Finally, when the conditions of Theorem 1 are satisfied so the regular QMLEs are also consistent, the robust variances of $\hat{\lambda}_n$ and $\hat{\beta}_n$ can easily be obtained from the results of Theorems 2-4. Some details are as follows. Starting with the concentrated score $\tilde{\psi}_n$ given in (7), one obtains $\tau^2(\hat{\lambda}_n)$ by simply replacing G_n° by $G_n - \frac{1}{n}\text{tr}(G_n)I_n$ in (11) and (12), and in Φ_n defined in Theorem 2. Similarly, replacing G_n° by $G_n - \frac{1}{n}\text{tr}(G_n)I_n$ in $\tau_n^2(\tilde{\beta}_n)$ given in Theorem 3 leads to $\tau_n^2(\hat{\beta}_n)$. The estimates of $\tau^2(\hat{\lambda}_n)$ and $\tau_n^2(\hat{\beta}_n)$ are obtained in the same way as those of $\tau^2(\tilde{\lambda}_n)$ and $\tau_n^2(\tilde{\beta}_n)$, and their consistency can be proved similarly to the results of Theorem 4.

4. Monte Carlo Study

Extensive Monte Carlo experiments were conducted to (i) investigate the finite sample behaviour of the original QMLE $\hat{\lambda}_n$ and the modified QMLE (MQMLE) $\tilde{\lambda}_n$ proposed in this paper, and their impacts on the estimators of β and σ^2, with respect to the changes in the sample size, spatial layouts, error distributions and the model parameters when the models are heteroskedastic; and (ii) compare the QMLE and the MQMLE with the non-robust generalized method of moments estimator (GMME) of Lee (2001), the robust GMME (RGMME) and the optimal RGMME (ORGMME) of Lin and Lee (2010), two stage least squares estimator (2SLSE) of Kelejian and Prucha (1998), and the root estimator (RE) of Jin and Lee (2012). We consider cases where the original QMLE are robust against heteroskedasticity and the cases it is not.

The simulations are carried out based on the following data generation process (DGP):

$$Y_n = \lambda W_n Y_n + \iota_n \beta_0 + X_{1n}\beta_1 + X_{2n}\beta_2 + \epsilon_n,$$

where ι_n is an $n\times 1$ vector of ones corresponding to the intercept term, X_{1n} and X_{2n} are the $n\times 1$ vectors containing the values of two fixed regressors, and $\epsilon_n = \sigma H_n e_n$. The regression coefficients β is set to either $(3, 1, 1)'$ or $(.3, .1, .1)'$, σ is set to 1, λ takes values form $\{-0.5, -0.25, 0, 0.25, 0.5\}$ and n take values from $\{100, 250, 500, 1000\}$. The ways of generating the values for (X_{1n}, X_{2n}), the spatial weights matrix W_n, the heteroskedasticity measure H_n, and the idiosyncratic errors e_n are described below. Each set of Monte Carlo results is based on $1,000$ Monte Carlo samples.

Spatial Weight Matrix: We use three different spatial layouts: (i) `Circular Neighbours`, (ii) `Group Interaction` and (iii) `Queen Contiguity`. In (i), neighbours occur in the positions immediately ahead and behind a particular spatial unit. For example, for the ith spatial unit with 6 neighbours, the ith row of W_n matrix has non-zero elements in the positions: $i-3, i-2, i-1, i+1, i+2$, and $i+3$. The weights matrix we consider has 2, 4, 6, 8 and 10 neighbours with equal proportion. In (ii), neighbours occur in groups where each group member is spatially related to one another resulting in a symmetric W_n matrix. In (iii), neighbours could occur in the eight cardinal and ordinal positions of each unit. To ensure the heteroskedasticity effect does not fade as n increases (so that the regular QMLE is inconsistent), the degree of spatial dependence is fixed with respect to n. This is attained by fixing the possible group sizes in the Group Interaction scheme, and fixing the number of neighbours behind and ahead in the Circular Neighbours scheme. The degree of spatial dependence is naturally bounded in the

Queen Contiguity weight matrix. To analyse the performance of the original QMLE when it is robust against heteroskedasticity, we use Queen Contiguity scheme and the `balanced Circular Neighbours` scheme where all spatial units have 6 peers each.

Heteroskedasticity: For the `unbalanced Circular Neighbour` scheme, $h_{n,i}$ is generated as the ratio of the total number of neighbours to the average number of neighbours for each i while for the Group Interaction scheme $h_{n,i}$ is generated as the ratio of the group size to mean group size. For the balanced Circular Neighbour and the Queen Contiguity schemes, we use $h_{n,i} = n[\sum_{i=1}^{n}(|X_{1n,i}| + |X_{2n,i}|)]^{-1}(|X_{1n,i}| + |X_{2n,i}|)$.

Regressors: The regressors are generated according to `REG1:` $\{x_{1i}, x_{2i}\} \overset{iid}{\sim} N(0,1)/\sqrt{2}$. For the Group Interaction scheme, the regressors can also be generated according to `REG2:` $\{x_{1,ir}, x_{2,ir}\} \overset{iid}{\sim} (2z_r + z_{ir})/\sqrt{10}$, where $(z_r, z_{ir}) \overset{iid}{\sim} N(0,1)$, for the ith observation in the rth group, to give a case of non-iid regressors taking into account the impact of group sizes on the regressors. Both schemes give a signal-to-noise ratio of 1 when $\beta_1 = \beta_2 = \sigma = 1$.

Error Distribution: To generate the e_n component of the disturbance term, three DGPs are considered: `DGP1:` $\{e_{n,i}\}$ are iid standard normal, `DGP2:` $\{e_{n,i}\}$ are iid standardized normal mixture with 10% of values from $N(0,4)$ and the remaining from $N(0,1)$ and `DGP3:` $\{e_{n,i}\}$ iid standardized log-normal with parameters 0 and 1. Thus, the error distribution from `DGP2` is leptokurtic, and that of `DGP3` is both skewed and leptokurtic.

