Appendix E: Supplementary Material

For "Fixed Effects Estimation of Spatial Panel Model with Missing Responses: An Application to US State Tax Competition"

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This `Supplementary Material` provides proofs of Lemmas B.2 and B.3 in Appendix B of the main text, detailed discussions on some important issues (literature, time/space invariant effects, computing) and technical assumptions, detailed proofs of the theoretical results in Sections 2 and 3, a complete set of Monte Carlo results, and an additional application using a simulated Boston housing price panel (obtained based on the popular Boston housing price data).

E.1. Proofs of Lemmas B.2 and B.3

Although $\mathbf{\Omega}_N(\delta)$ is defined differently in Sections 2 and 3, Lemmas B.2 and B.3 can be shown in a similar manner. In particular, Lemma B.2 (ii) is assumed to be true in Assumption F$'$ of Section 3. Therefore, the lemmas and following proofs are based on $\mathbf{\Omega}_N(\delta)$ in Section 2.

Proof of Lemma B.2:

Proof of (i). Let $\mathbf{M}_{nT}(\delta) = \mathbf{A}'_{nT}(\lambda)\mathbf{B}'_{nT}(\rho)\mathbf{B}_{nT}(\rho)\mathbf{A}_{nT}(\lambda)$. As $\mathbf{\Omega}_N(\delta)$ is the principal submatrix of $\mathbf{M}^{-1}_{nT}(\delta)$, it is sufficient to show that $\mathbf{M}^{-1}_{nT}(\delta)$ is uniformly bounded in both row and column sum norms, uniformly in $\delta \in \Delta$, which is directly implied by Assumption E (i) and Lemma B.1. Similarly, we have

$$\dot{\mathbf{\Omega}}_\lambda(\delta) = \mathcal{S}[(\tfrac{\partial}{\partial\lambda}\mathbf{A}^{-1}_{nT}(\lambda))\mathbf{B}^{-1}_{nT}(\rho)\mathbf{B}^{-1\prime}_{nT}(\rho)\mathbf{A}^{-1\prime}_{nT}(\lambda) + \mathbf{A}^{-1}_{nT}(\lambda)\mathbf{B}^{-1}_{nT}(\rho)\mathbf{B}^{-1\prime}_{nT}(\rho)(\tfrac{\partial}{\partial\lambda}\mathbf{A}^{-1\prime}_{nT}(\lambda))]\mathcal{S}',$$
$$\dot{\mathbf{\Omega}}_\rho(\delta) = \mathcal{S}\mathbf{A}^{-1}_{nT}(\lambda)[\tfrac{\partial}{\partial\rho}\mathbf{B}^{-1}_{nT}(\rho) + \tfrac{\partial}{\partial\rho}\mathbf{B}^{-1\prime}_{nT}(\rho)]\mathbf{A}^{-1\prime}_{nT}(\lambda)\mathcal{S}'.$$

By Assumption E (i) and Lemma B.1, both $\frac{\partial}{\partial\lambda}\mathbf{A}^{-1}_{nT}(\lambda) = \mathbf{A}^{-1}_{nT}(\lambda)\mathbf{W}\mathbf{A}^{-1}_{nT}(\lambda)$ and $\frac{\partial}{\partial\rho}\mathbf{B}^{-1}_{nT}(\rho) = \mathbf{B}^{-1}_{nT}(\rho)\mathbf{M}\mathbf{B}^{-1}_{nT}(\rho)$ are bounded in both row and column sums, uniformly in $\delta \in \Delta$. Thus, $\dot{\mathbf{\Omega}}_\lambda(\delta)$ and $\dot{\mathbf{\Omega}}_\rho(\delta)$ are also bounded in a row (column) sum, uniformly in $\delta \in \Delta$.

Note that $\mathbf{\Omega}^{-1}_N(\delta) = [\mathcal{S}\mathbf{M}^{-1}_{nT}(\delta)\mathcal{S}']^{-1}$. We first show $[S\mathbf{M}^{-1}_{nT}(\delta)S']^{-1}$, where S is a $(nT-1) \times nT$ selection matrix with a single observation omitted. Let r be the complement unit row such that $S'S + r'r = I_{nT}$. Thus, we have $\mathbf{M}_{nT}(\delta) = S'M_1(\delta)S + S'M_2(\delta)r + r'M_3(\delta)R + r'M_4(\delta)r$, where $M_1(\delta) = S\mathbf{M}_{nT}(\delta)S'$, $M_2(\delta) = S\mathbf{M}_{nT}(\delta)r'$, $M_3(\delta) = r\mathbf{M}_{nT}(\delta)S'$, and $M_4(\delta) = r\mathbf{M}_{nT}(\delta)r'$. As $\mathbf{M}_{nT}(\delta)$ is a positive definite (p.d.) matrix, all of its principal submatrices (including diagonal elements) are also p.d. matrices, uniformly in $\delta \in \Delta$. Thus, with the inverse formula of a partitioned matrix and $S'S + r'r = I_{nT}$, we have $\mathbf{M}^{-1}_{nT}(\delta) = S'[M_1(\delta) -$

$M_2(\delta)M_4^{-1}(\delta)M_3(\delta)]^{-1}S + S' \ldots r + r' \ldots R + r' \ldots r$. Using $SS' = I_{(n-1)T}$ and $rS' = 0$, we have $[S\mathbf{M}_{nT}^{-1}(\delta)S']^{-1} = M_1(\delta) - M_2(\delta)M_4^{-1}(\delta)M_3(\delta) = S\mathbf{M}_{nT}(\delta)S' - \frac{S\mathbf{M}_{nT}(\delta)r'r\mathbf{M}_{nT}(\delta)S'}{r\mathbf{M}_{nT}(\delta)r'}$. As S, r and $\mathbf{M}_{nT}(\delta)$ are all uniformly bounded in both row and column sums and $r\mathbf{M}_{nT}(\delta)r'$ is positive, we have $[S\mathbf{M}_{nT}^{-1}(\delta)S']^{-1}$ is also bounded in both row and column sums, uniformly in $\delta \in \Delta$, by Lemma B.1. Note that the selection matrix $\mathcal{S}$ can be written as the product of $nT - N$ selection matrices with a single observation deleted. Therefore, we can repeat the above procedures and show that $\mathbf{\Omega}_N^{-1}(\delta)$ is also bounded in both row and column sums, uniformly in $\delta \in \Delta$.

Proof of (ii)**.** Proof is simpler using a $\mathbf{D}_\alpha^\star$ under the constraint $\alpha_1 = 0$. Recall $\mathbb{D}(\delta) = [\mathbb{D}_\mu(\delta), \mathbb{D}_\alpha(\delta)]$ with $\mathbb{D}_\mu(\delta) = \mathbf{C}(\delta)\mathbf{D}_\mu$, $\mathbb{D}_\alpha(\delta) = \mathbf{C}(\delta)\mathbf{D}_\alpha^\star$ and $\mathbf{C}(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}(\lambda)$. Denote $\mathcal{D}_{11}(\delta) = \mathbb{D}_\mu'(\delta)\mathbb{D}_\mu(\delta)$, $\mathcal{D}_{12}(\delta) = \mathbb{D}_\mu'(\delta)\mathbb{D}_\alpha(\delta)$, $\mathcal{D}_{22}(\delta) = \mathbb{D}_\alpha'(\delta)\mathbb{D}_\alpha(\delta)$. Using the inverse formula of a partitioned matrix, we have

$$[\mathbb{D}'(\delta)\mathbb{D}(\delta)]^{-1} = \begin{bmatrix} \mathcal{F}^{-1}(\delta) & -\mathcal{F}^{-1}(\delta)\mathcal{D}_{12}(\delta)\mathcal{D}_{22}^{-1}(\delta) \\ -\mathcal{D}_{22}^{-1}(\delta)\mathcal{D}_{12}'(\delta)\mathcal{F}^{-1}(\delta) & \mathcal{D}_{22}^{-1}(\delta) + \mathcal{D}_{22}^{-1}(\delta)\mathcal{D}_{12}'(\delta)\mathcal{F}^{-1}(\delta)\mathcal{D}_{12}(\delta)\mathcal{D}_{22}^{-1}(\delta) \end{bmatrix},$$

where $\mathcal{F}(\delta) = \mathcal{D}_{11}(\delta) - \mathcal{D}_{12}(\delta)\mathcal{D}_{22}^{-1}(\delta)\mathcal{D}_{12}'(\delta)$. Plugging this into $\mathbb{Q}_{\mathbb{D}}(\delta)$, we obtain,

$$\mathbb{Q}_{\mathbb{D}}(\delta) = \mathbb{Q}_{\mathbb{D}_\alpha}(\delta) - \mathbb{Q}_{\mathbb{D}_\alpha}(\delta)\mathbb{D}_\mu(\delta)[\mathbb{D}_\mu'(\delta)\mathbb{Q}_{\mathbb{D}_\alpha}(\delta)\mathbb{D}_\mu(\delta)]^{-1}\mathbb{D}_\mu'(\delta)\mathbb{Q}_{\mathbb{D}_\alpha}(\delta).$$

Plugging this into $\Psi(\delta)$, we first show $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{Q}_{\mathbb{D}_\alpha}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$. Given the special structure of $\mathbb{D}_\alpha(\delta)$, one has $\mathbb{Q}_{\mathbb{D}_\alpha}(\delta) = \texttt{blkdiag}(Q_1(\delta), \ldots, Q_T(\delta))$, where $Q_1(\delta) = I_n$ and $Q_t(\delta) = I_n - \frac{1}{n}C_t(\delta)l_n[\frac{1}{n}l_n'C_t'(\delta)C_t(\delta)l_n]^{-1}l_n'C_t'(\delta)$ for $t = 2, \cdots, T$. Note that $\mathbf{\Omega}_N(\delta)$ is block diagonal and denote its tth block by $\Omega_t(\delta)$. Thus, we only need to show that $\Omega_t^{-\frac{1}{2}}(\delta)Q_t(\delta)\Omega_t^{-\frac{1}{2}}(\delta), t = 1, \cdots, T$ are uniformly bounded in both row and column sums, uniformly in $\delta \in \Delta$. It is trivial when $t = 1$. For $t = 2, \cdots, T$, by Assumption E and Lemma B.2(i), the limit of $\frac{1}{n}l_n'C_t'(\delta)C_t(\delta)l_n$ is bounded away from zero and the elements of $\Omega_t^{-\frac{1}{2}}(\delta)C_t(\delta)l_nl_n'C_t'(\delta)\Omega_t^{-\frac{1}{2}}(\delta)$ are uniformly bounded, uniformly in $\delta \in \Delta$. Therefore, $\Omega_t^{-\frac{1}{2}}(\delta)Q_t(\delta)\Omega_t^{-\frac{1}{2}}(\delta), t = 2, \cdots, T$ must be uniformly bounded in both row and column sums, uniformly in $\delta \in \Delta$.

We next consider $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{Q}_{\mathbb{D}_\alpha}(\delta)\mathbb{D}_\mu(\delta)[\mathbb{D}_\mu'(\delta)\mathbb{Q}_{\mathbb{D}_\alpha}(\delta)\mathbb{D}_\mu(\delta)]^{-1}\mathbb{D}_\mu'(\delta)\mathbb{Q}_{\mathbb{D}_\alpha}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$. Denote it as $\bar{\mathcal{Q}}(\delta)$, which can be partitioned into $T \times T$ blocks with (s,t)th block being

$$\bar{\mathcal{Q}}_{s,t}(\delta) = \tfrac{1}{T}\Omega_s^{-\frac{1}{2}}(\delta)Q_s(\delta)C_s(\delta)[\tfrac{1}{T}\textstyle\sum_{t=1}^T C_t'(\delta)Q_t(\delta)C_t(\delta)]^{-1}C_t'(\delta)Q_t(\delta)\Omega_t^{-\frac{1}{2}}(\delta).$$

By assuming $A_s^{-1}(\lambda)[\frac{1}{T}\sum_{t=1}^T C_t'(\delta)Q_t(\delta)C_t(\delta)]^{-1}A_t^{-1\prime}(\lambda)$ is uniformly bounded in both row and column sum norms, uniformly in $\delta \in \Delta$, for all s and t, we have that the row and column sums of each $\bar{\mathcal{Q}}_{s,t}(\delta)$ must have uniform order $O(1/T)$, uniformly in $\delta \in \Delta$. As there are T blocks in each row or in each column of $\bar{\mathcal{Q}}(\delta)$, we must have $\bar{\mathcal{Q}}(\delta)$ bounded in both row and column sum norms, uniformly in $\delta \in \Delta$. Consequently, $\Psi(\delta)$ is bounded in both row and column sum norms, uniformly in $\delta \in \Delta$.

Proof of *(iii)***.** Let $Z_N(\delta) = [\frac{1}{N}\tilde{\mathbb{X}}'(\delta)\tilde{\mathbb{X}}(\delta)]^{-1} = [\frac{1}{N}\mathbb{X}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta)]^{-1}$ with its (j,k)th element being denoted by $z_{jk}(\delta)$. From Assumption C, $Z_N(\delta)$ converges to a finite limit uniformly in $\delta \in \Delta$. Therefore, there exists a constant c_z such that $|z_{jk}(\delta)| \leq c_z$ uniformly in $\delta \in \Delta$ for large enough N. Note that $\mathbb{X}(\delta) = \mathbf{C}(\delta)\mathbf{X}$. As the elements of $\mathbf{X}$ are uniformly bounded (Assumption C), and $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbf{C}(\delta) \equiv \Psi(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}(\lambda)$ are bounded in both row and column sum norms, uniformly in $\delta \in \Delta$, the elements of $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta)$ are also uniformly bounded, uniformly in $\delta \in \Delta$. Hence, there exists a constant c_x such that $|x_{jk}(\delta)| \leq c_x$ uniformly in $\delta \in \Delta$, where $x_{jk}(\delta)$ is the (j,k)th element of $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta)$. Let $p_{jl}(\delta)$ be the (j,l)th element of $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$. It follows that uniformly in $\delta \in \Delta$, $\sum_{j=1}^{N}|p_{jl}(\delta)| \leq \frac{1}{N}\sum_{j=1}^{N}\sum_{r=1}^{k}\sum_{s=1}^{k}|z_{rs}(\delta)x_{jr}(\delta)x_{ls}(\delta)| \leq k^2 c_z c_x^2$ for all $l = 1, 2, \ldots, N$. Similarly, uniformly in $\delta \in \Delta$, we have $\sum_{l=1}^{nT}|p_{jl}(\delta)| \leq \frac{1}{N}\sum_{l=1}^{nT}\sum_{r=1}^{k}\sum_{s=1}^{k}|z_{rs}(\delta)x_{jr}(\delta)x_{ls}(\delta)| \leq k^2 c_z c_x^2$ for all $j = 1, 2, \ldots, N$. That is, both $\|\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\|_1$ and $\|\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\|_\infty$ are bounded, uniformly in $\delta \in \Delta$. ■

Proof of Lemma B.3: From the proof of Lemma B.2, the elements of $[\frac{1}{N}\mathbb{X}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta)]^{-1}$ are uniformly bounded, uniformly in $\delta \in \Delta$. If A_N and B_N are bounded in row (column) sum norm, then $A_N B_N$ is also bounded in row (column) sum norm. Thus, Lemma A.6 of Lee (2004) implies that the elements of $\frac{1}{N}\mathbf{X}' A_N B_N \mathbf{X}$ are uniformly bounded. It follows $\mathtt{tr}[A_N \mathbf{X}[\mathbb{X}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta)]^{-1}\mathbf{X}' B_N] = \mathtt{tr}[(\frac{1}{N}\mathbb{X}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta))^{-1}\frac{1}{N}\mathbf{X}'(\delta)A_N B_N \mathbf{X}(\delta)] = O(1)$, uniformly in $\delta \in \Delta$ because the number of regressors k is fixed. ■

E.2. Details on some important issues and technical assumptions

Literature. The model considered by Zhou et al. (2022) takes the following form:

$$Y_{it} = \alpha_i Y_{i(t-1)} + u_{it},$$

where $U_t = (u_{1t}, \ldots, u_n)$ follows a spatial autoregressive (SAR) process, that is, $U_t = \rho M U_t = V_t$ (to use our notation). They introduce an indicator Z_{it} for missingness in Y_{it} (1 for present and 0 for missing) and model missing probability by $p_{it} = P(Z_{it} = 1 | Y_{it}, X_{it}) = \frac{\exp(X_{it}'\beta)}{1+\exp(X_{it}'\beta)}$, where X_{it} is a vector of covariates, used only to model missing probabilities.

Clearly, this model differs substantially from our model (1.2), having a completely different mean structure and using a different way of handling missingness in response. Due to this major modelling difference, we are unable to carry out a more comprehensive comparison of the two methods, nor direct comparisons in both simulation studies and empirical applications.

Nevertheless, it is possible and interesting, as a future research, to extend our framework to include time lags $\alpha_i Y_{i(t-1)}$, but it may be difficult to extend the framework of Zhou et al. (2022) to include spatial lag, covariates, spatial Durbin effects, and fixed effects, since their estimation of temporal effect parameters α_i is based on weighted least squares.

Another related work, Liu et al. (2023), studied a dynamic SPD-MR model but without fixed effects and spatially and serially correlated errors, specified as:

$$Y_t = \lambda \mathbf{W} Y_t + \gamma Y_{t-1} + \rho \mathbf{W} Y_{t-1} + \mathbf{X}_t \beta + V_t,$$

where the missingness mechanism is designed following Zhou et al. (2022). This model also differs significantly from ours, as it does not control for unobserved individual and time heterogeneity that may correlate with the regressors (fixed effects). Furthermore, their objection function (5) appears not to account for missingness in the neighbors of observed units.

Time/space invariant covariate effects. As discussed in the main text, Model (2.1) in fact allows the time-invariant and space-invariant covariates effects Z, Z_t, such as *gender* and *policy*. Our view is that they are a part of the FEs and can be "decomposed" from $\mathbf{D}$ by adding further constraints on ϕ.

For example, suppose that the model includes a time-invariant `gender` covariate (taking value 1 for male and 0 for female). As *gender* is part of individual FE, to avoid the dummy variable trap, one can simply omit one column of $\mathbf{D}$ that corresponds to a male individual and drop the corresponding element of ϕ. Alternatively, one can impose the constraint that the sum of individual effects for males is zero. Denoting the resulting unobserved FEs as $\mathbf{D}^\circ \phi_0^\circ$ and merging *gender* variable into the regressor matrix $\mathbf{X}$, one proceeds with Model (2.1).

Similarly, if the model also includes a space-invariant `policy` variable (taking value 1 at or after policy implementation and 0 otherwise), then simply drop two columns in D (one before policy and one at or after) and the corresponding elements in ϕ_0. The first is to avoid multicolinearity between time dummies and individual FEs, and the second is to avoid multicolinearity between the time dummies at or after policy and the policy dummy.

More complicated time-invariant or space-invariant variable effects, such as `race` and `multiple policies`, can be handled in a similar spirit.

A computational note. Computation of our M-estimator boils down to solving the two-dimensional nonlinear equations $S_N^{*c}(\delta) = 0$, where $S_N^{*c}(\delta)$ is given in (2.9), which depends critically on the computation of $\mathbf{\Omega}_N^{-1}(\delta)$. Computation of QML estimator boils down to maximizing the concentrated loglikelihood of δ (concentrating out β and σ_v^2 from $\ell_N^c(\theta)$ given in (2.5)), which depends critically the computation of $|\mathbf{\Omega}_N(\delta)|$. Therefore, the times spent in computing M-estimator and QML estimator should be comparable.

To substantiate this, we conduct a simple Monte Carlo experiment by running Matlab on a computer equipped with 64.0 GB of RAM and a 13th Gen Intel(R) Core(TM) i7-13700K processor running at 3.40 GHz. The Matlab routine `fmincon` is used for QML estimation and the `lsqnonlin` for M-estimation. We choose the largest sample size ($n = 400, T = 10$) used in our Monte Carlo experiments with 10% missing, normal errors, and time-varying spatial weight matrices: (`Group Interaction` for spatial lag and `Queen Contiguity` for spatial error). Based on 500 replications, the results show that the average time spent for M-estimation is 87.1758 seconds and the average time spent for QML estimation is 96.6029 seconds.

For the **calculation of $\mathbf{\Omega}_N(\delta)^{-1/2}$**, because $\mathbf{\Omega}_N(\delta)$ is block diagonal, its inverse square root can be obtained block-by-block, which keeps the problem tractable even for large panels. When individual blocks are large, standard sparse-matrix tools, e.g., Cholesky decomposition and iterative methods such as conjugate gradients, allow fast evaluation or approximation of the inverse square root.

More on Assumption C. The strict exogeneity of $\mathcal{S}$ is consistent with the random missing mechanisms assumed throughout. The concept of random missingness in this paper refers to either missing completely at random (MCAR) or missing at random (MAR), as discussed in Little and Rubin (2019). The MAR condition allows missingness (represented by the selection matrix $\mathcal{S}$) to depend on the observed covariates and fixed effects.

In practice, one can first apply Little's (1988) test for missing completely at random (MCAR). If it is not supported by the data, we then recommend using other approaches to test the MAR assumption, such as the instrumental variables (IV) approach proposed by Breunig (2019).

To investigate the finite sample performance of the proposed M-estimation strategy under MAR mechanism, additional Monte Carlo experiments are carried out using $\mathcal{S}_t$ that are generated according to the values of covariates and fixed effects. See Monte Carlo section for details. The simulation results in Table 9 show that the proposed M-estimator performs excellently in finite sample, irrespective of data-generating processes, error distributions, or sample sizes.

More on Assumption F: Assumption F is not restrictive, as it always holds in the case of a balanced panel. Consider a balanced spatial panel data model with a time-invariant and row-normalized spatial weight matrix. In this setting, we have $n_t = n$ and $\mathcal{S}_t = I_n$ for all t, along with $M_t = M$ and $B_t(\rho) = I_n - \rho M \equiv B(\rho)$. Since $M \times l_n = l_n$, it follows that $Q_t(\delta) = I_n - \frac{1}{n} l_n l_n'$ for $t = 2, \ldots, T$, which is independent of the parameter δ. Consequently, the expression $A_s^{-1}(\lambda)[\frac{1}{T}\sum_{t=1}^{T} C_t'(\delta) Q_t(\delta) C_t(\delta)]^{-1} A_t^{-1\prime}(\lambda)$ simplifies to $(I_n - \frac{T-1}{nT} l_n l_n')^{-1}$. As $I_n - \frac{T-1}{nT} l_n l_n'$ is strictly diagonally dominant in both rows and columns, its inverse is bounded in both the row and column sum norms (Varah, 1975).

More on Assumption G. Let $\eta = \mathcal{S}\mathbf{A}_{nT}^{-1}(\mathbf{X}\beta_0 + \mathbf{D}\phi_0)$. Let $\mathbb{Q}_{\tilde{\mathbb{X}}}(\delta)$ be the projection matrices based on $\tilde{\mathbb{X}}(\delta) = \mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{X}(\delta)$. Denote $\mathcal{O}_N(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbf{\Omega}_N\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$, $\mathcal{Q}_N(\delta) = \mathbb{Q}_{\tilde{\mathbb{X}}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$, and $\mathbb{D}^\dagger(\delta) = \mathbf{D}[\mathbb{D}'(\delta)\mathbb{D}(\delta)]^{-1}\mathbb{D}'(\delta)$. A more primitive condition for Assumption G to hold is that either (a) or (b) holds for $\delta \neq \delta_0$:

(a) $\frac{1}{2\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\eta'\mathcal{Q}'_N(\delta)\mathbb{H}_\lambda(\delta)\mathcal{Q}_N(\delta)\eta + \frac{1}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\eta'\mathcal{Q}'_N(\delta)\mathbb{J}(\delta)[\mathbb{D}^\dagger(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\eta - \mathbb{X}(\delta)\bar{\beta}_{\mathtt{M}}(\delta) + \mathbf{X}\bar{\beta}_{\mathtt{M}}(\delta)]$
$+ \frac{\sigma^2_{v0}}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathtt{tr}[\mathbb{J}(\delta)\mathbb{D}^\dagger(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)] + \frac{1}{2}\mathtt{tr}[\frac{\sigma^2_{v0}}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathbb{H}_\lambda(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta) - \mathbb{H}_\lambda(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)] \neq 0,$

(b) $\frac{1}{2\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\eta'\mathcal{Q}'_N(\delta)\mathbb{H}_\rho(\delta)\mathcal{Q}_N(\delta)\eta + \frac{1}{2}\mathtt{tr}[\frac{\sigma^2_{v0}}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathbb{H}_\rho(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta) - \mathbb{H}_\rho(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)] \neq 0,$

where the expressions of $\bar{\beta}_{\mathtt{M}}(\delta)$ and $\bar{\sigma}^2_{v,\mathtt{M}}(\delta)$ are in (C.2) and (C.3). As $\mathcal{Q}_N(\delta_0)\eta = 0$, $\mathcal{O}_N(\delta_0) = I_N$, $\bar{\beta}_{\mathtt{M}}(\delta_0) = \beta_0$ and $\bar{\sigma}^2_{v,\mathtt{M}}(\delta_0) = \sigma^2_{v0}$, the two quantities in (a) and (b) are both 0 when $\delta = \delta_0$.

Additionally, Assumption G is analogous to identification conditions commonly discussed in the likelihood or GMM framework. Specifically, under a balanced panel with time-invariant, row-normalized spatial weights and i.i.d. errors, our M-estimation reduces exactly to the transformation method of Lee and Yu (2010), because our estimating equations correspond precisely to the first-order conditions derived from the likelihood function of their transformed model. Consequently, the above two primitive conditions then collapse to their identification Assumption 7′. Therefore, Assumption G just extends that familiar criterion to the more general incomplete-panel setting studied here.

More on the Additional Assumptions in Theorem 2.1. First, our asymptotic framework allows either T fixed or n fixed (but not both). In the former, n must be large, and in the latter, T must be large. It of course allows for the scenario of both n and T being large without constraints on their relative magnitude.

By $\frac{n_t}{n} \to c_t$ and $c_t \in (0, 1]$, we mean that when n goes to large, $\{n_t\}$ follow proportionally, and similarly for $\frac{T_i}{T} \to d_i$ with $d_i \in (0, 1]$. It is not sufficient to specify $min(n_t) \geq 1$ and $min(T_i) \geq 1$. To understand this, we need to separate the two issues: the identification of the FE parameters ϕ and the (subsequent) consistent estimation of θ.

Indeed, with only $min(n_t) \geq 1$, one can still have $c_t \in (0, 1]$ with fixed n, and the identification of ϕ is still possible, as long as $[\mathbb{X}(\delta), \mathbb{D}(\delta)]$ in (2.2) is of full column rank. However, it is not sufficient for the (subsequent) consistent estimation of the common parameters θ, because after concentrating ϕ out, any period with only one observed response will be excluded from the subsequent analysis, **causing additional missing responses**, incomplete spatial structure, and thus inconsistency of $\hat{\theta}_{\mathtt{M}}$. The same arguments apply when $min(T_i) \geq 1$ with fixed T.

The consistency of $\hat{\theta}_{\mathtt{M}}$ can be achieved if imposing $\min(n_t) \geq 2$. To substantiate this claim, we follow a suggestion from a referee and consider the extreme case with $n_t = 2, \forall t$. In this case, to have $c_t \in (0, 1]$, n must be fixed. It follows that T must be large. Let $n = 4$, and assume units 1 and 2 have observed responses in the first τ periods, while units 3 and 4 have observed responses in the remaining $T-\tau$ periods. Thus, the number of times each unit being observed over the entire T periods are $T_1 = T_2 = \tau$ and $T_3 = T_4 = T - \tau$.

To gain insights, consider a regular panel model ($\delta_0 = 0$) with one-way time FE. The proposed M-estimator of β reduces to a within estimator $\hat{\beta}_{\mathtt{M}}(0) = (\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{X})^{-1}(\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{Y})$, where $\mathbb{Q}_{\mathbb{D}}$ is the projection matrix based on $\mathbb{D} = \mathcal{S}\mathbf{D} = I_T \otimes l_2$ and $\mathcal{S}$ is the selection matrix based on this setup. We have $\hat{\beta}_{\mathtt{M}}(0) - \beta_0 = (\frac{1}{T}\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{X})^{-1}(\frac{1}{T}\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{V})$. Write

$$\frac{1}{T}\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{X} = \frac{1}{T}\sum_{t=1}^{\tau}\sum_{i=1}^{2}(x_{it} - \bar{x}_{\cdot t})(x_{it} - \bar{x}_{\cdot t})' + \frac{1}{T}\sum_{t=\tau+1}^{T}\sum_{i=3}^{4}(x_{it} - \bar{x}_{\cdot t})(x_{it} - \bar{x}_{\cdot t})', \tag{O.1}$$

$$\frac{1}{T}\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{V} = \frac{1}{T}\sum_{t=1}^{\tau}\sum_{i=1}^{2}(x_{it} - \bar{x}_{\cdot t})(v_{it} - \bar{v}_{\cdot t})' + \frac{1}{T}\sum_{t=\tau+1}^{T}\sum_{i=3}^{4}(x_{it} - \bar{x}_{\cdot t})(v_{it} - \bar{v}_{\cdot t})', \tag{O.2}$$

where $\bar{x}_{\cdot t} = \frac{1}{2}(x_{1t} + x_{2t})$ for $t = 1, \ldots, \tau$, $\bar{x}_{\cdot t} = \frac{1}{2}(x_{3t} + x_{4t})$ for $t = \tau+1, \ldots, T$ and $\bar{v}_{\cdot t}$ is defined similarly. Under our assumptions, the limit of (O.1) when T goes large is a non-singular matrix, and $\text{plim}_{T\to\infty}\frac{1}{T}\mathbf{X}'\mathcal{S}'\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{V} = 0$. Consistency of $\hat{\beta}_{\mathtt{M}}(0)$ thus follows.

In essence, the estimation relies on the increasing number of periods to gather enough information for a consistent estimation. Despite the fact that each period has only two observed responses, the 'observed' units are fully utilized as seen from the structure $\mathbb{Q}_{\mathbb{D}}\mathcal{S}\mathbf{Y}$. Interestingly, with $\delta_0 = 0$, X values are also "selected" and not fully utilized as seen from $\mathbb{X} = \mathbf{C}\mathbf{X} = \mathcal{S}\mathbf{X}$; see around (2.2) for the definitions of various quantities. In contrast, with $\lambda_0 \neq 0$, $\mathbb{X} = \mathbf{C}\mathbf{X} = \mathbf{\Omega}_N^{-\frac{1}{2}}\mathcal{S}A_{nT}^{-1}(\lambda_0)\mathbf{X}$, where $A_{nT}^{-1}(\lambda_0)\mathbf{X}$ picks all the X values including the period where Y were missing. Similarly, with $\rho_0 \neq 0$, $B_{nT}^{-1}(\rho_0)\mathbf{V}$ picks all the errors.

In general, when $\delta_0 \neq 0$, $\hat{\beta}_{\mathtt{M}}(\delta_0) - \beta_0$ again contains two terms analogous to (O.1) and (O.2), but the summations now involve all $\mathbf{X}$ values; see, e.g., $\mathbb{X} = \mathbf{\Omega}_N^{-\frac{1}{2}}\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{X}$. With the boundedness properties from Assumptions D to F and leveraging the expanding information in time dimension, the consistency of $\hat{\beta}_{\mathtt{M}}(\delta_0)$ can be established in this small n, large T setting. Along the same line, the consistency of $\hat{\sigma}^2_{v,\mathtt{M}}(\delta_0)$ can be proved.

Finally, the consistency of $\hat{\beta}_{\mathtt{M}} \equiv \hat{\beta}_{\mathtt{M}}(\hat{\delta}_{\mathtt{M}})$ and $\hat{\sigma}^2_{v,\mathtt{M}}(\hat{\delta}_{\mathtt{M}})$ depends on the consistency of $\hat{\delta}_{\mathtt{M}}$. To prove the consistency of $\hat{\delta}_{\mathtt{M}}$ with $n_t = 2, \forall t$, we simplify the primitive version of Assumption G, given above, and show that this condition is satisfied under this extreme case. Then, we verify conditions (a)-(d) in the proof of Theorem 2.1. Details are available upon request.

