in the sense that without this term Anselin's LM test does not converge to a correct level as sample size goes large; and (ii) if the elements of the weights matrix depend on sample size, this term poses a significant finite sample effect in the sense that without this term Anselin's LM test can have a large size distortion which gets smaller very slowly as sample size gets large.

Anselin and Bera (1998), Anselin (2001) and Florax and de Graaff (2004) provide excellent reviews on tests of spatial dependence in linear models. Section 2 introduces the spatial error components model and describes the existing test. Section 3 introduces a robust LM test for spatial error components. Section 4 presents Monte Carlo results. Section 5 concludes the paper.

2 The Spatial Error Components Model

The spatial error components (SEC) model proposed by Kelejian and Robinson (1995) takes the following form:

$$Y_n = X_n\beta + u_n \quad \text{with} \;\; u_n = W_n\nu_n + \varepsilon_n \tag{1}$$

where Y_n is an $n \times 1$ vector of observations on the response variable, X_n is an $n \times k$ matrix containing the values of explanatory (exogenous) variables, β is a $k \times 1$ vector of regression coefficients, W_n is an $n \times n$ spatial weights matrix, ν_n is an $n \times 1$ vector of errors that together with W_n incorporates the spatial dependence, and ε is an $n \times 1$ vector of location specific disturbance terms. The error components ν_n and ε_n are assumed to be independent, with independent and identically distributed (iid) elements of mean zero and variances σ_ν^2 and σ_ε^2, respectively. So, in this model the null hypothesis of no spatial effect can be either $H_0 : \sigma_\nu^2 = 0$, or $\theta = \sigma_\nu^2/\sigma_\varepsilon^2 = 0$. The alternative hypothesis can only be one-sided as σ_ν^2 is non-negative, i.e., $H_a : \sigma_\nu^2 > 0$, or $\theta > 0$. Anselin (2001) derived an LM test based on the assumptions that errors are normally distributed. The test is of the form

$$\text{LM}_{\text{SEC}} = \frac{\tilde{u}_n' W_n W_n' \tilde{u}_n / \tilde{\sigma}_\varepsilon^2 - T_{1n}}{(2T_{2n} - \frac{2}{n}T_{1n}^2)^{\frac{1}{2}}} \tag{2}$$

where $\tilde{\sigma}_\varepsilon^2 = \frac{1}{n}\tilde{u}_n'\tilde{u}_n$, $\tilde{u}_n$ is the vector of OLS residuals, $T_{1n} = \text{tr}(W_nW_n')$ and $T_{2n} = \text{tr}(W_nW_n'W_nW_n')$. Under H_0, the positive part of LM_{SEC} converges to that of $N(0,1)$. This means that the above one sided test can be carried out as per normal. Alternatively, if the squared version LM_{SEC}^2 is used, the reference null distribution of the test statistic for testing this one sided test is a chi-square mixture. See Verbeke and Molenberghs (2003) for a detailed discussion on tests where the parameter value under the null hypothesis falls on the boundary of parameter space.

Anselin and Moreno (2003) provide Monte Carlo evidence for the finite sample performance of LM_{SEC} and find that LM_{SEC} can be sensitive to distributional misspecifications and spatial layouts. Our Monte Carlo results given in Section 4 reinforce this point. However, the exact cause of this sensitivity is not clear, and also not clear is whether this sensitivity is of finite sample nature or large sample nature. The following heuristic arguments may help us gain some insights on these issus, and may shed light on the general way of robustifying the LM test given above, if necessary.

Note that $\tilde{\sigma}_\varepsilon^2 = \frac{1}{n}\tilde{u}_n'\tilde{u}_n = \frac{1}{n}u_n' M_n u_n$, where $M_n = I_n - X_n(X_n'X_n)X_n'$ and I_n is an $n \times n$ identity matrix. The statistic in (2) can be rewritten as

$$\text{LM}_{\text{SEC}} = \frac{\tilde{u}_n' W_n W_n' \tilde{u}_n - T_{1n}\tilde{\sigma}_\varepsilon^2}{\tilde{\sigma}_\varepsilon^2 (2T_{2n} - \frac{2}{N}T_{1n}^2)^{\frac{1}{2}}} \sim \frac{u_n' M_n (W_n W_n' - \frac{1}{n}T_{1n}I_n) M_n u_n}{\sigma_\varepsilon^2 (2T_{2n} - \frac{2}{n}T_{1n}^2)^{\frac{1}{2}}},$$

where $\sim$ denotes asymptotic equivalence. It follows from Lemma A.1 that under H_0,

$$\text{E}(\text{LM}_{\text{SEC}}) \approx \frac{\text{tr}(W_n W_n' M_n) - \frac{n-k}{n}T_{1n}}{(2T_{2n} - \frac{2}{n}T_{1n}^2)^{\frac{1}{2}}} \neq 0, \quad \text{and} \tag{3}$$

$$\text{Var}(\text{LM}_{\text{SEC}}) \approx \frac{\kappa_\varepsilon \sum_{i=1}^n c_{n,ii}^2 + 2\text{tr}(C_n^2)}{2T_{2n} - \frac{2}{n}T_{1n}^2} \neq 1, \tag{4}$$

where $C_n = M_n(W_n W_n' - \frac{1}{n}T_{1n}I_n)M_n$, $\{c_{n,ii}\}$ are the diagonal elements of C_n, and κ_ε is the excess kurtosis of ε_i. This shows the mean and variance of LM_{SEC} are both different from their nominal values. The intriguing questions are: (i) how big these differences can be, (ii) whether these differences shrink as n gets large, and (iii) what causes such differences.

Using Lemma A.3, one can easily see that $\text{tr}(W_n W_n' M_n) - \frac{n-k}{n}T_{1n} = O(1)$, and $\text{tr}(C_n^2) = T_{2n} - \frac{1}{n}T_{1n}^2 + O(1)$. It follows that

$$\begin{aligned}\text{E}(\text{LM}_{\text{SEC}}) &= O((T_{2n} - T_{1n}^2/n)^{-\frac{1}{2}}), \quad \text{and} \\ \text{Var}(\text{LM}_{\text{SEC}}) &= 1 + \frac{\kappa_\varepsilon \sum_{i=1}^n c_{n,ii}^2}{2T_{2n} - \frac{2}{n}T_{1n}^2} + O((T_{2n} - T_{1n}^2/n)^{-1}).\end{aligned}$$

Hence, the mean bias (of LM_{SEC}) depends on the magnitude of $(T_{2n} - \frac{1}{n}T_{1n}^2)^{-\frac{1}{2}}$, and the variance bias depends on the magnitude of both $(T_{2n} - \frac{1}{n}T_{1n}^2)^{-1}$ and $\kappa_\varepsilon \sum_{i=1}^n c_{n,ii}^2$. As n gets large, $(T_{2n} - \frac{1}{n}T_{1n}^2)^{-\frac{1}{2}}$ and $(T_{2n} - \frac{1}{n}T_{1n}^2)^{-1}$ shrink, but $\kappa_\varepsilon \sum_{i=1}^n c_{n,ii}^2/(T_{2n} - \frac{1}{n}T_{1n}^2)$ may not, leaving a permanent bias in variance, due to a non-zero κ_ε (non-normality) and a non-zero $\frac{1}{n}\sum_{i=1}^n c_{n,ii}^2$ (non-zero variability in the diagonal elements of $W_n W_n'$); see next section for formal results. In summary, LM_{SEC} has to be corrected for its general validity, and a natural way to correct LM_{SEC} is to find out the mean and variance of the quadratic form $u_n' C_n u_n$ and then normalize.[2]

[2]This is along the idea of Konenker (1981), and the resulted tests are robust against distributional misspecifications. A related problem, not considered in this paper, is to develop LM tests that are robust against local misspecification as in Bera and Yoon (1993), Bera et al. (2001), etc.

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

3 Robust LM Test for Spatial Error Components

We now present a robustified version of the LM test statistic given in (2). The following basic regularity conditions are necessary for studying the asymptotic behavior of the test statistics.

Assumption 1: *The innovations* $\{\varepsilon_i\}$ *are iid with mean zero, variance* σ_ε^2*, and excess kurtosis* κ_ε*. Also, the moment* $E|\varepsilon_i|^{4+\eta}$ *exists for some* $\eta > 0$.

Assumption 2: *The elements* $w_{n,ij}$ *of* W_n *are at most of order* h_n^{-1} *uniformly for all i and j, with the rate sequence* $\{h_n\}$*, bounded or divergent but satisfying* $h_n/n \to 0$ *as* $n \to \infty$*. The sequence* $\{W_n\}$ *are uniformly bounded in both row and column sums. As normalizations, the diagonal elements* $w_{n,ii} = 0$*, and* $\sum_j w_{n,ij} = 1$ *for all i.*

Assumption 3: *The elements of the* $n \times k$ *matrix* X_n *are uniformly bounded for all n, and* $\lim_{n\to\infty} \frac{1}{n} X_n' X_n$ *exists and is nonsingular.*

The Assumption 1 corresponds to one assumption of Kelejian and Prucha (2001) for their central limit theorem of linear-quadratic forms. Assumption 2 corresponds to one assumption in Lee (2004a) which identifies the different types of spatial dependence. Typically, one type of spatial dependence corresponds to the case where each unit has a fixed number of neighbors such as the Rook or Queen contiguity and in this case h_n is bounded, and the other type of spatial dependence corresponds to the case where the number of neighbors each spatial unit has grows as n goes to infinity, such as the case of group interaction. In this case h_n is divergent. To limit the spatial dependence to a manageable degree, it is thus required that $h_n/n \to 0$ as $n \to \infty$.

Theorem 1: *If* W_n*,* $\{\varepsilon_i\}$ *and* X_n *of Model (1) satisfy the Assumptions 1-3, then a robust LM test statistic for testing* $H_0 : \sigma_\nu^2 = 0$ *vs* $H_a : \sigma_\nu^2 > 0$ *takes the form*

$$\mathrm{LM}_{\mathrm{SEC}}^* = \frac{\tilde{u}_n' W_n W_n' \tilde{u}_n / \tilde{\sigma}_\varepsilon^2 - S_{1n}}{(\tilde{\kappa}_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}} \tag{5}$$

where $S_{1n} = \frac{n}{n-k}\mathrm{tr}(W_n W_n' M_n)$, $S_{2n} = \sum_i a_{n,ii}^2$ *with* $\{a_{n,ii}\}$ *being the diagonal elements of* $A_n = M_n(W_n W_n' - \frac{1}{n} S_{1n} I_n) M_n$, $S_{3n} = 2\mathrm{tr}(A_n^2)$*, and* $\tilde{\kappa}_\varepsilon$ *is the excess sample kurtosis of* $\tilde{u}_n$*. Under* H_0*, (i) the positive part of* LM_{SEC}^* *converges to that of* $N(0,1)$*, and (ii)* $\mathrm{LM}_{\mathrm{SEC}}^*$ *is asymptotically equivalent to* $\mathrm{LM}_{\mathrm{SEC}}$ *when* $\kappa_\varepsilon = 0$.