The GMM-type estimators are implemented by closely following Lin & Lee (2010). A GMM estimator is in general defined as a solution to the minimisation problem: $\min_{\theta \in \Theta} g_n'(\theta) a_n' a_n g_n(\theta)$ where $g_n(\theta) = \big(Q_n, P_{1n}\epsilon_n(\theta), \ldots, P_{mn}\epsilon_n(\theta)\big)'\epsilon_n(\theta)$ represents the linear and quadratic moment conditions, $Q_n = (X_n, W_n X_n)$ is the matrix of instrumental variables (IVs), and $a_n' a_n$ is the weighting matrix related to the distance function of the minimisation problem. The GMME (Kelejian & Prucha, 1999; Lee, 2001) under homoskedastic disturbances can be defined using the usual moment condition, $P_n = \big(G_n - \frac{\text{tr}(G_n)}{n} I_n\big)$ and the IVs, $(G_n X_n \beta, X_n)$. For the RGMME, the P_n matrix in the moment conditions changes to $G_n - \text{diag}(G_n)$. A first step GMME with $P_n = W_n$ is used to evaluate G_n. The weighting matrices of the distance functions are computed using the variance formula of the iid case using residual estimates given by the first step GMM estimate. The ORGMME is a variant of the RGMME in which the weighting matrix is robust to unknown heteroskedasticity. The ORGMME results given in the tables are computed using the RGMME as the initial estimate to compute the standard error estimates and the instruments. Finally, the 2SLSE uses the same IV matrix Q_n. Lin and Lee (2010) gives a detailed comparison of the finite sample performance of MLE, GMME, RGMME, ORGMME and 2SLSE for models with both homoskedastic and heteroskedastic errors. Our Monte Carlo experiments expand theirs by giving a detailed investigation on the effects of nonnormality, spatial layouts as well as negative values for the spatial parameter. The RE of Jin and Lee (2012) is also included.

To conserve space, only the partial results of QMLE, MQMLE, RGMME and ORGMME are reported. The full set of results are available from the authors upon request. The GMME and 2SLSE can perform very poorly. The root estimator performs equally well as the MQMLE when $|\lambda|$ is not large and n is not small but tends to give non-real roots otherwise. Tables 1-3 summarise the estimation results for λ and Tables 4-6 for β, where in each table, the Monte Carlo means, root mean square errors (rmse) and the standard errors (se) of the estimators are reported. To analyse the finite sample performance of the proposed OPG based robust standard error estimators, we also report the averaged se of the regular QMLE when it is heteroskedasticity robust and the averaged se of the MQMLE based on Theorem 4. The experiments with $\beta = (0.3, 0.1, 0.1)$ represent cases where the stochastic component is relatively more dominant than the deterministic component of the model. This allows a comparison between the QML-type estimators and the GMM-type estimators when the model suffers from relatively more severe heteroskedasticity and the IVs are weaker. The main observations made from the Monte Carlo results are summarized as follows:

(i) MQMLE of λ performs well in all cases considered, and it generally outperforms all other estimators in terms of bias and rmse.[8] Further, in cases where QMLE is consistent, MQMLE can be significantly less biased than QMLE, and is as efficient as QMLE.

(ii) RGMME and ORGMME of λ perform reasonably well when $\beta = (3, 1, 1)'$, but deteriorates significantly when $\beta = (.3, .1, .1)'$ and in this case GMME and 2SLSE can be very erratic. In contrast, MQMLE is much less affected by the magnitude of β, and is less biased and more efficient than RGMME and ORGMME more significantly when $\beta = (.3, .1, .1)'$.

(iii) RE of λ performs equally well as MQMLE when $|\lambda|$ is not big and n is not small, but otherwise tends to give imaginary roots. Thus, when one encounters a super large dataset and the QMLE or MQMLE run into computational difficulty, one may turn to RE and use its closed-form expression.

(iv) The GMM-type estimators can perform quite differently when the errors are normal as opposed to non-normal errors, especially when $\beta = (.3, .1, .1)'$. It is interesting to note that RGMME often outperforms the ORGMME.

(v) The OPG-based estimate of the robust standard errors of MQMLE of λ performs well in general with their values very close to their Monte Carlo counter parts.

(vi) Finally, the relative performance of various estimators of β is much less contrasting than that of various estimators of λ, although it can be seen that MQMLE of β is slightly more efficient than the competing RGMME and ORGMME.

5. Extension to More General Models

As discussed in the introduction and Remark 2 of Section 3.1, the modified QML estimation method can be easily extended to suit for more general models (spatial or non-spatial) where

[8] A referee points out that under homoskedasticity, the GMM estimator can be as efficient as the MLE when errors are normal, and can be more efficient than the QMLE when the errors are nonnormal. See also Lee and Liu (2010). However, under heteroskedasticity, the latter is not observed from our extensive Monte Carlo Experiments. It would be interesting to carry out a theoretical comparison on the efficiency of the heteroskedasticity robust GMM-type and QML-type estimators, but such a study is clearly beyond the scope of this paper.

there are two or more concentrated score elements that need to be modified to account for the unknown heteroskedasticity. One popular example is the so-called SARAR(1,1) model, which extends the SAR model considered above by allowing the disturbances ϵ_n to follow a heteroskedastic SAR process. In this section, we first present a full set of 'feasible' results for the SARAR(1,1) model to facilitate immediate practical applications, and then discuss further possible extensions of the proposed methods. The SARAR(1,1) model takes the form,

$$Y_n = \lambda W_{1n} Y_n + X_n \beta + \epsilon_n, \quad \epsilon_n = \rho W_{2n} \epsilon_n + v_n, \tag{22}$$

where $v_{n,i} \sim inid(0, \sigma^2 h_{n,i})$ such that and $\frac{1}{n}\sum_{i=1}^n h_{n,i} = 1$. Let $A_n(\lambda) = I_n - \lambda W_{1n}$ and $B_n(\rho) = I_n - \lambda W_{2n}$, then the concentrated Gaussian loglikelihood function for $\delta = (\lambda, \rho)'$ is,

$$\ell_n^c(\delta) = -\frac{n}{2}[\ln(2\pi) + 1] - \frac{n}{2}\ln(\hat{\sigma}_n^2(\delta)) + \ln|A_n(\lambda)| + \ln|B_n(\rho)|, \tag{23}$$