More on Assumption G′. Assumption G′ is a high-level assumption introduced for simplicity. We can demonstrate that it holds under certain primitive conditions. With the redefined δ and $\mathbf{\Omega}_N(\delta)$, update $\mathbb{Y}(\delta)$, $\mathbb{X}(\delta)$, $\mathbb{D}(\delta)$, and $\bar{\mathbb{V}}(\delta)$ in (C.2) and obtain $\bar{\beta}^{\diamond}_{\mathtt{M}}(\delta)$ and $\bar{\sigma}^{\diamond 2}_{v,\mathtt{M}}(\delta)$. Similarly, we can obtain the population counterpart $\bar{S}^{\diamond c}_N(\delta)$ of $S^{\diamond c}_N(\delta)$, corresponding to

$\bar{S}_N^{*c}(\delta)$ in (C.1). With the updated $\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)$, $\mathcal{Q}_N(\delta)$, $\mathcal{O}_N(\delta)$, $\mathbb{J}(\delta)$, $\mathbb{D}^{\dagger}(\delta)$ and $\mathbb{H}_{\omega}(\delta)$, $\omega=\lambda,\rho,\tau$, a more primitive condition for Assumption G$'$ to hold is that either (a), (b) or (c) holds for $\delta\neq\delta_0$, where

(*a*) $\frac{1}{2\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\eta'\mathcal{Q}_N'(\delta)\mathbb{H}_{\lambda}(\delta)\mathcal{Q}_N(\delta)\eta+\frac{1}{\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\eta'\mathcal{Q}_N'(\delta)\mathbb{J}(\delta)\big\{\mathbb{D}^{\dagger}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)[\eta-\mathcal{S}\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}\bar{\beta}_{\mathtt{M}}^{\diamond}(\delta)]+\mathbf{X}\bar{\beta}_{\mathtt{M}}^{\diamond}(\delta)\big\}+\frac{\sigma_{v0}^2}{\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\mathtt{tr}[\mathbb{J}(\delta)\mathbb{D}^{\dagger}(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)]+\frac{1}{2}\mathtt{tr}[\frac{\sigma_{v0}^2}{\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\mathbb{H}_{\lambda}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)-\mathbb{H}_{\lambda}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)]\neq 0$,

(*b*) $\frac{1}{2\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\eta'\mathcal{Q}_N'(\delta)\mathbb{H}_{\rho}(\delta)\mathcal{Q}_N(\delta)\eta+\frac{1}{2}\mathtt{tr}[\frac{\sigma_{v0}^2}{\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\mathbb{H}_{\rho}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)-\mathbb{H}_{\rho}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)]\neq 0$,

(*c*) $\frac{1}{2\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\eta'\mathcal{Q}_N'(\delta)\mathbb{H}_{\tau}(\delta)\mathcal{Q}_N(\delta)\eta+\frac{1}{2}\mathtt{tr}[\frac{\sigma_{v0}^2}{\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)}\mathbb{H}_{\tau}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)-\mathbb{H}_{\tau}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)]\neq 0$.

Again, as $\mathcal{Q}_N(\delta_0)\eta=0$, $\mathcal{O}_N(\delta_0)=I_N$, $\bar{\beta}_{\mathtt{M}}^{\diamond}(\delta_0)=\beta_0$ and $\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta_0)=\sigma_{v0}^2$, the quantities in (*a*), (*b*) and (*b*) are all 0 when $\delta=\delta_0$.

E.3. Proofs for Section 2

Proofs use the following facts: (*i*) the eigenvalues of a projection matrix are either 0 or 1; (*ii*) the eigenvalues of a positive definite (p.d.) matrix are strictly positive; (*iii*) $\gamma_{min}(A)\mathtt{tr}(B)\leq\mathtt{tr}(AB)\leq\gamma_{max}(A)\mathtt{tr}(B)$ for symmetric matrix A and positive semi-definite (p.s.d.) matrix B; (*iv*) $\gamma_{max}(A+B)\leq\gamma_{max}(A)+\gamma_{max}(B)$ for symmetric matrices A and B; (*v*) $\gamma_{max}(AB)\leq\gamma_{max}(A)\gamma_{max}(B)$ for p.s.d. matrices A and B; and (*vi*) Let $\gamma_k(\cdot)$ denote the k-th smallest eigenvalue of a matrix. Then $\gamma_k(A)\leq\gamma_k(A_r)\leq\gamma_{k+n-r}(A)$, $1\leq k\leq r$, for symmetric matrix A and its r-by-r principal submatrix A_r.

Proof of Theorem 2.1: By theorem 5.9 of Van der Vaart (1998), we only need to show $\sup_{\delta\in\delta}\frac{1}{N_1}\left\|S_N^{*c}(\delta)-\bar{S}_N^{*c}(\delta)\right\|\xrightarrow{p}0$ under the assumptions in Theorem 2.1. From (2.9) and (C.1), the consistency of $\hat{\delta}_{\mathtt{M}}$ follows from:

(*a*) $\inf_{\delta\in\Delta}\bar{\sigma}_{v,\mathtt{M}}^2(\delta)$ is bounded away from zero,

(*b*) $\sup_{\delta\in\Delta}\left|\hat{\sigma}_{v,\mathtt{M}}^2(\delta)-\bar{\sigma}_{v,\mathtt{M}}^2(\delta)\right|=o_p(1)$,

(*c*) $\sup_{\delta\in\Delta}\frac{1}{N_1}\left|\hat{\mathbb{V}}'(\delta)\mathbb{H}_{\omega}(\delta)\hat{\mathbb{V}}(\delta)-\mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{H}_{\omega}(\delta)\bar{\mathbb{V}}(\delta)]\right|=o_p(1)$, for $\omega=\lambda,\rho$,

(*d*) $\sup_{\delta\in\Delta}\frac{1}{N_1}\left|\hat{\mathbb{V}}'(\delta)\mathbb{J}(\delta)\boldsymbol{\varepsilon}(\hat{\beta}_{\mathtt{M}}(\delta),\delta)-\mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{J}(\delta)\boldsymbol{\varepsilon}(\bar{\beta}_{\mathtt{M}}(\delta),\delta)]\right|=o_p(1)$.

Proof of (a). Note that $\bar{\sigma}_{v,\mathtt{M}}^2(\delta)=\frac{1}{N_1}\eta'\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbf{Q}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\eta+\frac{\sigma_{v0}^2}{N_1}\mathtt{tr}[\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)]$. The first term can be written in the form of $a'(\delta)a(\delta)$ for an $N\times 1$ vector function of δ, and thus is non-negative, uniformly in $\delta\in\Delta$. For the second term,

$$\frac{\sigma_{v0}^2}{N_1}\mathtt{tr}[\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)]\geq\frac{\sigma_{v0}^2}{N_1}\gamma_{\min}[\mathcal{O}_N(\delta)]\mathtt{tr}[\mathbb{Q}_{\mathbb{D}}(\delta)]\geq\sigma_{v0}^2\gamma_{\max}(\mathbf{\Omega}_N)^{-1}\gamma_{\min}[\mathbf{\Omega}_N(\delta)]$$
$$\geq\sigma_{v0}^2\gamma_{\max}(\mathbf{A}_N'\mathbf{A}_N)^{-1}\gamma_{\max}(\mathbf{B}_N'\mathbf{B}_N)^{-1}\gamma_{\min}[\mathbf{A}_N'(\lambda)\mathbf{A}_N(\lambda)]\gamma_{\min}[\mathbf{B}_N'(\rho)\mathbf{B}_N(\rho)]>0,$$

uniformly in $\delta\in\Delta$, by Assumption E(*iii*). It follows that $\inf_{\delta\in\Delta}\bar{\sigma}_{v,\mathtt{M}}^2(\delta)>0$.

Proof of (b). From (2.8), $\hat{\mathbb{V}}(\delta) = \mathbb{Q}_{\mathbb{D}}(\delta)[\mathbb{Y}(\delta) - \mathbb{X}(\delta)\hat{\beta}_{\mathtt{M}}(\delta)] = \mathbb{Q}_{\tilde{\mathbb{X}}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{Y}(\delta)$ and $\hat{\sigma}^2_{v,\mathtt{M}}(\delta) = \frac{1}{N_1}\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta)$. From (C.3), $\bar{\sigma}^2_{v,\mathtt{M}}(\delta) = \frac{1}{N_1}\mathrm{E}[\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta)] + \frac{\sigma^2_{v0}}{N_1}\mathtt{tr}[\mathbf{P}(\delta)\mathcal{O}_N(\delta)]$. Thus, $\hat{\sigma}^2_{v,\mathtt{M}}(\delta) - \bar{\sigma}^2_{v,\mathtt{M}}(\delta) = \frac{1}{N_1}[\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta) - \mathrm{E}(\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta))] - \frac{\sigma^2_{v0}}{N_1}\mathtt{tr}[\mathbf{P}(\delta)\mathcal{O}_N(\delta)]$.

For the second term, $0 \le \frac{1}{N_1}\mathtt{tr}[\mathbf{P}(\delta)\mathcal{O}_N(\delta)] \le \frac{1}{N_1}\gamma_{\max}[\mathcal{O}_N(\delta)]\gamma^2_{\max}[\mathbb{Q}_{\mathbb{D}}(\delta)]\mathtt{tr}[\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)] = o(1)$, because $\mathtt{tr}[\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)] = k$, $\gamma_{\max}[\mathbb{Q}_{\mathbb{D}}(\delta)] = 1$ and, by Assumption E(iii),

$$\gamma_{\max}[\mathcal{O}_N(\delta)] \le \gamma_{\min}(\mathbf{A}'_N\mathbf{A}_N)^{-1}\gamma_{\min}(\mathbf{B}'_N\mathbf{B}_N)^{-1}\gamma_{\max}[\mathbf{A}'_N(\lambda)\mathbf{A}_N(\lambda)]\gamma_{\max}[\mathbf{B}'_N(\rho)\mathbf{B}_N(\rho)] < \infty.$$

Therefore, one has $\sup_{\delta\in\Delta}|\frac{\sigma^2_{v0}}{N_1}\mathtt{tr}[\mathbf{P}(\delta)\mathcal{O}_N(\delta)]| = o(1)$. For the first term, letting $\bar{\mathbf{Q}}(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbf{Q}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$ and using $\mathcal{S}\mathbf{Y} = \eta + \mathcal{S}\mathbf{A}^{-1}_{nT}\mathbf{B}^{-1}_{nT}\mathbf{V}$, we have

$$\begin{aligned}&\tfrac{1}{N_1}[\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta) - \mathrm{E}(\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta))] = \tfrac{1}{N_1}[\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y} - \mathrm{E}(\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y})]\\ &= \tfrac{2}{N_1}\eta'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{A}^{-1}_{nT}\mathbf{B}^{-1}_{nT}\mathbf{V} + \tfrac{1}{N_1}[\mathbf{V}'\mathbf{B}^{-1\prime}_{nT}\mathbf{A}^{-1\prime}_{nT}\mathcal{S}\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{A}^{-1}_{nT}\mathbf{B}^{-1}_{nT}\mathbf{V} - \sigma^2_{v0}\mathtt{tr}(\bar{\mathbf{Q}}(\delta)\mathbf{\Omega}_N)].\end{aligned}$$

Note that $\bar{\mathbf{Q}}(\delta) \equiv \Psi(\delta) - \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$, bounded in both row and column sum norms, uniformly in $\delta \in \Delta$, by Lemma B.2. Then, by Assumption E and Lemma B.1, $\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{A}^{-1}_{nT}\mathbf{B}^{-1}_{nT}$ and $\mathbf{B}^{-1\prime}_{nT}\mathbf{A}^{-1\prime}_{nT}\mathcal{S}\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{A}^{-1}_{nT}\mathbf{B}^{-1}_{nT}$ are also bounded in both row and column sum norms, uniformly in $\delta \in \Delta$. Further, the elements of η are uniformly bounded. Thus, the pointwise convergence of the first term follows from Lemma B.4 (v), and that of the second term follows from Lemma B.4 (iv). Therefore, $\frac{1}{N_1}[\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta) - \mathrm{E}(\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta))] \xrightarrow{p} 0$, for each $\delta \in \Delta$.

Next, let δ_1 and δ_2 be in Δ. By the mean value theorem (MVT):

$$\tfrac{1}{N_1}\mathbb{Y}'(\delta_1)\mathbf{Q}(\delta_1)\mathbb{Y}(\delta_1) - \tfrac{1}{N_1}\mathbb{Y}'(\delta_2)\mathbf{Q}(\delta_2)\mathbb{Y}(\delta_2) = \tfrac{1}{N_1}\mathbf{Y}'\mathcal{S}'[\tfrac{\partial}{\partial\delta'}\bar{\mathbf{Q}}(\bar{\delta})]\mathcal{S}\mathbf{Y}(\delta_2 - \delta_1),$$

where $\bar{\delta}$ lies between δ_1 and δ_2. It follows that $\frac{1}{N_1}\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta)$ is stochastically equicontinuous if $\sup_{\delta\in\Delta}\frac{1}{N_1}\mathbf{Y}'\mathcal{S}'[\frac{\partial}{\partial\varpi}\bar{\mathbf{Q}}(\delta)]\mathcal{S}\mathbf{Y} = O_p(1)$, $\varpi = \lambda, \rho$. We only show when $\varpi = \lambda$ as the proof of the other case is similar and simpler. To derive the expression of the partial derivative $\frac{\partial}{\partial\lambda}\bar{\mathbf{Q}}(\delta)$, write $\bar{\mathbf{Q}}(\delta) \equiv \Psi(\delta) - \Psi(\delta)\mathcal{X}(\lambda)[\mathcal{X}'(\lambda)\Psi(\delta)\mathcal{X}(\lambda)]^{-1}\mathcal{X}'(\lambda)\Psi(\delta)$, where $\mathcal{X}(\lambda) = \mathcal{S}\mathbf{A}^{-1}_{nT}(\lambda)\mathbf{X}$. For a conformable vector a and using (A.2) and $\mathbb{H}_\lambda(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)[\frac{\partial}{\partial\lambda}\mathbf{\Omega}_N(\delta)]\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$, we have,

$$\begin{aligned}a'\tfrac{\partial}{\partial\lambda}\bar{\mathbf{Q}}(\delta)a = &-a'\bar{\mathbf{Q}}(\delta)[\tfrac{\partial}{\partial\lambda}\mathbf{\Omega}_N(\delta)]\bar{\mathbf{Q}}(\delta)a - 2a'\bar{\mathbf{Q}}(\delta)\mathbb{K}(\delta)a\\ &-2a'\bar{\mathbf{Q}}(\delta)\mathcal{S}[\tfrac{\partial}{\partial\lambda}\mathbf{A}^{-1}_{nT}(\lambda)]\mathbf{X}[\mathcal{X}'(\lambda)\Psi(\delta)\mathcal{X}(\lambda)]^{-1}\mathcal{X}'(\lambda)\Psi(\delta)a.\end{aligned}$$

Again, Lemma B.2 implies $\bar{\mathbf{Q}}(\delta)$ is bounded in both row and column sum norms, uniformly in $\delta \in \Delta$. In addition, following exactly the same way of proving Lemma B.2(ii) and (iii), we show that $\mathbb{K}(\delta)$ and $\mathbf{X}[\mathcal{X}'(\lambda)\Psi(\delta)\mathcal{X}(\lambda)]^{-1}\mathcal{X}'(\lambda)$ are also bounded in both row and column sum norms, uniformly in $\delta \in \Delta$. For ease of presentation, we let $\bar{\mathbf{Q}}^\dagger_\lambda(\delta) = \bar{\mathbf{Q}}(\delta)[\frac{\partial}{\partial\lambda}\mathbf{\Omega}_N(\delta)]\bar{\mathbf{Q}}(\delta) + 2\bar{\mathbf{Q}}(\delta)\mathbb{K}(\delta) + 2\bar{\mathbf{Q}}(\delta)\mathcal{S}[\frac{\partial}{\partial\lambda}\mathbf{A}^{-1}_{nT}(\lambda)]\mathbf{X}[\mathcal{X}'(\lambda)\Psi(\delta)\mathcal{X}(\lambda)]^{-1}\mathcal{X}'(\lambda)\Psi(\delta)$ and then $a'\frac{\partial}{\partial\lambda}\bar{\mathbf{Q}}(\delta)a \equiv -a'\bar{\mathbf{Q}}^\dagger_\lambda(\delta)a$. With these, Lemma B.1 implies that $\|\bar{\mathbf{Q}}^\dagger_\lambda(\delta)\|_1$ and $\|\bar{\mathbf{Q}}^\dagger_\lambda(\delta)\|_\infty$ are bounded uniformly in $\delta \in \Delta$.

Thus, Lemma B.4 implies

$$\frac{1}{N_1}\mathbf{Y}'\mathcal{S}'[\frac{\partial}{\partial\lambda}\bar{\mathbf{Q}}(\delta)]\mathcal{S}\mathbf{Y} = -\frac{1}{N_1}\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}^{\dagger}_{\lambda}(\delta)\mathcal{S}\mathbf{Y} = -\frac{1}{N_1}(\eta+\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V})'\bar{\mathbf{Q}}^{\dagger}_{\lambda}(\delta)(\eta+\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V})$$
$$= -\frac{1}{N_1}\eta'\bar{\mathbf{Q}}^{\dagger}_{\lambda}(\delta)\eta - \frac{2}{N1}\eta'\bar{\mathbf{Q}}^{\dagger}_{\lambda}(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V} - \frac{1}{N_1}\mathbf{V}'\mathbf{B}_{nT}^{-1\prime}\mathbf{A}_{nT}^{-1\prime}\mathcal{S}'\bar{\mathbf{Q}}^{\dagger}_{\lambda}(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V} = O_p(1),$$

uniformly in $\delta\in\Delta$. Thus, $\sup_{\delta\in\Delta}\frac{1}{N_1}\mathbf{Y}'\mathcal{S}'[\frac{\partial}{\partial\lambda}\bar{\mathbf{Q}}(\delta)]\mathcal{S}\mathbf{Y} = O_p(1)$. Following a similar analysis, $\sup_{\delta\in\Delta}\frac{1}{N_1}\mathbf{Y}'\mathcal{S}'[\frac{\partial}{\partial\rho}\bar{\mathbf{Q}}(\delta)]\mathcal{S}\mathbf{Y} = O_p(1)$. With the pointwise convergence of $\frac{1}{N_1}[\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta) - \mathrm{E}(\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta))]$ to zero for each $\delta\in\Delta$ and the stochastic equicontinuity of $\frac{1}{N_1}\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta)$, the uniform convergence result, $\sup_{\delta\in\Delta}|\frac{1}{N_1}[\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta)-\mathrm{E}(\mathbb{Y}'(\delta)\mathbf{Q}(\delta)\mathbb{Y}(\delta))]| = o_p(1)$, follows (Andrews, 1992). Thus, (b) is shown.

Proof of (c). As the two results can be shown in a similar manner, we only show $\sup_{\delta\in\Delta}\frac{1}{N_1}\big|\hat{\mathbb{V}}'(\delta)\mathbb{H}_\lambda(\delta)\hat{\mathbb{V}}(\delta) - \mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{H}_\lambda(\delta)\bar{\mathbb{V}}(\delta)]\big| = o_p(1)$. By the expressions of $\mathbb{H}_\lambda(\delta)$, $\hat{\mathbb{V}}(\delta)$ and $\bar{\mathbb{V}}(\delta)$ given above, we have

$$\frac{1}{N_1}\hat{\mathbb{V}}'(\delta)\mathbb{H}_\lambda(\delta)\hat{\mathbb{V}}(\delta) - \frac{1}{N_1}\mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{H}_\lambda(\delta)\bar{\mathbb{V}}(\delta)]$$
$$= \frac{1}{N_1}[\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)(\frac{\partial}{\partial\lambda}\mathbf{\Omega}_N(\delta))\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y} - \mathrm{E}(\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)(\frac{\partial}{\partial\lambda}\mathbf{\Omega}_N(\delta))\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y})]$$
$$- \frac{\sigma_{v0}^2}{N_1}\mathtt{tr}[\bar{\mathbf{P}}(\delta)(\frac{\partial}{\partial\lambda}\mathbf{\Omega}_N(\delta))\bar{\mathbf{P}}(\delta)\mathbf{\Omega}_N],$$

where $\bar{\mathbf{P}}(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{P}_{\tilde{\mathbb{X}}}(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)$. We see that the first term is similar in form to $\frac{1}{N_1}[\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y} - \mathrm{E}(\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y})]$ from **(b)** and its uniform convergence is shown similarly. Furthermore, by Lemma B.3, it is easy to see the second term is $o(1)$ uniformly in $\delta\in\Delta$.

Proof of (d). Again, using the expressions of $\hat{\beta}_{\mathtt{M}}(\delta)$, $\bar{\beta}_{\mathtt{M}}(\delta)$, $\hat{\mathbb{V}}(\delta)$ and $\bar{\mathbb{V}}(\delta)$, we have

$$\frac{1}{N_1}\hat{\mathbb{V}}'(\delta)\mathbb{J}(\delta)\boldsymbol{\varepsilon}(\hat{\beta}_{\mathtt{M}}(\delta),\delta) - \frac{1}{N_1}\mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{J}(\delta)\boldsymbol{\varepsilon}(\bar{\beta}_{\mathtt{M}}(\delta),\delta)]$$
$$= \frac{1}{N_1}[\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)(\mathbb{M}(\delta)+\mathbb{K}(\delta))\mathcal{S}\mathbf{Y} - \mathrm{E}(\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)(\mathbb{M}(\delta)+\mathbb{K}(\delta))\mathcal{S}\mathbf{Y})]$$
$$- \frac{\sigma_{v0}^2}{N_1}\mathtt{tr}[\bar{\mathbf{P}}(\delta)\mathbb{K}(\delta)\mathbf{\Omega}_N] - \frac{\sigma_{v0}^2}{N_1}\mathtt{tr}[\bar{\mathbf{Q}}(\delta)\mathbb{M}(\delta)\mathbf{\Omega}_N],$$

where $\mathbb{M}(\delta) = [\mathcal{S}(\frac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\lambda))\mathbf{X} - \mathbb{K}(\delta)\mathcal{X}(\lambda)][\mathcal{X}'(\lambda)\Psi(\delta)\mathcal{X}(\lambda)]^{-1}\mathcal{X}'(\lambda)\Psi(\delta)$. Therefore, the uniform convergence of the first term can be shown in a similar way as we do for $\frac{1}{N_1}[\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y} - \mathrm{E}(\mathbf{Y}'\mathcal{S}'\bar{\mathbf{Q}}(\delta)\mathcal{S}\mathbf{Y})]$ from **(b)** due to their similar forms. By Lemma B.3, the remaining two terms are easily shown to be $o(1)$, uniformly in $\delta\in\Delta$. ∎

Proof of Theorem 2.2: Applying the MVT to each element of $S_N^*(\hat{\theta}_{\mathtt{M}})$, we have

$$0 = \frac{1}{\sqrt{N_1}}S_N^*(\hat{\theta}_{\mathtt{M}}) = \frac{1}{\sqrt{N_1}}S_N^*(\theta_0) + \big[\frac{1}{N_1}\frac{\partial}{\partial\theta'}S_N^*(\theta)\big|_{\theta=\bar{\theta}_r \text{ in } r\text{th row}}\big]\sqrt{N_1}(\hat{\theta}_{\mathtt{M}} - \theta_0), \tag{O.3}$$

where $\{\bar{\theta}_r\}$ are on the line segment between $\hat{\theta}_{\mathtt{M}}$ and θ_0. The result follows if

(*a*) $\frac{1}{\sqrt{N_1}}S_N^*(\theta_0) \xrightarrow{D} N[0, \lim_{N\to\infty}\Gamma_N^*(\theta_0)]$, (*b*) $\frac{1}{N_1}[\frac{\partial}{\partial\theta'}S_N^*(\theta)\big|_{\theta=\bar\theta_r \text{ in } r\text{th row}} - \frac{\partial}{\partial\theta'}S_N^*(\theta_0)] = o_p(1)$, and

(*c*) $\frac{1}{N_1}[\frac{\partial}{\partial\theta'}S_N^*(\theta_0) - \mathrm{E}(\frac{\partial}{\partial\theta'}S_N^*(\theta_0))] = o_p(1)$.

Proof of (a). As seen from (2.10), the elements of $S_N^*(\theta_0)$ are linear-quadratic forms in $\mathbf{V}$. Thus, for every non-zero $(k+3)\times 1$ constant vector a, $a'S_N^*(\theta_0)$ is of the form:

$$a'S_N^*(\theta_0) = b_N'\mathbf{V} + \mathbf{V}'\Phi_N\mathbf{V} - \sigma_v^2\mathtt{tr}(\Phi_N),$$

for suitably defined non-stochastic vector b_N and matrix Φ_N. Based on Assumptions A-F, it is easy to verify (by Lemma B.1 and Lemma B.2) that b_N and matrix Φ_N satisfy the conditions of the CLT for LQ form of Kelejian and Prucha (2001), and hence the asymptotic normality of $\frac{1}{\sqrt{N_1}}a'S_N^*(\theta_0)$ follows. By Cramér-Wold device, $\frac{1}{\sqrt{N_1}}S_N^*(\theta_0) \xrightarrow{D} N[0, \lim_{N\to\infty}\Gamma_N^*(\theta_0)]$, where elements of $\Gamma_N^*(\theta_0)$ are given in Appendix A.

Proof of (b). The Hessian matrix $H_N^*(\theta) = \frac{\partial}{\partial\theta'}S_N^*(\theta)$ is given in Appendix A. Note that we can rewrite $\dot\Psi_\lambda(\delta)$ in (A.2) and $\dot\Psi_\rho(\delta)$ in (A.3) as $-\Psi(\delta)\dot{\mathbf{\Omega}}_\lambda(\delta)\Psi(\delta) - \Psi(\delta)\mathbb{K}(\delta) - \mathbb{K}'(\delta)\Psi(\delta)$ and $-\Psi(\delta)\dot{\mathbf{\Omega}}_\rho(\delta)\Psi(\delta)$, respectively. Following exactly the same way of proving Lemma B.2(*ii*), we show that both $\mathbb{K}(\delta)$ and $\frac{\partial}{\partial\omega}\mathbb{K}(\delta)$,$\omega = \lambda, \rho$ are uniformly bounded in both row and column sums, uniformly in $\delta \in \Delta$. In addition, the proof of Lemma B.2(*i*) also implies $\ddot{\mathbf{\Omega}}_{\omega\varpi}(\delta)$, $\omega, \varpi = \lambda, \rho$ is bounded in row and column sum norms, uniformly in $\delta \in \Delta$. Thus, by Lemma B.1, we have $\dot{\mathbf{\Psi}}_\omega(\delta)$ and $\ddot{\mathbf{\Psi}}_{\omega\varpi}(\delta)$, $\omega, \varpi = \lambda, \rho$ are all bounded in row and column sum norms, uniformly in $\delta \in \Delta$. With these, $\tilde{\mathbb{V}}(\beta_0, \delta_0) = \mathbb{Q}_\mathbb{D}\mathbf{\Gamma}\mathbf{V}$ and $\mathcal{V}(\beta_0, \lambda_0) = \mathcal{S}\mathbf{A}_{nT}^{-1}[\mathbf{D}\phi_0 + \mathbf{B}_{nT}^{-1}\mathbf{V}]$, Lemma B.4 leads to $\frac{1}{N_1}H_N^*(\theta_0) = O_p(1)$. Thus, $\frac{1}{N_1}H_N^*(\bar\theta) = O_p(1)$ since $\bar\theta \xrightarrow{p} \theta_0$ due to $\hat\theta_{\mathtt{M}} \xrightarrow{p} \theta_0$, where for simplicity, $H_N^*(\bar\theta)$ is used to denote $\frac{\partial}{\partial\theta'}S_N^*(\theta)\big|_{\theta=\bar\theta_r \text{ in } r\text{th row}}$. As $\bar\sigma_v^2 \xrightarrow{p} \sigma_{v0}^2$, we have $\bar\sigma_v^{-r} = \sigma_{v0}^{-r} + o_p(1)$, for $r = 2, 4, 6$. As σ_v^{-r} appears in $H_N^*(\theta)$ multiplicatively, $\frac{1}{N_1}H_N^*(\bar\theta) = \frac{1}{N_1}H_N^*(\bar\beta, \bar\delta, \sigma_{v0}^2) + o_p(1)$. Thus, the proof of **(b)** is equivalent to the proof of

$$\frac{1}{N_1}[H_N^*(\bar\beta, \bar\delta, \sigma_{v0}^2) - H_N^*(\theta_0)] \xrightarrow{p} 0,$$

or the proofs of $\frac{1}{N_1}[H_N^{*\mathtt{S}}(\bar\beta, \bar\delta, \sigma_{v0}^2) - H_N^{*\mathtt{S}}(\theta_0)] \xrightarrow{p} 0$ and $\frac{1}{N_1}[H_N^{*\mathtt{NS}}(\bar\delta) - H_N^{*\mathtt{NS}}(\delta_0)] \xrightarrow{p} 0$, where $H_N^{*\mathtt{S}}$ and $H_N^{*\mathtt{NS}}$ denote, respectively, the stochastic and non-stochastic parts of H_N^*.

For the stochastic part, we see that all the components of $H_N^{*\mathtt{S}}(\beta, \delta, \sigma_{v0}^2)$ are linear or quadratic in β, but nonlinear in δ. Hence, with an application of the MVT on $H_N^{*\mathtt{S}}(\bar\beta, \bar\delta, \sigma_{v0}^2)$ w.r.t $\bar\delta$, we can write $\frac{1}{N_1}[H_N^{*\mathtt{S}}(\bar\beta, \bar\delta, \sigma_{v0}^2) - H_N^{*\mathtt{S}}(\theta_0)]$ as

$$\frac{1}{N_1}[\frac{\partial}{\partial\delta'}H_N^{*\mathtt{S}}(\bar\beta, \dot\delta, \sigma_{v0}^2)](\bar\delta - \delta_0) + \frac{1}{N_1}[H_N^{*\mathtt{S}}(\bar\beta, \delta_0, \sigma_{v0}^2) - H_N^{*\mathtt{S}}(\theta_0)],$$

where for simplicity, $\frac{\partial}{\partial\delta'}H_N^{*\mathtt{S}}(\bar\beta, \dot\delta, \sigma_{v0}^2)$ is used to denote $\frac{\partial}{\partial\delta'}H_N^{*\mathtt{S}}(\beta, \delta, \sigma_{v0}^2)\big|_{\beta=\bar\beta_s, \delta=\dot\delta_s \text{ in } s\text{th row}}$ and $\{\dot\delta_s\}$ are on the line segment between $\bar\delta$ and δ_0. Therefore, it suffices to show (*i*) $\frac{1}{N_1}\frac{\partial}{\partial\delta'}H_N^{*\mathtt{S}}(\bar\beta,\dot\delta,\sigma_{v0}^2)=O_p(1)$ and (*ii*) $\frac{1}{N_1}[H_N^{*\mathtt{S}}(\bar\beta,\delta_0,\sigma_{v0}^2)-H_N^{*\mathtt{S}}(\theta_0)]=o_p(1)$.

We do so for the most complicated term, $H_{\lambda\lambda}^{*\mathtt{S}}(\theta)$. As $\frac{\partial}{\partial\lambda}H_{\lambda\lambda}^{*\mathtt{S}}(\bar\beta,\dot\delta,\sigma_{v0}^2)$ and $\frac{\partial}{\partial\rho}H_{\lambda\lambda}^{*\mathtt{S}}(\bar\beta,\dot\delta,\sigma_{v0}^2)$ can be analyzed in a similar manner, we show the later case for instance. We have,

$$\begin{aligned}\frac{1}{N_1}\frac{\partial}{\partial\rho}H_{\lambda\lambda}^{*\mathtt{S}}(\bar\beta,\dot\delta,\sigma_{v0}^2)=&\frac{2}{N_1\sigma_{v0}^2}\mathcal{V}'(\bar\beta,\dot\lambda)\ddot\Psi_{\lambda\rho}(\dot\delta)\mathcal{S}[\tfrac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\dot\lambda)]\mathbf{X}\bar\beta\\&+\frac{2}{N_1\sigma_{v0}^2}\mathcal{V}'(\bar\beta,\dot\lambda)\dot\Psi_{\rho}(\dot\delta)[\tfrac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\dot\lambda)]\mathbf{W}_{nT}\mathbf{A}_{nT}^{-1}(\dot\lambda)\mathbf{X}\bar\beta\\&-\frac{1}{N_1\sigma_{v0}^2}\bar\beta'\mathbf{X}'[\tfrac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\dot\lambda)]'\dot\Psi_{\rho}(\dot\delta)[\tfrac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\dot\lambda)]\mathbf{X}\bar\beta\\&-\frac{1}{2N_1\sigma_{v0}^2}\mathcal{V}'(\bar\beta,\dot\lambda)[\tfrac{\partial}{\partial\rho}\ddot\Psi_{\lambda\lambda}(\dot\delta)]\mathcal{V}(\bar\beta,\dot\lambda)\end{aligned}$$

From (A.2), we note that the expression of $\ddot\Psi_{\lambda\lambda}(\delta)$ involves only $\ddot{\mathbf{\Omega}}_{\lambda\lambda}(\delta)$, $\dot\Psi_{\lambda}(\delta)$ and $\frac{\partial}{\partial\lambda}\mathbb{K}(\delta)$. The partial derivatives of these components w.r.t ρ are easily shown to be bounded in both row and column sums, uniformly in $\delta\in\Delta$. It follows by Lemmas B.1 and B.4 that the above equation is $O_p(N)$, and then the result (*i*) follows.