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Proof: Proof of the theorem needs the four lemmas given in Appendix. We have

$$\text{LM}^*_{\text{SEC}} = \frac{\tilde{u}_n' W_n W_n' \tilde{u}_n/\tilde{\sigma}_\varepsilon^2 - S_{1n}}{(\tilde{\kappa}_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}} = \frac{\tilde{u}_n' W_n W_n' \tilde{u}_n - \tilde{\sigma}_\varepsilon^2 S_{1n}}{\sigma_\varepsilon^2(\kappa_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}} \cdot \frac{\sigma_\varepsilon^2(\kappa_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}}{\tilde{\sigma}_\varepsilon^2(\tilde{\kappa}_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}}$$

and $\tilde{u}_n' W_n W_n' \tilde{u}_n - \tilde{\sigma}_\varepsilon^2 S_{1n} = \tilde{u}_n'(W_n W_n' - \frac{1}{n} S_{1n} I_n)\tilde{u}_n = u_n' M_n (W_n W_n' - \frac{1}{n} S_{1n} I_n) M_n u_n = u_n' A_n u_n$. Under H_0, the elements of u_n are iid, we have $\text{E}(u_n' A_n u_n) = \sigma_\varepsilon^2 \text{tr}(A_n) = 0$. By Assumption 1 and Lemma A.1 in Appendix and noticing that the matrix A_n is symmetric, we have $\text{Var}(u_n' A_n u_n) = \sigma_\varepsilon^4(\kappa_\varepsilon S_{2n} + S_{3n})$. Now, Assumption 2 ensures that the elements of $W_n W_n'$ are uniformly of order h_n^{-1}, and Lemma A.2 shows that M_n is uniformly bounded in both row and column sums. It follows from Lemma A.4(iii) that $\frac{1}{n} S_{1n} = O(h_n^{-1})$. Assumption 2 and Lemma A.4 lead to that $\{W_n W_n'\}$ are uniformly bounded in both row and column sums. Thus, $\{W_n W_n' - \frac{1}{n} S_{1n} I_n\}$ are uniformly bounded in both row and column sums. Finally, Lemma A.4(i) gives that $\{A_n\}$ are uniformly bounded in both row and column sums. It follows that the central limit theorem of linear-quadratic forms of Kelejian and Prucha (2001) is applicable, which gives

$$\frac{\tilde{u}_n' W_n W_n' \tilde{u}_n - \tilde{\sigma}_\varepsilon^2 S_{1n}}{\sigma_\varepsilon^2(\kappa_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}} \xrightarrow{D} N(0, 1).$$

Now, it is easy to show that under H_0 $\tilde{\sigma}_\varepsilon^2 \xrightarrow{p} \sigma_\varepsilon^2$ and that $\tilde{\kappa}_\varepsilon \xrightarrow{p} \kappa_\varepsilon$.[3] Thus,

$$\frac{\sigma_\varepsilon^2(\kappa_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}}{\tilde{\sigma}_\varepsilon^2(\tilde{\kappa}_\varepsilon S_{2n} + S_{3n})^{\frac{1}{2}}} \xrightarrow{p} 1.$$

This finishes the proof of Part (i).

For Part (ii), it suffices to show that $S_{1n} \sim T_{1n}$ and $S_{3n} \sim 2T_{2n} - \frac{2}{n} T_{1n}^2$. The former follows from Lemma A.3(i), i.e., $S_{1n} = \frac{n}{n-k} \text{tr}(W_n W_n' M_n) = \text{tr}(W_n W_n') + O(1)$. For the latter, write $A_n = M_n A_n^o M_n$, where $A_n^o = W_n W_n' - \frac{1}{n} S_{1n} I_n$. We have

$$\begin{aligned} \text{tr}(A_n^2) &= \text{tr}(M_n A_n^o M_n M_n A_n^o M_n) \\ &= \text{tr}[(A_n^o M_n)^2] \\ &= \text{tr}((A_n^o)^2) + O(1) \ \text{ by Lemma A.3(iii)} \end{aligned}$$

[3]Let $\varepsilon_n^* = (I_n - M_n)\varepsilon_n$. Then $\tilde{u}_n = \varepsilon_n - \varepsilon_n^*$. The 2nd and 4th sample moments of $\tilde{u}_n$ become $\frac{1}{n}\tilde{u}_n'\tilde{u}_n = \frac{1}{n}\varepsilon_n'\varepsilon_n - \frac{1}{n}\varepsilon_n^{*\prime}\varepsilon_n^*$ and $\frac{1}{n}(\tilde{u}_n \odot \tilde{u}_n)'(\tilde{u}_n \odot \tilde{u}_n) = \frac{1}{n}(\varepsilon_n \odot \varepsilon_n)'(\varepsilon_n \odot \varepsilon_n) + \frac{4}{n}(\varepsilon_n \odot \varepsilon_n^*)'(\varepsilon_n \odot \varepsilon_n^*) + \frac{1}{n}(\varepsilon_n^* \odot \varepsilon_n^*)'(\varepsilon_n^* \odot \varepsilon_n^*) - \frac{4}{n}(\varepsilon_n \odot \varepsilon_n)'(\varepsilon_n \odot \varepsilon_n^*) + \frac{2}{n}(\varepsilon_n \odot \varepsilon_n)'(\varepsilon_n^* \odot \varepsilon_n^*) - \frac{4}{n}(\varepsilon_n \odot \varepsilon_n^*)'(\varepsilon_n^* \odot \varepsilon_n^*)$, where $\odot$ denotes the Hadamard product. As the elements of $I_n - M_n$ are $O(n^{-1})$ (Assumption 3), all the terms involving ε_n^* are $o_p(1)$.

$$
\begin{aligned}
&= \operatorname{tr}[(W_nW_n' - \frac{1}{n}S_{1n}I_n)(W_nW_n' - \frac{1}{n}S_{1n}I_n)] + O(1) \\
&= \operatorname{tr}(W_nW_n'W_nW_n') - \frac{2}{n}S_{1n}\operatorname{tr}(W_nW_n') + \frac{1}{n^2}S_{1n}^2\operatorname{tr}(I_n) + O(1) \\
&= T_{2n} - \frac{1}{n}T_{1n}^2 + O(1),
\end{aligned}
$$

which shows that $S_{3n} = 2\operatorname{tr}(A_n^2) = 2T_2 - \frac{2}{n}T_1^2 + O(1)$. *Q.E.D.*

From Theorem 1 we see that when n is large LM^*_{SEC} differs from LM_{SEC} essentially by a term $\tilde{\kappa}_\varepsilon S_{n2}$ in the denominators of the test statistics. This term becomes (asymptotically) negligible when $\kappa_\varepsilon = 0$, which occurs when ε is normal. This is because $S_{2n} = \sum_{i=1}^n a_{ii}^2 \leq \sum_{i=1}^n \sum_{j=1}^n a_{ij}^2 = \frac{1}{2}S_{3n}$. When $\kappa_\varepsilon \neq 0$, which typically occurs when ε is non-normal, $\tilde{\kappa}_\varepsilon = O_p(1)$. In this case it becomes unclear whether $\tilde{\kappa}_\varepsilon S_{2n}$ is also asymptotically negligible. The key is the relative magnitudes of S_{2n} and S_{3n}, which depend on many factors. The following corollary summarizes the detailed results.

Corollary 1: *Under the assumptions of Theorem 1, we have*

(i) If h_n is bounded, then $S_{2n} \sim S_{3n}$;

(ii) If $h_n \to \infty$ as $n \to \infty$, then $S_{2n} = O(n/h_n^2)$ and $S_{3n} = O(n/h_n)$;

(iii) $\frac{1}{n}S_{2n} \sim \sigma_n^2(r) \equiv \frac{1}{n}\sum_{i=1}^n (r_{n,i} - \bar{r}_n)^2$, where $\bar{r}_n = \sum_{i=1}^n r_{n,i}$ with $\{r_{n,i}\}$ being the diagonal elements of W_nW_n'.

Proof: For (i) and (ii), note that the elements of $W_nW_n' - \frac{1}{n}S_{1n}I_n$ are at most of order $O(h_n^{-1})$ uniformly. By Lemma A.4 (iii), the elements of A_n are also at most of order $O(h_n^{-1})$ uniformly as $\{M_n\}$ are uniformly bounded in both row and column sums. This leads to $S_{2n} = O(n/h_n^2)$. Furthermore, as A_n itself is uniformly bounded in both row and column sums (see the proof of Theorem 1), Lemma A.4 (iii) shows that the elements of A_n^2 are at most of order $O(h_n^{-1})$ uniformly. This shows that $S_{3n} = O(n/h_n)$.

To prove (iii), note that W_n is row normalized. We have $\frac{1}{n}S_{1n} \sim \frac{1}{n}T_{1n} = \frac{1}{n}\operatorname{tr}(W_nW_n') = \frac{1}{n}\sum_{i=1}^n r_{n,i} = \bar{r}_n$. From Lemma A.3 (vi) we have

$$
\frac{1}{n}S_{2n} = \frac{1}{n}\sum_{i=1}^n a_{ii}^2 \sim \frac{1}{n}\sum_{i=1}^n \Big((W_nW_n' - \frac{1}{n}S_{1n}I_n)_{ii}\Big)^2 \sim \frac{1}{n}\sum_{i=1}^n (r_{n,i} - \bar{r}_n)^2 \equiv \sigma_n^2(r), \tag{6}
$$

which completes the proof of Corollary 1.

The results of Corollary 1 lead to some important conclusions. Firstly, when h_n is bounded, $S_{2n} \sim S_{3n}$. Hence, if $\kappa_\varepsilon \neq 0$, the asymptotic variance of LM_{SEC} will be larger than 1, leading to over-rejections of null hypothesis when errors are nonnormal. This point is confirmed by the Monte Carlo results given in Table 1 and, in particular, Table 3a, where we see that the empirical coverage of the LM_{SEC} test under non-normal errors increases with n, reaching to around {24%, 21%, 15%} when $n = 1512$ (Table 3a, dgp=3) for tests of nominal levels {10%, 5%, 1%}. In contrast, the LM^*_{SEC} test performs very well in general.

Secondly, when h_n increases with n, S_{3n} is generally of higher order in magnitude than S_{2n}. Hence, as n increases S_{3n} eventually becomes the dominate term in the denominator of the test statistic LM^*_{SEC}, and LM^*_{SEC} would eventually behave like LM_{SEC} even when there exists excess kurtosis or non-normality in general. However, a detailed examination shows that the finite sample difference between LM^*_{SEC} and LM_{SEC} could still be large even when the sample size is very large. Taking, for example, $h_n = n^{0.25}$, we have $S_{2n} = O(n^{0.5})$ and $S_{3n} = O(n^{0.75})$. It follows that with $\tilde{\kappa}_\varepsilon$ being $O_p(1)$ the excess kurtosis may have significant impact on the variance and hence on the test statistic even when n is very large. The Monte Carlo simulation results given in Table 4a indeed confirm this point, where we see huge size distortions of LM_{SEC} in the cases of non-normal errors. Although the magnitude of size distortion seems decreasing as n increases the empirical sizes of LM_{SEC} can still be around {16%, 11%, 6%} corresponding to nominal sizes {10%, 5%, 1%} even when n is 1500 (Table 4a, dgp=3). Taking another example with $h_n = n^{0.75}$, we have $S_{2n} = O(n^{-0.5})$ and $S_{3n} = O(n^{0.25})$. Apparently under this situation, the impact of the $\tilde{\kappa}_\varepsilon S_{2n}$ term is negligible. However, as shown in Corollary 2 and confirmed by the Monte Carlo results in Table 4c, the finite sample bias in the mean of LM_{SEC} starts to have a much bigger impact which distorts greatly the null distribution of LM_{SEC}, making the test severely under-sized.[4]

Thirdly and perhaps more importantly, the result (iii) of Corollary 1 shows that the variance $\sigma_n^2(r)$ (variability in general) of the diagonal elements of $W_n W_n'$ plays a key role in the behavior of the test statistics. When $\sigma_n^2(r) = 0$ or $\sigma_n^2(r) \to 0$ as $n \to \infty$, $\text{LM}_{\text{SEC}} \sim \text{LM}^*_{\text{SEC}}$. When $\sigma_n^2(r) \neq 0$ even when n is large, LM_{SEC} may differ from LM^*_{SEC}, and as n goes large the difference may grow (as in the case where h_n is bounded and errors

[4]Note that one spatial layout leading to $h_n = n^\delta, 0 < \delta < 1$, is the so-called group interaction (see, e.g., Lee, 2007). In this case h_n corresponds to the average group size. If $\delta = 0.25$, for example, then there are many groups but each group contains only a few members although the number of units in each group grows with n. If $\delta = 0.75$, however, then there are a few groups, but each group contains many members.

are nonnormal), or may shrink (as in the case where h_n is unbounded and the errors are nonnormal). It is interesting to note that in the framework of spatial contiguity the ith diagonal element $r_{n,i}$ of $W_n W_n'$ is the reciprocal of the number of neighbors the ith spatial unit has; and that in the framework of group interaction, $r_{n,i}$ is the reciprocal of the size of the ith group. Hence, in these situations, the variability of $\{r_{n,i}\}$ boils down to whether the number of neighbors or whether the group size varies across the spatial units, and whether these variations disappear as the sample size goes large.