where $\hat{\sigma}_n^2(\delta) = \frac{1}{n}Y_n'(\delta)M_n(\rho)Y_n(\delta)$, $M_n(\rho) = I_n - B_n(\rho)X_n[X_n'B_n'(\rho)B_n(\rho)X_n]^{-1}X_n'B_n'(\rho)$ and $Y_n(\delta) = B_n(\rho)A_n(\lambda)Y_n$. Maximizing (23) gives the QMLE $\hat{\delta}_n$ of δ, and thus the QMLE of β as $\hat{\beta}_n \equiv \hat{\beta}_n(\hat{\delta}_n)$ where $\hat{\beta}_n(\delta) = [X_n'B_n'(\rho)B_n(\rho)X_n]^{-1}X_n'B_n'(\rho)Y_n(\delta)$, and the QMLE of σ^2 as $\hat{\sigma}_n^2 \equiv \hat{\sigma}_n^2(\hat{\delta}_n)$. The concentrated score function upon dividing by n is,

$$\tilde{\psi}_n(\delta) = \begin{cases} -\frac{1}{n}\text{tr}(G_{1n}(\lambda)) + \dfrac{Y_n'(\delta)M_n(\rho)\bar{G}_{1n}(\delta)Y_n(\delta)}{Y_n'(\delta)M_n(\rho)Y_n(\delta)}, \\ -\frac{1}{n}\text{tr}(G_{2n}(\rho)) + \dfrac{Y_n'(\delta)M_n(\rho)\bar{G}_{2n}(\rho)Y_n(\delta)}{Y_n'(\lambda)M_n(\rho)Y_n(\delta)}, \end{cases} \tag{24}$$

where $\bar{G}_{1n}(\delta) = B_n(\rho)G_{1n}(\lambda)B_n^{-1}(\rho)$, $\bar{G}_{2n}(\rho) = G_{2n}(\rho)M_n(\rho)$, $G_{1n}(\lambda) = W_{1n}A_n^{-1}(\lambda)$, and $G_{2n}(\rho) = W_{2n}B_n^{-1}(\rho)$. Using similar arguments as given in Section 3, we have, after some algebraic manipulations, the following modified concentrated score function,

$$\tilde{\psi}_n^*(\delta) = \begin{cases} \dfrac{Y_n'(\delta)M_n(\rho)\bar{G}_{1n}^{\circ}(\delta)Y_n(\delta)}{Y_n'(\delta)M_n(\rho)Y_n(\delta)}, \\ \dfrac{Y_n'(\delta)M_n(\rho)\bar{G}_{2n}^{\circ}(\rho)Y_n(\delta)}{Y_n'(\delta)M_n(\rho)Y_n(\delta)}, \end{cases} \tag{25}$$

where $\bar{G}_{rn}^{\circ}(\delta) = \bar{G}_{rn}(\delta) - \text{diag}(M_n(\rho))^{-1}\text{diag}[M_n(\rho)\bar{G}_{rn}(\delta)], r = 1, 2$.

The modified QMLE of δ is defined as $\tilde{\delta}_n = \arg\{\tilde{\psi}_n^*(\delta) = 0\}$, and the modified QMLEs of β and σ^2 are $\tilde{\beta}_n \equiv \hat{\beta}_n(\tilde{\delta}_n)$ and $\tilde{\sigma}_n^2 \equiv \hat{\sigma}_n^2(\tilde{\delta}_n)$. To the best of our knowledge, the three-step estimator of Kelejian and Prucha (2010) may be the only heteroskedasticity robust estimator for the SARAR(1,1) model available in the literature.[9] Thus, it would be of a great interest to investigate and compare the finite sample properties of the three-step estimator and the proposed modified QMLE estimator for the SARAR(1,1) model. For brevity, Table 7 presents a small set of Monte Carlo results that serve such purposes, and more results are available from the authors. Both the reported and unreported Monte Carlo results show that the proposed

[9] Arraiz, et al. (2010) provide some additional details for this estimator including some Monte Carlo results.

modified QMLE has an excellent finite sample performance, and it outperforms the three-step estimator of Kelejian and Prucha (2010) from a combined consideration in terms of bias, consistency and efficiency.[10]

For heteroskedasticity robust inferences based on the SARAR(1,1) model, one needs the feasible heteroskedasticity robust estimators of the asymptotic variances of $\tilde{\delta}$ and $\tilde{\beta}_n$. Under an extended set of regularity conditions and using the multivariate CLT for linear-quadratic forms of Kelejian and Prucha (2010, Appendix A), we can show that as $n \to \infty$,

$$\sqrt{n}(\tilde{\delta}_n - \delta_0) \xrightarrow{D} N\big(0, \lim_{n\to\infty} \tau_n^2(\tilde{\delta}_n)\big), \quad \text{and} \quad \tau_n^2(\tilde{\delta}_n) = \Phi_n^{-1}\tau_n^2(\tilde{\psi}_n^*)\Phi_n^{-1}, \tag{26}$$

where Φ_n equals to $-\mathrm{E}[\frac{\partial}{\partial\delta_0'}\tilde{\psi}^*(\delta_0)]$ or its first-order term, and $\tau_n^2(\tilde{\psi}_n^*)$ is the first-order terms of $\mathrm{Var}[\sqrt{n}\tilde{\psi}^*(\delta_0)]$. Both Φ_n and $\tau_n^2(\tilde{\psi}_n^*)$ possess analytical expressions but are not needed for practical applications as the former can be estimated consistently by $\tilde{\Phi}_n = -\frac{\partial}{\partial\delta_0'}\tilde{\psi}^*(\delta_0)|_{\delta_0=\tilde{\delta}_n}$, and the latter by the following OPG estimator:

$$\tilde{\tau}_n^2(\tilde{\psi}_n^*) = \textstyle\sum_{i=1}^n \tilde{\epsilon}_{n,i}^2 \tilde{\Upsilon}_{n,i}\tilde{\Upsilon}_{n,i}', \tag{27}$$

where $\tilde{\Upsilon}_{n,i} = (\tilde{\zeta}_{1n,i} + \tilde{p}_{1n,ii}\tilde{\epsilon}_{n,i} + \tilde{c}_{1n,i},\, \tilde{\zeta}_{2n,i} + \tilde{p}_{2n,ii}\tilde{\epsilon}_{n,i} + \tilde{c}_{2n,i})'$, $\tilde{\zeta}_{rn} = (P_{rn}^{u\prime} + P_{rn}^l)\tilde{\epsilon}_n$, $r = 1,2$, $\tilde{\epsilon}_n = Y(\tilde{\delta}_n) - B_n(\tilde{\rho})X_n\tilde{\beta}_n$, and P_{rn} and c_{rn} are defined in the following asymptotic representation:

$$\sqrt{n}\tilde{\psi}_n^* = \begin{cases} \frac{1}{\sqrt{n}\sigma_0^2}\big(\epsilon_n' P_{1n}\epsilon_n + c_{1n}'\epsilon_n\big) + o_p(1), \\ \frac{1}{\sqrt{n}\sigma_0^2}\big(\epsilon_n' P_{2n}\epsilon_n + c_{2n}'\epsilon_n\big) + o_p(1), \end{cases} \tag{28}$$

where $P_{rn} = M_n\bar{G}_{rn}^\circ$ and $c_{rn} = M_n\bar{G}_{rn}^\circ B_n X_n\beta_0$, $r = 1,2$, with $p_{rn,ii}$, P_{rn}^u and P_{rn}^l denoting, respectively, the diagonal elements, the upper and lower triangular matrices of P_{rn}.

With the asymptotic results for $\tilde{\delta}_n$, one can easily derive the asymptotic results for $\tilde{\beta}_n$. Under a similar set of regularity conditions, we can show that as $n \to \infty$,

$$\sqrt{n}(\tilde{\beta}_n - \beta_0) \xrightarrow{D} N\big(0, \lim_{n\to\infty}(X_n'B_n'B_nX_n)^{-1}X_n'B_n'\mathbb{A}_nB_nX_n(X_n'B_n'B_nX_n)^{-1}\big), \tag{29}$$

where $\mathbb{A}_n = n\sigma_0^2 H_n + \tau_{n,11}^2(\tilde{\delta}_n)\eta_n\eta_n' + 2\sqrt{n}(\sigma_0^{-2}P_{1n}^d s_n + H_n c_{1n}, \sigma_0^{-2}P_{2n}^d s_n + H_n c_{2n})\Phi_n^{-1}(\eta_n, 0_n)'$, $s_n = \mathrm{E}(\epsilon_n^3)$, $P_{rn}^d = \mathrm{diag}(P_{rn})$, $\eta_n = B_nG_{1n}X_n\beta_0$, $\tau_{n,11}^2(\tilde{\delta}_n)$ is the top-right corner element of $\tau_n^2(\tilde{\delta}_n)$, and 0_n is an $n\times 1$ vector of 0's. With the estimates $\tilde{\Phi}_n$ and $\tilde{\tau}_n^2(\tilde{\psi}_n^*)$ defined above, the estimates $\tilde{s}_n = \tilde{\epsilon}_n^3$ and $\tilde{H}_n = \tilde{\sigma}_n^{-2}\mathrm{diag}(\tilde{\epsilon}_n^2)$ of s_n and H_n, and the plug-in estimates for the remaining quantities, a consistent estimate for $\tau_n^2(\tilde{\beta}_n^*)$ follows.

The proposed methods can be further extended. For example, the SARAR(p,q), which contains spatial lags of order p and spatial autoregressive errors of order q, can be dealt with in a similar manner as for the SARAR(1,1) model. To have an idea on how our methods can be extended

[10] A more rigorous comparison may be interesting but beyond the scope of this paper. The robust GMM approach of Lin and Lee (2010) may lead to a more efficient estimator than does the three-step approach of Kelejian and Prucha (2010), but from Lin and Lee (2010) it is not clear how to extend their robust GMM estimation approach for the SAR to the general SARAR(1,1) model.

to the SARAR(p, q) model, note that the Gaussian likelihood takes an identical form as (24) for SARAR(1,1), except that now $A_n(\lambda) = I_n - \sum_{j=1}^{p} \lambda_j W_{1,jn}$ and $B_n(\rho) = I_n - \sum_{j=1}^{q} \rho_j W_{2,jn}$, $\lambda = \{\lambda_1, \ldots, \lambda_p\}$ and $\rho = \{\rho_1, \ldots, \rho_q\}$, see Lee and Liu (2010). Thus, the concentrated scores and their modifications can be found in a similar manner, resulting modified QMLEs for the SARAR(p, q) model that are robust against unknown heteroskedasticity.[11] Moving further, our methods can be applied to give heteroskedasticity robust estimator for the fixed effects spatial panel data model. As argued in the introduction, heteroskedasticity is common particularly in spatial models. This makes it more desirable to develop heteroskedasticity robust inference methods for these models. The methods proposed in this paper shed much light on these intriguing research problems. However, formal studies on these models, including detailed proofs of the results (26)-(29) and the proofs of consistency of the variance estimates therein, are beyond the scope of this paper, and will be pursued in future research.

6. Conclusion

This paper looks at heteroskedasticity robust QML-type estimation for spatial autoregressive (SAR) models. We provide clear conditions for the regular QMLE to be consistent even when the disturbances suffer from heteroskedasticity of unknown form. When these conditions are violated, the regular QMLE becomes inconsistent and in this case we suggest a modified QMLE by making a simple adjustment to the score function so that it becomes robust to unknown heteroskedasticity. This method is proven to work well in the simulation studies and was shown to be robust to many situations including, deteriorated signal strength as well as nonnormal errors (besides the unknown heteroskedasticity). To provide inference methods robust to heteroskedasticity and nonnormality, OPG-based estimators of the variances of QMLE and modified QMLE are introduced. Monte Carlo results show that the proposed modified QMLE for the SAR model and the associated robust variance estimator work very well in finite samples.

The proposed methodology (modifying score for achieving heteroskedasticity robustness for parameter estimation and finding a suitable OPG for achieving heteroskedasticity robustness for variance estimation) has a great potential to be extended to more general models, not necessarily the spatial models, thus paving a simple way for developing heteroskedasticity robust inference methods for applied researchers.

[11] Lee and Liu (2010) proposed efficient GMM estimation of this model under homoskedasticity assumption. Badinger and Egger (2011) extend the estimation strategy of Kelejian and Prucha (2010) to give a heteroskedasticity robust three-step estimator of the SARAR(p, q) model, where some Monte Carlo results are presented under a SARAR(3,3) model and some special spatial weight matrices.

Appendix A: Some Useful Lemmas

The following lemmas are extended versions of the selected lemmas from Lee (2004), Kelejian and Prucha (2001) and Lin and Lee (2010), which are required in the proofs of the main results.