To prove (*ii*), we note that all the terms in $H_{\lambda\lambda}^{*\mathtt{S}}(\theta)$ are quadratic in β and therefore,

$$\frac{1}{N_1}[H_{\lambda\lambda}^{*\mathtt{S}}(\bar\beta,\delta_0,\sigma_{v0}^2)-H_{\lambda\lambda}^{*\mathtt{S}}(\theta_0)]=\frac{1}{N_1\sigma_{v0}^2}(\bar\beta+\beta_0)'\mathcal{H}(\delta_0)(\bar\beta-\beta_0),$$

where $\mathcal{H}(\delta)=2\mathbf{X}'\mathbf{A}_{nT}^{-1\prime}(\lambda)\dot\Psi_\lambda(\delta)\mathcal{S}[\frac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\lambda)]\mathbf{X}+2\mathbb{X}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{J}(\delta)\mathbf{W}_{nT}\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}-\mathbf{X}'\mathbb{J}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{J}(\delta)\mathbf{X}-\frac{1}{2}\mathbf{X}'\mathbf{A}_{nT}^{-1\prime}(\lambda)\ddot\Psi_{\lambda\lambda}(\delta)\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}$. By Lemmas B.1 and B.2, it is easy to show that $\mathcal{H}(\delta_0)$ are bounded in both row and column sums. Therefore, (*ii*) holds as $\bar\beta-\beta_0=o_p(1)$.

For the non-stochastic part, we still illustrate the proof using the most complicated $\lambda\lambda$-term. As the non-stochastic part is nonlinear in both $\bar\lambda$ and $\bar\rho$, we have by the MVT,

$$\begin{aligned}&\frac{1}{N_1}[H_{\lambda\lambda}^{*\mathtt{NS}}(\bar\delta)-H_{\lambda\lambda}^{*\mathtt{NS}}(\delta_0)]\\=&-(\bar\lambda-\lambda_0)\frac{1}{2N_1}\mathtt{tr}[\ddot{\mathbf{\Omega}}_{\lambda\lambda}(\dot\delta)\dot\Psi_\lambda(\dot\delta)+\dot{\mathbf{\Omega}}_{\lambda}(\dot\delta)\ddot\Psi_{\lambda\lambda}(\dot\delta)+[\tfrac{\partial}{\partial\lambda}\ddot{\mathbf{\Omega}}_{\lambda\lambda}(\dot\delta)]\Psi(\dot\delta)+\ddot{\mathbf{\Omega}}_{\lambda\lambda}(\dot\delta)\dot\Psi_\lambda(\dot\delta)]\\&-(\bar\rho-\rho_0)\frac{1}{2N_1}\mathtt{tr}[\ddot{\mathbf{\Omega}}_{\lambda\rho}(\dot\delta)\dot\Psi_\lambda(\dot\delta)+\dot{\mathbf{\Omega}}_{\lambda}(\dot\delta)\ddot\Psi_{\lambda\rho}(\dot\delta)+[\tfrac{\partial}{\partial\rho}\ddot{\mathbf{\Omega}}_{\lambda\lambda}(\dot\delta)]\Psi(\dot\delta)+\ddot{\mathbf{\Omega}}_{\lambda\lambda}(\dot\delta)\dot\Psi_\rho(\dot\delta)],\end{aligned}$$

where $\dot\lambda$ lies between $\bar\lambda$ and λ_0 and $\dot\rho$ lies between $\bar\rho$ and ρ_0. Again, by Lemmas B.1 and B.2, we conclude that both terms in the trace operator are uniformly bounded in both row and column sums. Therefore, the terms inside the trace both have elements that are uniformly bounded. As $\bar\delta-\delta_0=o_p(1)$, we have $\frac{1}{N_1}[H_{\lambda\lambda}^{*\mathtt{NS}}(\bar\delta)-H_{\lambda\lambda}^{*\mathtt{NS}}(\delta_0)]=o_p(1)$.

Proof of (c). Since $\tilde{\mathbb{V}}(\beta_0,\delta_0)=\mathbb{Q}_{\mathbb{D}}\mathbf{\Gamma}\mathbf{V}$ and $\mathcal{V}(\beta_0,\lambda_0)=\mathcal{S}\mathbf{A}_{nT}^{-1}[\mathbf{D}\phi_0+\mathbf{B}_{nT}^{-1}\mathbf{V}]$, the Hessian matrix at true θ_0 are seen to be linear combinations of terms linear or quadratic in $\mathbf{V}$, and constants. The constant terms are canceled out. Other terms are shown to be $o_p(1)$ based on Lemma B.4. For example,

$$\begin{aligned}&\frac{1}{N_1}[H_{\rho\rho}^{*}(\rho_0)-\mathrm{E}(H_{\rho\rho}^{*}(\rho_0))]\\=&\frac{1}{N_1\sigma_{v0}^2}[\mathbf{V}'\mathbf{B}_{nT}^{-1\prime}\mathbf{A}_{nT}^{-1\prime}\mathcal{S}'\ddot\Psi_{\rho\rho}(\delta_0)\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V}-\mathrm{E}(\mathbf{V}'\mathbf{B}_{nT}^{-1\prime}\mathbf{A}_{nT}^{-1\prime}\mathcal{S}'\ddot\Psi_{\rho\rho}(\delta_0)\mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V})]=o_p(1).\ \blacksquare\end{aligned}$$

Proof of Corollary 2.1: Note that $\Gamma_N^*(\hat{\theta}_{\mathtt{M}}) = \Gamma_N^*(\theta)|_{(\theta=\hat{\theta}_{\mathtt{M}},\phi=\hat{\phi}_{\mathtt{M}},\kappa_3=\hat{\kappa}_{3,N},\kappa_4=\hat{\kappa}_{4,N})}$. As $\hat{\theta}_{\mathtt{M}}$, $\hat{\kappa}_{3,N}$ and $\hat{\kappa}_{4,N}$ are consistent estimators for θ_0, κ_3 and κ_4, plugging these estimators into $\Gamma_N^*(\theta)$ will not bring additional bias to the estimation of $\Gamma_N^*(\theta_0)$. However, due to incidental parameters problem, the $\hat{\mu}_{\mathtt{M}}$ component of $\hat{\phi}_{\mathtt{M}}$ is not consistent for the estimation of μ_0 when T is fixed. To estimate the bias caused by replacing ϕ_0 by $\hat{\phi}_{\mathtt{M}}$, rewrite (2.4),

$$\hat{\phi}(\beta, \delta) = [\mathbb{D}'(\delta)\mathbb{D}(\delta)]^{-1}\mathbb{D}'(\delta)\mathbf{C}(\delta)[\mathbf{A}_N(\lambda)\mathbf{Y} - \mathbf{X}\beta].$$

Thus, the unconstrained estimate of ϕ_0 is just $\hat{\phi}_{\mathtt{M}} = \hat{\phi}(\hat{\beta}_{\mathtt{M}}, \hat{\delta}_{\mathtt{M}})$. From the expression of $\Gamma^*_{\lambda\lambda}(\theta_0)$, we see that ϕ_0 is embedded in $\mathbf{\Gamma}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0$ from Π_2, where we recall $\mathbf{\Gamma}(\delta) = \mathbf{C}(\delta)\mathbf{B}_{nT}^{-1}(\rho)$, $\mathbf{C}(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}(\lambda)$ and $\mathbb{J}(\delta) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\mathcal{S}[\frac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\lambda)]$. Thus, we have $\mathbf{\Gamma}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}\hat{\phi}_{\mathtt{M}} = \mathbb{M}(\hat{\delta}_{\mathtt{M}})[\mathbf{A}_N(\hat{\lambda}_{\mathtt{M}})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}]$, where $\mathbb{M}(\hat{\delta}_{\mathtt{M}}) = \mathbf{\Gamma}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}[\mathbb{D}'(\hat{\delta}_{\mathtt{M}})\mathbb{D}(\hat{\delta}_{\mathtt{M}})]^{-1}\mathbb{D}'(\hat{\delta}_{\mathtt{M}})\mathbf{C}(\hat{\delta}_{\mathtt{M}})$. Note $\mathbf{A}_N\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}} = \mathbf{A}_N\mathbf{Y} - \mathbf{X}\beta_0 - \mathbf{X}(\hat{\beta}_{\mathtt{M}} - \beta_0)$. Applying the MVT on each row of $\mathbb{M}(\hat{\delta}_{\mathtt{M}})[\mathbf{A}_N(\hat{\lambda}_{\mathtt{M}})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}]$ w.r.t δ, we have,

$$\begin{aligned}
&\mathbb{M}(\hat{\delta}_{\mathtt{M}})[\mathbf{A}_N(\hat{\lambda}_{\mathtt{M}})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}] \\
&= \mathbb{M}[\mathbf{A}_N\mathbf{Y} - \mathbf{X}\beta_0 - \mathbf{X}(\hat{\beta}_{\mathtt{M}} - \beta_0)] + [\tfrac{\partial}{\partial\rho}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}](\hat{\rho}_{\mathtt{M}} - \rho_0) \\
&\quad + \big\{[\tfrac{\partial}{\partial\lambda}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}] - \mathbb{M}(\bar{\delta})\mathbf{W}\mathbf{Y}\big\}(\hat{\lambda}_{\mathtt{M}} - \lambda_0) \\
&= \mathbf{\Gamma}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0 + \mathbb{M}\mathbf{B}_N^{-1}\mathbf{V} - \mathbb{M}\mathbf{X}(\hat{\beta}_{\mathtt{M}} - \beta_0) + [\tfrac{\partial}{\partial\rho}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}](\hat{\rho}_{\mathtt{M}} - \rho_0) \\
&\quad + \big\{[\tfrac{\partial}{\partial\lambda}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}] - \mathbb{M}(\bar{\delta})\mathbf{W}\mathbf{Y}\big\}(\hat{\lambda}_{\mathtt{M}} - \lambda_0), \qquad \text{(O.4)}
\end{aligned}$$

where $\bar{\delta}$ lies between $\hat{\rho}_{\mathtt{M}}$ and ρ_0 and changes over the rows of $[\frac{\partial}{\partial\rho}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}]$ and $[\frac{\partial}{\partial\lambda}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}] - \mathbb{M}(\bar{\delta})\mathbf{W}\mathbf{Y}$. Note that $\mathbb{M}(\delta) \equiv \mathbf{B}_{nT}^{-1\prime}(\rho)\mathbf{A}_{nT}^{-1\prime}(\lambda)\mathcal{S}'\Psi(\delta)\mathbb{K}(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}(\lambda)$, and thus it is easy to see that $\mathbb{M}(\delta)$ and $\frac{\partial}{\partial\omega}\mathbb{M}(\bar{\delta})$, $\omega = \lambda, \rho$, are uniformly bounded in both row and column sums, uniformly in $\delta \in \Delta$, by Lemma B.1. From the expression of $\Gamma_N^*(\hat{\theta}_{\mathtt{M}})$, it has components linear or quadratic in $\mathbf{\Gamma}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}\hat{\phi}_{\mathtt{M}}$. Let d_N be a non-stochastic N-vector with elements being of uniform order $O(1)$ or $O(h_n^{-1})$. Using (O.4), the terms of $\Gamma_N^*(\hat{\theta}_{\mathtt{M}})$ linear in $\mathbf{\Gamma}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}\hat{\phi}_{\mathtt{M}}$ are represented as

$$\begin{aligned}
&\tfrac{1}{N_1}d_N'\mathbf{\Gamma}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}\hat{\phi}_{\mathtt{M}} \\
&= \tfrac{1}{N_1}d_N'\mathbf{\Gamma}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0 + \tfrac{1}{N_1}d_N'\mathbb{M}\mathbf{B}_N^{-1}\mathbf{V} - \tfrac{1}{N_1}d_N'\mathbb{M}\mathbf{X}(\hat{\beta}_{\mathtt{M}} - \beta_0) \\
&\quad + \tfrac{1}{N_1}d_N'[\tfrac{\partial}{\partial\rho}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}](\hat{\rho}_{\mathtt{M}} - \rho_0) \\
&\quad + \tfrac{1}{N_1}d_N'\big\{[\tfrac{\partial}{\partial\lambda}\mathbb{M}(\bar{\delta})][\mathbf{A}_N(\bar{\lambda})\mathbf{Y} - \mathbf{X}\hat{\beta}_{\mathtt{M}}] - \mathbb{M}(\bar{\delta})\mathbf{W}\mathbf{Y}\big\}(\hat{\lambda}_{\mathtt{M}} - \lambda_0) \\
&= \tfrac{1}{N_1}d_N'\mathbf{\Gamma}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0 + o_p(1),
\end{aligned}$$

where the last equation holds because of the consistency of $\hat{\theta}_{\mathtt{M}}$ and Lemma B.4, using $\mathcal{S}\mathbf{Y} = \eta + \mathcal{S}\mathbf{A}_{nT}^{-1}\mathbf{B}_{nT}^{-1}\mathbf{V}$. Hence, we can conclude that the terms of $\Gamma_N^*(\theta_0)$ linear in ϕ_0 can be consistently estimated by simply replacing ϕ_0 with $\hat{\phi}_{\mathtt{M}}$.

Note that $\mathbf{\Gamma\Gamma}' = I_N$. Hence, the only term that is quadratic in ϕ_0 is contained in $\Gamma^*_{\lambda\lambda}(\theta_0)$, $\frac{1}{N_1\sigma^2_{v0}}\phi_0'\mathbf{D}'\mathbb{J}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0$. The plug-in estimator, $\frac{1}{N_1\hat{\sigma}^2_{v,\mathtt{M}}}\hat{\phi}_{\mathtt{M}}'\mathbf{D}'\mathbb{J}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}\hat{\phi}_{\mathtt{M}}$, estimates this term. Using (O.4), $\hat{\theta}^*_N - \theta_0 = o_p(1)$ and Lemma B.4, we show that this estimator is biased/inconsistent:

$$
\begin{aligned}
&\frac{1}{N_1\hat{\sigma}^2_{v,\mathtt{M}}}\hat{\phi}_{\mathtt{M}}'\mathbf{D}'\mathbb{J}'(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbb{J}(\hat{\delta}_{\mathtt{M}})\mathbf{D}\hat{\phi}_{\mathtt{M}} \\
&= \frac{1}{N_1\sigma^2_{v0}}\phi_0'\mathbf{D}'\mathbb{J}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0 + \frac{1}{N_1\sigma^2_{v0}}\mathbf{V}'\mathbf{B}^{-1\prime}_{nT}\mathbb{M}'\mathbb{M}\mathbf{B}^{-1}_{nT}\mathbf{V} + o_p(1) \\
&= \frac{1}{N_1\sigma^2_{v0}}\phi_0'\mathbf{D}'\mathbb{J}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}\phi_0 + \frac{1}{N_1}\mathtt{tr}[(\mathbb{D}'\mathbb{D})^{-1}\mathbf{D}'\mathbb{J}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}] + o_p(1).
\end{aligned}
$$

We see that the bias term, $\frac{1}{N_1}\mathtt{tr}[(\mathbb{D}'\mathbb{D})^{-1}\mathbf{D}'\mathbb{J}'\mathbb{Q}_{\mathbb{D}}\mathbb{J}\mathbf{D}]$, involves only the common parameters that can be consistently estimated. Thus, a bias correction can easily be made. Define

$$
\text{Bias}^*_{\lambda\lambda}(\delta) = \frac{1}{N_1}\mathtt{tr}[(\mathbb{D}'(\delta)\mathbb{D}(\delta))^{-1}\mathbf{D}'\mathbb{J}'(\delta)\mathbb{Q}_{\mathbb{D}}(\delta)\mathbb{J}(\delta)\mathbf{D}]. \tag{O.5}
$$

This gives the bias matrix of $\Gamma^*_N(\hat{\theta}_{\mathtt{M}})$, which is a matrix of the same dimension as $\Gamma^*_N(\hat{\theta}_{\mathtt{M}})$, and has the sole non-zero element $\text{Bias}^*_{\lambda\lambda}(\delta_0)$ corresponding to the $\Gamma^*_{\lambda\lambda}(\hat{\theta}_{\mathtt{M}})$ component. ■

Proof of Corollary 2.2.

Proof of (i). Recall $\bar{\mathbb{Q}}_{\mathbb{D}} \equiv \mathbf{\Omega}_N^{-\frac{1}{2}}\mathbb{Q}_{\mathbb{D}}\mathbf{\Gamma}$. Three vectors $\mathbf{V}$, $\mathbf{\Omega}_N^{-\frac{1}{2}}\tilde{\mathbb{V}} = \bar{\mathbb{Q}}_{\mathbb{D}}\mathbf{V}$ and $\mathbf{\Omega}_N^{-\frac{1}{2}}(\hat{\delta}_{\mathtt{M}})\hat{\mathbb{V}}(\hat{\beta}_{\mathtt{M}}, \hat{\delta}_{\mathtt{M}}) = \mathbf{\Omega}_N^{-\frac{1}{2}}(\hat{\delta}_{\mathtt{M}})\mathbb{Q}_{\mathbb{D}}(\hat{\delta}_{\mathtt{M}})\mathbf{\Omega}_N^{-\frac{1}{2}}(\hat{\delta}_{\mathtt{M}})\mathcal{S}[\mathbf{Y} - \mathbf{A}^{-1}_{nT}(\hat{\lambda}_{\mathtt{M}})\mathbf{X}\hat{\beta}_{\mathtt{M}}]$ are with respective elements $\{v_l\}$, $\{\tilde{v}_j\}$ and $\{\hat{v}_j\}$, and $\bar{\mathbb{Q}}_{\mathbb{D}}$ has elements $\{q_{jl}\}$, $l = 1, \ldots, nT$, $j = 1, \ldots, N$, where j and l are the combined indices of both cross-sectional and time dimensions.

Consistency of $\hat{\kappa}_{3,N}$. As $\hat{\sigma}_{v,\mathtt{M}} - \sigma_{v0} = o_p(1)$ and $\hat{\delta}^*_N - \delta_0 = o_p(1)$, the denominators of $\hat{\kappa}_{3,N}$ and κ_3 agree asymptotically. Thus, $\hat{\kappa}_{3,N}$ is consistent if $\frac{1}{N}\sum_{j=1}^N[\hat{v}_j^3 - \text{E}(\tilde{v}_j^3)] \xrightarrow{p} 0$, or (a) $\frac{1}{N}\sum_{j=1}^N[\tilde{v}_j^3 - \text{E}(\tilde{v}_j^3)] \xrightarrow{p} 0$ and (b) $\frac{1}{N}\sum_{j=1}^N(\hat{v}_j^3 - \tilde{v}_j^3) \xrightarrow{p} 0$.

To prove (a), noting that $\tilde{v}_j = \sum_{h=1}^{nT} q_{jh}v_h$, we have,

$$
\begin{aligned}
\frac{1}{N}\sum_{j=1}^N[\tilde{v}_j^3 - \text{E}(\tilde{v}_j^3)] &= \frac{1}{N}\sum_{j=1}^N\sum_{h=1}^{nT} q_{jh}^3[v_h^3 - \text{E}(v_h^3)] + \frac{3}{N}\sum_{j=1}^N\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT} q_{jl}^2 q_{jm}v_l^2 v_m \\
&\quad + \frac{6}{N}\sum_{j=1}^N\sum_{m=1}^{nT}\sum_{\substack{l\neq m\\ l=1}}^{nT}\sum_{\substack{h\neq m,l\\ h=1}}^{nT} q_{jm}q_{jl}q_{jh}v_m v_l v_h \equiv K_1 + K_2 + K_3.
\end{aligned}
$$

First, consider the K_1 term. By Lemmas B.1 and B.2, $\bar{\mathbb{Q}}_{\mathbb{D}}$ is uniformly bounded in both row and column sums. This implies that the elements of $\bar{\mathbb{Q}}_{\mathbb{D}}$ are uniformly bounded. Therefore, there exists a constant $\bar{q}$ such that $|q_{jh}| \leq \bar{q}$ for all j and h. Given these, we have $\sum_{j=1}^N q_{jh}^3 \leq \sum_{j=1}^N |q_{jh}|^3 \leq \bar{q}^2\sum_{j=1}^N |q_{jh}| < \infty$. Also, note $\{v_i\}$ are iid by Assumption A. Thus, Khinchine's weak law of large number (WLLN) (Feller, 1967, pp. 243-244) implies that K_1 converges to zero in probability as the sample size increases.

For the other two terms, we have by switching the order of summations when needed,

$K_2 = \frac{3}{N}\sum_{j=1}^{N}\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT} q_{jl}^2 q_{jm}(v_l^2-\sigma_v^2)v_m + \frac{3}{N}\sum_{j=1}^{N}\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT} q_{jl}^2 q_{jm}\sigma_v^2 v_m$
$= \frac{3}{N}\sum_{m=1}^{nT}(v_m^2-\sigma_v^2)(\sum_{j=1}^{N}\sum_{l=1}^{m-1} q_{jm}^2 q_{jl}v_l) + \frac{3}{N}\sum_{m=1}^{nT} v_m[\sum_{j=1}^{N}\sum_{l=1}^{m-1} q_{jl}^2 q_{jm}(v_l^2-\sigma_v^2)]$
$+\frac{3}{N}\sum_{m=1}^{nT}\sum_{j=1}^{N}\sum_{\substack{l\neq m\\ l=1}}^{N} q_{jl}^2 q_{jm}\sigma_v^2 v_m \equiv \frac{1}{N}\sum_{m=1}^{nT}(g_{1,m}+g_{2,m}+g_{3,m})$, and

$K_3 = \frac{18}{N}\sum_{m=1}^{nT} v_m(\sum_{j=1}^{N}\sum_{l=1}^{m-1}\sum_{\substack{h\neq l\\ h=1}}^{m-1} q_{jm}q_{jl}q_{jh}v_l v_h) \equiv \frac{1}{N}\sum_{m=1}^{nT} g_{4,m}$,

where $g_{1,m} = 3(v_m^2-\sigma_v^2)\sum_{j=1}^{N}\sum_{l=1}^{m-1} q_{jm}^2 q_{jl}v_l$, $g_{2,m} = 3v_m\sum_{j=1}^{N}\sum_{l=1}^{m-1} q_{jl}^2 q_{jm}(v_l^2-\sigma_v^2)$, $g_{3,m} = 3\sum_{j=1}^{N}\sum_{\substack{l\neq m\\ l=1}}^{nT} q_{jl}^2 q_{jm}\sigma_v^2 v_m$, and $g_{4,m} = v_m\sum_{j=1}^{N}\sum_{l=1}^{m-1}\sum_{\substack{h\neq l\\ h=1}}^{m-1} q_{jm}q_{jl}q_{jh}v_l v_h$.

Let $\{\mathcal{G}_m\}$ be the increasing sequence of σ-fields generated by $(v_1,\cdots,v_j, j=1,\cdots,m)$, $m=1,\cdots,nT$. Then, $\mathrm{E}[(g_{1,m},g_{2,m},g_{3,m},g_{4,m})|\mathcal{G}_{m-1}]=0$; hence, $\{(g_{1,m},g_{2,m},g_{3,m},g_{4,m})',\mathcal{G}_m\}$ form a vector martingale difference (M.D.) sequence. As $\bar{\mathbb{Q}}_{\mathbb{D}}$ is bounded in row and column sum norms, by Assumption A, it is easy to see that $\mathrm{E}|g_{s,m}|^{1+\epsilon}<\infty$, for $s=1,2,3,4$ and $\epsilon>0$. Hence, $\{g_{1,m}\}$, $\{g_{2,m}\}$, $\{g_{3,m}\}$ and $\{g_{4,m}\}$ are uniformly integrable, and the WLLN of Davidson (1994, Theorem 19.7) applies to give $K_2 \xrightarrow{p} 0$ and $K_3 \xrightarrow{p} 0$.

To prove (b), let $\tilde{v}_j(\xi)$ be the jth element of $\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\tilde{\mathbb{V}}(\xi) = \Psi(\delta)\mathcal{S}[\mathbf{Y}-\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}\beta]$, where $\xi=(\beta',\delta')'$. Thus, $\tilde{v}_j$ and $\hat{v}_j$ are just $\tilde{v}_j(\xi_0)$ and $\tilde{v}_j(\hat{\xi}_{\mathtt{M}})$, respectively. Let $\mathbf{S}(\xi) = \frac{\partial}{\partial\xi'}[\mathbf{\Omega}_N^{-\frac{1}{2}}(\delta)\tilde{\mathbb{V}}(\xi)]$, which has components $\mathbf{S}_\beta(\xi) = -\Psi(\delta)\mathcal{S}\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}$, $\mathbf{S}_\lambda(\xi) = \dot{\Psi}_\lambda(\delta)\mathcal{S}[\mathbf{Y}-\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}\beta] - \Psi(\delta)\mathcal{S}[\frac{\partial}{\partial\lambda}\mathbf{A}_{nT}^{-1}(\lambda)]\mathbf{X}\beta$, and $\mathbf{S}_\rho(\xi) = \dot{\Psi}_\rho(\delta)\mathcal{S}[\mathbf{Y}-\mathbf{A}_{nT}^{-1}(\lambda)\mathbf{X}\beta]$. Let $s_j'(\xi)$ be the jth row of $\mathbf{S}(\xi)$. We have by the MVT, for each $j=1,2,\ldots,N$,

$$\hat{v}_j \equiv \tilde{v}_j(\hat{\xi}_{\mathtt{M}}) = \tilde{v}_j(\xi_0) + s_j'(\bar{\xi})(\hat{\xi}_{\mathtt{M}}-\xi_0) = \tilde{v}_j + \psi_j'(\hat{\xi}_{\mathtt{M}}-\xi_0) + o_p(\|\hat{\xi}_{\mathtt{M}}-\xi_0\|), \tag{O.6}$$

where $\bar{\xi}$ lies between $\hat{\xi}_{\mathtt{M}}$ and ξ_0, and $\psi_j' = \mathrm{plim}_{N\to\infty}s_j'(\bar{\xi})$, which is easily shown to be $O_p(1)$ as follows. Consider the first k (the number of regressors) elements of ψ_j' first. They are the limits of the jth row of $-\mathbb{X}(\bar{\rho})$, which are just the jth row of $-\Psi(\bar{\delta})\mathcal{S}\mathbf{A}_{nT}^{-1}(\bar{\lambda})\mathbf{X}$ because $\bar{\delta} \xrightarrow{p} \delta_0$, implied by $\hat{\delta}_N^*-\delta_0=o_p(1)$. Hence, we conclude that the first k elements of ψ_j' are $O(1)$, for each $j=1,2,\ldots,N$. For the remaining two elements in each ψ_j', they are the limits of elements from the last two columns of $\mathbf{S}(\bar{\xi})$. It is easy to see the limits of the last two columns of $\mathbf{S}(\bar{\xi})$ are just $\mathbf{S}_\lambda(\xi_0)$ and $\mathbf{S}_\rho(\xi_0)$. Using $\mathcal{S}[\mathbf{Y}-\mathbf{A}_{nT}^{-1}\mathbf{X}\beta_0] = \mathcal{S}\mathbf{A}_{nT}^{-1}[\mathbf{D}\phi_0+\mathbf{B}_{nT}^{-1}\mathbf{V}]$, we can easily see that each element of $\mathbf{S}_\lambda(\xi_0)$ and $\mathbf{S}_\rho(\xi_0)$ are $O_p(1)$, i.e., the last two elements in ψ_j' are also $O_p(1)$, for each $j=1,2,\ldots,N$.

As $\tilde{v}_j=O_p(1)$, $\psi_j'=O_p(1)$ and $\hat{\xi}_{\mathtt{M}}-\xi_0=O_p(\frac{1}{\sqrt{N_1}})$, we have by (O.6), $\hat{v}_j^3 = \tilde{v}_j^3 + 3\tilde{v}_j^2\psi_j'(\hat{\xi}_{\mathtt{M}}-\xi_0) + o_p(\|\hat{\xi}_{\mathtt{M}}-\xi_0\|)$. It follows that

$$\frac{1}{N}\sum_{j=1}^{N}(\hat{v}_j^3-\tilde{v}_j^3) = \frac{3}{N}\sum_{j=1}^{N}\tilde{v}_j^2\psi_j'(\hat{\xi}_{\mathtt{M}}-\xi_0) + o_p(\|\hat{\xi}_{\mathtt{M}}-\xi_0\|)$$
$$= \frac{3\sigma_v^2}{N}\sum_{j=1}^{N}(\sum_{k=1}^{nT} q_{jk}^2\psi_j')(\hat{\xi}_{\mathtt{M}}-\xi_0) + o_p(\|\hat{\xi}_{\mathtt{M}}-\xi_0\|) = o_p(1),$$

as $\frac{1}{N}\sum_{j=1}^{N}(\sum_{k=1}^{nT} q_{jk}^2\psi_j') = (\sum_{k=1}^{nT} q_{jk}^2)\frac{1}{N}(\sum_{j=1}^{N}\psi_j') = O(1)$.

Consistency of $\hat{\kappa}_{4,N}$. As $\hat{\sigma}_{v,\mathtt{M}}-\sigma_{v0}=o_p(1)$ and $\hat{\delta}_N^*-\delta_0=o_p(1)$, the result follows if $\frac{1}{N}\sum_{j=1}^N[\hat{v}_j^4-\mathrm{E}(\tilde{v}_j^4)]\xrightarrow{p}0$. This shows that

$$(c)\ \tfrac{1}{N}\sum_{j=1}^N[\tilde{v}_j^4-\mathrm{E}(\tilde{v}_j^4)]\xrightarrow{p}0\ \text{ and }\ (d)\ \tfrac{1}{N}\sum_{j=1}^N(\hat{v}_j^4-\tilde{v}_j^4)\xrightarrow{p}0.$$

To prove (c), we have

$$\begin{aligned}&\tfrac{1}{N}\sum_{j=1}^N\tilde{v}_j^4-\tfrac{1}{N}\sum_{j=1}^N\mathrm{E}(\tilde{v}_j^4)\\ =\ &\tfrac{1}{N}\sum_{j=1}^N\sum_{h=1}^{nT}q_{jh}^4[v_h^4-\mathrm{E}(v_h^4)]+\tfrac{3}{N}\sum_{j=1}^N\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT}q_{jl}^2q_{jm}^2(v_l^2v_m^2-\sigma_v^4)\\ &+\tfrac{4}{N}\sum_{j=1}^N\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT}q_{jl}^3q_{jm}v_l^3v_m+\tfrac{6}{N}\sum_{j=1}^N\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT}\sum_{\substack{h\neq m,l\\ h=1}}^{nT}q_{jl}^2q_{jm}q_{jh}v_l^2v_mv_h\\ &+\tfrac{1}{N}\sum_{j=1}^N\sum_{l=1}^{nT}\sum_{\substack{m\neq l\\ m=1}}^{nT}\sum_{\substack{h\neq m,l\\ h=1}}^{nT}\sum_{\substack{p\neq m,l,h\\ p=1}}^{nT}q_{jl}q_{jm}q_{jh}q_{jp}v_lv_mv_hv_p\equiv\sum_{r=1}^5R_r.\end{aligned}$$

By using WLLN of Davidson (1994, Theorem 19.7) for M.D. arrays as in the proof of (a), we have $R_r=o_p(1)$ for $r=1,3,4,5$. For R_2, we have

$$\begin{aligned}R_2=\ &\tfrac{6}{N}\sum_{l=1}^{nT}(v_l^2-\sigma_v^2)[\sum_{j=1}^N\sum_{m=1}^{l-1}q_{jl}^2q_{jm}^2(v_m^2-\sigma_v^2)]\\ &+\tfrac{6}{N}\sum_{l=1}^{nT}[\sum_{j=1}^N\sum_{\substack{m\neq l\\ m=1}}^{nT}q_{jl}^2q_{jm}^2\sigma_v^2(v_l^2-\sigma_v^2)]\equiv\tfrac{6}{N}\sum_{l=1}^{nT}(f_{1,l}+f_{2,l}),\end{aligned}$$

noting that $v_l^2v_m^2-\sigma_v^4=(v_l^2-\sigma_v^2)(v_m^2-\sigma_v^2)+\sigma_v^2(v_m^2-\sigma_v^2)+\sigma_v^2(v_l^2-\sigma_v^2)$. Since $\mathrm{E}[f_{1,l}|\mathcal{G}_{l-1}]=0$ and $\{f_{2,l}\}$ are independent, both $\{f_l\}$ and $\{f_{2,l}\}$ form M.D. sequences. It is easy to see that $\mathrm{E}|f_{s,l}|^{1+\epsilon}<\infty$, for $s=1,2$ and $\epsilon>0$, so that $\{f_{1,l}\}$ and $\{f_{2,l}\}$ are uniformly integrable. Therefore, the WLLN of Davidson (1994, Theorem 19.7) implies that $\frac{6}{N}\sum_{l=1}^{nT}f_{1,l}=o_p(1)$ and $\frac{6}{N}\sum_{l=1}^{nT}f_{2,l}=o_p(1)$.