We close this section by formalizing the heuristic arguments given in Section 2 regarding the biases in the mean and variance of LM_{SEC}, which complements Corollary 1 for a more detailed understanding on why the existing LM test often under-performs.

Corollary 2: *Under the assumptions of Theorem 1, (i)* $\text{E}(\text{LM}_{\text{SEC}}) = -O((h_n/n)^{\frac{1}{2}})$, *and (ii)* $\text{Var}(\text{LM}_{\text{SEC}}) = 1 + \kappa_\varepsilon O(h_n^{-1}) - O(h_n/n)$, *where the three big O terms are all positive.*

Proof: The numerator of $\text{E}(\text{LM}_{\text{SEC}})$ given in (3) is $\text{tr}(W_n W_n' M_n) - \frac{n-k}{n} T_{1n} = \frac{k}{n} T_{1n} - \text{tr}[(X_n' X_n)^{-1}(X_n' W_n W_n' X_n)] = -O(1)$ as the two $k \times k$ matrices $\frac{1}{n} X_n' X_n$ and $\frac{1}{n} X_n' W_n W_n' X_n$ are either positive definite or positive semidefinite, and both have uniformly bounded elements. From the proof of Theorem 1, $T_{2n} - \frac{1}{n} T_{1n}^2 = O(n/h_n)$ and is strictly positive. Thus the result in (i) follows. For (ii), as $T_{1n} \sim S_{1n}$ (Lemma A.3(i)), $C_n \sim A_n$, and thus $\sum_{i=1}^n c_{ii}^2 = O(n/h_n^2) > 0$ (Corollary 1). Now, letting $C_n^o = W_n W' - \frac{1}{n} T_{1n} I_n$, we have,

$$
\begin{aligned}
\text{tr}(C_n^2) &= \text{tr}(M_n C_n^o M_n C_n^o) \\
&= \text{tr}(C_n^o M_n C_n^o) - \text{tr}[(X_n' X_n)^{-1}(X_n' C_n^o M_n C_n^o X_n)] \\
&= \text{tr}(C_n^{o2}) - \text{tr}[(X_n' X_n)^{-1}(X_n' C_n^{o2} X_n)] - \text{tr}[(X_n' X_n)^{-1}(X_n' C_n^o M_n C_n^o X_n)] \\
&= T_{2n} - \frac{1}{n} T_{1n}^2 - O(1),
\end{aligned}
$$

because the two $k \times k$ matrices $\frac{1}{n} X_n' C_n^{o2} X_n$ and $\frac{1}{n} X_n' C_n^o M_n C_n^o X_n$ are both positive semidefinite and have uniformly bounded elements. The result in (ii) thus follows.

It turns out that the results given in Corollary 2 are rather indicative on the finite sample behavior of LM_{SEC}. First, there is a sizable downward bias in the mean of LM_{SEC}, which shrinks with n but the speed of shrinkage can be quite slow when h_n is of order close to n. For example, when $h_n = n^{0.75}$, the bias in mean $= -O((h_n/n)^{\frac{1}{2}}) = -O(n^{-0.125})$ (see, e.g., Table 4c). Second, there is a bias in the variance of $\mathrm{LM_{SEC}}$, which can be permanent and upward when h_n is bounded, sizable and upward when h_n is unbounded but small, and sizable and downward when h_n is unbounded and big. For example, when h_n is bounded, the bias $= O(1)$ if errors are nonnormal (see, e.g., Table 3a); when $h_n = n^{0.25}$ the bias $= O(h_n^{-1}) = O(n^{-0.25})$ when errors are nonnormal (see Table 4a); when $h_n = n^{0.75}$ the bias $= -O(h_n/n) = -O(n^{-0.25})$, irrespective of the error distributions (see Table 4c).

4 Monte Carlo Results

The finite sample performance of the test statistics introduced in this paper is evaluated based on a series of Monte Carlo experiments under a number of different error distributions and a number of different spatial layouts. Comparisons are made between the newly introduced test $\mathrm{LM^*_{SEC}}$ and the existing $\mathrm{LM_{SEC}}$ of Anselin (2001) to see the improvement of the new tests in the situations where there is a distributional misspecification. The Monte Carlo experiments are carried out based on the following data generating process:

$$Y_i = \beta_0 + X_{1i}\beta_1 + X_{2i}\beta_2 + u_i,$$

where X_{1i}'s are drawn from $10U(0,1)$ and X_{2i}'s are drawn from $5N(0,1)+5$. Both are treated as fixed in the experiments. The parameters $\beta = \{5, 1, 0.5\}'$ and $\sigma = 1$. Seven different sample sizes are considered for each combination of error distributions and spatial layouts. Each set of Monte Carlo results (each row in the table) is based on 10,000 samples.

Spatial Weight Matrix. Three general spatial layouts are considered in the Monte Carlo experiments. The first is based on Rook contiguity, the second is based on Queen contiguity and the third is based on the notion of group interactions.

The detail for generating the W_n matrix under rook contiguity is as follows: (i) allocate symbolically the n spatial units indexed by $\{1, 2, \cdots, n\}$ randomly into a lattice of $\ell \times m (\geq n)$ squares, (ii) let $W_{ij} = 1$ if the unit j is in a square which is on immediate left, or right, or above, or below the square which contains the unit i, otherwise $W_{ij} = 0, i, j = 1, \cdots, n$, to form an $n \times n$ matrix, and (iii) divide each element of this matrix by its row sum to give W_n. So, under Rook contiguity there are 4 neighbors for each of the inner units, 3 for a unit on the edge, and 2 for a corner unit. The W_n matrix under Queen contiguity can be generated in a similar way as that under rook contiguity, but with additional neighbors which share a common vertex with the unit of interest. In this case a inner unit has 8 neighbors, an edge unit has 5, and a corner unit has 3. Thus the variability of the number of neighbors is greater under Queen than under Rook contiguity. For irregular spatial contiguity, the variation in number of neighbors is greater.

For both regular Rook and Queen spatial layouts, whether ℓ is fixed makes a difference. Thus, we consider two cases: (i) $\ell = 5$, and (ii) $\ell = m$. It is easy to show that for spatial units arranged in a regular $\ell \times m$ lattice, Rook contiguity leads to $\sigma_n^2(r)$ defined in (5) as

$$n\sigma_n^2(r) = \left(\frac{\ell + m + 2}{6\ell m}\right)^2 + 2\left(\frac{1}{12} - \frac{m + \ell + 2}{6\ell m}\right)^2 (k + \ell - 4) + 4\left(\frac{1}{4} - \frac{m + \ell + 2}{6\ell m}\right). \quad (7)$$

With $n = \ell m$, it is easy to see that, if ℓ is fixed, then $m = O(n)$ and $\sigma_n^2(r) = O(1)$ for $\ell > 2$; if both ℓ and m go large as $n \to \infty$, then $\sigma_n^2(r) = o(1)$. Thus the case of either $\ell > 2$ or $m > 2$ fixed leads to a permanent variablity in $\{r_{n,i}\}$, whereas the case of neither ℓ nor m fixed leads to a temporary or finite sample variability in $\{r_{n,i}\}$ which disappears as $n \to \infty$. Similarly, under Queen contiguity, we have

$$\begin{aligned} n\sigma_n^2(r) &= \left(\frac{9(\ell + m) - 14}{60\ell m}\right)^2 (\ell - 2)(m - 2) + 2\left(\frac{3}{40} - \frac{9(\ell + m) - 14}{60\ell m}\right)^2 (\ell + m - 4) \\ &\quad + 4\left(\frac{5}{24} - \frac{9(\ell + m) - 14}{60\ell m}\right), \end{aligned} \quad (8)$$

which gives $\sigma_n^2(r) = O(1)$ when either $k > 2$ or $m > 2$ is fixed, and $\sigma_n^2(r) = o(1)$ when neither ℓ nor m is fixed.

To generate the W_n matrix according to the group interaction scheme, suppose we have ℓ groups of sizes $\{m_1, m_2, \cdots, m_\ell\}$. Define $W_n = \text{diag}\{W_j/(m_j - 1), j = 1, \cdots, \ell\}$, a matrix formed by placing the submatrices W_j along the diagonal direction, where W_j is an $m_j \times m_j$ matrix with ones on the off-diagonal positions and zeros on the diagonal positions. Note that $n = \sum_{j=1}^{\ell} m_j$. We consider three different methods of generating the group sizes.

The first is that the group size is a constant across the groups and with respect to the sample size n, i.e., $m_1 = m_2 = \cdots = m_\ell = m$, where m is free of n. In this case increasing n means having more groups of the same size m. The second is that the group size changes across the groups but not with respect to the sample size n. In this case, increasing n means having more groups of sizes m_1 or $m_2, \cdots$. The third method is the most complicated one and is described as follows: (a) calculate the number of groups according to $\ell = \text{Round}(n^\epsilon)$, and the approximate average group size $m = n/\ell$, (b) generate the group sizes $(m_1, m_2, \cdots, m_\ell)$ according to a discrete uniform distribution from $m/2$ to $3m/2$, (c) adjust the group sizes so that $\sum_{j=1}^{\ell} m_j = n$, and (d) define $W = \text{diag}\{W_j/(m_j - 1), j = 1, \cdots, \ell\}$ as described above. In our Monte Carlo experiments, we choose $\epsilon = 0.25, 0.50$, and 0.75, representing respectively the situations where (i) there are few groups and many spatial units in a group, (ii) the number of groups and the sizes of the groups are of the same magnitude, and (iii) there are many groups of few elements in each. Clearly, the first method leads to $\sigma_n^2(r) = 0$, the second method leads to $\sigma_n^2(r) = O(1)$, and the third method leads to $\sigma_n^2(r) = o(1)$. In all spatial layouts described above, only the last one gives h_n unbounded with $h_n = n^{1-\epsilon}$.

Error Distributions. The reported Monte Carlo results correspond to the following four error distributions: (i) standard normal, (ii) mixture normal, (iii) log-normal, and (iv) chi-square, where (ii)-(iv) are all standardized to have mean zero and variance one, and (i)-(iii) correspond to those used in Kelejian and Prucha (1999) and Anselin and Moreno (2003). The standardized normal-mixture variates are generated according to

$$u_i = ((1 - \xi_i)Z_i + \xi_i \tau Z_i)/(1 - p + p * \tau^2)^{0.5},$$

where ξ is a Bernoulli random variable with probability of success p and Z_i is standard normal independent of ξ. The parameter p in this case also represents the proportion of mixing the two normal populations. In our experiments, we choose $p = 0.05$, meaning that 95% of the random variates are from standard normal and the remaining 5% are from another normal population with standard deviation τ. We choose $\tau = 5$, or 10 to simulate the situation where there are gross errors in the data. The standardized lognormal random variates are generated according to

$$u_i = [\exp(Z_i) - \exp(0.5)]/[\exp(2) - \exp(1)]^{0.5}.$$

This gives an error distribution that is both skewed and leptokurtic. The normal mixture gives an error distribution that is still symmetric like normal but leptokurtic. The standardized chi-square random variates are generated in a similar fashion. Other non-normal distributions, such as normal-gamma mixture, are also considered and the results are available from the author upon request.