Lemma A.1: *Suppose the matrix of independent variables X_n has uniformly bounded elements, then the projection matrices $P_n = X_n(X_n'X_n)^{-1}X_n'$ and $M_n = I_n - P_n$ are uniformly bounded in both row and column sums.*

Lemma A.2: *Let A_n be an $n \times n$ matrix, uniformly bounded in both row and column sums. Then for M_n defined in Lemma A.1,*
(i) $\text{tr}(A_n^m) = O(n)$ *for* $m \geq 1$,
(ii) $\text{tr}(A_n'A_n) = O(n)$,
(iii) $\text{tr}((M_nA_n)^m) = \text{tr}(A_n^m) + O(1)$ *for* $m \geq 1$ *and*
(iv) $\text{tr}((A_n'M_nA_n)^m) = \text{tr}((A_n'A_n)^m) + O(1)$ *for* $m \geq 1$.
Let B_n be another $n \times n$ matrix, uniformly bounded in both row and column sums. Then,
(iv) A_nB_n *is uniformly bounded in both row and column sums,*
(v) $\text{tr}(A_nB_n) = \text{tr}(B_nA_n) = O(n)$ *uniformly.*

Lemma A.3 (Moments and Limiting Distribution of Quadratic Forms): *For a given process of innovations $\{\epsilon_{n,i}\}$, let $\epsilon_{n,i} \sim inid(0, \sigma_{n,i}^2)$, where $\sigma_{n,i}^2 = \sigma_0^2 h_{n,i}$, $h_{n,i} > 0$ for $i = 1, \ldots, n$ such that $\frac{1}{n}\sum_{i=1}^n h_{n,i} = 1$. Let $H_n = diag(h_{n,1}, \ldots, h_{n,n})$, B_n be an $n \times n$ matrix of elements $b_{n,ij}$, and c_n an $n \times 1$ vector of elements $c_{n,i}$. For $Q_n = \epsilon_n'B_n\epsilon_n + c_n'\epsilon_n$,*
(i) $\text{E}(Q_n) = \sigma_0^2\text{tr}(H_nB_n)$ *and*
(ii) $\text{Var}(Q_n) = \sum_{i=1}^n(\sigma_{n,i}^4 b_{n,ii}^2\kappa_{n,i} + 2\sigma_{n,i}^3 b_{n,ii}c_{n,i}\gamma_{n,i}) + \sigma_0^4 tr[H_nB_n(H_nB_n + B_n'H_n)] + \sigma_0^2 c_n'H_nc_n$,
where $\gamma_{n,i}$ and $\kappa_{n,i}$ are, respectively, the skewness and excess kurtosis of $\epsilon_{n,i}$. Now, if B_n is uniformly bounded in either row or column sums then,
(iii) $\text{E}(Q_n) = O(n)$,
(iv) $\text{Var}(Q_n) = O(n)$,
(v) $Q_n = O_p(n)$,
(vi) $\frac{1}{n}Q_n - \frac{1}{n}\text{E}(Q_n) = O_p\big(n^{-\frac{1}{2}}\big)$ *and*
(vii) $\text{Var}(\frac{1}{n}Q_n) = O(n^{-1})$.
Further, if B_n is uniformly bounded in both row and column sums and Assumption 4 holds then,
(viii) $\frac{Q_n - \text{E}(Q_n)}{\sqrt{\text{Var}(Q_n)}} \xrightarrow{D} N(0, 1)$.

Appendix B: Proofs of Theorems and Corollaries

Proof of Theorem 1: We only prove the consistency of $\hat{\lambda}_n$ as the consistency of $\hat{\beta}_n$ and $\hat{\sigma}_n^2$ immediately follows from identities similar to (14) and (15). Define $\bar{\ell}_n^c(\lambda) = \max_{\beta,\sigma^2} \text{E}[\ell_n(\theta)]$. By Theorem 5.7 of van der Vaart (1998), it amounts to show, (a) identification uniqueness condition: $\sup_{\lambda:d(\lambda,\lambda_0)\geq\epsilon} \frac{1}{n}[\bar{\ell}_n^c(\lambda) - \bar{\ell}_n^c(\lambda_0)] < 0$ for any $\epsilon > 0$ and a distance measure $d(\lambda, \lambda_0)$ and (b) uniform convergence: $\frac{1}{n}[\ell_n^c(\lambda) - \bar{\ell}_n^c(\lambda)] \xrightarrow{p} 0$ uniformly in $\lambda \in \Lambda$.

It is easy to see that $\bar{\ell}^c_n(\lambda) = -\frac{n}{2}(\ln(2\pi)+1) - \frac{n}{2}\ln(\bar{\sigma}^2_n(\lambda)) + \ln|A_n(\lambda)|$, where $\bar{\sigma}^2_n(\lambda) = \frac{1}{n}\big[(\lambda_0-\lambda_n)^2\eta_n' M_n\eta_n + \sigma_0^2\text{tr}[H_n A_n'^{-1}A_n'(\lambda)A_n(\lambda)A_n^{-1}]\big]$. Recall $\ell^c_n(\lambda)$ defined in (3).

Condition (a)**:** Observe that $\bar{\sigma}^2_n(\lambda_0) = \sigma_0^2$, then,

$$
\begin{aligned}
& \lim_{n\to\infty} \tfrac{1}{n}\big[\bar{\ell}^c_n(\lambda) - \bar{\ell}^c_n(\lambda_0)\big] \\
= \;& \lim_{n\to\infty} \big[\tfrac{1}{2n}(\log|A_n'(\lambda)A_n(\lambda)| - \log|A_n'A_n|) + \tfrac{1}{2n}(\log|\sigma_n^{-2}(\lambda)I_n| - \log|\sigma_0^{-2}I_n|)\big] \\
\neq \;& 0 \text{ for } \lambda\neq\lambda_0, \text{ by Assumption 6.}
\end{aligned}
$$

Next, note that $p_n(\theta_0) = \exp[\ell_n(\theta_0)]$ is the *quasi* joint pdf of ϵ_n, which is $N(0,\sigma^2 I_n)$. Let $p^0_n(\theta_0)$ be the *true* joint pdf of $\epsilon_n \sim (0,\sigma^2 H_n)$. Let E^q denote the expectation with respect to $p_n(\theta_0)$, to differentiate from the usual notation E that corresponds to $p^0_n(\theta_0)$.