To prove (d), $\hat{v}_j^4=\tilde{v}_j^4+4\tilde{v}_j^3\psi_j'(\hat{\xi}_\mathtt{M}-\xi_0)+o_p(\|\hat{\xi}_\mathtt{M}-\xi_0\|)$ by (O.6). It follows that

$$\begin{aligned}\tfrac{1}{N}\sum_{j=1}^N(\hat{v}_j^4-\tilde{v}_j^4)&=\tfrac{4}{N}\sum_{j=1}^N\tilde{v}_j^3\psi_j'(\hat{\xi}_\mathtt{M}-\xi_0)+o_p(\|\hat{\xi}_\mathtt{M}-\xi_0\|)\\ &=\tfrac{4\sigma_v^3\kappa_3}{N}\sum_{j=1}^N(\sum_{k=1}^{nT}q_{jk}^3\psi_j')(\hat{\xi}_\mathtt{M}-\xi_0)+o_p(\|\hat{\xi}_\mathtt{M}-\xi_0\|)=o_p(1).\end{aligned}$$

Proof of (ii). The consistency of $\widehat{\Sigma}_N^*$ to $\Sigma_N^*(\theta_0)$ can be shown similarly as what we do in the proof of Theorem 2.2 for results (b) and (c). For $\widehat{\Gamma}_N^*-\Gamma_N^*(\theta_0)\xrightarrow{p}0$, we only need to show that $\mathrm{Bias}^*(\hat{\delta}_N^*)-\mathrm{Bias}^*(\delta_0)=o_p(1)$, based on Corollary 2.1. That is to show

$$\tfrac{1}{N_1}\{\mathtt{tr}[(\mathbb{D}'(\hat{\delta}_\mathtt{M})\mathbb{D}(\hat{\delta}_\mathtt{M}))^{-1}\mathbf{D}'\mathbb{J}'(\hat{\delta}_\mathtt{M})\mathbb{Q}_\mathbb{D}(\hat{\delta}_\mathtt{M})\mathbb{J}(\hat{\delta}_\mathtt{M})\mathbf{D}]-\mathtt{tr}[(\mathbb{D}'\mathbb{D})^{-1}\mathbf{D}'\mathbb{J}'\mathbb{Q}_\mathbb{D}\mathbb{J}\mathbf{D}]\}=o_p(1),$$

which can be proved as that for $\frac{1}{N_1}[H_{\lambda\lambda}^{*\mathtt{NS}}(\bar{\delta})-H_{\lambda\lambda}^{*\mathtt{NS}}(\delta_0)]$ in the proof of Theorem 2.2 (b). ∎

E.4. Proofs for Section 3

Proof of Theorem 3.1: Similar to the proof of Theorem 2.1 in Appendix C and with δ and $\Omega_N(\delta)$ being redefined, the consistency of $\hat{\delta}_\mathtt{M}^\diamond$ follows if:

(a) $\inf_{\delta\in\Delta}\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)$ is bounded away from zero,

(b) $\sup_{\delta\in\Delta}\left|\hat{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)-\bar{\sigma}_{v,\mathtt{M}}^{\diamond 2}(\delta)\right|=o_p(1)$,

(c) $\sup_{\delta\in\Delta}\frac{1}{N_1}\big|\hat{\mathbb{V}}'(\delta)\mathbb{H}_\omega(\delta)\hat{\mathbb{V}}(\delta)-\mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{H}_\omega(\delta)\bar{\mathbb{V}}(\delta)]\big|=o_p(1)$, for $\omega=\lambda,\rho,\tau$,

(d) $\sup_{\delta\in\Delta}\frac{1}{N_1}\big|\hat{\mathbb{V}}'(\delta)\mathbb{J}(\delta)\boldsymbol{\varepsilon}(\hat{\beta}^{\diamond}_{\mathtt{M}}(\delta),\delta)-\mathrm{E}[\bar{\mathbb{V}}'(\delta)\mathbb{J}(\delta)\boldsymbol{\varepsilon}(\bar{\beta}^{\diamond}_{\mathtt{M}}(\delta),\delta)]\big|=o_p(1)$.

Proof of (a). Note that $\bar{\sigma}^{\diamond 2}_{v,\mathtt{M}}(\delta)=\frac{1}{N_1}\eta'\boldsymbol{\Omega}_N^{-\frac{1}{2}}(\delta)\mathbf{Q}(\delta)\boldsymbol{\Omega}_N^{-\frac{1}{2}}(\delta)\eta+\frac{\sigma^2_{v0}}{N_1}\mathtt{tr}[\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)]$. The first term is still non-negative as it can be written in the form of $a'(\delta)a(\delta)$ for an $N\times 1$ vector function of δ, uniformly in $\delta\in\Delta$. For the second term, as $0<\underline{c}_\tau\le\inf_{\tau\in\Delta_\tau}\gamma_{\min}[\Upsilon(\tau)\Upsilon'(\tau)\otimes I_n]\le\sup_{\tau\in\Delta_\tau}\gamma_{\max}[\Upsilon(\tau)\Upsilon'(\tau)\otimes I_n]\le\bar{c}_\tau<\infty$,

$$\begin{aligned}&\tfrac{\sigma^2_{v0}}{N_1}\mathtt{tr}[\mathbb{Q}_{\mathbb{D}}(\delta)\mathcal{O}_N(\delta)]\ge\tfrac{\sigma^2_{v0}}{N_1}\gamma_{\min}[\mathcal{O}_N(\delta)]\mathtt{tr}[\mathbb{Q}_{\mathbb{D}}(\delta)]\ge\sigma^2_{v0}\gamma_{\max}[\boldsymbol{\Omega}_N(\delta)]^{-1}\gamma_{\min}(\boldsymbol{\Omega}_N)\\&\ge\frac{\underline{c}_\tau}{\bar{c}_\tau}\sigma^2_{v0}\gamma_{\max}(\mathbf{A}_N'\mathbf{A}_N)^{-1}\gamma_{\max}(\mathbf{B}_N'\mathbf{B}_N)^{-1}\gamma_{\min}[\mathbf{A}_N'(\lambda)\mathbf{A}_N(\lambda)]\gamma_{\min}[\mathbf{B}_N'(\rho)\mathbf{B}_N(\rho)]>0,\end{aligned}$$

uniformly in $\delta\in\Delta$, by Assumption E(iii). It follows that $\inf_{\delta\in\Delta}\bar{\sigma}^{\diamond 2}_{v,\mathtt{M}}(\delta)>0$.

Proofs of (b), (c) and (d) are quite similar to the proofs of (b), (c) and (d) of Theorem 2.1 (the results of Lemma B.2 still hold with the redefined $\Omega_N(\delta)$). Thus, they are omitted. ■

Proof of Theorem 3.2: Applying the MVT to each element of $S^{\diamond}_N(\hat{\theta}_{\mathtt{M}})$, we have

$$0=\frac{1}{\sqrt{N_1}}S^{\diamond}_N(\hat{\theta}^{\diamond}_{\mathtt{M}})=\frac{1}{\sqrt{N_1}}S^{\diamond}_N(\theta_0)+\Big[\frac{1}{N_1}\frac{\partial}{\partial\theta'}S^{\diamond}_N(\theta)\big|_{\theta=\bar{\theta}_r\text{ in }r\text{th row}}\Big]\sqrt{N_1}(\hat{\theta}^{\diamond}_{\mathtt{M}}-\theta_0),\tag{O.7}$$

where $\{\bar{\theta}_r\}$ are on the line segment between $\hat{\theta}^{\diamond}_{\mathtt{M}}$ and θ_0. The result follows if

(a) $\frac{1}{\sqrt{N_1}}S^{\diamond}_N(\theta_0)\xrightarrow{D}N[0,\lim_{N\to\infty}\Gamma^{\diamond}_N(\theta_0)]$,
(b) $\frac{1}{N_1}[\frac{\partial}{\partial\theta'}S^{\diamond}_N(\theta)\big|_{\theta=\bar{\theta}_r\text{ in }r\text{th row}}-\frac{\partial}{\partial\theta'}S^{\diamond}_N(\theta_0)]=o_p(1)$, and
(c) $\frac{1}{N_1}[\frac{\partial}{\partial\theta'}S^{\diamond}_N(\theta_0)-\mathrm{E}(\frac{\partial}{\partial\theta'}S^{\diamond}_N(\theta_0))]=o_p(1)$.

Proof of (a). Again, from (3.2), the elements of $S^{\diamond}_N(\theta_0)$ are linear-quadratic forms in $\mathcal{E}$. Thus, for every non-zero $(k+3)\times 1$ constant vector a, $a'S^{\diamond}_N(\theta_0)$ is of the form:

$$a'S^{\diamond}_N(\theta_0)=b_N'\mathcal{E}+\mathcal{E}'\Phi_N\mathcal{E}-\sigma^2_v\mathtt{tr}(\Phi_N),$$

for suitably defined non-stochastic vector b_N and matrix Φ_N. Based on Assumptions A$'$-F$'$, it is easy to verify (by Lemma B.1 and Lemma B.2) that b_N and matrix Φ_N satisfy the conditions of the CLT for LQ form of Kelejian and Prucha (2001), and hence the asymptotic normality of $\frac{1}{\sqrt{N_1}}a'S^{\diamond}_N(\theta_0)$ follows. By Cramér-Wold device, $\frac{1}{\sqrt{N_1}}S^{\diamond}_N(\theta_0)\xrightarrow{D}N[0,\lim_{N\to\infty}\Gamma^{\diamond}_N(\theta_0)]$, where elements of $\Gamma^{\diamond}_N(\theta_0)$ are given in Appendix A.

Proofs of (b) and (c) are similar to those of Theorem 2.2, and thus are omitted. ■

Proofs of the results in Corollaries 3.1 and 3.2 are similar to those of Corollaries 2.1 and 2.2 and thus are omitted to conserve space. They are available from the authors upon request.

E.5. The Full Set of Monte Carlo Results

This section reports the full set of Monte Carlo results, involving three data generating processes (DGPs), nine estimators, three different error distributions, various combinations of different types of spatial weight matrices, and various sample sizes.

The three DGPs are, for $t = 1, \ldots, T$,

$$\texttt{DGP1}: \ \mathcal{S}_t Y_t = \mathcal{S}_t A_t^{-1}(\lambda)(X_t\beta + \mu + \alpha_t l_n + U_t), \ \ U_t = \rho M_t U_t + V_t,$$

$$\texttt{DGP2}: \ \mathcal{S}_t Y_t = \mathcal{S}_t A_t^{-1}(\lambda)(X_t\beta + \mu + \alpha_t l_n + V_t),$$

$$\texttt{DGP3}: \ \mathcal{S}_t Y_t = \mathcal{S}_t A_t^{-1}(\lambda)(X_t\beta + \mu + \alpha_t l_n + U_t), \ \ U_t = \rho M_t U_t + V_t, \ \ V_t = \tau V_{t-1} + \epsilon_t,$$

We choose $n = 50, 100, 200, 400$, and $T = 5, 10$. The parameters values are set at $\beta = 1$, $\lambda = 0.2$, $\rho = 0.2$ and $\sigma_v^2 = 1$ for `DGP1`, $\beta = 1$, $\lambda = 0.2$ and $\sigma_v^2 = 1$ for `DGP2`, and $\beta = 1$, $\lambda = 0.2$, $\rho = 0.2$, $\tau = 0.5$ and $\sigma_e^2 = 1$ for `DGP3`. $X_t's$ are generated independently from $N(2, 2^2 I_n)$, and individual effects are set to be $\mu = \frac{1}{T}\Sigma_{t=1}^T X_t + e$, where $e \sim N(0, I_n)$. The time fixed effects α are generated from $N(0, I_T)$. The number of Monte Carlo runs is 1000.

The spatial weight matrices can be `Rook` contiguity, `Queen` contiguity, and `Group` Interaction. To generate W_t under `Rook`, randomly permute the indices $\{1, 2, \ldots, n\}$ for n spatial units and then allocate them into a lattice of $k \times m$ squares. Let $W_{nt,ij} = 1$ if the index j is in a square that is immediately left or right, above, or below the square that contains the index i. Similarly, W_{nt} under `Queen` is generated with additional neighbors sharing a common vertex with the unit i. To generate W_t under `Group`, we let $K(n) = Round(n^{0.5})$ be the number of groups and then generate $K(n)$ group sizes according to a uniform distribution.

The distribution of the idiosyncratic errors $\{v_{it}\}$ can be (*i*) `normal`, (*ii*) standardized `normal mixture` (10% $N(0, 4^2)$ and 90% $N(0, 1)$), or (*iii*) standardized `chi-square` with 3 degrees of freedom. See Yang (2015) for details.

The selection matrices, $\mathcal{S}_t$, are generated in two ways. In the first approach, we assume MCAR (Missing Completely At Random) missingness. For each t, we associate each row of I_n with a uniform (0, 1) random number, and rows with random numbers smaller than $p_t \in (0, 1)$ are deleted, resulting in $100p_t\%$ missing responses. We consider two missingness levels, 10% and 30%, to assess the effect of missing data on estimation. In the second approach, we consider MAR (Missing At Random) missingness, where missingness depends on both covariates and fixed effects. Each individual's fixed effect μ_i is first standardized to $\tilde{\mu}_i = \frac{\mu_i - \text{mean}(\mu)}{\text{std}(\mu)}$, and the missingness probability for each observation is defined by $\Phi(\tilde{\mu}_i)$, where Φ is the CDF of the standard normal distribution. A uniform (0, 1) random number v_{it} is then generated for each individual and period, and a data point is marked as missing if $v_{it} \leq \Phi(\tilde{\mu}_i)$ and $x_{it} > \text{mean}(\mathbf{X})$. This approach results in an approximate overall missing percentage of 25%. To generate the MR data, a full sample of size n is first generated for each period, and then the "observed" responses are selected based on the generated selection matrix.

The estimators. Beside the three estimators reported in the main text, `QMLE-MR`, `ME-GU` and `ME-MR`, six additional estimators are also included in our Monte Carlo experiments: `Naïve`, `NLSE`, `QMLE-GU`, `Impu-I`, `Impu-II`, and `Impu-III`, which are, respectively, the naïve to missingness estimator (M-estimator based on a balanced panel formed by list-wise deletion; the nonlinear least square estimator of Wang and Lee (2013); the QMLE assuming GU, and the three M-estimators based on balanced panels obtained through impuation of Efron (1994), Honaker and King (2010), and Wang and Lee (2013).

The detail of $\texttt{Impu} - \texttt{III}$ is worth noting. Let $\hat{\theta}_{\mathtt{M}} = (\hat{\beta}'_{\mathtt{M}}, \hat{\sigma}^2_{v,\mathtt{M}}, \hat{\lambda}_{\mathtt{M}}, \hat{\rho}_{\mathtt{M}})'$ and $\hat{\phi}_{\mathtt{M}}$ be the M-estimators of θ_0 and ϕ_0. Define C_t as a diagonal matrix of dimension $n \times n$ for each time period t, where the diagonal entries are zero for missing units and one for observed units. Let $\mathbf{C} = \texttt{bdiag}(C_1, \ldots, C_T)$ be the block diagonal matrix constructed from the C_t matrices across all time periods. Then, the 'completed responses' through prediction can be expressed as:

$$\mathbf{Y}_{\texttt{Impu}} = \mathbf{C} * \mathbf{Y} + (I_{nT} - \mathbf{C})\mathbf{A}^{-1}_{nT}(\hat{\lambda}_{\mathtt{M}})[\mathbf{X}\hat{\beta}_{\mathtt{M}} + \mathbf{D}\hat{\phi}_{\mathtt{M}}], \tag{O.8}$$

where the first term selects the observed responses, and the second imputes the missing ones.

Tables 1a and 1b present Monte Carlo results for the MR model under `DGP1` with $T = 5$ and 10, respectively, when the missing percentage is 10% under MCAR, $\{W_t\}$ are `Queen` and $\{M_t\}$ are `Rook`. As expected, the results show an excellent performance of `ME-MR` and its inference methods. `ME-MR` performs well even when the sample size is quite small, and shows convergence to their true values as the sample size increases. Their corresponding standard error estimates are also close to Monte Carlo standard deviations. In contrast, the finite sample performance of the `QMLE-MR` of the spatial estimates is not as good as that of the proposed M-estimation. By the results of `QMLE-GU` and `ME-GU`, we can see the consequences of treating MR models as GU models in that both of them cannot provide consistent estimation for spatial parameters even when the sample size is large enough. Similarly, the `Naïve` cannot give us unbiased estimation for spatial parameters as well, as it completely ignores the spatial effects of deleted units on remaining units. Besides, the finite sample performance of `Impu-I` and `Impu-II` can be very poor and they cannot provide unbiased estimation for spatial parameters even when the sample size is large enough. This is due to the ignorance of spatial interaction effects when imputing the data. `Impu-I` has an even worse performance than `Impu-II` as the former does not consider the panel data feature. These together show again the serious consequences when models are misspecified and wrong estimation methods are applied.

Tables 2a and 2b report Monte Carlo results with different spatial weight matrices, where $\{W_t\}$ are `Group` and $\{M_t\}$ are `Queen`. We see a similar pattern as in Tables 1a and 1b. The results again show an excellent performance of the proposed set of estimation and inference methods. In contrast, all the other estimators can perform poorly.

Tables 3a, 3b, 4a, and 4b report the Monte Carlo results with missing percentages 30% under MCAR. As the balanced panel formed by omitting missing units may not be possible when the missing percentage is high, `Naïve` is not considered in these cases. We see, when the missing percentage is higher, `QMLE-GU`, `ME-GU`, `Impu-I` and `Impu-II` become more biased. This is consistent with our expectations as the consequences of misspecification and wrong estimation methods are more serious. In contrast, `QMLE-MR` and `ME-MR` both can provide consistent estimation, but our proposed `ME-MR` is less biased in terms of finite sample performance.

Tables 5a and 5b show the Monte Carlo results for the MR model under `DGP2` with $T = 5$ and 10, respectively, when the missing percentage is 10% under MCAR and $\{W_t\}$ are `Group`. In this experiment, we consider the existing `NLSE` method proposed by Wang and Lee (2013), instead of `Naïve`. The results also show an excellent finite sample performance of the proposed `ME-MR` and their estimated standard errors. In contrast, `QMLE-GU`, `ME-GU`, `Impu-I` and `Impu-II` still cannot provide unbiased estimations even when the sample size is large enough. `QMLE-MR` is more biased than the proposed `ME-MR` in finite sample. `NLSE` can provide unbiased estimation but their standard errors are larger than those of `ME-MR`. Besides, `NLSE` is not feasible for the MR model under `DGP1`.

Tables 6a, 6b, 7a, and 7b present the Monte Carlo results for the MR model under `DGP3`, evaluated with different spatial weights and assuming MCAR. We only report `Impu-I`, `Impu-II`, `QMLE-MR` and `ME-MR`, as the estimation based on GU under this setup is unavailable and `Naïve` is expected to have a very poor performance. The `ME-MR` of all the parameters has a good finite sample performance. Their corresponding standard error estimates are also close to Monte Carlo standard deviations. In contrast, `QMLE-MR` typically provides much worse estimates for error variance parameter σ^2, spatial error parameter ρ, and serial correlation parameter τ. The performances of `Impu-I` and `Impu-II` are very poor, regardless of spatial weights, error distributions, and sample sizes.

Table 8 provides a horizontal comparison of the simulation results for M-estimators based on `Impu-I`, `Impu-II`, and `Impu-III` under `DGP1` and `DGP3`, assuming MCAR. The time-varying spatial weight matrices are constructed using group interactions for the spatial lag term and queen contiguity for the spatial error term. The results indicate that M-estimates based on `Impu-II` generally outperform those based on `Impu-I`, which does not account for the panel structure considered by `Impu-II`. However, the performance of the `Impu-II` estimator remains significantly inferior to that of `Impu-III`, regardless of error distributions, sample sizes, or spatial weights, largely due to the failure of current imputation methods to account for spatial interaction effects. Interestingly, although the `Impu-III` approach is valid only when both n and T are large, ensuring consistency in fixed effects estimates, it still surpasses the other two methods. This underscores the serious consequences of ignoring spatial dependence when imputing network data. In cases of serially correlated errors, both `Impu-I` and `Impu-II` estimators exhibit considerable bias, particularly with inaccurate estimates of the serial correlation parameter τ. In contrast, while the `Impu-III`-based estimators show improved performance, they remain biased when compared to the results from the proposed M-estimation method.

Table 9 reports the Monte Carlo results for the MR model under `DGP1` and `DGP3`, with approximately 25% missing data under the MAR mechanism. Table 10 summarizes the results under a heavier-tailed error distribution, specifically t-distributed errors with 5 degrees of freedom. The results indicate that `ME-MR` consistently outperforms `QMLE-MR` in terms of estimation accuracy, regardless of the data-generating processes, error distributions, or sample sizes.

E.6. Analyses of Simulated Housing Price Panel

Lastly, we generate an *incomplete* housing price panel by mimicking the housing price environment using the popular *Boston housing price data*. The data is given by Harrison Jr and Rubinfeld (1978) and is corrected and augmented with longitude and latitude by Gilley and Pace (1996). It is cross-sectional data with the median housing price (for each of the 506 census tracts in the Boston metropolitan statistical area) as the response. The explanatory variables include per capita crime rate by town (`crime`), proportion of residential land zoned for lots over 25,000 square feet (`zoning`), proportion of non-retail business acres per town (`industry`), tract bounding river Charles River dummy (`charlesr`), nitric oxide concentration (`noxsq`), average number of rooms per dwelling (`rooms`), proportion of owner-occupied units built prior to 1940 (`houseage`), weighted distances to five Boston employment centers (`distance`), index of accessibility to radial highways (`access`), full-value property-tax rate per 10,000 (`taxrate`), pupil-teacher ratio by town (`ptratio`), $1000(Bk-0.63)$ where Bk is the proportion of blacks by town (`blackpop`), and lower status of the population proportion (`lowclass`).

Housing price panels are usually formed through aggregation (e.g., median, mean) and may be incomplete (with missing observations on response) when researchers aggregate the data to the least possible level (e.g., the census tracts as in the *Boston housing price data*) to keep as much information as possible. However, the characteristics (values of the explanatory variables) of the aggregated spatial units are usually completely available, giving rise to incomplete spatial panel data with missing responses (MR). The spatial weight matrix is constructed using the Euclidean distance with longitude and latitude.

We first estimate a spatial cross-sectional model with spatial lag and spatial error based on the original data and using the QML method. Then, we use the estimated model (QMLE-CS) as the true model to generate data for other new periods. For the time-invariant variables (`charlesr` and `distance`), we can simply repeat them for each period. For the time-varying variables, we perturb them to give their observations for different periods. With error distribution assumed to be log-normally distributed, we generate values of the dependent variable using the generated panel of explanatory variables, and individual and time-fixed effects in a similar way as in the Monte Carlo experiments. We generate five new periods of data to give a 506×5 panel. Clearly, the characteristics of the census tracts are always available but the median price may not be available at every census tract and in every time period if the time periods are short, e.g., month, or quarter. The random missing percentage for median housing price is set as 10% and thus the selection matrix $\mathcal{S}_t$ can be generated in a similar way as before.

Table 11 presents estimation results for this simulated housing price panel. The true parameter values, used to generate the panel, are given under QMLE-CS. We see that `Naïve` gives a largely biased and inefficient estimation for spatial parameters, as a lot of units are deleted whose spatial effects on remaining units are ignored. `ME-GU` cannot give us unbiased estimation for spatial lag parameter either, as treating an `FE-ISPD-MR` model as an `FE-USPD-GU` model completely ignores the spatial interaction effects from those spatial units in the missing periods. In contrast, we can see much more reasonable results from `ME-MR`.

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Table 1a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP1, 10% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, W = Queen and M = Rook, and $\boldsymbol{T} = \mathbf{5}$.

	Naïve	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			$n = 50$; error = 1, 2, 3, for the three panels below				
β	-.0028(.047)	.0002(.041)	.0044(.038)[.038]	.0145(.040)[.049]	-.0017(.040)[.043]	-.0008(.037)	-.0017(.040)[.040]
λ	-.1198(.053)	-.0777(.067)	-.0646(.060)[.058]	-.0424(.064)[.082]	-.0360(.063)[.074]	-.0213(.065)	-.0020(.061)[.062]
ρ	-.1708(.122)	-.2395(.287)	.0039(.122)[.125]	-.0905(.078)[.117]	-.0763(.086)[.111]	-.0105(.153)	-.0022(.122)[.119]
σ_v^2	.0225(.152)	-.2312(.088)	.0059(.113)[.110]	.5107(.178)[.280]	.2370(.128)[.169]	-.2597(.085)	-.0157(.115)[.107]
β	.0000(.048)	.0026(.038)	.0045(.037)[.038]	.0142(.041)[.049]	-.0022(.040)[.043]	-.0009(.037)	-.0022(.039)[.041]
λ	-.1230(.057)	-.0849(.070)	-.0630(.055)[.057]	-.0488(.064)[.082]	-.0409(.063)[.074]	-.0198(.060)	-.0007(.065)[.062]
ρ	-.1664(.127)	-.2080(.269)	.0032(.121)[.125]	-.0927(.082)[.118]	-.0778(.090)[.111]	-.0092(.156)	-.0017(.115)[.119]
σ_v^2	-.0017(.321)	-.2323(.182)	.0011(.230)[.216]	.5256(.281)[.334]	.2426(.236)[.240]	-.2634(.173)	-.0034(.231)[.219]
β	-.0030(.050)	.0061(.041)	.0059(.038)[.038]	.0158(.039)[.048]	-.0013(.038)[.043]	.0005(.038)	.0001(.039)[.040]
λ	-.1192(.057)	-.0755(.066)	-.0611(.057)[.058]	-.0517(.064)[.082]	-.0457(.064)[.074]	-.0181(.062)	-.0024(.061)[.061]
ρ	-.1691(.119)	-.2296(.286)	.0009(.127)[.125]	-.0958(.082)[.117]	-.0781(.085)[.111]	-.0155(.160)	-.0028(.116)[.119]
σ_v^2	.0111(.219)	-.2252(.131)	.0063(.176)[.162]	.5100(.223)[.301]	.2372(.185)[.198]	-.2592(.131)	-.0250(.165)[.158]
			$n = 100$; error = 1, 2, 3, for the three panels below				
β	.0013(.035)	.0024(.028)	.0044(.027)[.028]	.0173(.031)[.039]	-.0066(.031)[.034]	.0001(.027)	.0016(.030)[.029]
λ	-.0846(.048)	-.0514(.048)	-.0341(.040)[.040]	-.0558(.048)[.066]	-.0488(.048)[.060]	-.0081(.038)	-.0010(.041)[.041]
ρ	-.1714(.101)	-.1722(.185)	-.0037(.084)[.084]	-.1036(.053)[.084]	-.0880(.058)[.079]	.0283(.107)	-.0044(.079)[.082]
σ_v^2	.0511(.097)	-.1971(.064)	.0194(.079)[.078]	.7012(.148)[.247]	.3791(.099)[.151]	-.2440(.060)	-.0063(.082)[.076]
β	.0025(.034)	.0040(.029)	.0049(.028)[.028]	.0194(.032)[.039]	-.0072(.032)[.034]	.0005(.028)	.0008(.029)[.029]
λ	-.0873(.046)	-.0478(.048)	-.0350(.040)[.040]	-.0513(.048)[.066]	-.0427(.048)[.059]	-.0078(.039)	-.0051(.040)[.041]
ρ	-.1607(.107)	-.1803(.187)	-.0042(.083)[.084]	-.1014(.055)[.084]	-.0824(.060)[.079]	.0266(.106)	-.0015(.085)[.083]
σ_v^2	.0582(.212)	-.1971(.130)	.0222(.161)[.161]	.6869(.206)[.278]	.3669(.172)[.195]	-.2423(.121)	-.0037(.164)[.161]
β	.0005(.035)	.0050(.029)	.0036(.026)[.028]	.0182(.032)[.039]	-.0070(.033)[.034]	-.0008(.026)	.0024(.029)[.029]
λ	-.0891(.050)	-.0484(.049)	-.0335(.041)[.040]	-.0540(.046)[.066]	-.0478(.047)[.061]	-.0077(.039)	-.0043(.040)[.041]
ρ	-.1701(.100)	-.1662(.180)	-.0056(.084)[.084]	-.1015(.053)[.084]	-.0875(.057)[.079]	.0251(.107)	-.0001(.082)[.083]
σ_v^2	.0508(.151)	-.1953(.099)	.0163(.124)[.118]	.7100(.186)[.262]	.3907(.141)[.171]	-.2463(.094)	-.0114(.120)[.117]
			$n = 200$; error = 1, 2, 3, for the three panels below				
β	-.0013(.023)	-.0017(.020)	-.0009(.019)[.019]	.0229(.022)[.026]	.0015(.021)[.021]	.0001(.019)	-.0006(.020)[.020]
λ	-.1096(.025)	-.0435(.032)	-.0324(.026)[.027]	-.0492(.035)[.046]	-.0358(.034)[.040]	-.0040(.027)	.0024(.029)[.029]
ρ	-.1724(.063)	-.1416(.127)	-.0016(.058)[.059]	-.0846(.039)[.057]	-.0551(.043)[.053]	.0516(.074)	-.0011(.057)[.058]
σ_v^2	.0702(.076)	-.1885(.044)	.0195(.056)[.055]	.4533(.083)[.126]	.0945(.058)[.065]	-.2407(.043)	-.0040(.054)[.054]
β	-.0005(.024)	-.0010(.019)	-.0006(.019)[.019]	.0219(.022)[.025]	.0002(.021)[.021]	.0004(.019)	-.0001(.020)[.020]
λ	-.1111(.025)	-.0424(.031)	-.0326(.026)[.027]	-.0448(.035)[.045]	-.0339(.035)[.040]	-.0042(.026)	.0020(.027)[.028]
ρ	-.1767(.061)	-.1461(.125)	-.0054(.057)[.059]	-.0875(.040)[.057]	-.0600(.043)[.053]	.0484(.072)	-.0032(.056)[.058]
σ_v^2	.0548(.155)	-.1873(.096)	.0204(.117)[.116]	.4566(.137)[.158]	.1001(.112)[.111]	-.2395(.089)	-.0065(.112)[.114]
β	-.0009(.024)	-.0013(.020)	-.0010(.019)[.019]	.0209(.023)[.025]	.0007(.022)[.021]	.0000(.019)	-.0004(.020)[.020]
λ	-.1103(.027)	-.0455(.031)	-.0330(.028)[.027]	-.0459(.036)[.045]	-.0329(.035)[.039]	-.0038(.028)	-.0007(.028)[.028]
ρ	-.1755(.061)	-.1406(.129)	-.0047(.058)[.059]	-.0862(.041)[.057]	-.0590(.044)[.053]	.0491(.073)	-.0005(.061)[.058]
σ_v^2	.0733(.117)	-.1838(.068)	.0216(.087)[.085]	.4512(.106)[.142]	.0946(.086)[.087]	-.2391(.066)	-.0082(.087)[.084]
			$n = 400$; error = 1, 2, 3, for the three panels below				
β	.0061(.018)	-.0013(.014)	.0015(.014)[.014]	.0191(.014)[.017]	-.0012(.013)[.014]	-.0002(.014)	.0000(.013)[.014]
λ	-.1047(.021)	-.0429(.022)	-.0353(.019)[.019]	-.0433(.024)[.030]	-.0314(.023)[.026]	-.0033(.020)	-.0009(.020)[.020]
ρ	-.1702(.047)	-.1274(.084)	-.0013(.040)[.041]	-.0838(.027)[.040]	-.0547(.029)[.037]	.0599(.051)	.0006(.040)[.040]
σ_v^2	.0668(.051)	-.1824(.031)	.0213(.040)[.039]	.4152(.056)[.086]	.0620(.039)[.043]	-.2335(.031)	-.0036(.039)[.038]
β	.0063(.018)	-.0017(.013)	.0022(.014)[.014]	.0192(.015)[.017]	-.0011(.015)[.014]	.0006(.014)	-.0008(.014)[.014]
λ	-.1031(.021)	-.0444(.024)	-.0341(.020)[.019]	-.0422(.022)[.030]	-.0307(.022)[.026]	-.0025(.020)	-.0014(.020)[.020]
ρ	-.1681(.047)	-.1188(.089)	-.0034(.041)[.041]	-.0829(.029)[.040]	-.0535(.030)[.037]	.0572(.051)	.0017(.039)[.041]
σ_v^2	.0675(.102)	-.1832(.068)	.0186(.083)[.082]	.4156(.092)[.109]	.0642(.080)[.078]	-.2351(.064)	-.0051(.081)[.082]
β	.0079(.019)	-.0022(.014)	.0014(.013)[.014]	.0193(.014)[.017]	-.0015(.013)[.014]	-.0003(.013)	.0000(.014)[.014]
λ	-.1037(.020)	-.0431(.023)	-.0333(.020)[.019]	-.0427(.022)[.030]	-.0324(.023)[.026]	-.0020(.020)	-.0005(.020)[.019]
ρ	-.1700(.047)	-.1227(.087)	-.0005(.041)[.041]	-.0837(.028)[.040]	-.0531(.029)[.037]	.0601(.051)	.0047(.041)[.040]
σ_v^2	.0607(.076)	-.1829(.049)	.0177(.059)[.061]	.4109(.076)[.096]	.0581(.063)[.059]	-.2358(.045)	-.0055(.061)[.060]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 1b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP1, 10% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, W = Queen and M = Rook, and $\boldsymbol{T = 10}$.