Size of the Tests. Tables 1-4 summarize the empirical sizes of the two tests. The results in Table 1 correspond to Rook or Queen contiguity with ℓ fixed at 5. In this case, h_n is bounded, $\sigma_n^2(r) = O(1)$ as shown in (7) and (8), and $\tilde{\kappa} = O_p(1)$ if errors are nonnormal, resulting in $\tilde{\kappa} S_{2n} \sim S_{3n}$. This means that the difference between the two tests may not vanish as n goes large. Monte Carlo results in Table 1 indeed show that when errors are non-normal, the null distribution of the LM_{SEC} (except the mean) diverges away from $N(0,1)$ as n increases; in contrast, the null distribution of LM^*_{SEC} converges to $N(0,1)$. Comparing the results in Table 1b with the results in Table 1a, we see that the null performance of LM_{SEC} gets poorer because in this case $\sigma_n^2(r)$ is larger. This is not the case for the new test LM^*_{SEC}. It is interesting to note that LM^*_{SEC} seems perform better than LM_{SEC} even when the errors are drawn from a normal population. The results reported in Table 2 also correspond to Rook or Queen contiguity but with $\ell = m = \sqrt{n}$. In this case $\sigma_n^2(r) = o(1)$ as shown in (7) and (8), and hence the term $\tilde{\kappa} S_{2n}$ is negligible relative to S_{3n} when n is large, and the two statistics should behave similarly. The results confirm this theoretical finding although the comparative advantage seems go to the new statistic.

The results reported in Table 3 correspond to group interaction spatial layout with group sizes fixed at $\{2,3,4,5,6,7\}$ for Table 3a, and at 5 for Table 3b. In these situations, increasing n means having more groups of the same sizes, and thus h_n is bounded. The first case gives $\sigma_n^2(r) = O(1)$ where our theory predicts that the variance of LM_{SEC} has a bias of order $O(1)$. Indeed, the results in Table 3a show that when errors are non-normal, the LM_{SEC} test can perform quite badly with its variance and empirical sizes increasing with n quite rapidly and being far above their nominal levels. In contrast, the LM^*_{SEC} test performs very well in all situations. The second case gives $\sigma_n^2(r) = 0$. As our theory predicted, the results given in Table 3b confirm that LM_{SEC} performs well when n is large.

The results given in Table 4 correspond to group interaction again, but this time the group size varies across groups and increases with n, which results in an unbounded h_n. Although the theory predicts that the two tests should behave similarly when n is large and the results indeed show some signs of convergence in size, the LM_{SEC} can still perform badly even when $n = 1500$, in particular when h_n is either small (e.g., there are many small groups) or large (e.g., there are a few large groups). In contrast, the proposed test LM^*_{SEC}

performs very well in general. Table 4a corresponds to $\ell = n^{0.75}(h_n = n^{0.25})$, i.e., there are many small groups. As the theory predicted, the bias in mean of $\mathrm{LM}_{\mathrm{SEC}}$ is small, but the bias in variance (upward) can be large when errors are non-normal, resulting an overall poor performance of $\mathrm{LM}_{\mathrm{SEC}}$ when errors are non-normal. This point is clearly shown by the Monte Carlo results in Table 4a. Table 4b corresponds $\ell = n^{0.5}(h_n = n^{0.5})$. As h_n becomes larger, the effect of downward mean bias of $\mathrm{LM}_{\mathrm{SEC}}$ starts to show. Also, when n is not large, the downward bias term in the variance of $\mathrm{LM}_{\mathrm{SEC}}$ overtakes the upward bias term, making the overall variance of $\mathrm{LM}_{\mathrm{SEC}}$ being smaller than 1 and the empirical frequencies of rejection lower than the nominal levels. Table 4c corresponds $\ell = n^{0.25}(h_n = n^{0.75})$, i.e., there are a few large groups. In this case, the downward biases in the mean and variance of $\mathrm{LM}_{\mathrm{SEC}}$ become more severe, resulting in the empirical frequencies of rejection much lower than the nominal levels. As n increases, the convergence of $\mathrm{LM}_{\mathrm{SEC}}$ to $N(0,1)$ is very slow. In contrast, the overall performance of $\mathrm{LM}^*_{\mathrm{SEC}}$ is quite acceptable, except that the heavy spatial dependence causes the distribution $\mathrm{LM}^*_{\mathrm{SEC}}$ being flatter in far tails than $N(0,1)$.

Power of the proposed test. The Monte Carlo results given above show that the null distribution of $\mathrm{LM}_{\mathrm{SEC}}$ depends very much on the error distribution and on the spatial layouts and can be far from the nominal distribution, namely $N(0,1)$, even when sample size is large. This means that $\mathrm{LM}_{\mathrm{SEC}}$ is generally invalid unless one is certain that the errors are normally distributed. In contrast, the null distribution of $\mathrm{LM}^*_{\mathrm{SEC}}$ is generally quite close to $N(0,1)$, hence should be recommended for practical applications. One issue left is the power of the test. Given the non-robust nature and poor finite sample performance of $\mathrm{LM}_{\mathrm{SEC}}$, it is only necessary to investigate the power property of the proposed test. As seen from the Monte Carlo results given above, the empirical sizes of $\mathrm{LM}^*_{\mathrm{SEC}}$ at the 10% nominal level are most stable, thus the power study will be carried at the 10% nominal size.

Table 5a reports the empirical powers of the proposed test under Rook and Queen spatial contiguity, and Table 5b reports the results under group interaction. Some observations are in order: (i) power increases as σ_v increases from 0, (ii) power decreases as the degree of spatial dependence increases (Table 5a, Rook vs Queen; and Table 5b, $\delta = .25$ vs $\delta = .5$ and .75), (iii) variation in group sizes does not affect much on the power (Table 5a, $\ell = 5$ vs $\ell = \sqrt{n}$; Table 5b, panel 1 vs panel 2), and (iv) power increases sharply with n.

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

5 Conclusions and Discussions

A modified LM test of spatial error components is proposed, which is shown to be robust against non-normality and spatial layout. Monte Carlo results show that the proposed test performs very well in general and clearly outperforms the existing LM test. It is shown that the variation in the diagonal elements of $W_n W_n'$ (reciprocals of group sizes in case of group interaction) plays a key role in the robustness of the LM test. When this variation is zero or approaches to zero as n goes large, the existing test is robust asymptotically, but otherwise it is not. However, our results show that even when the existing test is asymptotically robust, its finite sample performance can be quite sensitive to error distributions and spatial layouts. In contrast, the proposed test performs well in all situations. It is interesting to note that the proposed test statistic is free of the skewness measure, showing that skewness of the error distribution does not affect the asymptotic behavior of the LM tests.

Naturally, one may think that the same idea may be applied to a closely related but more popular models, such as the linear regression with a SAR or SMA error process, to obtain a robust LM test. However, it turns out that the LM test for this model is already asymptotically robust against the distributional misspecifications. This follows either from the results of Kelejian and Prucha (2001), or can be seen directly as follows. Burridge's (1980) LM test against either a SAR or SMA error in linear models is of the form:

$$\mathrm{LM}_{\mathrm{BUR}} = \frac{n}{\sqrt{T_n}} \frac{\tilde{u}_n' W_n \tilde{u}_n}{\tilde{u}_n' \tilde{u}_n} = \frac{n}{\sqrt{T_n}} \frac{u_n' M_n W_n M_n u_n}{u_n' M_n u_n} \sim \frac{u_n' M_n W_n M_n u_n}{\sqrt{T_n}\sigma_\varepsilon^2},$$

where $\tilde{u}_n$ is again the OLS residuals and $T_n = \mathrm{tr}(W_n' W_n + W_n^2)$. This shows that $\mathrm{LM}_{\mathrm{BUR}}$ has a similar structure as $\mathrm{LM}_{\mathrm{SEC}}$, but with W_n in place of $W_n W_n' - \frac{1}{n} T_{1n} I_n$ and $M_n W_n M_n$ in place of C_n. The key difference is that the diagonal elements of W_n are all zero, but those of $W_n W_n' - \frac{1}{n} T_{1n} I_n$ are not necessarily so. As a result, $\mathrm{E}(\mathrm{LM}_{\mathrm{BUR}}) \approx 0$ and $\mathrm{Var}(\mathrm{LM}_{\mathrm{BUR}}) \approx 1$ for n large, irrespective of the error distributions. Similar phenomenon may be observed for other models such as the linear regression models with a SAR lag, or more generally with a SARAR(p, q) (Kelejian and Prucha, 2001) or SARMA(p, q) (Anselin, 2001) effect. A general and important issue left is the finite sample dependence of these LM tests on the spatial layouts. While the issue of finite sample corrections is important and can be handled along the same idea, the scope of it is quite wide and a separate future work is necessary.

Appendix: Some Useful Lemmas

Lemma A.1 (Lee, 2004a, p. 1918): *Let v_n be an $n\times 1$ random vector of iid elements with mean zero, variance σ_v^2, and finite excess kurtosis κ_v. Let A_n be an n dimensional square matrix. Then* $\mathrm{E}(v_n' A_n v_n) = \sigma_v^2 \mathrm{tr}(A_n)$ *and* $\mathrm{Var}(v_n' A_n v_n) = \sigma_v^4 \kappa_v \sum_{i=1}^n a_{n,ii}^2 + \sigma_v^4 \mathrm{tr}(A_n A_n' + A_n^2)$*, where $\{a_{n,ii}\}$ are the diagonal elements of A_n.*

Lemma A.2 (Lee, 2004a, p. 1918): *Suppose that the elements of the $n\times k$ matrix X_n are uniformly bounded; and $\lim_{n\to\infty} \frac{1}{n} X_n' X_n$ exists and is nonsingular. Then the projectors $P_n = X_n(X_n'X_n)^{-1}X_n'$ and $M_n = I_n - X_n(X_n'X_n)^{-1}X_n'$ are uniformly bounded in both row and column sums.*

Lemma A.3 (Lemma A.9, Lee, 2004b): *Suppose that A_n represents a sequence of $n\times n$ matrices that are uniformly bounded in both row and column sums. The elements of the $n \times k$ matrix X_n are uniformly bounded; and $\lim_{n\to\infty} \frac{1}{n} X_n' X_n$ exists and is nonsingular. Let $M_n = I_n - X_n(X_n'X_n)^{-1}X_n'$. Then*

(i) $\mathrm{tr}(M_n A_n) = \mathrm{tr}(A_n) + O(1)$

(ii) $\mathrm{tr}(A_n' M_n A_n) = \mathrm{tr}(A_n' A_n) + O(1)$

(iii) $\mathrm{tr}[(M_n A_n)^2] = \mathrm{tr}(A_n^2) + O(1)$*, and*

(iv) $\mathrm{tr}[(A_n' M_n A_n)^2] = \mathrm{tr}[(M_n A_n' A_n)^2] = \mathrm{tr}[A_n' A_n)^2] + O(1)$

Furthermore, if $a_{n,ij} = O(h_n^{-1})$ for all i and j, then

(v) $\mathrm{tr}^2(M_n A_n) = \mathrm{tr}^2(A_n) + O(\frac{n}{h_n})$*, and*

(vi) $\sum_{i=1}^n [(M_n A_n)_{ii}]^2 = \sum_{i=1}^n (a_{n,ii})^2 + O(h_n^{-1})$,

where $(M_n A_n)_{ii}$ are the diagonal elements of $M_n A_n$, and $a_{n,ii}$ the diagonal elements of A_n.

Lemma A.4 (Kelejian and Prucha, 1999; Lee, 2002): *Let $\{A_n\}$ and $\{B_n\}$ be two sequences of $n \times n$ matrices that are uniformly bounded in both row and column sums. Let C_n be a sequence of conformable matrices whose elements are uniformly $O(h_n^{-1})$. Then*

(i) the sequence $\{A_n B_n\}$ are uniformly bounded in both row and column sums,

(ii) the elements of A_n are uniformly bounded and tr$(A_n) = O(n)$, and

(iii) the elements of $A_n C_n$ and $C_n A_n$ are uniformly $O(h_n^{-1})$.