Now consider $\epsilon_n(\beta,\lambda) = A_n(\lambda)Y_n - X_n\beta = B_n(\lambda)\epsilon_n + b_n(\beta,\lambda)$, where $B_n(\lambda) = A_n(\lambda)A_n^{-1}$ and $b_n(\beta,\lambda) = A_n(\lambda)A_n^{-1}X_n\beta_0 - X_n\beta$. Then, with $\ell_n(\theta)$ given in (2), we have

$$
\begin{aligned}
\text{E}^q[\ell_n(\theta_0)] &= -\tfrac{n}{2}\ln(2\pi\sigma^2) + \ln|A_n| - \tfrac{n}{2}, \\
\text{E}[\ell_n(\theta_0)] &= -\tfrac{n}{2}\ln(2\pi\sigma^2) + \ln|A_n| - \tfrac{n}{2}, \text{ as } \tfrac{1}{n}\textstyle\sum_{i=1}^n h_{n,i} = 1 \\
\text{E}^q[\ell_n(\theta)] &= -\tfrac{n}{2}\ln(2\pi\sigma^2) + \ln|A_n(\lambda)| - \tfrac{1}{2\sigma^2}[\sigma_0^2\text{tr}(B_n'(\lambda)B_n(\lambda)) + b_n'(\beta,\lambda)b_n(\beta,\lambda)], \\
\text{E}[\ell_n(\theta)] &= -\tfrac{n}{2}\ln(2\pi\sigma^2) + \ln|A_n(\lambda)| - \tfrac{1}{2\sigma^2}[\sigma_0^2\text{tr}(H_nB_n'(\lambda)B_n(\lambda)) + b_n'(\beta,\lambda)b_n(\beta,\lambda)],
\end{aligned}
$$

where we have used the identities, $B_n(\lambda_0) = I_n$ and $b_n(\beta_0,\lambda_0) = 0$. Now using the identities $A_n(\lambda) = A_n + (\lambda_0-\lambda)W_n$ and $B_n(\lambda) = I_n + (\lambda_0-\lambda)G_n$, we have,

$$
\begin{aligned}
& \text{E}[\ell_n(\theta)] - \text{E}^q[\ell_n(\theta)] \\
& = 2(\lambda_0-\lambda)[\text{tr}(H_nG_n) - tr(G_n)] + (\lambda_0-\lambda)^2[\text{tr}(H_nG_n'G_n) - \text{tr}(G_n'G_n)] = o(1),
\end{aligned}
$$

where the last equality holds by assumptions $\text{Cov}(g_n,h_n) = o(1)$ and $\text{Cov}(q_n,h_n) = o(1)$.

Now by Jensen's inequality, $0 = \log\text{E}^q\big(\frac{p_n(\theta)}{p_n(\theta_0)}\big) \geq \text{E}^q\big[\log\big(\frac{p_n(\theta)}{p_n(\theta_0)}\big)\big]$, and the above results, we conclude that $\text{E}\big[\log\big(\frac{p_n(\theta)}{p_n(\theta_0)}\big)\big] \leq 0$ or $\text{E}[\log p_n(\theta)] \leq \text{E}[\log p_n(\theta_0)]$, for large enough n. Thus,

$$
\bar{\ell}_n(\lambda) = \max_{\beta,\sigma^2}\text{E}[\log p_n(\theta)] \leq \max_{\beta,\sigma^2}\text{E}[\log p_n(\theta_0)] = \text{E}[\log p_n(\theta_0)] = \bar{\ell}_n(\lambda_0), \quad \text{for } \lambda\neq\lambda_0,
$$

and n large enough. The identification uniqueness condition thus follows.

Condition (b)**:** Note that $\frac{1}{n}[\ell^c_n(\lambda) - \bar{\ell}^c_n(\lambda)] = -\frac{1}{2}[\log(\hat{\sigma}^2_n(\lambda)) - \log(\bar{\sigma}^2_n(\lambda))]$. By the mean value theorem, $\log(\hat{\sigma}^2_n(\lambda)) - \log(\bar{\sigma}^2_n(\lambda)) = \frac{1}{\dot{\sigma}^2_n(\lambda)}[\hat{\sigma}^2_n(\lambda) - \bar{\sigma}^2_n(\lambda)]$, where $\dot{\sigma}^2_n(\lambda)$ lies between $\hat{\sigma}^2_n(\lambda)$ and $\bar{\sigma}^2_n(\lambda)$. Using $M_nA_n(\lambda)Y_n = (\lambda_0-\lambda)M_n\eta_n + M_nA_n(\lambda)A_n^{-1}\epsilon_n$ we can write,

$$
\hat{\sigma}^2_n(\lambda) = (\lambda_0-\lambda)^2\tfrac{1}{n}\eta_n'M_n\eta_n + 2(\lambda_0-\lambda)T_{1n}(\lambda) + T_{2n}(\lambda), \tag{B-1}
$$

where $T_{1n}(\lambda) = \frac{1}{n}\eta_n'M_nA_n(\lambda)A_n^{-1}\epsilon$ and $T_{2n}(\lambda) = \frac{1}{n}\epsilon_n'A_n^{-1}A_n'(\lambda)M_nA_n(\lambda)A_n^{-1}\epsilon_n$.

Using $A_n(\lambda) = A_n + (\lambda_0-\lambda)W_n$, we have, $T_{1n}(\lambda) = o_p(1)$ uniformly. Further, $T_{2n}(\lambda) = \frac{1}{n}\epsilon_n'A_n^{-1}A_n'(\lambda)A_n(\lambda)A_n^{-1}\epsilon_n + o_p(1)$, since, $\frac{1}{n}\epsilon_n'A_n^{-1}A_n'(\lambda)P_nA_n(\lambda)A_n^{-1}\epsilon_n = \frac{1}{n}[\epsilon_n'P_n\epsilon + 2\epsilon_n'G_n'P_n\epsilon_n + \epsilon_n'G_n'P_nG_n\epsilon_n] = o_p(1)$ uniformly, using the condition $\text{Cov}(h_n,g_n) = o(1)$. Now, Lemmas A.1 -

A.3 imply, $\frac{1}{n^2}\mathrm{Var}(\epsilon_n' A_n^{-1} A_n'(\lambda) A_n(\lambda) A_n^{-1}\epsilon_n) = o(1)$. Then, together with Chebyshev inequality, $T_{2n}(\lambda) - \sigma_0^2 \frac{1}{n}\mathrm{tr}[H_n A_n'^{-1} A_n'(\lambda) A_n(\lambda) A_n^{-1}] = o_p(1)$, uniformly for $\lambda \in \Lambda$.