	Naïve	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			$n = 50$; error = 1, 2, 3, for the three panels below				
β	-.0004(.040)	-.0001(.026)	-.0003(.024)[.024]	.0097(.029)[.033]	.0053(.028)[.028]	-.0005(.024)	.0013(.026)[.026]
λ	-.1411(.035)	-.0348(.042)	-.0287(.037)[.036]	-.0451(.044)[.058]	-.0342(.043)[.052]	-.0156(.038)	.0018(.040)[.042]
ρ	-.1457(.089)	-.2361(.132)	-.0054(.071)[.074]	-.0859(.054)[.076]	-.0613(.059)[.072]	-.0375(.078)	-.0013(.071)[.073]
σ_v^2	.0694(.114)	-.1014(.069)	.0129(.075)[.072]	.4251(.102)[.168]	.1452(.080)[.099]	-.1333(.064)	-.0125(.072)[.071]
β	.0019(.038)	-.0008(.023)	-.0006(.024)[.024]	.0091(.030)[.033]	.0040(.028)[.028]	-.0007(.023)	-.0006(.026)[.025]
λ	-.1428(.034)	-.0380(.041)	-.0270(.037)[.036]	-.0449(.050)[.059]	-.0362(.049)[.053]	-.0140(.036)	-.0048(.042)[.042]
ρ	-.1406(.090)	-.2341(.129)	-.0032(.073)[.074]	-.0831(.055)[.076]	-.0631(.058)[.073]	-.0323(.079)	.0006(.073)[.074]
σ_v^2	.0695(.249)	-.1028(.146)	.0112(.165)[.158]	.4278(.184)[.214]	.1542(.167)[.161]	-.1349(.143)	-.0144(.158)[.154]
β	-.0021(.041)	.0007(.024)	-.0003(.023)[.024]	.0098(.028)[.033]	.0047(.027)[.028]	-.0004(.023)	-.0015(.027)[.026]
λ	-.1416(.034)	-.0374(.041)	-.0278(.037)[.036]	-.0431(.044)[.059]	-.0337(.043)[.052]	-.0148(.037)	-.0010(.042)[.042]
ρ	-.1458(.093)	-.2431(.124)	-.0012(.071)[.074]	-.0829(.054)[.076]	-.0591(.055)[.073]	-.0314(.078)	-.0009(.072)[.073]
σ_v^2	.0603(.176)	-.1089(.104)	.0060(.115)[.113]	.4247(.147)[.190]	.1486(.124)[.128]	-.1395(.099)	-.0067(.119)[.115]
			$n = 100$; error = 1, 2, 3, for the three panels below				
β	.0001(.026)	-.0022(.018)	-.0021(.018)[.018]	.0124(.021)[.024]	.0011(.019)[.021]	.0002(.018)	.0008(.019)[.018]
λ	-.1459(.026)	-.0534(.031)	-.0393(.027)[.027]	-.0473(.030)[.041]	-.0384(.029)[.037]	-.0102(.028)	-.0002(.029)[.029]
ρ	-.1473(.063)	-.1688(.088)	-.0064(.051)[.051]	-.0950(.036)[.055]	-.0767(.039)[.052]	-.0061(.056)	.0000(.052)[.051]
σ_v^2	.0681(.076)	-.0821(.049)	.0172(.050)[.051]	.5951(.092)[.153]	.3006(.072)[.092]	-.1243(.043)	-.0097(.048)[.050]
β	-.0006(.024)	-.0023(.017)	-.0018(.017)[.018]	.0120(.020)[.024]	.0002(.019)[.021]	.0005(.017)	-.0007(.017)[.018]
λ	-.1461(.026)	-.0531(.032)	-.0395(.027)[.027]	-.0462(.032)[.041]	-.0373(.032)[.036]	-.0087(.028)	.0005(.031)[.029]
ρ	-.1500(.061)	-.1670(.095)	-.0025(.051)[.051]	-.0981(.036)[.055]	-.0775(.038)[.052]	-.0014(.056)	.0000(.049)[.051]
σ_v^2	.0599(.166)	-.0844(.106)	.0164(.118)[.113]	.5877(.147)[.177]	.2923(.127)[.130]	-.1262(.103)	-.0063(.118)[.112]
β	-.0010(.026)	-.0025(.017)	-.0018(.018)[.018]	.0106(.021)[.024]	-.0002(.020)[.021]	.0004(.018)	.0007(.018)[.018]
λ	-.1438(.027)	-.0506(.031)	-.0395(.026)[.027]	-.0493(.031)[.041]	-.0394(.031)[.037]	-.0090(.028)	-.0003(.030)[.029]
ρ	-.1506(.063)	-.1672(.093)	-.0028(.052)[.051]	-.0953(.037)[.055]	-.0777(.040)[.052]	-.0019(.057)	-.0024(.050)[.051]
σ_v^2	.0641(.129)	-.0776(.075)	.0189(.086)[.082]	.5971(.120)[.164]	.3005(.098)[.110]	-.1240(.074)	-.0060(.084)[.081]
			$n = 200$; error = 1, 2, 3, for the three panels below				
β	-.0001(.019)	.0020(.012)	.0021(.012)[.012]	.0115(.015)[.018]	.0015(.014)[.016]	.0001(.012)	.0016(.012)[.013]
λ	-.1528(.018)	-.0391(.023)	-.0280(.020)[.020]	-.0492(.023)[.030]	-.0422(.023)[.028]	-.0043(.020)	-.0027(.020)[.021]
ρ	-.1593(.042)	-.1586(.066)	-.0042(.036)[.036]	-.0996(.026)[.039]	-.0855(.028)[.037]	.0098(.040)	-.0026(.034)[.036]
σ_v^2	.0780(.056)	-.0752(.033)	.0185(.036)[.036]	.6277(.068)[.110]	.3977(.053)[.076]	-.1198(.032)	-.0008(.035)[.036]
β	.0003(.020)	.0007(.012)	.0020(.012)[.012]	.0103(.015)[.018]	.0004(.015)[.016]	.0000(.012)	.0008(.012)[.013]
λ	-.1532(.019)	-.0404(.024)	-.0276(.020)[.020]	-.0479(.023)[.031]	-.0416(.023)[.028]	-.0043(.020)	.0000(.021)[.021]
ρ	-.1575(.042)	-.1524(.062)	-.0057(.036)[.036]	-.0985(.025)[.039]	-.0852(.026)[.037]	.0085(.040)	.0019(.037)[.036]
σ_v^2	.0913(.132)	-.0733(.079)	.0184(.083)[.081]	.6351(.104)[.130]	.4049(.094)[.101]	-.1197(.073)	-.0032(.082)[.081]
β	.0008(.020)	.0013(.012)	.0024(.012)[.012]	.0099(.015)[.018]	.0005(.014)[.016]	.0004(.012)	-.0009(.013)[.013]
λ	-.1524(.018)	-.0386(.024)	-.0274(.019)[.020]	-.0484(.023)[.030]	-.0412(.023)[.028]	-.0045(.020)	-.0007(.020)[.021]
ρ	-.1645(.043)	-.1560(.067)	-.0035(.037)[.036]	-.0978(.026)[.039]	-.0831(.026)[.037]	.0103(.040)	.0004(.035)[.036]
σ_v^2	.0806(.094)	-.0760(.055)	.0193(.060)[.059]	.6248(.082)[.119]	.3939(.069)[.087]	-.1188(.052)	-.0089(.060)[.058]
			$n = 400$; error = 1, 2, 3, for the three panels below				
β	.0026(.015)	.0007(.009)	.0025(.009)[.009]	.0088(.010)[.013]	-.0007(.010)[.011]	-.0002(.009)	-.0001(.009)[.009]
λ	-.1560(.013)	-.0549(.017)	-.0422(.013)[.013]	-.0520(.016)[.022]	-.0412(.016)[.020]	-.0028(.013)	-.0002(.014)[.014]
ρ	-.1646(.029)	-.1245(.044)	-.0037(.026)[.026]	-.0948(.017)[.027]	-.0754(.019)[.026]	.0184(.029)	.0001(.026)[.025]
σ_v^2	.0880(.046)	-.0704(.023)	.0253(.027)[.026]	.5917(.045)[.074]	.3092(.032)[.047]	-.1170(.023)	-.0006(.027)[.025]
β	.0037(.015)	.0005(.009)	.0026(.009)[.009]	.0077(.010)[.013]	-.0020(.010)[.011]	-.0001(.009)	-.0008(.009)[.009]
λ	-.1564(.013)	-.0548(.016)	-.0415(.014)[.013]	-.0523(.016)[.022]	-.0415(.017)[.020]	-.0018(.014)	.0005(.014)[.014]
ρ	-.1659(.030)	-.1270(.045)	-.0042(.025)[.026]	-.0908(.018)[.028]	-.0729(.018)[.025]	.0181(.028)	-.0010(.027)[.025]
σ_v^2	.0842(.095)	-.0721(.053)	.0241(.062)[.058]	.5860(.069)[.089]	.3026(.055)[.065]	-.1183(.054)	.0005(.055)[.058]
β	.0027(.015)	.0014(.009)	.0026(.009)[.009]	.0080(.010)[.012]	-.0023(.009)[.011]	.0000(.009)	.0001(.009)[.009]
λ	-.1567(.013)	-.0542(.017)	-.0417(.014)[.013]	-.0494(.016)[.022]	-.0390(.017)[.020]	-.0024(.014)	.0004(.016)[.014]
ρ	-.1620(.029)	-.1290(.046)	-.0034(.025)[.026]	-.0934(.016)[.027]	-.0752(.018)[.025]	.0187(.028)	-.0002(.025)[.025]
σ_v^2	.0862(.072)	-.0703(.037)	.0258(.044)[.042]	.5864(.057)[.079]	.3088(.047)[.056]	-.1166(.038)	.0026(.040)[.042]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 2a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under `DGP1`, 10% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, `W = Group` and `M = Queen`, and $\boldsymbol{T} = \mathbf{5}$.

	`Naïve`	`QMLE-GU`	`ME-GU`	`Impu-I`	`Impu-II`	`QMLE-MR`	`ME-MR`
			$n = 50$; error = 1, 2, 3, for the three panels below				
β	.0060(.055)	.0016(.045)	.0024(.044)[.042]	.0156(.039)[.045]	.0018(.039)[.039]	-.0015(.044)	-.0031(.044)[.042]
λ	-.1133(.069)	-.0616(.063)	-.0473(.058)[.055]	-.0447(.070)[.076]	-.0389(.070)[.069]	-.0196(.064)	-.0049(.063)[.061]
ρ	-.1752(.116)	-.2283(.216)	-.0131(.163)[.162]	-.0644(.102)[.149]	-.0332(.098)[.139]	-.0917(.223)	.0096(.215)[.151]
σ_v^2	.0157(.142)	-.2336(.084)	-.0043(.111)[.109]	.3341(.134)[.216]	.0604(.111)[.127]	-.2597(.084)	-.0284(.111)[.106]
β	.0050(.053)	.0020(.043)	.0038(.041)[.042]	.0120(.040)[.045]	-.0010(.040)[.039]	-.0004(.041)	-.0016(.041)[.042]
λ	-.1216(.072)	-.0633(.064)	-.0472(.056)[.055]	-.0481(.059)[.077]	-.0432(.055)[.068]	-.0178(.061)	-.0034(.060)[.059]
ρ	-.1670(.121)	-.2311(.230)	-.0214(.159)[.163]	-.0864(.114)[.153]	-.0634(.105)[.143]	-.1002(.214)	-.0100(.179)[.154]
σ_v^2	-.0010(.314)	-.2394(.187)	-.0084(.237)[.216]	.3451(.274)[.278]	.0652(.220)[.210]	-.2630(.179)	-.0316(.235)[.213]
β	.0095(.053)	.0066(.044)	.0046(.042)[.042]	.0152(.042)[.046]	.0034(.042)[.039]	.0008(.043)	-.0005(.043)[.042]
λ	-.1217(.068)	-.0663(.064)	-.0489(.058)[.055]	-.0385(.063)[.075]	-.0328(.066)[.066]	-.0233(.064)	-.0084(.063)[.060]
ρ	-.1723(.123)	-.2337(.220)	-.0112(.159)[.161]	-.0813(.113)[.154]	-.0521(.116)[.140]	-.0884(.216)	.0017(.185)[.152]
σ_v^2	.0075(.226)	-.2380(.131)	.0014(.177)[.163]	.3505(.181)[.243]	.0736(.162)[.160]	-.2538(.134)	-.0199(.175)[.161]
			$n = 100$; error = 1, 2, 3, for the three panels below				
β	.0025(.030)	.0017(.026)	.0042(.027)[.027]	.0228(.032)[.035]	.0043(.031)[.029]	.0010(.027)	.0007(.027)[.027]
λ	-.0879(.062)	-.0701(.063)	-.0438(.054)[.051]	-.0338(.043)[.061]	-.0308(.042)[.054]	-.0120(.052)	-.0009(.051)[.051]
ρ	-.1730(.111)	-.1682(.178)	-.0147(.105)[.106]	-.0944(.075)[.106]	-.0716(.080)[.100]	-.0182(.133)	-.0095(.114)[.103]
σ_v^2	.0189(.097)	-.2168(.061)	.0068(.076)[.076]	.5098(.104)[.199]	.1659(.087)[.105]	-.2408(.058)	-.0122(.075)[.075]
β	.0012(.032)	.0002(.028)	.0028(.027)[.027]	.0271(.029)[.035]	.0033(.025)[.029]	-.0003(.027)	-.0005(.027)[.027]
λ	-.0872(.064)	-.0750(.064)	-.0496(.054)[.051]	-.0504(.048)[.062]	-.0398(.044)[.054]	-.0190(.055)	-.0077(.054)[.052]
ρ	-.1709(.124)	-.1545(.179)	-.0158(.103)[.106]	-.0842(.073)[.107]	-.0633(.078)[.099]	-.0200(.131)	-.0121(.108)[.103]
σ_v^2	.0200(.195)	-.2190(.123)	.0130(.167)[.159]	.5174(.186)[.234]	.1574(.159)[.158]	-.2354(.127)	-.0052(.165)[.158]
β	.0038(.031)	.0019(.027)	.0030(.027)[.027]	.0224(.030)[.036]	.0005(.027)[.030]	-.0002(.027)	-.0004(.027)[.027]
λ	-.0845(.061)	-.0721(.066)	-.0471(.051)[.051]	-.0455(.047)[.062]	-.0371(.047)[.055]	-.0181(.053)	-.0068(.052)[.052]
ρ	-.1791(.110)	-.1594(.168)	-.0084(.104)[.105]	-.0956(.072)[.109]	-.0731(.077)[.100]	-.0125(.133)	-.0049(.114)[.102]
σ_v^2	.0184(.149)	-.2167(.097)	.0023(.125)[.116]	.5342(.141)[.220]	.1692(.115)[.131]	-.2433(.095)	-.0154(.124)[.115]
			$n = 200$; error = 1, 2, 3, for the three panels below				
β	.0038(.022)	.0033(.019)	.0055(.019)[.020]	.0181(.023)[.026]	-.0001(.024)[.022]	.0006(.019)	.0004(.019)[.020]
λ	-.0681(.049)	-.0441(.054)	-.0261(.042)[.041]	-.0551(.049)[.066]	-.0487(.051)[.061]	-.0098(.041)	-.0031(.040)[.039]
ρ	-.1797(.101)	-.1686(.150)	-.0066(.075)[.073]	-.0861(.047)[.075]	-.0632(.051)[.068]	.0191(.093)	-.0075(.074)[.071]
σ_v^2	.0188(.063)	-.2119(.041)	.0035(.057)[.053]	.4994(.080)[.134]	.2112(.063)[.082]	-.2352(.043)	-.0081(.056)[.053]
β	.0034(.023)	.0037(.020)	.0054(.019)[.020]	.0131(.023)[.026]	-.0040(.022)[.022]	.0006(.019)	.0003(.019)[.020]
λ	-.0676(.045)	-.0460(.050)	-.0243(.043)[.041]	-.0539(.051)[.066]	-.0419(.050)[.060]	-.0089(.040)	-.0023(.040)[.039]
ρ	-.1802(.097)	-.1585(.144)	-.0013(.073)[.072]	-.0876(.048)[.073]	-.0727(.054)[.069]	.0269(.088)	-.0013(.071)[.071]
σ_v^2	.0181(.139)	-.2134(.092)	.0026(.116)[.114]	.4788(.134)[.161]	.1964(.121)[.118]	-.2356(.089)	-.0086(.115)[.114]
β	.0042(.022)	.0048(.020)	.0052(.020)[.020]	.0199(.022)[.025]	.0025(.022)[.022]	.0004(.020)	.0001(.020)[.020]
λ	-.0636(.045)	-.0369(.050)	-.0257(.042)[.041]	-.0547(.049)[.065]	-.0456(.048)[.060]	-.0097(.040)	-.0030(.039)[.039]
ρ	-.1898(.098)	-.1773(.145)	-.0020(.070)[.072]	-.0750(.051)[.075]	-.0560(.059)[.070]	.0253(.084)	-.0027(.068)[.071]
σ_v^2	.0218(.106)	-.2091(.068)	.0066(.084)[.084]	.4844(.112)[.152]	.1966(.097)[.099]	-.2327(.065)	-.0049(.084)[.083]
			$n = 400$; error = 1, 2, 3, for the three panels below				
β	.0013(.017)	.0007(.013)	.0008(.013)[.013]	.0204(.014)[.018]	-.0006(.013)[.015]	.0004(.013)	.0003(.013)[.013]
λ	-.0951(.039)	-.0683(.043)	-.0457(.036)[.035]	-.0448(.036)[.049]	-.0370(.036)[.044]	-.0083(.036)	-.0033(.036)[.036]
ρ	-.1687(.079)	-.1132(.123)	-.0005(.053)[.053]	-.0814(.040)[.052]	-.0571(.038)[.049]	.0426(.064)	-.0025(.052)[.051]
σ_v^2	.0239(.052)	-.2057(.032)	.0092(.038)[.039]	.4776(.062)[.095]	.1570(.043)[.053]	-.2286(.030)	-.0005(.038)[.039]
β	.0004(.017)	.0002(.013)	.0002(.013)[.013]	.0207(.014)[.018]	-.0001(.014)[.015]	-.0002(.013)	-.0002(.013)[.013]
λ	-.0979(.040)	-.0697(.043)	-.0464(.037)[.035]	-.0424(.034)[.048]	-.0337(.034)[.043]	-.0092(.037)	-.0041(.037)[.036]
ρ	-.1693(.075)	-.1133(.121)	.0011(.051)[.053]	-.0851(.034)[.052]	-.0634(.038)[.049]	.0446(.061)	-.0012(.050)[.051]
σ_v^2	.0206(.109)	-.2071(.067)	.0063(.082)[.083]	.4735(.084)[.115]	.1494(.069)[.081]	-.2308(.063)	-.0034(.082)[.083]
β	.0013(.017)	.0004(.014)	.0005(.014)[.013]	.0193(.015)[.018]	-.0011(.015)[.015]	.0001(.014)	.0001(.014)[.013]
λ	-.0921(.039)	-.0658(.043)	-.0460(.035)[.035]	-.0369(.039)[.049]	-.0289(.039)[.043]	-.0081(.035)	-.0030(.034)[.036]
ρ	-.1686(.069)	-.1120(.116)	-.0003(.052)[.053]	-.0904(.040)[.052]	-.0695(.043)[.049]	.0435(.063)	-.0021(.051)[.051]
σ_v^2	.0287(.082)	-.2074(.049)	.0073(.063)[.061]	.4846(.076)[.103]	.1652(.064)[.067]	-.2301(.049)	-.0025(.063)[.061]

Note: error = 1(`normal`), 2(`normal mixture`), 3(`chi-square`).

Table 2b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP1, 10% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, W = Group and M = Queen, and $\boldsymbol{T} = \mathbf{10}$.

	Naïve	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			$n = 50$; error = 1, 2, 3, for the three panels below				
β	.0018(.045)	.0045(.028)	.0053(.026)[.026]	.0124(.026)[.032]	.0038(.027)[.029]	.0014(.026)	.0006(.026)[.026]
λ	-.1521(.056)	-.0669(.048)	-.0422(.042)[.041]	-.0454(.044)[.053]	-.0385(.042)[.048]	-.0190(.042)	-.0029(.041)[.040]
ρ	-.1417(.103)	-.1815(.123)	-.0133(.096)[.096]	-.0732(.069)[.099]	-.0537(.071)[.093]	-.0810(.105)	-.0100(.093)[.093]
σ_v^2	.0325(.118)	-.1084(.064)	.0140(.071)[.072]	.3440(.096)[.146]	.1076(.069)[.091]	-.1365(.060)	-.0101(.069)[.070]
β	.0026(.046)	.0039(.025)	.0042(.025)[.026]	.0094(.031)[.032]	.0022(.029)[.029]	.0003(.025)	-.0006(.025)[.026]
λ	-.1515(.054)	-.0638(.049)	-.0435(.044)[.041]	-.0365(.041)[.052]	-.0275(.040)[.047]	-.0197(.042)	-.0038(.041)[.041]
ρ	-.1439(.100)	-.1786(.124)	-.0129(.094)[.097]	-.0771(.075)[.101]	-.0478(.080)[.092]	-.0816(.104)	-.0104(.092)[.094]
σ_v^2	.0334(.273)	-.1057(.151)	.0171(.160)[.158]	.3448(.173)[.198]	.1154(.157)[.157]	-.1337(.139)	-.0069(.159)[.157]
β	.0019(.044)	.0032(.026)	.0052(.026)[.026]	.0079(.024)[.032]	.0016(.025)[.028]	.0013(.026)	.0005(.026)[.026]
λ	-.1503(.056)	-.0633(.050)	-.0415(.043)[.041]	-.0355(.043)[.052]	-.0264(.041)[.047]	-.0199(.041)	-.0039(.041)[.041]
ρ	-.1424(.105)	-.1747(.119)	-.0117(.097)[.097]	-.0802(.067)[.100]	-.0543(.074)[.093]	-.0800(.105)	-.0090(.093)[.093]
σ_v^2	.0377(.196)	-.1068(.104)	.0085(.116)[.113]	.3241(.134)[.170]	.0989(.115)[.118]	-.1401(.100)	-.0143(.114)[.112]
			$n = 100$; error = 1, 2, 3, for the three panels below				
β	.0038(.032)	.0028(.019)	.0025(.018)[.018]	.0098(.019)[.024]	.0008(.018)[.021]	-.0005(.018)	-.0007(.018)[.018]
λ	-.1325(.040)	-.0351(.042)	-.0221(.034)[.034]	-.0433(.041)[.045]	-.0358(.041)[.040]	-.0122(.033)	-.0016(.033)[.033]
ρ	-.1679(.073)	-.1867(.099)	-.0018(.065)[.066]	-.0822(.042)[.071]	-.0586(.042)[.067]	-.0271(.071)	-.0044(.064)[.064]
σ_v^2	.0309(.089)	-.1044(.045)	.0083(.049)[.051]	.4878(.082)[.129]	.1870(.060)[.074]	-.1238(.043)	-.0050(.049)[.050]
β	.0035(.029)	.0027(.019)	.0033(.019)[.018]	.0090(.020)[.024]	.0005(.020)[.020]	.0003(.019)	.0000(.019)[.018]
λ	-.1331(.040)	-.0350(.040)	-.0206(.034)[.034]	-.0393(.032)[.044]	-.0300(.032)[.040]	-.0111(.032)	-.0006(.032)[.032]
ρ	-.1678(.072)	-.1838(.103)	-.0025(.062)[.066]	-.0868(.050)[.070]	-.0647(.058)[.066]	-.0278(.068)	-.0051(.061)[.064]
σ_v^2	.0477(.185)	-.0954(.101)	.0013(.120)[.113]	.4803(.118)[.162]	.1710(.110)[.115]	-.1295(.105)	-.0115(.119)[.112]
β	.0035(.028)	.0034(.018)	.0037(.018)[.018]	.0097(.021)[.024]	-.0012(.020)[.020]	.0007(.018)	.0005(.018)[.018]
λ	-.1305(.038)	-.0389(.041)	-.0241(.035)[.034]	-.0398(.035)[.044]	-.0309(.033)[.039]	-.0148(.034)	-.0041(.033)[.033]
ρ	-.1718(.072)	-.1839(.096)	-.0044(.066)[.066]	-.0870(.049)[.072]	-.0601(.051)[.066]	-.0296(.072)	-.0066(.064)[.064]
σ_v^2	.0437(.143)	-.0981(.075)	.0118(.084)[.082]	.4880(.108)[.144]	.1833(.091)[.095]	-.1201(.074)	-.0007(.084)[.082]
			$n = 200$; error = 1, 2, 3, for the three panels below				
β	.0037(.022)	.0011(.012)	.0026(.012)[.012]	.0071(.016)[.016]	-.0008(.016)[.014]	.0004(.012)	.0003(.012)[.012]
λ	-.1625(.029)	-.0498(.033)	-.0308(.029)[.028]	-.0446(.028)[.039]	-.0369(.029)[.036]	-.0088(.030)	-.0006(.029)[.029]
ρ	-.1646(.052)	-.1445(.083)	-.0020(.046)[.046]	-.0735(.036)[.049]	-.0520(.035)[.045]	-.0034(.050)	-.0034(.045)[.045]
σ_v^2	.0382(.066)	-.0944(.034)	.0066(.035)[.036]	.4050(.055)[.084]	.1577(.043)[.050]	-.1188(.031)	-.0032(.035)[.036]
β	.0055(.022)	.0020(.012)	.0031(.012)[.012]	.0091(.014)[.016]	.0003(.014)[.014]	.0009(.012)	.0008(.012)[.012]
λ	-.1613(.029)	-.0498(.034)	-.0305(.030)[.028]	-.0387(.030)[.039]	-.0295(.029)[.035]	-.0090(.030)	-.0008(.030)[.029]
ρ	-.1662(.050)	-.1571(.085)	.0026(.045)[.046]	-.0797(.034)[.049]	-.0624(.035)[.045]	.0016(.048)	.0011(.043)[.045]
σ_v^2	.0384(.151)	-.0966(.071)	.0063(.085)[.081]	.3935(.094)[.104]	.1484(.082)[.080]	-.1190(.075)	-.0035(.085)[.080]
β	.0042(.022)	.0019(.013)	.0015(.012)[.012]	.0082(.016)[.016]	-.0004(.015)[.014]	-.0006(.012)	-.0008(.012)[.012]
λ	-.1632(.028)	-.0495(.034)	-.0305(.030)[.029]	-.0391(.032)[.039]	-.0321(.031)[.036]	-.0078(.030)	.0003(.030)[.029]
ρ	-.1630(.053)	-.1552(.086)	-.0027(.045)[.046]	-.0814(.032)[.049]	-.0619(.034)[.046]	-.0038(.048)	-.0038(.044)[.045]
σ_v^2	.0320(.112)	-.0966(.054)	.0076(.059)[.059]	.3934(.074)[.091]	.1506(.064)[.064]	-.1181(.052)	-.0025(.059)[.058]
			$n = 400$; error = 1, 2, 3, for the three panels below				
β	.0029(.015)	.0011(.009)	-.0005(.009)[.009]	.0120(.011)[.012]	-.0008(.011)[.011]	.0000(.009)	-.0001(.009)[.009]
λ	-.1435(.027)	-.0439(.031)	-.0487(.022)[.022]	-.0490(.030)[.034]	-.0408(.028)[.031]	-.0081(.024)	-.0022(.024)[.024]
ρ	-.1680(.044)	-.1482(.074)	.0011(.033)[.032]	-.0951(.025)[.035]	-.0753(.029)[.034]	.0111(.035)	-.0006(.032)[.031]
σ_v^2	.0315(.043)	-.0938(.024)	.0067(.025)[.025]	.5621(.044)[.073]	.2676(.033)[.044]	-.1152(.023)	-.0010(.025)[.025]
β	.0027(.014)	.0012(.009)	-.0008(.009)[.009]	.0105(.010)[.013]	-.0014(.009)[.011]	-.0004(.009)	-.0005(.009)[.009]
λ	-.1423(.025)	-.0468(.033)	-.0472(.023)[.022]	-.0493(.027)[.034]	-.0378(.026)[.031]	-.0064(.025)	-.0005(.025)[.024]
ρ	-.1734(.044)	-.1415(.073)	.0009(.032)[.032]	-.0880(.025)[.035]	-.0694(.025)[.033]	.0114(.034)	-.0004(.031)[.031]
σ_v^2	.0353(.099)	-.0854(.054)	.0096(.058)[.058]	.5558(.056)[.087]	.2608(.048)[.064]	-.1128(.051)	.0017(.058)[.057]
β	.0019(.014)	.0011(.009)	-.0005(.009)[.009]	.0117(.011)[.012]	-.0005(.010)[.011]	-.0001(.009)	-.0002(.009)[.009]
λ	-.1411(.028)	-.0420(.030)	-.0486(.023)[.022]	-.0468(.025)[.034]	-.0346(.026)[.031]	-.0080(.025)	-.0021(.025)[.024]
ρ	-.1657(.052)	-.1458(.074)	.0016(.032)[.032]	-.0923(.021)[.035]	-.0741(.024)[.033]	.0120(.034)	.0002(.031)[.031]
σ_v^2	.0204(.066)	-.0963(.039)	.0078(.043)[.042]	.5591(.056)[.079]	.2635(.044)[.052]	-.1143(.038)	.0001(.043)[.041]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 3a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP1, 30% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, W = Queen and M = Rook, and $\boldsymbol{T = 5}$.