References

[1] Anselin L. (2001). Rao's score test in spatial econometrics. *Journal of Statistical Planning and Inferences* **97**, 113-139.

[2] Anselin L. and Bera, A. K. (1998). Spatial dependence in linear regression models with an introduction to spatial econometrics. In: *Handbook of Applied Economic Statistics, Edited by Aman Ullah and David E. A. Giles*. New York: Marcel Dekker.

[3] Anselin L. and Moreno, R. (2003). Properties of tests for spatial error components. *Regional Science and Urban Economics* **33**, 595-618.

[4] Bera, A. K. and Yoon, M. J. (1993). Specification testing with locally misspecified alternatives. *Econometric Theory* **9**, 649-658.

[5] Bera, A. K. Sossa-Escudero, W. and Yoon, M. J. (2001). Tests for the error component model in the presence of local misspecification. *Journal of Econometrics* **101** 1-23.

[6] Florax, R. J. G. M. and de Graaff, T. (2004). The performance of diagnostic tests for spatial dependence in linear regression models: a meta-analysis of simulation studies. In: *Advances in Spatial Econometrics, Edited by L. Anselin, R. J. G. M. Florax and S. J. Rey*. Berlin: Springer-Verlag.

[7] Kelejian, H. H. and Robinson, D. P. (1995). Spatial correlation: a suggested alternative to the autoregressive models. In: *New Directions in Spatial Econometrics, Edited by L. Anselin and R. J. G. M. Florax*. Berlin: Springer-Verlag.

[8] Kelejian H. H. and Prucha, I. R. (1999). A generalized moments estimator for the autoregressive parameter in a spatial model. *International Economic Review* **40**, 509-533.

[9] Kelejian H. H. and Prucha, I. R. (2001). On the asymptotic distribution of the Moran I test statistic with applications. *Journal of Econometrics* **104**, 219-257.

[10] Koenker, R. (1981). A note on studentising a test for heterscedasticity. *Journal of Econometrics* **17**, 107-112.

[11] Lee, L. F. (2002). Consistency and efficiency of least squares estimation for mixed regressive, spatial autoregressive models. *Econometric Theory* **18**, 252-277.

[12] Lee, L. F. (2004a). Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models. *Econometrica* **72**, 1899-1925.

[13] Lee, L. F. (2004b). A supplement to 'Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models'. Working paper, Department of Economics, Ohio State University.

[14] Lee, L. F. (2007). Identification and estimation of econometric models with group interaction, contextual factors and fixed effects. *Journal of Econometrics* **140**, 333-374.

[15] Moran, P.A.P. (1950). Notes on continuous stochastic phenomena. *Biometrika* **37**, 17-33.

[16] Verbeke, G. and Molenberghs, G. (2003). The use of score test for inference on variance components. *Biometrics* **59**, 254-262.

Table 1a. Empirical Means, SDs and Sizes of the Tests, Rook Contiguity, $\ell = 5^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	20	-0.2552	0.8854	0.0541	.0255	.0047	-0.0061	1.1108	.1284	.0811	.0280
	50	-0.1949	0.9508	0.0668	.0327	.0069	-0.0015	1.0363	.1158	.0624	.0182
	100	-0.1333	0.9784	0.0797	.0408	.0092	-0.0001	1.0226	.1098	.0606	.0157
	200	-0.1149	0.9971	0.0845	.0440	.0100	-0.0026	1.0204	.1085	.0577	.0148
	500	-0.0639	0.9899	0.0880	.0435	.0080	0.0133	0.9998	.1057	.0529	.0105
	1000	-0.0384	1.0064	0.0968	.0506	.0105	0.0136	1.0113	.1089	.0555	.0126
	1500	-0.0358	0.9994	0.0916	.0485	.0130	0.0073	1.0026	.1002	.0533	.0148
2	20	-0.2512	0.8399	0.0440	.0177	.0021	-0.0013	1.0224	.1115	.0650	.0178
	50	-0.1952	1.0170	0.0814	.0496	.0171	-0.0017	0.9642	.0994	.0591	.0179
	100	-0.1186	1.0746	0.0906	.0544	.0206	0.0136	0.9606	.0912	.0518	.0171
	200	-0.1126	1.0959	0.0952	.0562	.0201	-0.0003	0.9491	.0842	.0460	.0140
	500	-0.0949	1.1261	0.1093	.0626	.0196	-0.0158	0.9678	.0902	.0476	.0100
	1000	-0.0401	1.1499	0.1185	.0711	.0225	0.0112	0.9879	.0946	.0513	.0132
	1500	-0.0249	1.1613	0.1266	.0737	.0219	0.0154	0.9968	.0977	.0513	.0117
3	20	-0.2586	0.8531	0.0468	.0193	.0037	-0.0109	1.0391	.1180	.0682	.0194
	50	-0.1981	0.9962	0.0797	.0458	.0158	-0.0058	0.9873	.1052	.0608	.0191
	100	-0.1286	1.0269	0.0873	.0531	.0203	0.0046	0.9678	.0970	.0553	.0198
	200	-0.1253	1.0685	0.0944	.0574	.0199	-0.0095	0.9687	.0931	.0542	.0179
	500	-0.0641	1.1399	0.1173	.0700	.0259	0.0107	0.9870	.0995	.0528	.0168
	1000	-0.0614	1.1851	0.1250	.0787	.0307	-0.0067	0.9916	.0989	.0544	.0162
	1500	-0.0361	1.2124	0.1365	.0840	.0319	0.0043	0.9920	.1007	.0532	.0151
4	20	-0.2533	0.8763	0.0524	.0256	.0045	-0.0036	1.0883	.1286	.0782	.0274
	50	-0.2120	0.9612	0.0719	.0372	.0101	-0.0192	1.0205	.1050	.0621	.0202
	100	-0.1139	0.9962	0.0901	.0465	.0127	0.0197	1.0181	.1169	.0647	.0200
	200	-0.1203	1.0025	0.0847	.0480	.0113	-0.0078	1.0036	.1016	.0565	.0151
	500	-0.0762	1.0096	0.0910	.0465	.0121	0.0007	0.9997	.1019	.0513	.0134
	1000	-0.0639	1.0217	0.0982	.0524	.0115	-0.0120	1.0076	.1036	.0555	.0126
	1500	-0.0426	1.0124	0.0939	.0495	.0118	0.0005	0.9971	.0976	.0516	.0117

*The n spatial units are randomly placed on a lattice of $\ell \times m$ squares.
dgp1=normal, dgp2=normal mixture($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Table 1b. Empirical Means, SDs and Sizes of the Tests, Queen Contiguity, $\ell = 5^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	20	-0.5543	0.6913	.0189	.0089	.0018	0.0120	1.1381	.1299	.0924	.0441
	50	-0.3213	0.8662	.0475	.0262	.0082	-0.0029	1.0362	.1139	.0691	.0276
	100	-0.2562	0.9330	.0628	.0349	.0107	-0.0002	1.0253	.1124	.0681	.0238
	200	-0.1735	0.9793	.0761	.0399	.0110	0.0019	1.0240	.1114	.0621	.0189
	500	-0.1092	0.9893	.0845	.0432	.0117	0.0107	1.0082	.1055	.0583	.0150
	1000	-0.0853	0.9890	.0890	.0457	.0109	0.0036	0.9989	.1060	.0555	.0145
	1500	-0.0823	0.9903	.0898	.0462	.0113	-0.0142	0.9964	.1031	.0547	.0129
2	20	-0.5627	0.6953	.0175	.0070	.0012	-0.0038	1.0550	.1248	.0820	.0307
	50	-0.3175	0.9750	.0708	.0427	.0162	-0.0009	0.9800	.1056	.0675	.0244
	100	-0.2649	1.0840	.0840	.0520	.0220	-0.0079	0.9633	.0962	.0570	.0223
	200	-0.1637	1.1503	.1025	.0643	.0262	0.0085	0.9664	.0938	.0532	.0186
	500	-0.1167	1.1792	.1176	.0719	.0243	0.0021	0.9684	.0952	.0519	.0116
	1000	-0.0838	1.2209	.1291	.0804	.0286	0.0040	0.9987	.1007	.0547	.0144
	1500	-0.0681	1.2278	.1335	.0863	.0296	0.0001	1.0026	.1031	.0554	.0138
3	20	-0.5558	0.7032	.0208	.0084	.0011	0.0066	1.0780	.1271	.0841	.0370
	50	-0.3218	0.9259	.0597	.0361	.0135	-0.0050	0.9837	.1039	.0646	.0241
	100	-0.2656	1.0171	.0732	.0452	.0174	-0.0099	0.9689	.0978	.0567	.0226
	200	-0.1799	1.1241	.0974	.0612	.0246	-0.0056	0.9968	.0968	.0600	.0222
	500	-0.1250	1.2040	.1216	.0764	.0308	-0.0102	0.9886	.1015	.0560	.0167
	1000	-0.0955	1.2372	.1330	.0859	.0337	-0.0025	0.9828	.0996	.0558	.0138
	1500	-0.0888	1.2869	.1394	.0911	.0384	-0.0133	0.9968	.1002	.0543	.0155
4	20	-0.5558	0.6889	.0178	.0080	.0013	0.0070	1.1046	.1285	.0860	.0399
	50	-0.3259	0.8918	.0545	.0303	.0103	-0.0083	1.0317	.1082	.0674	.0287
	100	-0.2621	0.9370	.0644	.0359	.0104	-0.0065	0.9956	.1054	.0629	.0217
	200	-0.1689	0.9973	.0814	.0453	.0138	0.0066	1.0125	.1077	.0636	.0195
	500	-0.1288	1.0082	.0875	.0488	.0142	-0.0091	1.0000	.1020	.0577	.0169
	1000	-0.0825	1.0190	.0963	.0509	.0130	0.0062	1.0026	.1073	.0571	.0137
	1500	-0.0558	1.0247	.1019	.0541	.0156	0.0121	1.0050	.1088	.0582	.0162

*The n spatial units are randomly placed on a lattice of $\ell \times m$ squares.
dgp1=normal, dgp2=normal mixture($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 2a. Empirical Means, SDs and Sizes of the Tests, Rook Contiguity, $\ell = m^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	5^2	-0.3575	0.8954	.0456	.0227	.0041	0.0007	1.0987	.1298	.0794	.0280
	7^2	-0.2155	0.9602	.0707	.0350	.0090	0.0050	1.0597	.1194	.0715	.0222
	10^2	-0.1358	0.9741	.0810	.0421	.0098	-0.0006	1.0206	.1100	.0625	.0171
	15^2	-0.1155	0.9787	.0823	.0411	.0075	-0.0030	1.0007	.1037	.0540	.0123
	23^2	-0.0873	0.9880	.0834	.0447	.0095	-0.0062	0.9980	.1006	.0547	.0123
	32^2	-0.0383	1.0032	.0958	.0498	.0104	0.0175	1.0085	.1056	.0567	.0128
	39^2	-0.0639	0.9955	.0895	.0460	.0100	-0.0174	0.9990	.0977	.0515	.0119
2	5^2	-0.3546	0.8840	.0471	.0208	.0028	0.0035	1.0106	.1127	.0675	.0189
	7^2	-0.2112	1.0039	.0794	.0491	.0172	0.0064	0.9615	.1001	.0611	.0174
	10^2	-0.1435	1.0727	.0905	.0544	.0232	-0.0074	0.9526	.0886	.0515	.0189
	15^2	-0.0982	1.1033	.0995	.0626	.0240	0.0124	0.9624	.0903	.0539	.0185
	23^2	-0.0951	1.1076	.1028	.0614	.0213	-0.0128	0.9882	.0921	.0509	.0153
	32^2	-0.0612	1.0895	.1082	.0597	.0187	-0.0050	0.9941	.0989	.0509	.0146
	39^2	-0.0318	1.0787	.1088	.0625	.0168	0.0139	0.9976	.1008	.0546	.0123
3	5^2	-0.3556	0.8998	.0502	.0264	.0053	0.0021	1.0398	.1150	.0730	.0249
	7^2	-0.2206	0.9827	.0765	.0427	.0145	-0.0003	0.9863	.1053	.0632	.0199
	10^2	-0.1237	1.0340	.0936	.0569	.0215	0.0115	0.9716	.0996	.0604	.0206
	15^2	-0.1092	1.0686	.1011	.0596	.0203	0.0030	0.9700	.0980	.0545	.0178
	23^2	-0.0623	1.1088	.1116	.0679	.0270	0.0172	0.9905	.1033	.0579	.0200
	32^2	-0.0887	1.0851	.1052	.0655	.0247	-0.0286	0.9766	.0958	.0560	.0179
	39^2	-0.0649	1.1092	.1099	.0687	.0270	-0.0169	1.0000	.1001	.0591	.0189
4	5^2	-0.3540	0.8962	.0492	.0228	.0050	0.0042	1.0770	.1260	.0785	.0270
	7^2	-0.2230	0.9522	.0668	.0356	.0099	-0.0031	1.0215	.1092	.0638	.0194
	10^2	-0.1460	0.9800	.0817	.0420	.0112	-0.0110	1.0015	.1046	.0569	.0155
	15^2	-0.1083	0.9990	.0884	.0462	.0128	0.0043	1.0006	.1029	.0587	.0154
	23^2	-0.0758	1.0009	.0912	.0490	.0113	0.0052	0.9965	.1030	.0554	.0135
	32^2	-0.0518	1.0085	.0951	.0499	.0104	0.0039	1.0030	.1042	.0545	.0115
	39^2	-0.0405	1.0063	.0951	.0483	.0135	0.0060	1.0008	.1021	.0518	.0144