It left to show $\sigma_n^2(\lambda)$ (defined in Assumption 6 and the main part of $\bar{\sigma}_n^2(\lambda)$) is uniformly bounded away from zero. Suppose $\sigma_n^2(\lambda)$ is not uniformly bounded away from zero. Then $\exists\{\lambda_n\} \subset \Lambda$ such that $\sigma_n^2(\lambda_n) \to 0$. Consider the model with $\beta_0 = 0$. The Gaussian log-likelihood is $\ell_{t,n}(\theta) = -\frac{n}{2}\log(2\pi\sigma^2) + \log|A_n(\lambda)| - \frac{1}{2\sigma^2} Y_n' A_n'(\lambda) A_n(\lambda) Y_n$ and $\bar{\ell}_{t,n}(\lambda) = \max_{\sigma^2} \mathrm{E}[\ell_{t,n}(\theta)]$. By Jensen's inequality, we have $\bar{\ell}_{t,n}(\lambda) \le \max_{\sigma^2} \mathrm{E}[\ell_{t,n}(\theta_0)] = \bar{\ell}_{t,n}(\lambda_0)$. Then together with Lemma A.2, we have $\frac{1}{n}[\bar{\ell}_{t,n}(\lambda) - \bar{\ell}_{t,n}(\lambda_0)] \le 0$, and $-\frac{n}{2}\log(\sigma_n^2(\lambda)) \le -\frac{n}{2}\log(\sigma_0^2) + \frac{1}{n}(\log|A_n(\lambda_0)| - \log|A_n(\lambda)|) = O(1)$. That is, $-\frac{n}{2}\log(\sigma_n^2(\lambda))$ is bounded from above which is a contradiction. Hence, $\sigma_n^2(\lambda)$ is bounded away from zero uniformly, and $\frac{n}{2}\log(\sigma_n^2(\lambda))$ is well defined $\forall \lambda \in \Lambda$.

Collecting all these results we have, $\sup_{\lambda\in\Lambda}\frac{1}{n}|[\ell_n^c(\lambda) - \bar{\ell}_n^c(\lambda)]| = o_p(1)$, completing the proof of consistency part.

To prove the asymptotic normality, first note that $\mathrm{tr}(H_n) = n$. By the mean value theorem, $\sqrt{n}(\hat{\theta}_n - \theta_0) = -\left[\frac{1}{n}\frac{\partial^2}{\partial\theta\partial\theta'}\ell_n(\tilde{\theta})\right]^{-1}\frac{1}{\sqrt{n}}\frac{\partial}{\partial\theta}\ell_n(\theta_0)$, where $\tilde{\theta}_n$ lies elementwise between $\hat{\theta}_n$ and θ_0. By Assumptions 1-6, the condition $\mathrm{Cov}(g_n, h_n) = o(n^{-1/2})$, and the CLT for vector linear-quadratic forms of Kelejian and Prucha (2010, p. 63), we have $\frac{1}{\sqrt{n}}\frac{\partial}{\partial\theta}\ell_n(\theta_0) \xrightarrow{D} N(0, \Sigma)$, where Σ is defined in the theorem.

Let $\mathcal{H}_n(\theta) = \frac{\partial^2}{\partial\theta\partial\theta'}\ell_n(\theta)$. It left to show (i) $\frac{1}{n}\mathcal{H}_n(\tilde{\theta}_n) - \mathcal{H}_n = o_p(1)$ and (ii) $\mathcal{H}_n - \mathbb{I}_n = o_p(1)$.

Condition (i)**:** By Assumptions 3-5 and the assumption that $\mathrm{Cov}(h_n, g_n) = o(1)$ stated in the theorem, Lemma A.2-A.3, $\tilde{\theta}_n - \theta_0 = o_p(1)$, $\epsilon_n(\tilde{\beta}_n, \tilde{\lambda}_n) = X_n(\beta_0 - \tilde{\beta}_n) + (\lambda_0 - \tilde{\lambda}_n) W_n Y_n + \epsilon_n$ and $\frac{1}{n}\epsilon_n'(\tilde{\beta}_n, \tilde{\lambda}_n)\epsilon_n(\tilde{\beta}_n, \tilde{\lambda}_n) = \frac{1}{n}\epsilon_n'\epsilon_n + o_p(1)$, we have,

$$
\begin{aligned}
\mathcal{H}_{n,\beta\beta}(\tilde{\theta}_n) - \mathcal{H}_{n,\beta\beta} &= \left(\tfrac{1}{\sigma_0^2} - \tfrac{1}{\tilde{\sigma}_n^2}\right)\tfrac{1}{n} X_n' X_n = o_p(1),\\
\mathcal{H}_{n,\sigma^2\beta}(\tilde{\theta}_n) - \mathcal{H}_{n,\sigma^2\beta} &= \tfrac{1}{\sigma_0^4 n}\epsilon_n' X_n - \tfrac{1}{\tilde{\sigma}_0^4 n}(X_n(\beta_0 - \tilde{\beta}_n) + (\lambda_0 - \tilde{\lambda}_n) W_n Y_n + \epsilon_n)' X_n = o_p(1),\\
\mathcal{H}_{n,\sigma^2\sigma^2}(\tilde{\theta}_n) - \mathcal{H}_{n,\sigma^2\sigma^2} &= \tfrac{1}{n}\left(\tfrac{1}{\sigma_0^6}\epsilon_n'\epsilon_n - \tfrac{1}{\tilde{\sigma}_n^6}\epsilon_n'(\tilde{\delta}_n)\epsilon_n(\tilde{\delta}_n)\right) - \tfrac{1}{2}\left(\tfrac{1}{\sigma_0^4} - \tfrac{1}{\tilde{\sigma}_n^4}\right) = o_p(1),\\
\mathcal{H}_{n,\lambda\beta}(\tilde{\theta}_n) - \mathcal{H}_{n,\lambda\beta} &= \left(\tfrac{1}{\sigma_0^2} - \tfrac{1}{\tilde{\sigma}_n^2}\right)\tfrac{1}{n} Y_n' W_n' X_n = o_p(1),\\
\mathcal{H}_{n,\lambda\sigma^2}(\tilde{\theta}_n) - \mathcal{H}_{n,\lambda\sigma^2} &= \tfrac{1}{\sigma_0^4 n} Y_n' W_n' \epsilon_n - \tfrac{1}{\tilde{\sigma}_0^4 n} Y_n' W_n'(X_n(\beta_0 - \tilde{\beta}_n) + (\lambda_0 - \tilde{\lambda}_n) W_n Y_n + \epsilon_n) = o_p(1),\\
\mathcal{H}_{n,\lambda\lambda}(\tilde{\theta}_n) - \mathcal{H}_{n,\lambda\lambda} &= \left(\tfrac{1}{\sigma_0^2} - \tfrac{1}{\tilde{\sigma}_n^2}\right)\tfrac{1}{n} Y_n' W_n' W_n Y_n + \tfrac{1}{n}\mathrm{tr}(G_n^2) - \mathrm{tr}(G_n^2(\tilde{\lambda}_n)) = o_p(1),
\end{aligned}
$$