	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			$n = 50$; error = 1, 2, 3, for the three panels below			
β	-.0042(.049)	.0011(.046)[.047]	.0406(.061)[.070]	-.0075(.056)[.058]	-.0042(.046)	-.0023(.047)[.046]
λ	-.1160(.072)	-.0973(.064)[.064]	-.1004(.062)[.105]	-.0927(.061)[.089]	-.0206(.078)	-.0018(.079)[.072]
ρ	-.1685(.385)	-.0138(.170)[.171]	-.1579(.065)[.128]	-.1452(.078)[.125]	-.0476(.232)	-.0113(.162)[.166]
σ_v^2	.0204(.145)	.0239(.132)[.130]	1.3740(.245)[.419]	.4946(.172)[.203]	-.0419(.130)	-.0399(.127)[.126]
β	.0051(.047)	.0054(.045)[.047]	.0470(.056)[.070]	-.0013(.056)[.058]	.0006(.045)	.0011(.044)[.046]
λ	-.1223(.060)	-.1027(.065)[.063]	-.1094(.061)[.105]	-.1004(.060)[.089]	-.0283(.074)	-.0065(.076)[.072]
ρ	-.2540(.411)	-.0022(.159)[.171]	-.1627(.063)[.129]	-.1491(.069)[.123]	-.0223(.223)	.0038(.158)[.170]
σ_v^2	.0206(.276)	.0341(.280)[.250]	1.3826(.361)[.454]	.5023(.271)[.247]	-.0302(.275)	-.0334(.261)[.240]
β	-.0097(.050)	.0040(.047)[.047]	.0450(.059)[.070]	-.0020(.061)[.058]	-.0011(.047)	-.0021(.046)[.046]
λ	-.1309(.079)	-.0940(.064)[.064]	-.1078(.063)[.104]	-.1010(.062)[.089]	-.0185(.073)	.0013(.080)[.073]
ρ	-.0566(.347)	-.0122(.161)[.173]	-.1561(.065)[.129]	-.1396(.070)[.123]	-.0382(.228)	.0008(.157)[.165]
σ_v^2	.0138(.192)	.0292(.200)[.191]	1.3672(.294)[.429]	.4873(.210)[.220]	-.0362(.195)	-.0300(.196)[.185]
			$n = 100$; error = 1, 2, 3, for the three panels below			
β	-.0056(.035)	-.0039(.037)[.037]	.0489(.045)[.057]	-.0249(.047)[.048]	-.0022(.037)	-.0012(.035)[.036]
λ	-.1203(.046)	-.1010(.045)[.044]	-.1180(.048)[.084]	-.1059(.050)[.074]	-.0077(.049)	-.0022(.048)[.049]
ρ	-.0839(.297)	-.0068(.121)[.128]	-.1666(.045)[.091]	-.1561(.054)[.087]	.0420(.176)	-.0052(.127)[.121]
σ_v^2	.0485(.099)	.0511(.102)[.098]	1.8662(.224)[.373]	.8129(.148)[.183]	-.0367(.103)	-.0168(.090)[.095]
β	-.0015(.032)	-.0029(.034)[.037]	.0476(.044)[.057]	-.0273(.046)[.049]	-.0013(.034)	.0003(.037)[.036]
λ	-.1210(.043)	-.1058(.043)[.044]	-.1192(.047)[.084]	-.1080(.050)[.074]	-.0099(.047)	.0024(.051)[.049]
ρ	-.1069(.291)	-.0120(.124)[.127]	-.1643(.043)[.091]	-.1559(.054)[.087]	.0345(.192)	.0007(.122)[.124]
σ_v^2	.0488(.177)	.0476(.192)[.188]	1.8632(.274)[.392]	.8016(.198)[.212]	-.0420(.187)	-.0187(.190)[.183]
β	-.0038(.038)	-.0009(.035)[.037]	.0476(.044)[.057]	-.0267(.045)[.048]	.0007(.035)	.0003(.036)[.036]
λ	-.1195(.049)	-.1016(.044)[.043]	-.1131(.046)[.085]	-.1038(.050)[.073]	-.0056(.049)	-.0012(.052)[.049]
ρ	-.1028(.294)	-.0150(.124)[.127]	-.1675(.045)[.091]	-.1588(.055)[.087]	.0306(.185)	.0016(.117)[.121]
σ_v^2	.0344(.156)	.0406(.146)[.137]	1.8744(.251)[.386]	.8160(.167)[.195]	-.0481(.143)	-.0219(.143)[.138]
			$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0016(.023)	.0027(.023)[.024]	.0534(.031)[.038]	-.0098(.032)[.032]	-.0014(.023)	-.0008(.023)[.024]
λ	-.0966(.034)	-.0799(.028)[.031]	-.1141(.036)[.059]	-.1045(.035)[.051]	-.0047(.029)	-.0002(.032)[.032]
ρ	-.0258(.198)	-.0077(.084)[.083]	-.1615(.033)[.064]	-.1534(.037)[.061]	.0646(.114)	-.0020(.077)[.080]
σ_v^2	.0670(.065)	.0458(.066)[.067]	1.6491(.144)[.242]	.6808(.093)[.117]	-.0333(.067)	-.0122(.065)[.065]
β	.0043(.023)	.0056(.024)[.024]	.0523(.029)[.038]	-.0069(.031)[.031]	.0014(.025)	.0007(.023)[.024]
λ	-.0951(.035)	-.0801(.030)[.031]	-.1135(.034)[.059]	-.1037(.033)[.051]	-.0038(.033)	-.0002(.033)[.032]
ρ	-.0326(.194)	-.0154(.082)[.083]	-.1622(.034)[.064]	-.1487(.037)[.061]	.0558(.115)	-.0017(.080)[.081]
σ_v^2	.0602(.139)	.0564(.147)[.134]	1.6551(.194)[.259]	.6884(.134)[.142]	-.0233(.141)	-.0105(.132)[.132]
β	.0033(.025)	.0030(.025)[.024]	.0542(.032)[.038]	-.0082(.032)[.032]	-.0014(.025)	-.0018(.023)[.024]
λ	-.0955(.035)	-.0801(.030)[.031]	-.1134(.033)[.059]	-.1027(.035)[.051]	-.0043(.032)	.0023(.033)[.032]
ρ	-.0322(.190)	-.0060(.082)[.083]	-.1614(.031)[.063]	-.1505(.038)[.061]	.0694(.110)	-.0031(.078)[.081]
σ_v^2	.0583(.101)	.0504(.100)[.100]	1.6624(.165)[.251]	.6878(.114)[.128]	-.0307(.101)	-.0080(.104)[.098]
			$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0023(.015)	.0029(.017)[.017]	.0553(.019)[.025]	.0003(.020)[.020]	.0007(.017)	.0007(.015)[.016]
λ	-.0958(.026)	-.0796(.022)[.022]	-.1072(.023)[.040]	-.0963(.024)[.033]	-.0034(.024)	-.0004(.024)[.024]
ρ	-.0112(.118)	-.0108(.057)[.057]	-.1530(.022)[.045]	-.1370(.026)[.042]	.0857(.076)	-.0001(.054)[.055]
σ_v^2	.0614(.049)	.0553(.047)[.047]	1.2344(.084)[.136]	.3101(.055)[.061]	-.0264(.049)	-.0055(.049)[.046]
β	.0040(.018)	.0026(.016)[.017]	.0552(.020)[.025]	.0005(.021)[.020]	.0006(.016)	-.0001(.016)[.016]
λ	-.0925(.025)	-.0798(.022)[.022]	-.1085(.025)[.040]	-.0949(.024)[.034]	-.0028(.024)	-.0023(.025)[.024]
ρ	-.0363(.124)	-.0074(.054)[.057]	-.1530(.023)[.044]	-.1359(.028)[.043]	.0908(.072)	-.0011(.056)[.055]
σ_v^2	.0748(.094)	.0563(.092)[.096]	1.2368(.122)[.154]	.3187(.088)[.084]	-.0269(.090)	-.0109(.098)[.092]
β	.0036(.016)	.0014(.016)[.017]	.0561(.020)[.025]	.0008(.019)[.020]	-.0006(.016)	-.0005(.016)[.016]
λ	-.0919(.026)	-.0791(.022)[.022]	-.1049(.023)[.040]	-.0923(.024)[.033]	-.0008(.023)	.0006(.025)[.024]
ρ	-.0265(.129)	-.0111(.057)[.057]	-.1530(.022)[.045]	-.1366(.028)[.043]	.0857(.076)	.0030(.054)[.055]
σ_v^2	.0647(.081)	.0542(.073)[.071]	1.2272(.102)[.143]	.3061(.070)[.070]	-.0290(.073)	-.0052(.073)[.069]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 3b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP1, 30% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, W = Queen and M = Rook, and $\boldsymbol{T} = \mathbf{10}$.

	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			$n = 50$; error = 1, 2, 3, for the three panels below			
β	-.0053(.042)	.0033(.027)[.028]	.0263(.038)[.046]	.0097(.034)[.036]	.0019(.027)	-.0006(.027)[.027]
λ	-.1818(.099)	-.0699(.042)[.041]	-.1040(.043)[.074]	-.0891(.042)[.062]	-.0178(.046)	-.0010(.043)[.044]
ρ	-.0020(.663)	-.0002(.101)[.104]	-.1558(.042)[.085]	-.1354(.047)[.083]	-.0502(.110)	-.0016(.098)[.100]
σ_v^2	.0360(.082)	.0375(.083)[.086]	1.1186(.151)[.249]	.3294(.104)[.120]	-.0059(.080)	-.0149(.080)[.084]
β	-.0086(.044)	.0026(.029)[.028]	.0216(.039)[.046]	.0071(.036)[.036]	.0012(.029)	.0004(.027)[.027]
λ	-.1792(.096)	-.0680(.043)[.041]	-.1074(.046)[.074]	-.0921(.045)[.062]	-.0140(.043)	.0005(.043)[.044]
ρ	.0349(.673)	.0031(.100)[.103]	-.1512(.045)[.084]	-.1285(.048)[.081]	-.0424(.114)	.0107(.102)[.101]
σ_v^2	-.0035(.184)	.0408(.196)[.180]	1.1325(.225)[.282]	.3433(.185)[.165]	-.0056(.193)	-.0139(.178)[.176]
β	-.0109(.041)	.0020(.027)[.028]	.0255(.039)[.047]	.0132(.034)[.036]	.0003(.027)	-.0028(.028)[.027]
λ	-.1885(.104)	-.0681(.043)[.041]	-.1050(.045)[.073]	-.0884(.045)[.062]	-.0145(.044)	.0016(.045)[.044]
ρ	.0496(.700)	-.0071(.093)[.104]	-.1503(.044)[.085]	-.1290(.045)[.082]	-.0587(.105)	.0010(.092)[.101]
σ_v^2	.0216(.135)	.0426(.134)[.134]	1.1335(.197)[.260]	.3395(.150)[.143]	-.0010(.131)	-.0086(.144)[.132]
			$n = 100$; error = 1, 2, 3, for the three panels below			
β	-.0059(.022)	-.0011(.021)[.021]	.0314(.028)[.035]	-.0014(.027)[.029]	.0009(.021)	.0002(.021)[.021]
λ	-.0924(.036)	-.0705(.031)[.031]	-.1032(.031)[.052]	-.0913(.030)[.046]	-.0087(.032)	.0010(.034)[.033]
ρ	-.0943(.117)	-.0012(.068)[.067]	-.1594(.031)[.061]	-.1458(.035)[.058]	-.0079(.077)	-.0041(.064)[.065]
σ_v^2	.0640(.066)	.0346(.060)[.059]	1.5778(.139)[.225]	.7076(.096)[.120]	-.0111(.058)	-.0059(.058)[.058]
β	.0008(.020)	-.0038(.021)[.021]	.0312(.028)[.035]	-.0014(.027)[.029]	-.0018(.021)	-.0010(.022)[.021]
λ	-.0908(.034)	-.0731(.033)[.032]	-.1061(.030)[.052]	-.0927(.028)[.045]	-.0101(.034)	-.0029(.032)[.033]
ρ	-.1076(.115)	.0022(.064)[.067]	-.1627(.031)[.060]	-.1472(.034)[.059]	-.0043(.071)	.0005(.063)[.065]
σ_v^2	.0557(.141)	.0468(.136)[.128]	1.5778(.184)[.245]	.7050(.145)[.146]	.0004(.132)	-.0144(.129)[.125]
β	-.0035(.022)	-.0023(.021)[.021]	.0313(.028)[.035]	-.0010(.026)[.029]	-.0003(.021)	.0005(.021)[.021]
λ	-.0854(.038)	-.0696(.031)[.031]	-.1064(.032)[.052]	-.0941(.029)[.046]	-.0111(.031)	-.0020(.033)[.033]
ρ	-.1050(.127)	-.0048(.068)[.067]	-.1599(.031)[.061]	-.1458(.036)[.058]	-.0122(.075)	-.0041(.065)[.065]
σ_v^2	.0504(.098)	.0374(.099)[.094]	1.5781(.160)[.236]	.7112(.116)[.132]	-.0054(.098)	-.0058(.098)[.091]
			$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0020(.015)	.0023(.014)[.015]	.0245(.019)[.022]	-.0004(.018)[.017]	.0000(.014)	.0003(.015)[.014]
λ	-.0847(.027)	-.0653(.024)[.024]	-.0965(.021)[.035]	-.0826(.020)[.029]	-.0057(.024)	-.0014(.023)[.023]
ρ	-.0825(.090)	-.0069(.050)[.049]	-.1495(.022)[.041]	-.1262(.025)[.040]	.0148(.055)	-.0023(.046)[.047]
σ_v^2	.0709(.045)	.0480(.045)[.043]	.9921(.073)[.114]	.2403(.047)[.055]	-.0058(.043)	-.0055(.042)[.042]
β	-.0002(.015)	.0024(.014)[.015]	.0247(.018)[.022]	-.0009(.016)[.017]	.0001(.014)	.0004(.014)[.014]
λ	-.0865(.029)	-.0649(.024)[.024]	-.0957(.021)[.035]	-.0821(.019)[.029]	-.0049(.022)	.0004(.023)[.023]
ρ	-.0648(.086)	-.0065(.048)[.049]	-.1489(.021)[.042]	-.1248(.024)[.040]	.0148(.054)	.0016(.049)[.048]
σ_v^2	.0668(.101)	.0511(.096)[.094]	.9980(.115)[.132]	.2441(.097)[.079]	-.0030(.094)	.0031(.087)[.093]
β	-.0008(.015)	.0031(.014)[.015]	.0239(.019)[.022]	-.0010(.017)[.017]	.0007(.014)	.0001(.014)[.014]
λ	-.0835(.027)	-.0624(.025)[.024]	-.0973(.020)[.035]	-.0827(.021)[.029]	-.0020(.022)	-.0013(.023)[.023]
ρ	-.0857(.087)	-.0045(.050)[.049]	-.1497(.022)[.042]	-.1256(.024)[.041]	.0187(.056)	.0025(.046)[.047]
σ_v^2	.0635(.071)	.0497(.070)[.069]	.9871(.093)[.122]	.2396(.072)[.066]	-.0062(.068)	-.0007(.068)[.067]
			$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0011(.011)	.0030(.010)[.011]	.0272(.014)[.017]	-.0023(.012)[.013]	-.0003(.010)	-.0004(.011)[.010]
λ	-.1021(.016)	-.0828(.016)[.015]	-.1024(.015)[.026]	-.0887(.015)[.023]	-.0023(.017)	-.0009(.016)[.017]
ρ	-.0365(.058)	-.0045(.034)[.034]	-.1530(.016)[.030]	-.1359(.020)[.029]	.0243(.038)	.0022(.032)[.034]
σ_v^2	.0677(.030)	.0458(.030)[.030]	1.2340(.062)[.092]	.4429(.039)[.048]	-.0078(.029)	-.0014(.031)[.030]
β	.0015(.010)	.0031(.011)[.011]	.0272(.013)[.017]	-.0015(.011)[.013]	-.0002(.011)	.0001(.011)[.010]
λ	-.1034(.016)	-.0830(.016)[.015]	-.1046(.015)[.026]	-.0902(.014)[.023]	-.0030(.017)	-.0002(.017)[.017]
ρ	-.0328(.058)	-.0052(.033)[.034]	-.1520(.015)[.030]	-.1351(.018)[.029]	.0237(.038)	.0004(.032)[.033]
σ_v^2	.0576(.073)	.0442(.068)[.066]	1.2331(.087)[.104]	.4376(.077)[.062]	-.0091(.066)	-.0052(.064)[.065]
β	-.0003(.010)	.0030(.011)[.011]	.0259(.013)[.017]	-.0028(.011)[.013]	-.0004(.011)	-.0005(.011)[.010]
λ	-.1020(.016)	-.0830(.015)[.015]	-.1070(.016)[.026]	-.0920(.014)[.022]	-.0019(.016)	-.0010(.016)[.017]
ρ	-.0340(.055)	-.0047(.035)[.034]	-.1522(.016)[.030]	-.1332(.019)[.028]	.0238(.039)	-.0023(.031)[.034]
σ_v^2	.0655(.046)	.0456(.048)[.048]	1.2367(.073)[.098]	.4410(.054)[.055]	-.0083(.047)	-.0004(.047)[.048]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 4a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP1, 30% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, W = Group and M = Queen, and $\boldsymbol{T = 5}$.

	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			$n = 50$; error = 1, 2, 3, for the three panels below			
β	-.0086(.057)	-.0076(.053)[.051]	.0364(.049)[.063]	-.0069(.053)[.048]	-.0025(.054)	-.0044(.054)[.050]
λ	-.1002(.067)	-.0776(.059)[.059]	-.0909(.060)[.097]	-.0872(.060)[.081]	-.0187(.071)	-.0039(.067)[.068]
ρ	-.1962(.327)	-.0222(.218)[.230]	-.1435(.088)[.175]	-.1256(.099)[.167]	-.1845(.362)	-.0244(.208)[.213]
σ_v^2	-.3033(.092)	.0079(.130)[.128]	.9413(.234)[.331]	.1254(.156)[.150]	-.3414(.103)	-.0339(.127)[.125]
β	-.0048(.048)	-.0024(.050)[.051]	.0392(.055)[.064]	-.0039(.056)[.050]	.0023(.049)	-.0021(.048)[.050]
λ	-.0937(.065)	-.0812(.059)[.059]	-.1044(.058)[.097]	-.0961(.060)[.080]	-.0204(.070)	-.0088(.069)[.068]
ρ	-.2217(.291)	-.0157(.217)[.225]	-.1535(.088)[.174]	-.1309(.113)[.165]	-.1674(.359)	-.0238(.210)[.214]
σ_v^2	-.3260(.182)	.0161(.268)[.243]	.9511(.348)[.363]	.1248(.278)[.204]	-.3364(.203)	-.0201(.275)[.247]
β	-.0016(.051)	-.0032(.051)[.051]	.0439(.044)[.063]	-.0021(.045)[.050]	.0016(.052)	.0001(.051)[.050]
λ	-.1025(.066)	-.0818(.058)[.059]	-.1104(.064)[.100]	-.0922(.062)[.080]	-.0263(.068)	-.0120(.068)[.069]
ρ	-.1950(.295)	-.0103(.223)[.224]	-.1390(.088)[.170]	-.1275(.096)[.165]	-.1703(.369)	-.0286(.210)[.210]
σ_v^2	-.3154(.133)	-.0007(.206)[.185]	.9932(.277)[.341]	.1694(.188)[.181]	-.3458(.154)	-.0326(.194)[.182]
			$n = 100$; error = 1, 2, 3, for the three panels below			
β	-.0003(.035)	.0019(.035)[.035]	.0469(.038)[.047]	.0034(.042)[.036]	-.0002(.035)	-.0003(.034)[.035]
λ	-.1276(.065)	-.0994(.055)[.055]	-.1068(.056)[.093]	-.0934(.053)[.077]	-.0182(.064)	-.0087(.064)[.061]
ρ	-.1104(.267)	-.0098(.158)[.161]	-.1451(.070)[.122]	-.1233(.073)[.116]	-.0515(.246)	-.0099(.145)[.145]
σ_v^2	-.2830(.067)	.0227(.098)[.093]	.9959(.153)[.247]	.1293(.103)[.104]	-.3128(.073)	-.0202(.092)[.091]
β	-.0013(.034)	.0015(.034)[.035]	.0356(.040)[.047]	-.0048(.038)[.037]	-.0003(.034)	-.0008(.036)[.035]
λ	-.1204(.065)	-.1003(.055)[.055]	-.1111(.059)[.092]	-.0947(.054)[.076]	-.0161(.064)	-.0040(.064)[.061]
ρ	-.0937(.281)	-.0011(.163)[.161]	-.1412(.054)[.119]	-.1276(.075)[.114]	-.0393(.259)	-.0170(.144)[.147]
σ_v^2	-.2918(.130)	.0153(.184)[.182]	1.0237(.249)[.272]	.1518(.190)[.149]	-.3205(.140)	-.0087(.200)[.182]
β	-.0011(.036)	-.0013(.035)[.035]	.0325(.045)[.047]	-.0111(.039)[.036]	-.0033(.036)	-.0017(.035)[.035]
λ	-.1263(.063)	-.0989(.057)[.055]	-.1198(.060)[.095]	-.0987(.056)[.075]	-.0140(.066)	-.0049(.061)[.062]
ρ	-.1022(.312)	-.0224(.157)[.162]	-.1463(.058)[.120]	-.1157(.065)[.113]	-.0683(.246)	-.0082(.144)[.148]
σ_v^2	-.2874(.097)	.0161(.138)[.133]	.9718(.136)[.250]	.1127(.113)[.121]	-.3183(.102)	-.0174(.141)[.136]
			$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0018(.023)	.0054(.022)[.023]	.0599(.033)[.042]	-.0119(.039)[.036]	.0002(.022)	-.0010(.023)[.024]
λ	-.1209(.061)	-.0855(.049)[.047]	-.1124(.048)[.081]	-.1046(.050)[.072]	-.0146(.047)	-.0051(.057)[.058]
ρ	-.0409(.211)	-.0022(.105)[.103]	-.1658(.049)[.086]	-.1682(.055)[.083]	.0265(.141)	-.0010(.107)[.099]
σ_v^2	-.2705(.045)	.0127(.064)[.064]	2.1946(.170)[.302]	1.1872(.117)[.161]	-.3077(.049)	-.0102(.064)[.065]
β	.0021(.022)	.0056(.023)[.023]	.0570(.030)[.043]	-.0122(.031)[.037]	.0003(.023)	-.0007(.025)[.024]
λ	-.1160(.057)	-.0835(.047)[.047]	-.1144(.044)[.082]	-.1060(.044)[.071]	-.0119(.045)	-.0043(.059)[.057]
ρ	-.0394(.191)	-.0001(.104)[.103]	-.1625(.040)[.087]	-.1541(.045)[.083]	.0319(.137)	-.0097(.099)[.101]
σ_v^2	-.2712(.097)	.0229(.130)[.131]	2.2063(.207)[.318]	1.2221(.153)[.182]	-.3011(.100)	-.0196(.136)[.130]
β	.0015(.023)	.0047(.024)[.023]	.0538(.031)[.041]	-.0223(.037)[.036]	-.0008(.024)	-.0003(.023)[.024]
λ	-.1153(.058)	-.0802(.052)[.047]	-.1197(.045)[.083]	-.1060(.051)[.074]	-.0100(.050)	-.0077(.063)[.058]
ρ	-.0416(.183)	-.0044(.103)[.104]	-.1679(.034)[.084]	-.1683(.047)[.082]	.0261(.136)	-.0159(.096)[.102]
σ_v^2	-.2686(.074)	.0191(.095)[.097]	2.1858(.184)[.307]	1.1972(.121)[.169]	-.3035(.072)	-.0121(.098)[.097]
			$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0009(.016)	-.0014(.016)[.016]	.0494(.021)[.027]	-.0080(.021)[.022]	.0001(.016)	-.0003(.016)[.016]
λ	-.1343(.056)	-.0978(.039)[.037]	-.1114(.040)[.068]	-.0990(.045)[.058]	-.0087(.046)	-.0050(.044)[.042]
ρ	.0180(.162)	.0035(.073)[.072]	-.1577(.032)[.061]	-.1460(.035)[.058]	.0631(.091)	-.0009(.064)[.067]
σ_v^2	-.2679(.034)	.0127(.047)[.045]	1.6372(.096)[.171]	.6356(.059)[.081]	-.2990(.036)	-.0037(.044)[.045]
β	-.0008(.016)	-.0022(.016)[.016]	.0548(.018)[.027]	-.0033(.019)[.022]	-.0007(.016)	.0001(.016)[.016]
λ	-.1399(.052)	-.0975(.039)[.037]	-.1153(.045)[.068]	-.1032(.043)[.058]	-.0100(.049)	-.0020(.041)[.042]
ρ	.0118(.166)	.0049(.073)[.072]	-.1570(.030)[.061]	-.1452(.037)[.057]	.0649(.092)	-.0030(.065)[.067]
σ_v^2	-.2699(.073)	.0162(.093)[.095]	1.6400(.128)[.184]	.6437(.096)[.099]	-.2967(.071)	-.0096(.087)[.092]
β	-.0006(.017)	-.0007(.016)[.016]	.0549(.020)[.027]	-.0035(.019)[.023]	.0008(.016)	-.0012(.016)[.016]
λ	-.1400(.053)	-.0984(.040)[.037]	-.1074(.038)[.070]	-.1012(.043)[.059]	-.0100(.048)	-.0061(.044)[.042]
ρ	.0299(.158)	.0051(.071)[.072]	-.1602(.030)[.061]	-.1463(.035)[.057]	.0673(.088)	-.0045(.066)[.067]
σ_v^2	-.2726(.050)	.0129(.068)[.069]	1.6495(.114)[.175]	.6497(.083)[.087]	-.2992(.052)	-.0063(.070)[.069]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 4b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under `DGP1`, 30% MCAR missing, $(\beta, \lambda, \rho, \sigma_v^2) = (1, 0.2, 0.2, 1)$, `W = Group` and `M = Queen`, and $\boldsymbol{T} = \mathbf{10}$.

	`QMLE-GU`	`ME-GU`	`Impu-I`	`Impu-II`	`QMLE-MR`	`ME-MR`
			$n = 50$; error = 1, 2, 3, for the three panels below			
β	.0035(.030)	.0034(.030)[.030]	.0342(.039)[.044]	.0064(.039)[.036]	-.0009(.030)	-.0005(.030)[.029]
λ	-.1010(.059)	-.0685(.054)[.053]	-.1012(.043)[.069]	-.0846(.039)[.056]	-.0225(.052)	-.0054(.054)[.052]
ρ	-.1422(.173)	-.0036(.129)[.139]	-.1568(.057)[.114]	-.1420(.069)[.111]	-.1196(.151)	-.0020(.115)[.115]
σ_v^2	-.1443(.068)	.0305(.084)[.087]	.9203(.130)[.218]	.2553(.101)[.110]	-.1746(.071)	-.0128(.077)[.080]
β	.0054(.031)	.0071(.030)[.030]	.0269(.039)[.046]	.0010(.037)[.037]	.0028(.030)	-.0014(.030)[.029]
λ	-.0913(.059)	-.0622(.053)[.053]	-.0898(.044)[.065]	-.0814(.043)[.057]	-.0175(.047)	-.0062(.053)[.052]
ρ	-.1393(.178)	.0005(.127)[.139]	-.1497(.057)[.113]	-.1192(.065)[.107]	-.1149(.144)	-.0191(.111)[.118]
σ_v^2	-.1430(.168)	.0322(.187)[.180]	.9592(.224)[.255]	.2943(.188)[.163]	-.1728(.161)	-.0103(.189)[.173]
β	.0032(.030)	.0033(.032)[.030]	.0298(.034)[.044]	.0095(.032)[.036]	-.0008(.032)	-.0014(.029)[.029]
λ	-.0939(.059)	-.0652(.051)[.053]	-.1021(.040)[.068]	-.0891(.037)[.058]	-.0185(.049)	-.0030(.054)[.051]
ρ	-.1422(.174)	-.0026(.144)[.138]	-.1553(.061)[.115]	-.1369(.070)[.107]	-.1194(.159)	-.0070(.117)[.117]
σ_v^2	-.1470(.112)	.0338(.140)[.134]	.9481(.200)[.244]	.2812(.151)[.138]	-.1735(.119)	-.0158(.137)[.126]
			$n = 100$; error = 1, 2, 3, for the three panels below			
β	-.0005(.022)	.0045(.020)[.021]	.0469(.030)[.037]	.0153(.027)[.033]	.0014(.021)	-.0006(.021)[.021]
λ	-.1031(.044)	-.0615(.035)[.039]	-.1196(.039)[.064]	-.1068(.040)[.059]	-.0117(.036)	-.0020(.039)[.039]
ρ	-.1068(.143)	.0015(.086)[.090]	-.1629(.041)[.083]	-.1515(.044)[.080]	-.0408(.095)	-.0145(.089)[.087]
σ_v^2	-.1333(.055)	.0272(.058)[.060]	1.9020(.158)[.261]	1.0469(.090)[.154]	-.1576(.050)	-.0046(.063)[.059]
β	-.0025(.021)	.0045(.020)[.021]	.0465(.029)[.039]	.0128(.030)[.032]	.0014(.020)	-.0004(.021)[.021]
λ	-.1111(.046)	-.0632(.040)[.039]	-.1214(.039)[.066]	-.1085(.037)[.061]	-.0156(.039)	-.0042(.039)[.039]
ρ	-.0719(.129)	-.0052(.093)[.091]	-.1668(.042)[.082]	-.1555(.047)[.076]	-.0475(.103)	-.0057(.088)[.086]
σ_v^2	-.1401(.107)	.0381(.133)[.132]	1.8880(.193)[.283]	1.0435(.171)[.172]	-.1473(.115)	-.0137(.125)[.128]
β	-.0035(.021)	.0015(.022)[.021]	.0470(.032)[.038]	.0163(.030)[.032]	-.0016(.022)	.0016(.020)[.021]
λ	-.1068(.045)	-.0638(.041)[.039]	-.1097(.041)[.065]	-.0980(.039)[.057]	-.0159(.039)	-.0018(.041)[.039]
ρ	-.0808(.140)	.0060(.090)[.089]	-.1572(.039)[.081]	-.1520(.044)[.080]	-.0350(.098)	-.0058(.082)[.086]
σ_v^2	-.1329(.083)	.0258(.100)[.095]	1.9004(.180)[.272]	1.0184(.134)[.163]	-.1573(.088)	-.0103(.100)[.092]
			$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0003(.014)	.0013(.014)[.014]	.0224(.017)[.023]	-.0052(.018)[.018]	-.0008(.014)	.0007(.014)[.015]
λ	-.1062(.036)	-.0822(.031)[.030]	-.1063(.031)[.053]	-.0931(.032)[.046]	-.0106(.033)	-.0009(.033)[.032]
ρ	-.0511(.101)	.0008(.061)[.062]	-.1503(.028)[.057]	-.1313(.031)[.054]	-.0059(.064)	-.0018(.054)[.057]
σ_v^2	-.1261(.034)	.0205(.043)[.042]	1.1612(.081)[.134]	.4094(.055)[.066]	-.1521(.037)	-.0034(.040)[.041]
β	.0000(.015)	.0022(.015)[.014]	.0215(.017)[.023]	-.0067(.016)[.019]	.0001(.015)	.0001(.015)[.015]
λ	-.1046(.035)	-.0829(.030)[.030]	-.1025(.026)[.053]	-.0904(.027)[.045]	-.0110(.031)	-.0012(.032)[.032]
ρ	-.0566(.094)	.0030(.060)[.062]	-.1476(.029)[.057]	-.1253(.035)[.052]	-.0031(.062)	-.0008(.062)[.057]
σ_v^2	-.1269(.082)	.0236(.093)[.092]	1.1808(.122)[.146]	.4223(.096)[.089]	-.1495(.081)	-.0019(.092)[.091]
β	-.0003(.014)	.0019(.014)[.014]	.0227(.017)[.023]	-.0061(.017)[.019]	-.0002(.014)	-.0002(.014)[.015]
λ	-.1075(.034)	-.0822(.031)[.030]	-.0946(.029)[.053]	-.0810(.031)[.045]	-.0096(.034)	-.0019(.031)[.032]
ρ	-.0552(.102)	.0074(.062)[.061]	-.1533(.029)[.056]	-.1358(.033)[.054]	.0015(.066)	-.0031(.056)[.057]
σ_v^2	-.1225(.060)	.0213(.070)[.067]	1.1715(.100)[.137]	.4186(.080)[.079]	-.1523(.062)	-.0042(.063)[.066]
			$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0000(.011)	-.0005(.010)[.010]	.0283(.014)[.017]	.0012(.012)[.014]	-.0001(.010)	-.0002(.010)[.010]
λ	-.1096(.032)	-.0989(.028)[.024]	-.1020(.028)[.043]	-.0882(.024)[.037]	-.0066(.030)	-.0008(.027)[.027]
ρ	-.0402(.085)	.0033(.044)[.044]	-.1553(.021)[.041]	-.1350(.023)[.038]	.0127(.046)	-.0041(.041)[.040]
σ_v^2	-.1222(.026)	.0189(.029)[.030]	1.2752(.055)[.096]	.5049(.037)[.052]	-.1478(.025)	-.0016(.030)[.029]
β	-.0005(.010)	-.0012(.010)[.010]	.0286(.014)[.017]	.0009(.014)[.014]	-.0008(.010)	-.0003(.010)[.010]
λ	-.1087(.031)	-.0974(.025)[.024]	-.1001(.027)[.042]	-.0878(.025)[.037]	-.0074(.028)	-.0014(.026)[.026]
ρ	-.0368(.087)	.0061(.043)[.044]	-.1534(.022)[.040]	-.1360(.024)[.039]	.0153(.044)	.0015(.041)[.040]
σ_v^2	-.1240(.055)	.0119(.067)[.065]	1.2658(.076)[.108]	.4947(.061)[.064]	-.1535(.059)	-.0027(.067)[.065]
β	.0005(.011)	.0003(.011)[.010]	.0252(.013)[.017]	-.0031(.013)[.014]	.0007(.011)	.0004(.010)[.010]
λ	-.1092(.031)	-.0968(.024)[.024]	-.1005(.026)[.041]	-.0878(.026)[.037]	-.0056(.028)	-.0017(.026)[.026]
ρ	-.0432(.084)	.0023(.045)[.044]	-.1499(.021)[.040]	-.1340(.023)[.038]	.0116(.047)	.0005(.039)[.040]
σ_v^2	-.1215(.042)	.0184(.047)[.048]	1.2731(.070)[.103]	.5038(.054)[.057]	-.1483(.041)	-.0040(.046)[.047]

Note: error = 1(`normal`), 2(`normal mixture`), 3(`chi-square`).

Table 5a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP2, 10% MCAR missing, $(\beta, \lambda, \sigma_v^2) = (1, 0.2, 1)$, W = Group, and $\boldsymbol{T} = \mathbf{5}$.