*The n spatial units are randomly placed on a lattice of $\ell \times m$ squares.
dgp1=normal, dgp2=normal mixture($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 2b. Empirical Means, SDs and Sizes of the Tests, Queen Contiguity, $\ell = m^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	5^2	-0.4613	0.7557	.0277	.0136	.0034	0.0142	1.1079	.1306	.0858	.0379
	7^2	-0.3936	0.8373	.0398	.0201	.0055	-0.0010	1.0419	.1128	.0714	.0279
	10^2	-0.3157	0.9107	.0549	.0301	.0076	-0.0038	1.0216	.1124	.0672	.0237
	15^2	-0.1847	0.9669	.0749	.0408	.0105	0.0111	1.0134	.1121	.0643	.0198
	23^2	-0.1285	0.9641	.0787	.0405	.0105	-0.0026	0.9837	.1025	.0554	.0158
	32^2	-0.1000	0.9897	.0882	.0447	.0088	-0.0055	1.0006	.1052	.0555	.0122
	39^2	-0.0722	1.0030	.0930	.0462	.0089	0.0056	1.0105	.1092	.0561	.0126
2	5^2	-0.4941	0.7753	.0253	.0101	.0017	-0.0298	1.0220	.1180	.0708	.0240
	7^2	-0.3931	0.9389	.0566	.0354	.0138	0.0012	0.9892	.1044	.0686	.0269
	10^2	-0.3188	1.0812	.0786	.0499	.0230	-0.0062	0.9792	.0979	.0611	.0266
	15^2	-0.1891	1.1445	.0985	.0627	.0261	0.0051	0.9650	.0961	.0545	.0192
	23^2	-0.1024	1.1558	.1197	.0760	.0285	0.0200	0.9893	.1075	.0624	.0185
	32^2	-0.1197	1.1282	.1066	.0655	.0236	-0.0218	0.9885	.0955	.0540	.0157
	39^2	-0.0765	1.1091	.1097	.0642	.0221	0.0010	0.9879	.0977	.0515	.0161
3	5^2	-0.4730	0.7733	.0280	.0140	.0030	-0.0035	1.0339	.1231	.0754	.0283
	7^2	-0.3983	0.9043	.0514	.0316	.0106	-0.0049	1.0027	.1028	.0668	.0283
	10^2	-0.3061	1.0278	.0703	.0453	.0187	0.0059	0.9957	.1007	.0645	.0271
	15^2	-0.1932	1.1029	.0955	.0600	.0237	0.0036	0.9815	.1012	.0588	.0221
	23^2	-0.1178	1.1370	.1080	.0684	.0278	0.0059	0.9783	.1007	.0567	.0183
	32^2	-0.0974	1.1449	.1076	.0657	.0264	-0.0017	0.9782	.0965	.0540	.0179
	39^2	-0.0798	1.1433	.1126	.0722	.0292	-0.0013	0.9841	.1026	.0598	.0185
4	5^2	-0.4611	0.7627	.0277	.0133	.0036	0.0138	1.0841	.1292	.0827	.0336
	7^2	-0.3760	0.8622	.0447	.0245	.0069	0.0198	1.0380	.1158	.0747	.0300
	10^2	-0.3240	0.9324	.0568	.0314	.0104	-0.0125	1.0109	.1038	.0628	.0218
	15^2	-0.1912	0.9868	.0782	.0441	.0141	0.0046	1.0052	.1076	.0638	.0212
	23^2	-0.1360	1.0122	.0874	.0489	.0137	-0.0100	1.0107	.1047	.0596	.0188
	32^2	-0.0957	1.0006	.0924	.0477	.0133	-0.0011	0.9948	.1045	.0564	.0156
	39^2	-0.0650	1.0007	.0931	.0485	.0129	0.0127	0.9940	.1028	.0563	.0146

*The n spatial units are randomly placed on a lattice of $\ell \times m$ squares.
dgp1=normal, dgp2=normal mixture($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Table 3a. Empirical Means, SDs and Sizes of the Tests, Group Interaction
Group sizes = $\{2, 3, 4, 5, 6, 7\}$ repeated m times, $n = 27m^*$

		Anselin's Test					Proposed Test				
dgp	m	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	1	-0.2621	0.9313	.0650	.0363	.0079	0.0117	1.0988	.1367	.0860	.0304
	2	-0.1685	0.9739	.0775	.0406	.0111	-0.0074	1.0489	.1144	.0681	.0208
	4	-0.1239	0.9882	.0838	.0462	.0104	-0.0071	1.0247	.1054	.0623	.0167
	8	-0.0782	0.9890	.0848	.0434	.0119	-0.0046	1.0059	.1001	.0531	.0145
	19	-0.0589	0.9911	.0896	.0450	.0106	-0.0069	0.9982	.0994	.0504	.0122
	37	-0.0423	1.0038	.0914	.0482	.0115	-0.0030	1.0080	.0987	.0525	.0131
	56	-0.0346	0.9911	.0945	.0483	.0110	-0.0035	0.9936	.1001	.0514	.0118
2	1	-0.2913	1.1282	.0850	.0572	.0290	-0.0173	1.1070	.1256	.0863	.0417
	2	-0.1470	1.3015	.1137	.0779	.0441	0.0138	1.0439	.1156	.0755	.0378
	4	-0.1481	1.4938	.1413	.1040	.0616	-0.0208	1.0194	.1066	.0727	.0328
	8	-0.0429	1.6957	.1808	.1402	.0890	0.0166	1.0242	.1171	.0783	.0325
	19	-0.0365	1.7564	.2036	.1558	.0984	0.0093	1.0032	.1118	.0695	.0251
	37	-0.0546	1.8010	.2120	.1656	.0961	-0.0072	1.0036	.1059	.0625	.0204
	56	-0.0124	1.8028	.2247	.1716	.1017	0.0108	0.9943	.1076	.0612	.0185
3	1	-0.2805	1.2401	.1044	.0760	.0447	-0.0077	1.1166	.1309	.0929	.0527
	2	-0.1724	1.4587	.1284	.0968	.0605	-0.0067	1.0547	.1151	.0824	.0455
	4	-0.1291	1.7277	.1593	.1242	.0779	-0.0044	1.0288	.1105	.0793	.0392
	8	-0.0914	2.0160	.1851	.1489	.0993	-0.0040	1.0038	.1091	.0756	.0370
	19	-0.0465	2.4965	.2141	.1765	.1292	-0.0003	1.0203	.1105	.0785	.0371
	37	-0.0396	2.7348	.2372	.1988	.1404	0.0038	1.0046	.1083	.0755	.0349
	56	-0.0242	2.9737	.2439	.2060	.1510	0.0057	1.0107	.1087	.0744	.0362
4	1	-0.2711	1.0474	.0854	.0554	.0239	0.0012	1.1201	.1350	.0932	.0425
	2	-0.1546	1.1314	.1104	.0727	.0295	0.0081	1.0667	.1287	.0838	.0301
	4	-0.1074	1.1764	.1208	.0795	.0330	0.0091	1.0304	.1174	.0741	.0238
	8	-0.0686	1.2178	.1351	.0863	.0372	0.0046	1.0188	.1126	.0663	.0215
	19	-0.0537	1.2086	.1327	.0852	.0339	-0.0005	0.9870	.1008	.0591	.0156
	37	-0.0385	1.2351	.1459	.0939	.0338	0.0013	0.9980	.1090	.0588	.0139
	56	-0.0417	1.2515	.1440	.0889	.0357	-0.0091	1.0065	.1018	.0560	.0160

*dgp1 = normal, dgp2=normal mixture($\tau = 5, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Table 3b. Empirical Means, SDs and Sizes of the Tests, Group Interaction
Group sizes = 5 repeated m times, $n = 5m^*$

		Anselin's Test					Proposed Test				
dgp	m	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	5	-0.3704	0.8699	.0500	.0295	.0081	-0.0160	1.0739	.1230	.0793	.0349
	10	-0.3109	0.9189	.0587	.0314	.0078	0.0066	1.0359	.1170	.0730	.0259
	20	-0.1377	0.9812	.0834	.0461	.0124	0.0009	1.0215	.1110	.0646	.0187
	40	-0.1059	0.9870	.0898	.0440	.0111	0.0015	1.0083	.1095	.0596	.0151
	100	-0.0642	0.9994	.0944	.0471	.0120	0.0017	1.0078	.1064	.0549	.0143
	200	-0.0434	0.9993	.0959	.0503	.0115	0.0027	1.0034	.1042	.0556	.0126
	300	-0.0280	0.9970	.0961	.0514	.0116	0.0098	0.9997	.1027	.0551	.0126
2	5	-0.3483	0.8279	.0433	.0231	.0064	0.0112	1.0162	.1140	.0718	.0273
	10	-0.3046	0.8822	.0494	.0262	.0054	0.0135	0.9868	.1071	.0631	.0208
	20	-0.1457	0.9285	.0703	.0369	.0093	-0.0074	0.9653	.0974	.0530	.0157
	40	-0.1053	0.9433	.0753	.0390	.0105	0.0022	0.9630	.0962	.0515	.0142
	100	-0.0633	0.9826	.0863	.0449	.0105	0.0026	0.9906	.0980	.0522	.0129
	200	-0.0439	0.9818	.0898	.0472	.0117	0.0023	0.9858	.0980	.0512	.0130
	300	-0.0447	0.9894	.0943	.0463	.0084	-0.0070	0.9921	.1019	.0501	.0101
3	5	-0.3587	0.8002	.0412	.0211	.0069	-0.0016	0.9780	.1050	.0639	.0257
	10	-0.3190	0.8437	.0461	.0257	.0068	-0.0026	0.9406	.0944	.0569	.0198
	20	-0.1604	0.8975	.0680	.0397	.0117	-0.0227	0.9325	.0905	.0532	.0187
	40	-0.1100	0.9375	.0770	.0452	.0163	-0.0027	0.9566	.0932	.0557	.0201
	100	-0.0573	0.9655	.0875	.0505	.0170	0.0087	0.9732	.0974	.0561	.0186
	200	-0.0351	0.9880	.0940	.0563	.0187	0.0111	0.9920	.0995	.0615	.0214
	300	-0.0340	0.9811	.0926	.0520	.0151	0.0037	0.9837	.0973	.0555	.0162
4	5	-0.3509	0.8535	.0492	.0270	.0079	0.0082	1.0502	.1200	.0799	.0330
	10	-0.3094	0.9001	.0529	.0281	.0075	0.0083	1.0115	.1114	.0671	.0230
	20	-0.1476	0.9541	.0793	.0448	.0128	-0.0094	0.9929	.1038	.0631	.0191
	40	-0.1041	0.9723	.0845	.0442	.0125	0.0033	0.9931	.1044	.0569	.0168
	100	-0.0832	0.9917	.0891	.0488	.0126	-0.0174	0.9999	.1022	.0554	.0152
	200	-0.0605	1.0075	.0977	.0527	.0138	-0.0144	1.0116	.1060	.0580	.0154
	300	-0.0615	0.9836	.0880	.0452	.0106	-0.0238	0.9863	.0944	.0481	.0116