	NLSE	QMLE-GU	ME-GU	Impu-I	Impu-II	QMLE-MR	ME-MR
			***n* = 50; error = 1, 2, 3, for the three panels below**				
β	.0002(.041)	.0066(.038)	.0067(.041)[.039]	.0192(.040)[.046]	.0015(.037)[.037]	.0022(.041)	.0014(.041)[.039]
λ	-.0043(.069)	-.0303(.071)	-.0215(.063)[.066]	-.0457(.062)[.075]	-.0315(.059)[.063]	-.0227(.058)	-.0088(.057)[.059]
σ_v^2	-.0045(.110)	-.2421(.084)	.0034(.109)[.109]	.4233(.167)[.247]	.0266(.100)[.115]	-.2502(.082)	-.0143(.107)[.107]
β	.0013(.040)	.0093(.038)	.0081(.039)[.039]	.0190(.038)[.046]	-.0003(.034)[.037]	.0035(.039)	.0027(.039)[.038]
λ	-.0003(.073)	-.0307(.071)	-.0203(.071)[.065]	-.0496(.061)[.075]	-.0400(.057)[.064]	-.0196(.065)	-.0060(.064)[.058]
σ_v^2	-.0153(.239)	-.2341(.169)	-.0063(.238)[.217]	.4335(.276)[.307]	.0297(.238)[.205]	-.2590(.180)	-.0259(.236)[.215]
β	-.0024(.042)	.0077(.039)	.0051(.041)[.039]	.0206(.043)[.046]	.0014(.038)[.037]	.0003(.041)	-.0006(.041)[.039]
λ	.0108(.065)	-.0301(.071)	-.0111(.063)[.065]	-.0450(.061)[.075]	-.0331(.058)[.064]	-.0129(.060)	.0009(.059)[.058]
σ_v^2	-.0022(.177)	-.2473(.129)	.0077(.173)[.161]	.4426(.207)[.280]	.0395(.168)[.159]	-.2482(.131)	-.0117(.172)[.159]
			***n* = 100; error = 1, 2, 3, for the three panels below**				
β	.0003(.029)	.0038(.031)	.0011(.028)[.028]	.0121(.029)[.035]	-.0072(.027)[.029]	.0017(.028)	.0012(.028)[.028]
λ	.0005(.055)	-.0442(.049)	-.0405(.048)[.044]	-.0437(.056)[.072]	-.0325(.057)[.063]	-.0131(.049)	-.0047(.049)[.045]
σ_v^2	-.0012(.077)	-.2289(.056)	.0030(.076)[.076]	.4632(.107)[.186]	.1371(.085)[.100]	-.2390(.058)	-.0106(.075)[.075]
β	-.0002(.030)	.0016(.030)	.0007(.030)[.028]	.0155(.031)[.035]	-.0054(.029)[.030]	.0012(.030)	.0006(.030)[.028]
λ	.0000(.050)	-.0361(.051)	-.0413(.043)[.044]	-.0564(.059)[.072]	-.0431(.059)[.064]	-.0134(.044)	-.0050(.043)[.046]
σ_v^2	.0068(.167)	-.2283(.123)	.0114(.165)[.163]	.4755(.187)[.230]	.1431(.157)[.159]	-.2326(.126)	-.0024(.164)[.162]
β	-.0001(.028)	.0025(.029)	.0009(.027)[.028]	.0164(.031)[.035]	-.0051(.031)[.030]	.0015(.027)	.0010(.027)[.028]
λ	.0018(.053)	-.0382(.048)	-.0376(.045)[.044]	-.0499(.060)[.072]	-.0379(.061)[.064]	-.0115(.047)	-.0030(.047)[.045]
σ_v^2	.0021(.127)	-.2243(.093)	.0062(.127)[.118]	.4728(.148)[.206]	.1419(.123)[.128]	-.2362(.097)	-.0071(.126)[.117]
			***n* = 200; error = 1, 2, 3, for the three panels below**				
β	.0004(.019)	-.0033(.019)	-.0017(.019)[.019]	.0212(.022)[.027]	-.0045(.022)[.023]	.0009(.019)	.0008(.019)[.019]
λ	.0037(.048)	-.0458(.045)	-.0478(.039)[.037]	-.0621(.051)[.068]	-.0496(.051)[.061]	-.0106(.043)	-.0031(.043)[.042]
σ_v^2	.0033(.057)	-.2226(.042)	.0032(.056)[.053]	.6125(.096)[.159]	.2586(.064)[.087]	-.2296(.043)	-.0041(.056)[.053]
β	-.0015(.019)	-.0032(.018)	-.0036(.019)[.019]	.0224(.024)[.028]	-.0036(.024)[.024]	-.0011(.019)	-.0011(.019)[.019]
λ	.0037(.052)	-.0439(.046)	-.0457(.039)[.037]	-.0586(.048)[.068]	-.0472(.049)[.061]	-.0087(.044)	-.0012(.043)[.042]
σ_v^2	.0055(.116)	-.2214(.091)	.0051(.115)[.115]	.6227(.139)[.187]	.2628(.120)[.126]	-.2281(.089)	-.0022(.115)[.114]
β	-.0003(.018)	-.0043(.019)	-.0024(.019)[.019]	.0225(.024)[.027]	-.0042(.023)[.023]	.0001(.019)	.0001(.019)[.019]
λ	.0016(.050)	-.0440(.042)	-.0495(.040)[.037]	-.0569(.049)[.068]	-.0439(.050)[.061]	-.0125(.044)	-.0050(.044)[.042]
σ_v^2	.0023(.085)	-.2219(.071)	.0022(.084)[.083]	.6153(.121)[.172]	.2600(.088)[.104]	-.2303(.065)	-.0049(.084)[.083]
			***n* = 400; error = 1, 2, 3, for the three panels below**				
β	-.0002(.014)	.0001(.014)	-.0004(.014)[.013]	.0186(.016)[.019]	-.0020(.016)[.017]	.0001(.014)	.0000(.014)[.013]
λ	.0031(.042)	-.0587(.038)	-.0622(.032)[.033]	-.0557(.042)[.054]	-.0481(.040)[.049]	-.0080(.034)	-.0025(.034)[.037]
σ_v^2	.0060(.038)	-.2170(.030)	.0084(.038)[.038]	.7258(.075)[.130]	.4354(.051)[.080]	-.2242(.029)	.0002(.038)[.038]
β	-.0004(.013)	-.0006(.014)	-.0006(.013)[.013]	.0174(.015)[.019]	-.0038(.015)[.017]	-.0001(.013)	-.0001(.013)[.013]
λ	.0053(.043)	-.0621(.038)	-.0602(.034)[.033]	-.0550(.038)[.054]	-.0479(.039)[.049]	-.0086(.037)	-.0032(.037)[.036]
σ_v^2	-.0053(.079)	-.2244(.071)	-.0029(.078)[.081]	.7281(.098)[.145]	.4368(.080)[.103]	-.2325(.061)	-.0106(.078)[.081]
β	-.0004(.013)	-.0001(.012)	-.0007(.013)[.013]	.0182(.016)[.019]	-.0032(.016)[.017]	-.0002(.013)	-.0002(.013)[.013]
λ	.0026(.042)	-.0547(.035)	-.0609(.033)[.033]	-.0549(.039)[.054]	-.0466(.040)[.049]	-.0088(.036)	-.0033(.035)[.037]
σ_v^2	.0053(.063)	-.2182(.044)	.0073(.062)[.060]	.7350(.091)[.136]	.4424(.068)[.090]	-.2247(.048)	-.0004(.062)[.060]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 5b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under `DGP2`, 10% MCAR missing, $(\beta, \lambda, \sigma_v^2) = (1, 0.2, 1)$, `W = Group`, and $\boldsymbol{T} = \mathbf{10}$.

	`NLSE`	`QMLE-GU`	`ME-GU`	`Impu-I`	`Impu-II`	`QMLE-MR`	`ME-MR`
				$n = 50$; error = 1, 2, 3, for the three panels below			
β	-.0013(.024)	.0071(.027)	.0048(.024)[.025]	.0122(.031)[.038]	.0080(.032)[.036]	.0004(.024)	-.0006(.024)[.025]
λ	-.0020(.049)	-.0510(.049)	-.0459(.041)[.041]	-.0545(.043)[.058]	-.0502(.043)[.054]	-.0232(.043)	-.0064(.042)[.042]
σ_v^2	.0096(.073)	-.1205(.060)	.0160(.073)[.072]	.7255(.157)[.252]	.5681(.133)[.198]	-.1321(.061)	-.0021(.071)[.071]
β	-.0017(.025)	.0039(.025)	.0045(.024)[.025]	.0129(.031)[.038]	.0087(.030)[.036]	.0000(.024)	-.0010(.024)[.025]
λ	-.0007(.050)	-.0492(.046)	-.0463(.041)[.040]	-.0494(.043)[.057]	-.0466(.045)[.054]	-.0213(.042)	-.0046(.042)[.041]
σ_v^2	.0028(.168)	-.1169(.137)	.0103(.169)[.156]	.7229(.200)[.287]	.5611(.185)[.236]	-.1379(.146)	-.0088(.168)[.155]
β	-.0015(.024)	.0091(.024)	.0048(.024)[.025]	.0124(.030)[.038]	.0084(.031)[.036]	.0003(.024)	-.0006(.024)[.025]
λ	-.0006(.049)	-.0440(.047)	-.0466(.042)[.040]	-.0503(.043)[.057]	-.0474(.045)[.055]	-.0219(.042)	-.0052(.042)[.041]
σ_v^2	-.0002(.117)	-.1236(.101)	.0072(.116)[.113]	.7219(.172)[.264]	.5669(.151)[.218]	-.1401(.101)	-.0115(.116)[.112]
				$n = 100$; error = 1, 2, 3, for the three panels below			
β	-.0011(.019)	.0019(.018)	.0000(.019)[.018]	.0134(.020)[.025]	.0024(.019)[.021]	-.0006(.019)	-.0008(.019)[.018]
λ	-.0002(.041)	-.0453(.041)	-.0454(.035)[.035]	-.0482(.034)[.046]	-.0351(.033)[.040]	-.0149(.036)	-.0024(.035)[.036]
σ_v^2	.0078(.051)	-.1074(.047)	.0090(.051)[.051]	.5782(.092)[.151]	.1942(.059)[.077]	-.1235(.044)	-.0034(.050)[.050]
β	-.0009(.019)	.0015(.017)	.0003(.019)[.018]	.0145(.021)[.025]	.0026(.019)[.021]	-.0003(.019)	-.0005(.019)[.018]
λ	-.0007(.045)	-.0418(.041)	-.0465(.037)[.034]	-.0448(.037)[.046]	-.0328(.034)[.040]	-.0160(.038)	-.0037(.038)[.036]
σ_v^2	.0028(.119)	-.1067(.100)	.0045(.119)[.113]	.5743(.141)[.177]	.1946(.117)[.119]	-.1276(.103)	-.0080(.118)[.113]
β	-.0019(.017)	.0018(.017)	-.0008(.017)[.018]	.0129(.020)[.025]	.0019(.019)[.021]	-.0014(.017)	-.0016(.017)[.018]
λ	.0003(.043)	-.0419(.040)	-.0452(.037)[.034]	-.0473(.035)[.046]	-.0345(.034)[.040]	-.0141(.037)	-.0017(.036)[.036]
σ_v^2	.0077(.084)	-.1161(.076)	.0090(.083)[.082]	.5790(.115)[.162]	.2014(.091)[.098]	-.1237(.073)	-.0036(.083)[.082]
				$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0000(.014)	.0003(.013)	.0012(.014)[.013]	.0093(.015)[.016]	.0001(.014)[.014]	.0006(.014)	.0004(.014)[.013]
λ	.0014(.034)	-.0354(.032)	-.0403(.027)[.026]	-.0391(.029)[.039]	-.0293(.028)[.035]	-.0101(.028)	-.0023(.028)[.028]
σ_v^2	.0072(.038)	-.1101(.032)	.0071(.037)[.036]	.4630(.053)[.090]	.1702(.039)[.052]	-.1166(.033)	-.0008(.037)[.035]
β	-.0001(.013)	-.0004(.011)	.0011(.013)[.013]	.0082(.014)[.016]	-.0004(.013)[.014]	.0005(.013)	.0002(.013)[.013]
λ	.0025(.032)	-.0364(.032)	-.0404(.027)[.026]	-.0419(.030)[.039]	-.0323(.029)[.035]	-.0094(.028)	-.0017(.028)[.028]
σ_v^2	.0078(.088)	-.1112(.070)	.0078(.088)[.081]	.4606(.094)[.112]	.1670(.083)[.082]	-.1162(.078)	-.0003(.088)[.080]
β	-.0010(.014)	-.0003(.014)	.0001(.013)[.013]	.0102(.014)[.016]	.0014(.013)[.014]	-.0005(.013)	-.0007(.013)[.013]
λ	.0006(.034)	-.0345(.029)	-.0405(.026)[.026]	-.0422(.030)[.039]	-.0325(.030)[.035]	-.0103(.028)	-.0025(.028)[.028]
σ_v^2	.0057(.060)	-.1184(.051)	.0055(.059)[.058]	.4662(.074)[.101]	.1722(.062)[.066]	-.1181(.052)	-.0025(.059)[.058]
				$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0003(.009)	.0002(.008)	.0002(.009)[.009]	.0064(.010)[.012]	-.0023(.009)[.011]	.0004(.009)	.0004(.009)[.009]
λ	.0010(.030)	-.0378(.028)	-.0360(.024)[.024]	-.0400(.022)[.031]	-.0324(.023)[.028]	-.0088(.024)	-.0028(.024)[.025]
σ_v^2	.0058(.025)	-.1072(.023)	.0045(.025)[.025]	.4530(.039)[.062]	.2113(.029)[.039]	-.1139(.022)	-.0005(.025)[.025]
β	.0003(.009)	.0011(.009)	.0003(.009)[.009]	.0089(.010)[.012]	-.0003(.009)[.010]	.0004(.009)	.0004(.009)[.009]
λ	.0043(.029)	-.0356(.025)	-.0324(.023)[.024]	-.0392(.025)[.031]	-.0322(.025)[.029]	-.0047(.024)	.0013(.024)[.025]
σ_v^2	.0048(.059)	-.1055(.054)	.0034(.058)[.058]	.4605(.063)[.079]	.2158(.056)[.060]	-.1150(.052)	-.0018(.058)[.057]
β	-.0001(.009)	-.0001(.009)	-.0001(.009)[.009]	.0066(.009)[.012]	-.0021(.009)[.010]	.0001(.009)	.0001(.009)[.009]
λ	.0015(.031)	-.0384(.027)	-.0344(.025)[.024]	-.0474(.024)[.031]	-.0402(.023)[.029]	-.0066(.026)	-.0006(.026)[.025]
σ_v^2	.0060(.042)	-.1087(.035)	.0045(.041)[.042]	.4635(.053)[.070]	.2182(.043)[.049]	-.1141(.036)	-.0007(.041)[.041]

Note: error = 1(`normal`), 2(`normal mixture`), 3(`chi-square`).

Table 6a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under `DGP3`, 10% MCAR missing, $(\beta, \lambda, \rho, \tau, \sigma_v^2) = (1, 0.2, 0.2, 0.5, 1)$, `W = Queen` and `M = Rook`, and $\boldsymbol{T = 5}$.

	`Impu-I`	`Impu-II`	`QMLE-MR`	`ME-MR`
	$n = 50$; error = 1, 2, 3, for the three panels below			
β	-.7568(.272)[.309]	-.4203(.349)[.351]	-.0013(.037)	-.0015(.036)[.034]
λ	-.1624(.053)[.055]	-.1080(.071)[.081]	-.0172(.054)	-.0021(.059)[.056]
ρ	-.1684(.042)[.060]	-.1239(.067)[.098]	.0070(.162)	.0032(.116)[.111]
τ	-.4142(.100)[.148]	-.3018(.130)[.234]	-.2127(.124)	.0872(.114)[.130]
σ_v^2	-.6548(.385)[.536]	-.2579(.458)[.717]	-.2496(.089)	-.0606(.120)[.117]
β	-.7354(.301)[.301]	-.4354(.347)[.362]	-.0013(.034)	-.0013(.032)[.034]
λ	-.1585(.050)[.057]	-.1100(.066)[.082]	-.0162(.059)	.0010(.063)[.057]
ρ	-.1708(.043)[.058]	-.1339(.073)[.098]	.0074(.146)	-.0027(.107)[.111]
τ	-.4105(.105)[.154]	-.3126(.126)[.246]	-.1924(.157)	.1045(.139)[.150]
σ_v^2	-.6394(.406)[.539]	-.3194(.424)[.727]	-.2540(.172)	-.0702(.223)[.215]
β	-.7082(.298)[.325]	-.3750(.353)[.315]	-.0007(.037)	.0005(.036)[.033]
λ	-.1562(.051)[.060]	-.0982(.075)[.080]	-.0212(.060)	-.0035(.061)[.056]
ρ	-.1678(.045)[.062]	-.1312(.069)[.098]	.0175(.147)	.0081(.107)[.110]
τ	-.3953(.110)[.162]	-.2812(.136)[.242]	-.2000(.144)	.0901(.128)[.136]
σ_v^2	-.5760(.427)[.597]	-.2073(.456)[.724]	-.2700(.136)	-.0849(.177)[.164]
	$n = 100$; error = 1, 2, 3, for the three panels below			
β	.0174(.032)[.040]	-.0042(.032)[.034]	.0020(.027)	.0018(.026)[.025]
λ	-.0514(.044)[.067]	-.0443(.049)[.060]	-.0106(.035)	-.0048(.037)[.035]
ρ	-.1100(.049)[.085]	-.0957(.057)[.080]	.0325(.101)	-.0092(.073)[.076]
τ	-.2540(.058)[.091]	-.1946(.057)[.087]	-.2675(.096)	.0291(.066)[.074]
σ_v^2	.8582(.170)[.281]	.4617(.115)[.163]	-.2221(.063)	-.0177(.080)[.076]
β	.0182(.034)[.040]	-.0030(.031)[.034]	.0023(.026)	.0020(.025)[.025]
λ	-.0571(.052)[.067]	-.0459(.049)[.059]	-.0119(.038)	-.0029(.039)[.035]
ρ	-.1158(.050)[.085]	-.0919(.056)[.081]	.0370(.105)	-.0087(.075)[.076]
τ	-.2585(.063)[.092]	-.1892(.072)[.092]	-.2424(.118)	.0471(.098)[.091]
σ_v^2	.8686(.247)[.308]	.4786(.210)[.207]	-.2258(.124)	-.0265(.160)[.154]
β	.0173(.033)[.040]	-.0086(.027)[.034]	.0002(.026)	-.0002(.025)[.025]
λ	-.0576(.048)[.066]	-.0459(.047)[.060]	-.0052(.038)	.0023(.038)[.035]
ρ	-.1088(.061)[.085]	-.0935(.062)[.080]	.0425(.102)	-.0028(.074)[.075]
τ	-.2479(.055)[.091]	-.1776(.060)[.087]	-.2553(.106)	.0387(.076)[.082]
σ_v^2	.8607(.177)[.286]	.4550(.170)[.178]	-.2231(.095)	-.0205(.121)[.115]
	$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0214(.020)[.026]	.0040(.019)[.023]	.0000(.019)	.0001(.018)[.017]
λ	-.0586(.033)[.048]	-.0489(.029)[.042]	-.0073(.026)	-.0029(.025)[.025]
ρ	-.1060(.039)[.057]	-.0882(.046)[.054]	.0635(.067)	.0009(.050)[.052]
τ	-.2769(.036)[.061]	-.2468(.040)[.054]	-.2679(.078)	.0200(.047)[.051]
σ_v^2	.7790(.134)[.185]	.4259(.082)[.111]	-.2108(.042)	-.0072(.052)[.055]
β	.0205(.018)[.027]	.0022(.018)[.023]	-.0020(.020)	-.0023(.019)[.017]
λ	-.0563(.032)[.047]	-.0452(.033)[.042]	-.0050(.026)	-.0002(.026)[.025]
ρ	-.1026(.034)[.056]	-.0868(.038)[.056]	.0628(.071)	.0004(.052)[.052]
τ	-.2733(.037)[.063]	-.2423(.041)[.055]	-.2555(.079)	.0235(.054)[.060]
σ_v^2	.7719(.150)[.201]	.4255(.135)[.136]	-.2066(.094)	-.0026(.118)[.113]
β	.0184(.018)[.027]	-.0032(.019)[.023]	-.0011(.019)	-.0009(.018)[.017]
λ	-.0540(.035)[.049]	-.0447(.034)[.043]	-.0068(.026)	-.0018(.027)[.025]
ρ	-.1045(.034)[.061]	-.0886(.038)[.056]	.0574(.070)	-.0031(.052)[.052]
τ	-.2855(.049)[.059]	-.2502(.045)[.055]	-.2658(.081)	.0242(.052)[.055]
σ_v^2	.7721(.149)[.198]	.4251(.103)[.121]	-.2109(.063)	-.0092(.079)[.082]
	$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0197(.013)[.018]	.0023(.011)[.015]	.0010(.012)	.0009(.012)[.012]
λ	-.0524(.024)[.032]	-.0425(.025)[.028]	-.0038(.017)	-.0015(.017)[.017]
ρ	-.0975(.026)[.041]	-.0764(.030)[.038]	.0714(.050)	.0009(.037)[.037]
τ	-.2377(.030)[.041]	-.1770(.027)[.038]	-.2960(.057)	.0072(.032)[.036]
σ_v^2	.6247(.064)[.104]	.2514(.049)[.059]	-.2144(.029)	-.0081(.036)[.038]
β	.0193(.011)[.017]	.0006(.011)[.015]	-.0010(.013)	-.0008(.012)[.012]
λ	-.0540(.023)[.032]	-.0461(.023)[.029]	-.0034(.020)	-.0009(.020)[.017]
ρ	-.0958(.031)[.041]	-.0754(.029)[.039]	.0686(.049)	-.0026(.035)[.037]
τ	-.2446(.040)[.042]	-.1766(.037)[.041]	-.2929(.069)	.0118(.037)[.042]
σ_v^2	.5909(.102)[.122]	.2278(.091)[.082]	-.2180(.062)	-.0137(.078)[.079]
β	.0205(.015)[.018]	.0022(.014)[.015]	-.0012(.014)	-.0015(.014)[.012]
λ	-.0563(.019)[.032]	-.0445(.020)[.029]	-.0015(.019)	.0010(.017)[.017]
ρ	-.0938(.021)[.040]	-.0695(.023)[.038]	.0709(.050)	-.0002(.036)[.037]
τ	-.2397(.027)[.043]	-.1774(.030)[.041]	-.2909(.061)	.0093(.038)[.038]
σ_v^2	.6050(.082)[.115]	.2338(.053)[.069]	-.2129(.047)	-.0066(.058)[.058]

Note: error = 1(`normal`), 2(`normal mixture`), 3(`chi-square`).

Table 6b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP3, 10% MCAR missing, $(\beta, \lambda, \rho, \tau, \sigma_v^2) = (1, 0.2, 0.2, 0.5, 1)$, W = Queen and M = Rook, and $\boldsymbol{T} = \mathbf{10}$.

	Impu-I	Impu-II	QMLE-MR	ME-MR
	$n = 50$; error = 1, 2, 3, for the three panels below			
β	-.1090(.206)[.197]	.0056(.032)[.033]	-.0002(.020)	-.0007(.020)[.021]
λ	-.0633(.052)[.072]	-.0355(.043)[.052]	-.0120(.034)	-.0005(.035)[.032]
ρ	-.1047(.047)[.081]	-.0729(.059)[.075]	-.0130(.068)	.0009(.059)[.065]
τ	-.2575(.074)[.202]	-.1832(.053)[.174]	-.0766(.067)	.0071(.057)[.058]
σ_v^2	.3598(.319)[.751]	.2404(.088)[.447]	-.1300(.062)	-.0123(.071)[.071]
β	-.0818(.193)[.155]	.0037(.033)[.033]	.0000(.021)	-.0003(.020)[.021]
λ	-.0604(.053)[.067]	-.0421(.042)[.053]	-.0117(.033)	-.0010(.035)[.032]
ρ	-.1076(.056)[.079]	-.0797(.057)[.073]	-.0223(.069)	-.0074(.061)[.065]
τ	-.2497(.071)[.202]	-.1834(.051)[.174]	-.0717(.066)	.0105(.058)[.062]
σ_v^2	.4331(.355)[.742]	.2742(.185)[.469]	-.1256(.136)	-.0073(.155)[.154]
β	-.1200(.227)[.193]	-.0013(.036)[.038]	.0001(.022)	-.0004(.022)[.021]
λ	-.0576(.061)[.068]	-.0313(.047)[.053]	-.0117(.032)	.0005(.034)[.032]
ρ	-.1045(.052)[.080]	-.0677(.053)[.074]	-.0109(.071)	.0037(.063)[.065]
τ	-.2565(.080)[.201]	-.1787(.053)[.174]	-.0761(.068)	.0073(.058)[.060]
σ_v^2	.3742(.379)[.762]	.2746(.141)[.467]	-.1297(.106)	-.0121(.120)[.111]
	$n = 100$; error = 1, 2, 3, for the three panels below			
β	.0103(.020)[.025]	.0001(.021)[.021]	.0003(.016)	.0000(.015)[.016]
λ	-.0463(.029)[.043]	-.0391(.030)[.038]	-.0078(.024)	-.0007(.025)[.025]
ρ	-.1026(.038)[.055]	-.0837(.042)[.051]	.0048(.049)	-.0047(.043)[.045]
τ	-.2629(.031)[.050]	-.2150(.029)[.046]	-.0755(.042)	.0041(.038)[.039]
σ_v^2	.7413(.096)[.172]	.4103(.067)[.105]	-.1176(.045)	-.0070(.051)[.050]
β	.0117(.021)[.025]	.0025(.020)[.022]	.0012(.015)	.0008(.015)[.016]
λ	-.0475(.029)[.042]	-.0377(.030)[.037]	-.0077(.026)	.0001(.028)[.025]
ρ	-.1017(.035)[.057]	-.0858(.037)[.052]	.0124(.052)	.0022(.045)[.045]
τ	-.2622(.038)[.050]	-.2140(.036)[.048]	-.0717(.051)	.0075(.045)[.043]
σ_v^2	.7352(.147)[.189]	.3978(.124)[.134]	-.1174(.092)	-.0067(.104)[.109]
β	.0099(.019)[.025]	-.0007(.018)[.022]	.0001(.016)	.0000(.016)[.016]
λ	-.0568(.032)[.042]	-.0477(.030)[.038]	-.0087(.025)	-.0013(.026)[.025]
ρ	-.1013(.040)[.055]	-.0857(.038)[.054]	.0132(.050)	.0025(.044)[.045]
τ	-.2726(.036)[.049]	-.2232(.037)[.047]	-.0767(.048)	.0031(.042)[.041]
σ_v^2	.7548(.110)[.172]	.4122(.092)[.119]	-.1148(.078)	-.0036(.088)[.080]
	$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0107(.013)[.016]	.0018(.010)[.014]	.0003(.011)	.0002(.011)[.011]
λ	-.0370(.021)[.027]	-.0292(.022)[.024]	-.0046(.017)	-.0015(.018)[.018]
ρ	-.0860(.030)[.039]	-.0632(.030)[.036]	.0200(.036)	-.0019(.031)[.032]
τ	-.2232(.023)[.033]	-.1596(.021)[.032]	-.0791(.032)	.0010(.027)[.027]
σ_v^2	.5162(.061)[.090]	.2050(.038)[.051]	-.1114(.031)	-.0030(.035)[.036]
β	.0106(.015)[.016]	.0024(.014)[.014]	.0008(.011)	.0007(.011)[.011]
λ	-.0456(.021)[.028]	-.0361(.021)[.024]	-.0040(.017)	-.0009(.018)[.017]
ρ	-.0851(.025)[.039]	-.0607(.025)[.036]	.0202(.037)	-.0018(.032)[.032]
τ	-.2283(.024)[.034]	-.1601(.024)[.033]	-.0762(.035)	.0038(.031)[.030]
σ_v^2	.5272(.105)[.113]	.2099(.089)[.080]	-.1182(.071)	-.0108(.080)[.079]
β	.0055(.012)[.016]	-.0005(.012)[.013]	.0004(.012)	.0002(.012)[.011]
λ	-.0434(.019)[.028]	-.0378(.019)[.025]	-.0041(.018)	-.0003(.018)[.017]
ρ	-.0833(.022)[.039]	-.0639(.024)[.036]	.0211(.037)	-.0011(.033)[.032]
τ	-.2258(.027)[.035]	-.1669(.023)[.032]	-.0784(.033)	.0016(.029)[.029]
σ_v^2	.4929(.090)[.098]	.1867(.064)[.065]	-.1120(.054)	-.0038(.061)[.057]
	$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0120(.011)[.012]	.0018(.011)[.010]	-.0011(.009)	-.0011(.009)[.008]
λ	-.0483(.015)[.021]	-.0390(.015)[.019]	-.0058(.011)	-.0041(.012)[.013]
ρ	-.0938(.017)[.027]	-.0718(.013)[.026]	.0233(.019)	-.0050(.016)[.023]
τ	-.2498(.016)[.023]	-.1950(.015)[.023]	-.0871(.021)	-.0047(.017)[.020]
σ_v^2	.6183(.047)[.072]	.2952(.035)[.044]	-.1082(.017)	-.0014(.019)[.025]
β	.0082(.011)[.012]	.0001(.009)[.010]	-.0030(.008)	-.0031(.008)[.008]
λ	-.0460(.016)[.021]	-.0365(.015)[.019]	-.0017(.012)	.0019(.013)[.013]
ρ	-.0919(.015)[.027]	-.0730(.018)[.026]	.0329(.020)	.0043(.018)[.022]
τ	-.2486(.015)[.024]	-.1965(.019)[.023]	-.0832(.028)	-.0023(.025)[.022]
σ_v^2	.6145(.074)[.085]	.2990(.063)[.061]	-.1193(.040)	-.0136(.044)[.056]
β	.0082(.011)[.012]	-.0006(.011)[.010]	.0002(.009)	.0002(.009)[.008]
λ	-.0459(.016)[.021]	-.0369(.015)[.019]	-.0030(.014)	-.0025(.014)[.013]
ρ	-.0877(.016)[.027]	-.0647(.018)[.026]	.0366(.025)	.0065(.022)[.023]
τ	-.2461(.018)[.024]	-.1929(.016)[.023]	-.0718(.021)	.0072(.018)[.020]
σ_v^2	.6126(.057)[.080]	.2921(.062)[.051]	-.1086(.035)	-.0014(.039)[.040]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 7a: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP3, 10% MCAR missing, $(\beta, \lambda, \rho, \tau, \sigma_v^2) = (1, 0.2, 0.2, 0.5, 1)$, W = Group and M = Queen, and $\boldsymbol{T} = \mathbf{5}$.