*dgp1 = normal, dgp2=normal mixture($\tau = 5, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 4a. Empirical Means, SDs and Sizes of the Tests, Group Interaction
Number of groups $\ell \propto n^{.75}$, Group sizes $m \sim U(.5n^{.25}, 1.5n^{.25})^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	20	-0.1115	0.9529	.0758	.0308	.0018	-0.0015	1.1489	.1405	.0809	.0150
	50	-0.0878	0.9946	.0865	.0369	.0056	0.0057	1.0608	.1169	.0608	.0115
	100	-0.0758	0.9849	.0879	.0429	.0078	0.0074	1.0176	.1102	.0551	.0120
	200	-0.0880	0.9975	.0870	.0449	.0090	-0.0166	1.0148	.1029	.0547	.0127
	500	-0.0669	0.9990	.0958	.0489	.0109	-0.0050	1.0071	.1079	.0581	.0135
	1000	-0.0597	0.9954	.0890	.0440	.0116	-0.0114	0.9998	.0983	.0493	.0136
	1500	-0.0379	0.9968	.0941	.0489	.0106	0.0024	0.9999	.1016	.0544	.0122
2	20	-0.1230	1.3587	.1569	.1167	.0582	-0.0150	1.1944	.1859	.1121	.0108
	50	-0.1199	1.9904	.2418	.2051	.1472	-0.0074	1.0853	.1799	.0704	.0048
	100	-0.1266	2.3663	.2421	.2097	.1591	-0.0166	1.0398	.1502	.0923	.0182
	200	-0.0613	2.1549	.2092	.1708	.1201	0.0037	1.0275	.1119	.0836	.0373
	500	-0.0680	1.7972	.1954	.1519	.0918	-0.0023	0.9985	.0988	.0608	.0242
	1000	-0.0728	1.4726	.1750	.1195	.0571	-0.0156	0.9886	.0972	.0535	.0145
	1500	-0.0415	1.4200	.1697	.1145	.0541	-0.0011	1.0026	.1008	.0573	.0178
3	20	-0.1327	1.3252	.1606	.1144	.0381	-0.0249	1.2033	.1839	.1038	.0108
	50	-0.0631	1.7203	.2239	.1778	.1064	0.0235	1.0846	.1698	.0775	.0068
	100	-0.0699	2.0036	.2207	.1838	.1239	0.0064	1.0500	.1504	.0921	.0155
	200	-0.0738	1.8680	.1814	.1438	.0851	-0.0016	1.0179	.1084	.0728	.0360
	500	-0.0586	1.7801	.1731	.1337	.0779	0.0004	1.0056	.1006	.0604	.0242
	1000	-0.0394	1.5745	.1695	.1234	.0676	0.0027	0.9955	.1065	.0622	.0236
	1500	-0.0426	1.5613	.1634	.1148	.0640	-0.0037	1.0063	.0998	.0612	.0243
4	20	-0.0963	1.0741	.1133	.0591	.0080	0.0129	1.1744	.1616	.0930	.0164
	50	-0.0785	1.1981	.1376	.0853	.0274	0.0034	1.0719	.1356	.0720	.0160
	100	-0.0668	1.2344	.1416	.0914	.0341	-0.0021	1.0357	.1203	.0662	.0154
	200	-0.0559	1.1970	.1345	.0836	.0325	0.0262	1.0172	.1165	.0669	.0191
	500	-0.0490	1.1242	.1209	.0706	.0271	0.0064	1.0014	.1069	.0584	.0184
	1000	-0.0466	1.0585	.1111	.0604	.0161	0.0021	0.9986	.1065	.0561	.0139
	1500	-0.0321	1.0356	.1056	.0596	.0149	0.0078	0.9860	.1022	.0551	.0133

*dgp1 = normal, dgp2=normal mixture ($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 4b. Empirical Means, SDs and Sizes of the Tests, Group Interaction
Number of groups $\ell \propto n^{.5}$, Group sizes $m \sim U(.5n^{.5}, 1.5n^{.5})^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	20	-0.3578	0.8713	.0550	.0310	.0084	-0.0099	1.1168	.1317	.0879	.0413
	50	-0.2694	0.9177	.0661	.0371	.0107	0.0036	1.0348	.1204	.0764	.0289
	100	-0.2032	0.9620	.0748	.0433	.0152	0.0055	1.0271	.1136	.0701	.0256
	200	-0.1568	0.9785	.0812	.0473	.0163	0.0173	1.0196	.1135	.0672	.0266
	500	-0.1378	0.9859	.0875	.0481	.0150	0.0117	1.0126	.1129	.0672	.0211
	1000	-0.1186	0.9792	.0865	.0478	.0130	0.0037	0.9962	.1078	.0628	.0190
	1500	-0.1019	0.9933	.0904	.0500	.0156	0.0121	1.0077	.1093	.0627	.0201
2	20	-0.3479	0.8240	.0424	.0200	.0055	0.0034	1.0231	.1178	.0719	.0261
	50	-0.2721	0.8545	.0498	.0254	.0075	0.0004	0.9174	.0889	.0515	.0173
	100	-0.1958	0.9479	.0668	.0382	.0124	0.0126	0.9334	.0915	.0511	.0165
	200	-0.1763	0.9917	.0787	.0448	.0145	-0.0037	0.9428	.0930	.0506	.0157
	500	-0.1511	0.9917	.0810	.0437	.0145	-0.0017	0.9696	.0979	.0539	.0167
	1000	-0.1151	0.9934	.0866	.0485	.0152	0.0070	0.9778	.1001	.0575	.0183
	1500	-0.1457	0.9949	.0836	.0465	.0131	-0.0314	0.9803	.0967	.0524	.0157
3	20	-0.3483	0.8186	.0439	.0236	.0063	0.0023	1.0172	.1128	.0724	.0290
	50	-0.2629	0.9016	.0610	.0366	.0131	0.0107	0.9834	.1014	.0651	.0262
	100	-0.2082	0.9444	.0704	.0411	.0164	0.0000	0.9567	.0976	.0582	.0223
	200	-0.1681	0.9967	.0850	.0517	.0208	0.0058	0.9731	.1023	.0628	.0245
	500	-0.1287	1.0011	.0876	.0507	.0201	0.0199	0.9817	.1047	.0612	.0226
	1000	-0.1309	0.9993	.0847	.0513	.0168	-0.0078	0.9782	.0989	.0581	.0184
	1500	-0.1311	1.0038	.0856	.0510	.0170	-0.0166	0.9800	.0962	.0563	.0188
4	20	-0.3674	0.8465	.0482	.0255	.0071	-0.0217	1.0735	.1225	.0846	.0346
	50	-0.2822	0.9108	.0640	.0376	.0115	-0.0106	1.0170	.1125	.0701	.0281
	100	-0.2022	0.9465	.0724	.0438	.0151	0.0066	0.9992	.1065	.0651	.0246
	200	-0.1703	0.9743	.0811	.0460	.0168	0.0034	1.0039	.1055	.0659	.0250
	500	-0.1631	0.9825	.0826	.0462	.0157	-0.0142	1.0038	.1062	.0632	.0210
	1000	-0.1172	0.9857	.0842	.0484	.0156	0.0050	0.9993	.1060	.0607	.0198
	1500	-0.0974	0.9891	.0898	.0510	.0139	0.0166	1.0004	.1096	.0619	.0188

*dgp1 = normal, dgp2=normal mixture ($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared(df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 4c. Empirical Means, SDs and Sizes of the Tests, Group Interaction
Number of groups $\ell \propto n^{.25}$, Group sizes $m \sim U(.5n^{.75}, 1.5n^{.75})^*$

		Anselin's Test					Proposed Test				
dgp	n	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
1	20	-0.5368	0.6713	.0295	.0173	.0058	-0.0091	1.1024	.1142	.0842	.0488
	50	-0.3820	0.8296	.0505	.0306	.0114	0.0042	1.0276	.1109	.0759	.0380
	100	-0.4075	0.8141	.0462	.0298	.0123	0.0111	1.0254	.1064	.0740	.0382
	200	-0.3466	0.8753	.0554	.0361	.0151	0.0078	1.0199	.1081	.0716	.0361
	500	-0.3329	0.8852	.0592	.0355	.0141	-0.0143	0.9965	.1037	.0707	.0315
	1000	-0.2914	0.8988	.0602	.0363	.0141	-0.0042	0.9858	.1027	.0665	.0291
	1500	-0.3082	0.9051	.0612	.0363	.0136	-0.0214	0.9927	.1063	.0676	.0278
2	20	-0.5235	0.6229	.0206	.0105	.0039	0.0120	1.0058	.1079	.0764	.0386
	50	-0.3844	0.7241	.0299	.0164	.0052	0.0011	0.8787	.0893	.0539	.0204
	100	-0.4101	0.7467	.0385	.0210	.0063	0.0076	0.9108	.0955	.0624	.0271
	200	-0.3544	0.8200	.0457	.0260	.0100	-0.0014	0.9371	.0948	.0606	.0248
	500	-0.3084	0.9144	.0640	.0395	.0147	0.0122	0.9700	.1024	.0679	.0270
	1000	-0.2966	0.9014	.0621	.0355	.0142	-0.0097	0.9804	.1039	.0668	.0273
	1500	-0.2822	0.9016	.0615	.0382	.0142	0.0071	0.9865	.1040	.0659	.0285
3	20	-0.5355	0.6213	.0206	.0126	.0029	-0.0071	1.0021	.1062	.0743	.0396
	50	-0.3955	0.7724	.0420	.0227	.0088	-0.0126	0.9426	.0964	.0655	.0276
	100	-0.4280	0.7598	.0363	.0225	.0079	-0.0141	0.9385	.0946	.0610	.0288
	200	-0.3584	0.8277	.0470	.0282	.0104	-0.0059	0.9522	.0978	.0632	.0273
	500	-0.3212	0.9021	.0585	.0368	.0152	-0.0009	0.9611	.0960	.0628	.0277
	1000	-0.2884	0.9036	.0598	.0390	.0151	-0.0009	0.9810	.1039	.0639	.0294
	1500	-0.2980	0.8884	.0599	.0338	.0136	-0.0103	0.9714	.0992	.0650	.0254
4	20	-0.5366	0.6529	.0246	.0153	.0048	-0.0089	1.0653	.1153	.0810	.0445
	50	-0.3940	0.8274	.0489	.0318	.0110	-0.0104	1.0208	.1062	.0758	.0366
	100	-0.4217	0.8103	.0437	.0270	.0110	-0.0066	1.0161	.1037	.0749	.0357
	200	-0.3442	0.8638	.0537	.0334	.0133	0.0106	1.0043	.1086	.0706	.0333
	500	-0.3290	0.8876	.0581	.0362	.0142	-0.0098	0.9927	.1041	.0684	.0311
	1000	-0.2892	0.9227	.0636	.0393	.0157	-0.0017	1.0111	.1097	.0697	.0312
	1500	-0.3012	0.9028	.0603	.0369	.0142	-0.0137	0.9900	.1041	.0651	.0299