	Impu-I	Impu-II	QMLE-MR	ME-MR
	$n = 50$; error = 1, 2, 3, for the three panels below			
β	.0061(.042)[.047]	-.0037(.039)[.038]	.0008(.049)	.0001(.040)[.040]
λ	-.0411(.062)[.078]	-.0309(.061)[.068]	-.0238(.068)	-.0088(.056)[.056]
ρ	-.0916(.109)[.153]	-.0644(.122)[.147]	-.1046(.221)	-.0158(.134)[.144]
τ	-.2238(.091)[.124]	-.1532(.078)[.114]	-.7694(.629)	.0183(.111)[.124]
σ_v^2	.4859(.151)[.260]	.1357(.131)[.140]	-.3727(.178)	-.0361(.109)[.112]
β	.0108(.038)[.046]	-.0024(.037)[.038]	.0000(.050)	.0000(.041)[.040]
λ	-.0401(.056)[.077]	-.0360(.055)[.067]	-.0195(.070)	-.0050(.053)[.056]
ρ	-.0977(.103)[.159]	-.0749(.101)[.145]	-.0959(.230)	-.0138(.138)[.143]
τ	-.2254(.078)[.127]	-.1670(.083)[.119]	-.8118(.641)	.0170(.129)[.152]
σ_v^2	.4487(.281)[.302]	.1123(.234)[.204]	-.3816(.222)	-.0262(.225)[.217]
β	.0158(.040)[.047]	.0030(.038)[.038]	.0018(.049)	.0022(.038)[.039]
λ	-.0601(.059)[.079]	-.0537(.062)[.068]	-.0233(.071)	-.0084(.058)[.056]
ρ	-.0954(.105)[.156]	-.0508(.114)[.144]	-.0979(.229)	-.0082(.142)[.142]
τ	-.2197(.095)[.124]	-.1554(.096)[.116]	-.7904(.632)	.0146(.118)[.135]
σ_v^2	.4998(.209)[.288]	.1419(.163)[.175]	-.3818(.193)	-.0398(.169)[.162]
	$n = 100$; error = 1, 2, 3, for the three panels below			
β	.0268(.029)[.036]	.0063(.028)[.029]	.0024(.028)	.0011(.024)[.024]
λ	-.0560(.053)[.066]	-.0456(.053)[.057]	-.0150(.052)	-.0038(.043)[.044]
ρ	-.1028(.074)[.108]	-.0624(.075)[.100]	-.0098(.147)	-.0039(.096)[.094]
τ	-.2715(.061)[.087]	-.2121(.062)[.076]	-.6263(.573)	.0183(.082)[.079]
σ_v^2	.6661(.131)[.236]	.2447(.087)[.119]	-.3089(.156)	-.0195(.077)[.077]
β	.0302(.030)[.036]	.0098(.030)[.029]	.0005(.032)	.0007(.025)[.024]
λ	-.0441(.047)[.063]	-.0346(.044)[.055]	-.0123(.052)	-.0028(.044)[.044]
ρ	-.0840(.069)[.107]	-.0594(.082)[.099]	-.0054(.139)	-.0034(.094)[.094]
τ	-.2571(.060)[.086]	-.1988(.057)[.083]	-.6840(.594)	.0163(.095)[.096]
σ_v^2	.6479(.214)[.258]	.2274(.177)[.162]	-.3171(.182)	-.0084(.153)[.157]
β	.0221(.026)[.036]	.0040(.024)[.030]	.0002(.029)	-.0008(.023)[.024]
λ	-.0420(.054)[.065]	-.0319(.050)[.056]	-.0154(.053)	-.0036(.045)[.044]
ρ	-.1037(.073)[.111]	-.0782(.081)[.101]	-.0173(.145)	-.0092(.095)[.095]
τ	-.2549(.060)[.086]	-.2062(.061)[.076]	-.6763(.590)	.0140(.080)[.085]
σ_v^2	.6648(.179)[.251]	.2578(.134)[.142]	-.3189(.175)	-.0148(.123)[.114]
	$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0177(.022)[.026]	.0008(.020)[.022]	-.0003(.020)	-.0007(.017)[.016]
λ	-.0639(.052)[.067]	-.0517(.050)[.059]	-.0062(.040)	-.0002(.034)[.034]
ρ	-.1049(.048)[.076]	-.0799(.056)[.072]	.0238(.087)	-.0059(.061)[.064]
τ	-.2362(.036)[.060]	-.1696(.033)[.057]	-.6248(.557)	.0042(.051)[.051]
σ_v^2	.6419(.109)[.154]	.2920(.078)[.089]	-.2943(.146)	-.0085(.053)[.054]
β	.0152(.020)[.026]	-.0008(.018)[.022]	.0001(.019)	.0002(.016)[.016]
λ	-.0599(.054)[.066]	-.0437(.051)[.058]	-.0077(.042)	-.0029(.033)[.034]
ρ	-.0954(.056)[.075]	-.0783(.059)[.071]	.0263(.091)	-.0007(.064)[.064]
τ	-.2307(.046)[.063]	-.1691(.046)[.061]	-.6382(.569)	.0082(.059)[.061]
σ_v^2	.6352(.136)[.181]	.2869(.106)[.123]	-.3006(.164)	-.0096(.122)[.110]
β	.0172(.020)[.026]	.0003(.019)[.022]	.0003(.019)	.0007(.016)[.016]
λ	-.0539(.047)[.066]	-.0443(.047)[.058]	-.0105(.043)	-.0033(.035)[.034]
ρ	-.0966(.048)[.078]	-.0789(.055)[.070]	.0264(.093)	-.0027(.065)[.064]
τ	-.2342(.040)[.062]	-.1687(.044)[.059]	-.6507(.565)	.0028(.054)[.056]
σ_v^2	.6204(.120)[.166]	.2751(.093)[.103]	-.3049(.152)	-.0132(.083)[.081]
	$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0195(.013)[.018]	-.0005(.011)[.015]	-.0007(.014)	-.0008(.012)[.012]
λ	-.0423(.043)[.049]	-.0343(.042)[.043]	-.0094(.036)	-.0035(.031)[.032]
ρ	-.0928(.041)[.053]	-.0659(.045)[.049]	.0478(.063)	-.0027(.046)[.046]
τ	-.2530(.029)[.043]	-.1892(.026)[.037]	-.4706(.399)	.0029(.036)[.037]
σ_v^2	.6203(.062)[.113]	.2411(.045)[.061]	-.2466(.106)	-.0070(.041)[.038]
β	.0196(.015)[.018]	-.0006(.015)[.015]	-.0003(.014)	-.0002(.013)[.012]
λ	-.0435(.030)[.050]	-.0379(.031)[.042]	-.0070(.036)	.0002(.031)[.032]
ρ	-.0987(.042)[.053]	-.0767(.047)[.050]	.0469(.061)	-.0030(.045)[.045]
τ	-.2541(.034)[.044]	-.1847(.033)[.041]	-.5007(.438)	.0052(.042)[.045]
σ_v^2	.6317(.090)[.128]	.2341(.074)[.082]	-.2566(.122)	-.0066(.078)[.080]
β	.0207(.018)[.018]	.0008(.015)[.015]	-.0001(.013)	-.0001(.012)[.012]
λ	-.0429(.040)[.049]	-.0352(.042)[.043]	-.0081(.038)	-.0014(.034)[.032]
ρ	-.0911(.037)[.054]	-.0655(.036)[.050]	.0505(.063)	.0005(.045)[.046]
τ	-.2579(.032)[.042]	-.1951(.037)[.040]	-.4567(.395)	.0063(.041)[.040]
σ_v^2	.6267(.078)[.114]	.2364(.068)[.070]	-.2429(.107)	-.0043(.059)[.059]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 7b: Empirical bias(sd)[$\hat{se}$] of various estimators: **MR** model under DGP3, 10% MCAR missing, $(\beta,\lambda,\rho,\tau,\sigma_v^2)=(1,0.2,0.2,0.5,1)$, W = Group and M = Queen, and $\boldsymbol{T}=\mathbf{10}$.

	Impu-I	Impu-II	QMLE-MR	ME-MR
	$n = 50$; error = 1, 2, 3, for the three panels below			
β	.0066(.024)[.033]	-.0006(.023)[.029]	.0003(.021)	-.0001(.021)[.022]
λ	-.0427(.039)[.053]	-.0361(.040)[.049]	-.0182(.040)	-.0039(.039)[.037]
ρ	-.0834(.077)[.102]	-.0682(.076)[.096]	-.0478(.093)	.0022(.080)[.083]
τ	-.2158(.043)[.068]	-.1673(.043)[.064]	-.0829(.071)	.0047(.062)[.060]
σ_v^2	.4619(.127)[.168]	.2143(.097)[.106]	-.1279(.064)	-.0139(.072)[.071]
β	.0119(.027)[.033]	.0047(.025)[.028]	.0012(.023)	.0006(.023)[.022]
λ	-.0415(.045)[.054]	-.0335(.040)[.049]	-.0160(.039)	-.0017(.038)[.037]
ρ	-.0833(.068)[.099]	-.0675(.071)[.097]	-.0573(.091)	-.0072(.077)[.084]
τ	-.2213(.053)[.068]	-.1703(.049)[.066]	-.0809(.073)	.0067(.063)[.066]
σ_v^2	.4940(.222)[.213]	.2250(.185)[.161]	-.1260(.140)	-.0117(.158)[.153]
β	.0119(.029)[.034]	.0034(.028)[.029]	.0006(.022)	.0002(.022)[.022]
λ	-.0385(.040)[.054]	-.0309(.038)[.049]	-.0177(.038)	-.0032(.037)[.037]
ρ	-.0726(.071)[.099]	-.0502(.071)[.095]	-.0535(.095)	-.0040(.081)[.083]
τ	-.2281(.050)[.067]	-.1767(.044)[.065]	-.0766(.068)	.0104(.060)[.063]
σ_v^2	.5167(.180)[.196]	.2339(.151)[.134]	-.1268(.100)	-.0128(.113)[.112]
	$n = 100$; error = 1, 2, 3, for the three panels below			
β	.0086(.020)[.024]	-.0004(.019)[.021]	-.0002(.015)	-.0004(.015)[.016]
λ	-.0379(.033)[.044]	-.0307(.034)[.039]	-.0089(.030)	-.0001(.029)[.029]
ρ	-.0836(.053)[.070]	-.0645(.053)[.066]	-.0142(.065)	-.0051(.057)[.057]
τ	-.2370(.032)[.049]	-.1777(.030)[.048]	-.0829(.045)	.0019(.038)[.040]
σ_v^2	.6287(.084)[.151]	.2890(.068)[.084]	-.1159(.044)	-.0080(.049)[.050]
β	.0094(.020)[.025]	-.0014(.020)[.021]	.0006(.016)	.0003(.016)[.016]
λ	-.0358(.035)[.045]	-.0275(.036)[.039]	-.0097(.030)	-.0003(.029)[.029]
ρ	-.0966(.046)[.072]	-.0787(.049)[.070]	-.0158(.066)	-.0064(.057)[.057]
τ	-.2486(.032)[.049]	-.1849(.033)[.048]	-.0785(.049)	.0061(.043)[.045]
σ_v^2	.6532(.138)[.180]	.2953(.112)[.122]	-.1186(.097)	-.0113(.109)[.108]
β	.0090(.019)[.025]	.0012(.016)[.021]	.0005(.016)	.0001(.016)[.016]
λ	-.0427(.035)[.045]	-.0344(.032)[.039]	-.0117(.032)	-.0021(.032)[.029]
ρ	-.0899(.054)[.071]	-.0707(.052)[.067]	-.0145(.066)	-.0055(.058)[.057]
τ	-.2480(.032)[.049]	-.1868(.033)[.047]	-.0803(.047)	.0042(.042)[.042]
σ_v^2	.6287(.131)[.160]	.2815(.094)[.102]	-.1185(.072)	-.0111(.081)[.079]
	$n = 200$; error = 1, 2, 3, for the three panels below			
β	.0101(.015)[.016]	.0013(.011)[.014]	-.0007(.012)	-.0007(.012)[.011]
λ	-.0452(.030)[.040]	-.0390(.031)[.035]	-.0069(.027)	.0002(.026)[.026]
ρ	-.0891(.032)[.050]	-.0729(.036)[.047]	.0102(.046)	-.0008(.041)[.040]
τ	-.2358(.023)[.033]	-.1822(.021)[.032]	-.0830(.030)	.0012(.026)[.028]
σ_v^2	.5504(.065)[.097]	.2589(.052)[.058]	-.1095(.032)	-.0039(.036)[.036]
β	.0073(.011)[.017]	-.0019(.012)[.014]	.0002(.012)	.0002(.012)[.011]
λ	-.0362(.029)[.039]	-.0273(.029)[.035]	-.0090(.027)	-.0016(.026)[.026]
ρ	-.0806(.031)[.049]	-.0593(.036)[.046]	.0127(.048)	.0017(.043)[.040]
τ	-.2448(.023)[.034]	-.1887(.028)[.032]	-.0838(.036)	.0009(.031)[.032]
σ_v^2	.5654(.111)[.116]	.2673(.096)[.082]	-.1122(.073)	-.0068(.081)[.078]
β	.0063(.014)[.016]	-.0019(.012)[.014]	-.0004(.012)	-.0004(.012)[.011]
λ	-.0433(.034)[.040]	-.0310(.034)[.035]	-.0086(.027)	-.0013(.027)[.026]
ρ	-.0839(.033)[.050]	-.0607(.038)[.046]	.0106(.046)	-.0006(.041)[.040]
τ	-.2425(.028)[.034]	-.1869(.028)[.032]	-.0796(.035)	.0041(.030)[.030]
σ_v^2	.5546(.083)[.106]	.2504(.074)[.070]	-.1109(.051)	-.0055(.057)[.058]
	$n = 400$; error = 1, 2, 3, for the three panels below			
β	.0123(.013)[.013]	.0011(.012)[.011]	.0000(.008)	-.0001(.008)[.008]
λ	-.0488(.025)[.035]	-.0375(.025)[.031]	-.0068(.024)	-.0005(.024)[.024]
ρ	-.0981(.022)[.035]	-.0783(.023)[.033]	.0209(.031)	-.0001(.028)[.028]
τ	-.2649(.015)[.024]	-.2142(.016)[.023]	-.0803(.022)	.0028(.020)[.020]
σ_v^2	.7066(.045)[.078]	.3727(.038)[.050]	-.1065(.023)	-.0033(.025)[.025]
β	.0115(.011)[.013]	-.0006(.010)[.011]	.0000(.008)	.0000(.008)[.008]
λ	-.0449(.026)[.034]	-.0349(.027)[.030]	-.0067(.025)	-.0007(.025)[.024]
ρ	-.0968(.022)[.035]	-.0801(.020)[.033]	.0204(.031)	-.0002(.028)[.028]
τ	-.2696(.019)[.025]	-.2185(.020)[.023]	-.0839(.026)	-.0002(.022)[.022]
σ_v^2	.7157(.075)[.094]	.3741(.063)[.065]	-.1087(.050)	-.0056(.056)[.055]
β	.0124(.013)[.013]	-.0003(.011)[.011]	.0006(.008)	.0007(.008)[.008]
λ	-.0511(.026)[.035]	-.0435(.022)[.031]	-.0079(.023)	-.0016(.023)[.024]
ρ	-.0942(.029)[.035]	-.0741(.028)[.034]	.0172(.030)	-.0036(.026)[.028]
τ	-.2657(.016)[.024]	-.2186(.014)[.024]	-.0807(.023)	.0029(.020)[.021]
σ_v^2	.7049(.068)[.088]	.3779(.054)[.057]	-.1051(.038)	-.0018(.042)[.041]

Note: error = 1(normal), 2(normal mixture), 3(chi-square).

Table 8: Empirical bias(sd)[$\hat{se}$] of estimators based on various imputation methods: 10% MCAR missing, $(\beta, \lambda, \rho, \tau, \sigma_v^2) = (1, 0.2, 0.2, 0.5, 1)$, and `W = Group` and `M = Queen`.

	T=5			T=10		
	Impu-I	Impu-II	Impu-III	Impu-I	Impu-II	Impu-III
DGP 1; $n = 100$; error = 1, 2, 3, for the three panels below						
β	.0228(.032)	.0043(.031)	-.0010(.028)	.0098(.019)	.0008(.018)	.0003(.018)
λ	-.0338(.043)	-.0308(.042)	-.0097(.051)	-.0433(.041)	-.0358(.041)	-.0046(.031)
ρ	-.0944(.075)	-.0716(.080)	-.0287(.094)	-.0822(.042)	-.0586(.042)	-.0181(.057)
σ_v^2	.5098(.104)	.1659(.087)	-.1258(.068)	.4878(.082)	.1870(.060)	-.1126(.045)
β	.0271(.029)	.0033(.025)	.0008(.027)	.0090(.020)	.0005(.020)	.0003(.019)
λ	-.0504(.048)	-.0398(.044)	-.0054(.051)	-.0393(.032)	-.0300(.032)	-.0041(.031)
ρ	-.0842(.073)	-.0633(.078)	-.0223(.091)	-.0868(.050)	-.0647(.058)	-.0141(.059)
σ_v^2	.5174(.186)	.1574(.159)	-.1301(.141)	.4803(.118)	.1710(.110)	-.1052(.098)
β	.0224(.030)	.0005(.027)	.0001(.027)	.0097(.021)	-.0012(.020)	.0001(.018)
λ	-.0455(.047)	-.0371(.047)	-.0094(.051)	-.0398(.035)	-.0309(.033)	-.0064(.033)
ρ	-.0956(.072)	-.0731(.077)	-.0169(.098)	-.0870(.049)	-.0601(.051)	-.0151(.057)
σ_v^2	.5342(.141)	.1692(.115)	-.1332(.105)	.4880(.108)	.1833(.091)	-.1067(.076)
DGP 1; $n = 400$; error = 1, 2, 3, for the three panels below						
β	.0204(.014)	-.0006(.013)	.0006(.014)	.0120(.011)	-.0008(.011)	-.0003(.008)
λ	-.0448(.036)	-.0370(.036)	-.0062(.039)	-.0490(.030)	-.0408(.028)	-.0039(.022)
ρ	-.0814(.040)	-.0571(.038)	-.0203(.048)	-.0951(.025)	-.0753(.029)	-.0190(.030)
σ_v^2	.4776(.062)	.1570(.043)	-.1174(.033)	.5621(.044)	.2676(.033)	-.1111(.020)
β	.0207(.014)	-.0001(.014)	-.0001(.013)	.0105(.010)	-.0014(.009)	-.0005(.008)
λ	-.0424(.034)	-.0337(.034)	-.0062(.038)	-.0493(.027)	-.0378(.026)	-.0057(.025)
ρ	-.0851(.034)	-.0634(.038)	-.0124(.045)	-.0880(.025)	-.0694(.025)	-.0174(.031)
σ_v^2	.4735(.084)	.1494(.069)	-.1202(.074)	.5558(.056)	.2608(.048)	-.1144(.050)
β	.0193(.015)	-.0011(.015)	.0007(.015)	.0117(.011)	-.0005(.010)	.0000(.008)
λ	-.0369(.039)	-.0289(.039)	-.0027(.039)	-.0468(.025)	-.0346(.026)	-.0049(.023)
ρ	-.0904(.040)	-.0695(.043)	-.0199(.051)	-.0923(.021)	-.0741(.024)	-.0170(.029)
σ_v^2	.4846(.076)	.1652(.064)	-.1177(.050)	.5591(.056)	.2635(.044)	-.1082(.039)
DGP 3; $n = 100$; error = 1, 2, 3, for the three panels below						
β	.0268(.029)	.0063(.028)	.0007(.023)	.0086(.020)	-.0004(.019)	-.0005(.017)
λ	-.0560(.053)	-.0456(.053)	-.0073(.044)	-.0379(.033)	-.0307(.034)	-.0047(.028)
ρ	-.1028(.074)	-.0624(.075)	-.0275(.095)	-.0836(.053)	-.0645(.053)	-.0207(.052)
τ	-.2715(.061)	-.2121(.062)	-.0325(.071)	-.2370(.032)	-.1777(.030)	-.0602(.032)
σ_v^2	.6661(.131)	.2447(.087)	.0087(.082)	.6287(.084)	.2890(.068)	.0435(.051)
β	.0302(.030)	.0098(.030)	.0028(.022)	.0094(.020)	-.0014(.020)	.0003(.017)
λ	-.0441(.047)	-.0346(.044)	-.0117(.041)	-.0358(.035)	-.0275(.036)	-.0066(.027)
ρ	-.0840(.069)	-.0594(.082)	-.0210(.091)	-.0966(.046)	-.0787(.049)	-.0169(.055)
τ	-.2571(.060)	-.1988(.057)	-.0245(.085)	-.2486(.032)	-.1849(.033)	-.0609(.041)
σ_v^2	.6479(.214)	.2274(.177)	.0023(.160)	.6532(.138)	.2953(.112)	.0559(.111)
β	.0221(.026)	.0040(.024)	-.0013(.023)	.0090(.019)	.0012(.016)	-.0010(.018)
λ	-.0420(.054)	-.0319(.050)	-.0032(.046)	-.0427(.035)	-.0344(.032)	-.0039(.031)
ρ	-.1037(.073)	-.0782(.081)	-.0237(.085)	-.0899(.054)	-.0707(.052)	-.0189(.046)
τ	-.2549(.060)	-.2062(.061)	-.0399(.067)	-.2480(.032)	-.1868(.033)	-.0630(.037)
σ_v^2	.6648(.179)	.2578(.134)	.0272(.126)	.6287(.131)	.2815(.094)	.0410(.089)
DGP 3; $n = 400$; error = 1, 2, 3, for the three panels below						
β	.0195(.013)	-.0005(.011)	.0007(.013)	.0123(.013)	.0011(.012)	.0003(.008)
λ	-.0423(.043)	-.0343(.042)	-.0059(.035)	-.0488(.025)	-.0375(.025)	-.0050(.021)
ρ	-.0928(.041)	-.0659(.045)	-.0160(.043)	-.0981(.022)	-.0783(.023)	-.0220(.026)
τ	-.2530(.029)	-.1892(.026)	-.0541(.029)	-.2649(.015)	-.2142(.016)	-.0674(.016)
σ_v^2	.6203(.062)	.2411(.045)	.0290(.039)	.7066(.045)	.3727(.038)	.0491(.024)
β	.0196(.015)	-.0006(.015)	.0011(.012)	.0115(.011)	-.0006(.010)	-.0002(.008)
λ	-.0435(.030)	-.0379(.031)	-.0082(.037)	-.0449(.026)	-.0349(.027)	-.0056(.021)
ρ	-.0987(.042)	-.0767(.047)	-.0139(.044)	-.0968(.022)	-.0801(.020)	-.0182(.029)
τ	-.2541(.034)	-.1847(.033)	-.0552(.037)	-.2696(.019)	-.2185(.020)	-.0674(.020)
σ_v^2	.6317(.090)	.2341(.074)	.0264(.086)	.7157(.075)	.3741(.063)	.0409(.055)
β	.0207(.018)	.0008(.015)	.0011(.012)	.0124(.013)	-.0003(.011)	-.0012(.008)
λ	-.0429(.040)	-.0352(.042)	-.0090(.038)	-.0511(.026)	-.0435(.022)	-.0049(.023)
ρ	-.0911(.037)	-.0655(.036)	-.0175(.040)	-.0942(.029)	-.0741(.028)	-.0189(.025)
τ	-.2579(.032)	-.1951(.037)	-.0529(.032)	-.2657(.016)	-.2186(.014)	-.0665(.019)
σ_v^2	.6267(.078)	.2364(.068)	.0350(.058)	.7049(.068)	.3779(.054)	.0520(.043)

Note: error = 1(`normal`), 2(`normal mixture`), 3(`chi-square`).

Table 9: Empirical bias(sd)[$\hat{se}$] of `QMLE-MR` and `ME-MR`: 25% MAR missing, $(\beta, \lambda, \rho, \tau, \sigma_v^2) = (1, 0.2, 0.2, 0.5, 1)$, and `W = Group` and `M = Queen`.

	T=5		T=10	
	`QMLE-MR`	`ME-MR`	`QMLE-MR`	`ME-MR`
DGP 1; $n = 100$; error = 1, 2, 3, for the three panels below				
β	.0032(.034)	.0021(.034)[.035]	-.0003(.023)	-.0007(.023)[.021]
λ	-.0134(.053)	-.0014(.052)[.054]	-.0154(.040)	-.0026(.039)[.039]
ρ	-.0358(.210)	-.0125(.136)[.138]	-.0391(.092)	-.0051(.079)[.079]
σ_v^2	-.3094(.066)	-.0210(.092)[.088]	-.1515(.050)	-.0058(.059)[.057]
β	.0018(.035)	.0008(.035)[.035]	-.0012(.022)	-.0016(.022)[.022]
λ	-.0205(.054)	-.0081(.053)[.055]	-.0159(.041)	-.0029(.040)[.040]
ρ	-.0262(.214)	-.0056(.137)[.140]	-.0408(.096)	-.0064(.083)[.080]
σ_v^2	-.2991(.132)	-.0065(.187)[.181]	-.1462(.115)	.0002(.134)[.128]
β	-.0028(.037)	-.0040(.037)[.035]	.0002(.020)	-.0002(.020)[.021]
λ	-.0159(.059)	-.0035(.058)[.055]	-.0175(.041)	-.0047(.040)[.040]
ρ	-.0454(.213)	-.0175(.135)[.141]	-.0376(.087)	-.0040(.075)[.080]
σ_v^2	-.3055(.091)	-.0156(.127)[.130]	-.1524(.079)	-.0068(.093)[.091]
DGP 1; $n = 400$; error = 1, 2, 3, for the three panels below				
β	-.0001(.016)	-.0003(.016)[.016]	.0001(.011)	.0001(.011)[.011]
λ	-.0122(.042)	-.0037(.040)[.041]	-.0067(.029)	-.0005(.028)[.028]
ρ	.0591(.088)	-.0031(.066)[.065]	.0144(.041)	-.0003(.036)[.039]
σ_v^2	-.2869(.033)	-.0010(.045)[.045]	-.1419(.025)	-.0020(.030)[.028]
β	.0013(.016)	.0010(.016)[.016]	-.0003(.011)	-.0003(.011)[.010]
λ	-.0094(.042)	-.0013(.041)[.041]	-.0080(.028)	-.0018(.028)[.028]
ρ	.0607(.085)	-.0023(.064)[.065]	.0169(.042)	.0019(.037)[.039]
σ_v^2	-.2873(.071)	-.0013(.098)[.093]	-.1427(.056)	-.0028(.065)[.064]
β	.0005(.016)	.0003(.016)[.016]	-.0008(.011)	-.0009(.011)[.011]
λ	-.0097(.041)	-.0017(.040)[.041]	-.0056(.028)	.0007(.028)[.028]
ρ	.0595(.085)	-.0031(.064)[.065]	.0151(.044)	.0003(.039)[.039]
σ_v^2	-.2884(.048)	-.0033(.066)[.068]	-.1423(.037)	-.0023(.044)[.046]
DGP 3; $n = 100$; error = 1, 2, 3, for the three panels below				
β	-.0007(.041)	-.0011(.031)[.031]	.0012(.020)	.0006(.020)[.020]
λ	-.0089(.065)	-.0001(.048)[.050]	-.0140(.039)	-.0024(.038)[.036]
ρ	-.0459(.207)	-.0078(.117)[.122]	-.0300(.085)	-.0114(.071)[.072]
τ	-1.2780(.478)	.0060(.100)[.100]	-.1027(.060)	.0099(.052)[.052]
σ_v^2	-.5213(.133)	-.0321(.087)[.090]	-.1346(.052)	-.0126(.059)[.058]
β	.0004(.043)	.0006(.032)[.031]	.0003(.020)	.0000(.020)[.020]
λ	-.0095(.064)	-.0022(.048)[.050]	-.0144(.037)	-.0025(.036)[.037]
ρ	-.0549(.228)	-.0095(.123)[.122]	-.0259(.091)	-.0075(.076)[.072]
τ	-1.1527(.574)	.0300(.136)[.132]	-.1086(.067)	.0045(.058)[.061]
σ_v^2	-.4942(.159)	-.0342(.176)[.175]	-.1310(.110)	-.0079(.126)[.122]
β	.0009(.044)	.0002(.030)[.031]	.0024(.020)	.0021(.020)[.020]
λ	-.0159(.069)	-.0063(.050)[.051]	-.0144(.040)	-.0021(.038)[.037]
ρ	-.0630(.203)	-.0153(.118)[.123]	-.0294(.079)	-.0102(.066)[.072]
τ	-1.2246(.523)	.0201(.113)[.117]	-.1065(.062)	.0054(.054)[.055]
σ_v^2	-.5076(.145)	-.0290(.133)[.131]	-.1337(.079)	-.0112(.090)[.089]
DGP 3; $n = 400$; error = 1, 2, 3, for the three panels below				
β	-.0027(.020)	-.0014(.014)[.015]	-.0007(.010)	-.0007(.010)[.010]
λ	-.0058(.051)	-.0012(.036)[.036]	-.0075(.027)	-.0021(.027)[.025]
ρ	.0339(.098)	-.0011(.058)[.057]	.0256(.044)	.0003(.038)[.035]
τ	-1.3636(.368)	.0042(.049)[.046]	-.1102(.029)	-.0005(.024)[.025]
σ_v^2	-.5178(.104)	-.0071(.044)[.045]	-.1215(.024)	-.0004(.027)[.029]
β	.0015(.019)	.0007(.014)[.015]	-.0012(.009)	-.0014(.009)[.010]
λ	-.0054(.051)	-.0010(.036)[.036]	-.0064(.026)	-.0013(.026)[.025]
ρ	.0275(.102)	-.0059(.059)[.058]	.0306(.042)	.0048(.036)[.035]
τ	-1.3035(.433)	.0078(.056)[.057]	-.1071(.034)	.0030(.030)[.029]
σ_v^2	-.5045(.123)	-.0113(.089)[.088]	-.1240(.058)	-.0035(.066)[.062]
β	.0003(.021)	-.0001(.014)[.015]	-.0004(.010)	-.0004(.010)[.010]
λ	-.0072(.051)	-.0020(.035)[.036]	-.0044(.025)	.0014(.025)[.025]
ρ	.0343(.095)	-.0035(.058)[.058]	.0262(.041)	.0017(.036)[.035]
τ	-1.3214(.415)	.0020(.052)[.051]	-.1064(.032)	.0037(.028)[.027]
σ_v^2	-.5076(.117)	-.0056(.064)[.067]	-.1255(.032)	-.0056(.037)[.046]

Note: error = 1(`normal`), 2(`normal mixture`), 3(`chi-square`).

Table 10: Empirical bias(sd)[$\hat{se}$] of `QMLE-MR` and `ME-MR`: 10% MCAR missing, t-distributed errors (df = 5), and `W = Group` and `M = Queen`.

			T=5		T=10	
	n		QMLE-MR	ME-MR	QMLE-MR	ME-MR
DGP1	100	β	-.0001(.028)	-.0003(.028)[.027]	.0014(.018)	.0012(.018)[.018]
		λ	-.0171(.054)	-.0059(.053)[.051]	-.0140(.032)	-.0034(.032)[.033]
		ρ	-.0113(.126)	-.0067(.098)[.103]	-.0282(.075)	-.0053(.067)[.064]
		σ_v^2	-.2419(.094)	-.0148(.122)[.114]	-.1217(.080)	-.0029(.091)[.082]
	200	β	.0002(.020)	-.0001(.020)[.020]	-.0006(.012)	-.0007(.012)[.012]
		λ	-.0094(.041)	-.0027(.041)[.039]	-.0086(.030)	-.0004(.029)[.029]
		ρ	.0240(.085)	-.0037(.069)[.071]	.0008(.051)	.0004(.047)[.045]
		σ_v^2	-.2328(.069)	-.0092(.089)[.082]	-.1198(.060)	-.0044(.067)[.060]
	400	β	-.0003(.014)	-.0004(.014)[.013]	-.0004(.009)	-.0004(.009)[.009]
		λ	-.0065(.036)	-.0014(.036)[.036]	-.0098(.024)	-.0039(.024)[.025]
		ρ	.0441(.063)	-.0013(.051)[.051]	.0158(.036)	.0036(.033)[.031]
		σ_v^2	-.2316(.056)	.0007(.072)[.062]	-.1164(.038)	-.0023(.042)[.044]
DGP3	100	β	.0001(.031)	.0000(.024)[.023]	-.0011(.015)	-.0014(.015)[.016]
		λ	-.0174(.058)	-.0064(.047)[.045]	-.0128(.030)	-.0027(.030)[.031]
		ρ	-.0270(.144)	-.0106(.091)[.098]	-.0092(.064)	-.0015(.057)[.056]
		τ	-.8224(.617)	.0055(.086)[.088]	-.0791(.048)	.0046(.042)[.042]
		σ_v^2	-.3639(.179)	-.0180(.128)[.114]	-.1197(.078)	-.0127(.087)[.082]
	200	β	-.0004(.021)	-.0010(.017)[.017]	.0001(.011)	.0000(.011)[.011]
		λ	-.0096(.044)	-.0025(.036)[.039]	-.0075(.025)	-.0006(.024)[.026]
		ρ	.0205(.096)	-.0063(.065)[.065]	.0103(.049)	-.0006(.043)[.040]
		τ	-.6506(.555)	.0020(.058)[.058]	-.0843(.033)	.0005(.030)[.030]
		σ_v^2	-.3042(.155)	-.0136(.080)[.080]	-.1127(.056)	-.0074(.062)[.059]
	400	β	-.0002(.013)	.0002(.012)[.012]	.0012(.009)	.0010(.009)[.008]
		λ	-.0112(.036)	-.0047(.035)[.032]	-.0103(.024)	-.0041(.023)[.024]
		ρ	.0487(.065)	-.0047(.049)[.046]	.0197(.033)	-.0011(.029)[.028]
		τ	-.3154(.063)	.0049(.037)[.040]	-.0814(.022)	.0022(.019)[.021]
		σ_v^2	-.2029(.053)	-.0021(.069)[.062]	-.1083(.036)	-.0053(.040)[.042]

Table 11: Naïve-Est, ME-GU and ME-MR results of simulated housing price panel with MR, $n = 506$ and $T = 5$, 10% MCAR missing .

	QMLE-CS	Naïve-Est	ME-GU	ME-MR
Time-varying variables				
crime	-1.0982[.245]	-1.1056[.053]	-1.0605[.044]	-1.0640[.045]
zoning	.9163[.329]	.8729[.063]	.8607[.046]	.8730[.047]
industry	-.0143[.492]	-.0119[.063]	.0665[.048]	.0675[.048]
noxsq	-2.0339[.587]	-1.9226[.063]	-1.9739[.049]	-1.9746[.049]
rooms2	2.9906[.261]	2.9764[.066]	2.9504[.050]	2.9637[.050]
houseage	-.6105[.392]	-.6289[.061]	-.6814[.047]	-.6890[.047]
access	2.7855[.648]	2.8398[.057]	2.7514[.046]	2.7667[.046]
taxrate	-2.2076[.610]	-2.3243[.061]	-2.2808[.048]	-2.2955[.048]
ptratio	-1.3875[.326]	-1.3284[.062]	-1.3134[.048]	-1.3200[.048]
blackpop	.9257[.279]	.9102[.060]	.9222[.049]	.9184[.049]
lowclass	-3.0573[.373]	-3.0487[.061]	-3.0377[.049]	-3.0543[.049]
Spatial dependence				
SL(λ)	.0827[.066]	.0451[.019]	.0284[.021]	.0631[.023]
SE(ρ)	.6277[.063]	.4118[.033]	.6016[.027]	.6303[.029]
Variance parameter				
σ_v^2	3.9233[1.891]	4.3331[.279]	3.9595[.219]	3.8819[.214]

Standard error estimates are in the square brackets.