*dgp1 = normal, dgp2=normal mixture ($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared (df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 5a. Empirical Powers of the Proposed Test, Rook or Queen Contiguity

	$n = 25$				$n = 100$				$n = 225$			
σ_v	dgp1	dgp2	dgp3	dgp4	dgp1	dgp2	dgp3	dgp4	dgp1	dgp2	dgp3	dgp4
	Rook Contiguity, $\ell = 5$											
0.00	0.129	0.112	0.115	0.119	0.107	0.087	0.098	0.113	0.107	0.082	0.097	0.110
0.25	0.144	0.149	0.136	0.138	0.132	0.124	0.136	0.126	0.143	0.135	0.141	0.140
0.50	0.183	0.253	0.223	0.189	0.239	0.258	0.248	0.228	0.297	0.299	0.308	0.296
0.75	0.249	0.379	0.321	0.261	0.403	0.444	0.452	0.413	0.590	0.580	0.609	0.581
1.00	0.333	0.477	0.446	0.350	0.620	0.637	0.646	0.615	0.849	0.795	0.825	0.849
1.25	0.420	0.564	0.535	0.443	0.795	0.776	0.801	0.791	0.964	0.924	0.928	0.962
1.50	0.513	0.635	0.609	0.527	0.898	0.871	0.886	0.898	0.995	0.976	0.972	0.993
1.75	0.604	0.682	0.679	0.604	0.958	0.926	0.940	0.955	1.000	0.993	0.988	0.999
2.00	0.664	0.723	0.723	0.667	0.981	0.957	0.963	0.978	1.000	0.999	0.994	1.000
	Rook Contiguity, $\ell = \sqrt{n}$											
0.00	0.131	0.127	0.123	0.125	0.108	0.088	0.100	0.110	0.105	0.084	0.094	0.108
0.25	0.137	0.161	0.150	0.139	0.135	0.121	0.133	0.133	0.140	0.133	0.134	0.146
0.50	0.181	0.266	0.226	0.187	0.231	0.257	0.254	0.225	0.297	0.291	0.308	0.295
0.75	0.253	0.398	0.334	0.265	0.401	0.434	0.459	0.398	0.570	0.551	0.586	0.565
1.00	0.346	0.487	0.438	0.340	0.609	0.621	0.653	0.611	0.832	0.795	0.814	0.830
1.25	0.441	0.568	0.538	0.444	0.791	0.764	0.795	0.777	0.962	0.922	0.926	0.956
1.50	0.516	0.623	0.618	0.537	0.896	0.865	0.881	0.892	0.992	0.973	0.968	0.990
1.75	0.610	0.690	0.676	0.608	0.954	0.921	0.927	0.952	0.999	0.994	0.986	0.999
2.00	0.665	0.725	0.719	0.670	0.983	0.954	0.958	0.979	1.000	0.998	0.992	1.000
	Queen Contiguity, $\ell = 5$											
0.00	0.125	0.125	0.126	0.121	0.105	0.097	0.100	0.110	0.107	0.098	0.100	0.107
0.25	0.126	0.158	0.143	0.135	0.125	0.121	0.125	0.124	0.138	0.124	0.128	0.140
0.50	0.162	0.232	0.193	0.167	0.201	0.213	0.212	0.199	0.255	0.244	0.255	0.246
0.75	0.202	0.311	0.268	0.210	0.317	0.345	0.356	0.331	0.460	0.439	0.470	0.452
1.00	0.273	0.400	0.350	0.278	0.479	0.505	0.528	0.486	0.689	0.650	0.688	0.699
1.25	0.330	0.467	0.424	0.351	0.633	0.633	0.667	0.635	0.863	0.815	0.832	0.855
1.50	0.401	0.522	0.490	0.412	0.763	0.742	0.777	0.761	0.951	0.910	0.912	0.949
1.75	0.457	0.577	0.548	0.478	0.852	0.829	0.852	0.847	0.986	0.960	0.956	0.984
2.00	0.521	0.613	0.613	0.533	0.914	0.880	0.891	0.911	0.996	0.983	0.977	0.995
	Queen Contiguity, $\ell = \sqrt{n}$											
0.00	0.121	0.126	0.120	0.125	0.113	0.095	0.100	0.105	0.113	0.090	0.103	0.107
0.25	0.132	0.152	0.141	0.137	0.134	0.123	0.134	0.127	0.139	0.120	0.126	0.128
0.50	0.166	0.234	0.209	0.166	0.201	0.219	0.213	0.191	0.249	0.224	0.243	0.239
0.75	0.208	0.318	0.283	0.214	0.305	0.343	0.352	0.311	0.432	0.407	0.436	0.427
1.00	0.273	0.411	0.357	0.282	0.474	0.499	0.510	0.465	0.665	0.618	0.652	0.657
1.25	0.342	0.473	0.447	0.350	0.606	0.621	0.645	0.607	0.839	0.788	0.808	0.835
1.50	0.414	0.535	0.512	0.422	0.743	0.731	0.752	0.740	0.938	0.892	0.900	0.932
1.75	0.474	0.590	0.560	0.496	0.838	0.808	0.833	0.834	0.978	0.949	0.948	0.976
2.00	0.531	0.623	0.609	0.552	0.900	0.869	0.884	0.895	0.994	0.977	0.971	0.990

*dgp1 = normal, dgp2=normal mixture ($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared (df=3).

Appeared in: Regional Science and Urban Economics, 2010, 40, 299-310.

Table 5b. Empirical Power of the Proposed Test, Group Interaction

	$n = 25$				$n = 100$				$n = 225$			
σ_v	dgp1	dgp2	dgp3	dgp4	dgp1	dgp2	dgp3	dgp4	dgp1	dgp2	dgp3	dgp4
	Group Size = {2 3 5 7 8} for Panel 1;							Group Size = 5 for Panel 2				
0.00	0.138	0.123	0.128	0.140	0.116	0.116	0.113	0.114	0.103	0.116	0.115	0.113
0.25	0.159	0.173	0.163	0.155	0.148	0.137	0.137	0.148	0.165	0.143	0.145	0.157
0.50	0.209	0.306	0.250	0.216	0.271	0.227	0.246	0.250	0.387	0.217	0.265	0.332
0.75	0.305	0.435	0.371	0.300	0.508	0.377	0.442	0.455	0.738	0.400	0.504	0.653
1.00	0.420	0.534	0.500	0.424	0.750	0.539	0.625	0.694	0.950	0.631	0.728	0.902
1.25	0.529	0.617	0.603	0.532	0.902	0.691	0.778	0.864	0.994	0.832	0.864	0.983
1.50	0.641	0.674	0.685	0.637	0.967	0.808	0.863	0.944	1.000	0.939	0.932	0.998
1.75	0.713	0.728	0.742	0.717	0.990	0.891	0.914	0.979	1.000	0.983	0.964	1.000
2.00	0.780	0.776	0.788	0.777	0.997	0.939	0.941	0.992	1.000	0.996	0.980	1.000
0.00	0.127	0.096	0.100	0.115	0.115	0.078	0.089	0.111	0.106	0.075	0.094	0.110
0.25	0.138	0.146	0.133	0.141	0.146	0.130	0.141	0.140	0.163	0.138	0.152	0.158
0.50	0.193	0.269	0.243	0.196	0.268	0.317	0.315	0.270	0.365	0.409	0.398	0.366
0.75	0.261	0.404	0.353	0.275	0.475	0.564	0.557	0.487	0.686	0.734	0.723	0.691
1.00	0.355	0.517	0.459	0.368	0.699	0.736	0.750	0.700	0.920	0.902	0.907	0.921
1.25	0.453	0.589	0.562	0.470	0.857	0.857	0.868	0.858	0.988	0.969	0.970	0.983
1.50	0.533	0.651	0.632	0.546	0.937	0.923	0.926	0.937	0.998	0.993	0.990	0.998
1.75	0.610	0.696	0.688	0.631	0.975	0.959	0.961	0.974	1.000	0.999	0.996	0.999
2.00	0.674	0.745	0.734	0.679	0.990	0.980	0.977	0.987	1.000	0.999	0.998	1.000
	Group sizes $\sim U(.5n^{\delta}, 1.5n^{\delta})^*$, $\delta = .25, .5$ and $.75$, respectively, for Panels 3, 4 and 5 below											
0.00	0.126	0.195	0.182	0.147	0.111	0.162	0.155	0.122	0.112	0.120	0.121	0.111
0.25	0.152	0.231	0.206	0.170	0.147	0.187	0.173	0.152	0.159	0.145	0.140	0.159
0.50	0.203	0.331	0.286	0.213	0.285	0.267	0.284	0.249	0.394	0.234	0.280	0.340
0.75	0.268	0.428	0.380	0.285	0.522	0.402	0.461	0.469	0.749	0.444	0.533	0.666
1.00	0.353	0.516	0.463	0.390	0.762	0.548	0.645	0.695	0.948	0.683	0.758	0.909
1.25	0.458	0.572	0.540	0.468	0.913	0.687	0.778	0.868	0.995	0.861	0.881	0.988
1.50	0.542	0.622	0.604	0.542	0.972	0.807	0.861	0.948	1.000	0.953	0.944	0.999
1.75	0.610	0.655	0.661	0.613	0.991	0.881	0.913	0.984	1.000	0.988	0.965	1.000
2.00	0.669	0.691	0.708	0.671	0.998	0.934	0.942	0.993	1.000	0.996	0.980	1.000
0.00	0.135	0.117	0.115	0.123	0.117	0.095	0.101	0.105	0.112	0.092	0.103	0.108
0.25	0.145	0.166	0.146	0.145	0.138	0.134	0.136	0.135	0.142	0.139	0.140	0.143
0.50	0.198	0.295	0.246	0.202	0.236	0.267	0.258	0.236	0.274	0.286	0.281	0.269
0.75	0.278	0.430	0.382	0.294	0.386	0.437	0.441	0.388	0.479	0.512	0.518	0.484
1.00	0.390	0.536	0.497	0.398	0.578	0.609	0.617	0.578	0.714	0.712	0.729	0.703
1.25	0.495	0.623	0.598	0.512	0.734	0.728	0.756	0.742	0.863	0.851	0.860	0.862
1.50	0.579	0.683	0.676	0.594	0.845	0.824	0.846	0.849	0.944	0.928	0.935	0.943
1.75	0.664	0.724	0.732	0.668	0.922	0.890	0.902	0.914	0.979	0.966	0.964	0.978
2.00	0.722	0.770	0.771	0.729	0.956	0.928	0.937	0.953	0.992	0.983	0.984	0.993
0.00	0.111	0.096	0.100	0.107	0.108	0.090	0.097	0.107	0.113	0.097	0.094	0.102
0.25	0.127	0.130	0.126	0.124	0.115	0.114	0.113	0.127	0.123	0.118	0.124	0.123
0.50	0.148	0.198	0.175	0.154	0.162	0.186	0.178	0.171	0.176	0.192	0.186	0.181
0.75	0.185	0.270	0.251	0.208	0.223	0.270	0.261	0.235	0.272	0.290	0.286	0.271
1.00	0.242	0.353	0.314	0.263	0.315	0.355	0.356	0.311	0.384	0.397	0.409	0.383
1.25	0.297	0.414	0.375	0.310	0.400	0.443	0.441	0.403	0.503	0.504	0.519	0.488
1.50	0.348	0.458	0.424	0.370	0.480	0.529	0.524	0.479	0.588	0.605	0.608	0.598
1.75	0.401	0.490	0.472	0.416	0.542	0.598	0.588	0.548	0.671	0.683	0.694	0.671
2.00	0.454	0.540	0.518	0.469	0.614	0.646	0.651	0.619	0.743	0.745	0.764	0.749

*dgp1 = normal, dgp2=normal mixture ($\tau = 10, p = 0.05$), dgp3=lognormal, dgp4=chi-squared (df=3).