where the last equality holds since $\mathrm{tr}(G_n^2) - \mathrm{tr}(G_n^2(\tilde{\lambda}_n)) = 2\mathrm{tr}(G_n^2(\bar{\lambda}_n))(\lambda_0 - \tilde{\lambda}_n)$ by the mean value theorem for some $\bar{\lambda}_n$ between λ_0 and $\tilde{\lambda}_n$.

Condition (ii)**:** Given $\mathrm{E}(\epsilon_n'\epsilon_n) = \sigma_0^2\mathrm{tr}(H_n)$, $\mathrm{E}(\epsilon_n' G_n \epsilon_n) = \sigma_0^2\mathrm{tr}(H_n G_n)$, $\mathrm{E}(\epsilon_n' G_n' G_n \epsilon_n) = \sigma_0^2\mathrm{tr}(H_n G_n' G_n)$ and Lemma A.1-A.3, we have, $\mathrm{Var}(\frac{1}{n}\epsilon_n'\epsilon_n) = \frac{1}{n^2}\left(\mathrm{E}(\epsilon_{n,i}^4) - \sigma_0^4\mathrm{tr}(H_n^2)\right) = o(1)$, $\mathrm{Var}(\frac{1}{n}\epsilon_n' G_n \epsilon_n) = \frac{1}{n^2}\sum_{i=1}^n g_{n,ii}^2[\mathrm{E}(\epsilon_{n,i}^4) - 3\sigma_0^4 h_i^2] + \frac{1}{n^2}\sigma_0^4\mathrm{tr}[H_n G_n(G_n' H_n + H_n G_n)] = o(1)$ and similarly $\mathrm{Var}(\frac{1}{n}\epsilon_n' G_n' G_n \epsilon_n) = o_p(1)$. By these results and Chebyshev inequality, we have,

$$
\begin{aligned}
\mathcal{H}_{n,\beta\beta} - \mathbb{I}_{n,\beta\beta} &= 0,\\
\mathcal{H}_{n,\sigma^2\beta} - \mathbb{I}_{n,\sigma^2\beta} &= O_p\left(\tfrac{1}{\sqrt{n}}\right) = o_p(1),\\
\mathcal{H}_{n,\sigma^2\sigma^2} - \mathbb{I}_{n,\sigma^2\sigma^2} &= \tfrac{1}{\sigma_0^6}\left(\tfrac{\epsilon_n'\epsilon_n}{n} - \sigma_0^2\right) = o_p(1),\\
\mathcal{H}_{n,\lambda\beta} - \mathbb{I}_{n,\lambda\beta} &= \tfrac{1}{n} X_n' G_n \epsilon_n = O_p\left(\tfrac{1}{\sqrt{n}}\right) = o_p(1),
\end{aligned}
$$

$$\begin{aligned}\mathcal{H}_{n,\lambda\sigma^2} - \mathbb{I}_{n,\lambda\sigma^2} &= \tfrac{1}{\sigma_0^4 n}\epsilon_n' G_n \epsilon_n - \tfrac{1}{\sigma_0^2 n}\text{tr}(H_n G_n) + O_p(\tfrac{1}{\sqrt{n}}) = o_p(1) \text{ and}\\ \mathcal{H}_{n,\lambda\lambda} - \mathbb{I}_{n,\lambda\lambda} &= \tfrac{1}{n}\epsilon_n' G_n' G_n \epsilon_n - \tfrac{1}{n}\text{tr}(H_n G_n' G_n) + O_p(\tfrac{1}{\sqrt{n}}) = o_p(1).\end{aligned}$$

Proof of Theorem 2: Let $\text{E}(\tilde{\psi}_n^*(\lambda)) = \bar{\psi}^*(\lambda)$. By Theorem 5.9 of van der Vaart (1998), the proof of consistency of $\tilde{\lambda}_n$ requires (a) Convergence: $\sup_{\lambda\in\Lambda}|\tilde{\psi}_n^*(\lambda) - \bar{\psi}^*(\lambda)| = o_p(1)$ and (b) Identification uniqueness: for $\epsilon > 0$, $\inf_{\lambda:d(\lambda,\lambda_0)\geq\epsilon}|\bar{\psi}^*(\lambda)| > 0 = |\bar{\psi}^*(\lambda_0)|$.

The proof of Theorem 1 implies that $\hat{\sigma}_n^2(\lambda)$ is bounded away from 0 with probability one for large enough n. Thus, the modified QML estimator $\tilde{\lambda}_n = \arg\{\tilde{\psi}_n^*(\lambda) = 0\}$ is equivalently defined as $\tilde{\lambda}_n = \arg\{Y_n' A_n'(\lambda) M_n G_n^\circ(\lambda) A_n(\lambda) Y_n = 0\}$, suggesting that we can work purely with the numerator $T_n(\lambda) = Y_n' A_n'(\lambda) M_n G_n^\circ(\lambda) A_n(\lambda) Y_n$ of $\tilde{\psi}_n^*(\lambda)$ to establish consistency. Note $T_n(\lambda) = Y_n' A_n'(\lambda) M_n G_n(\lambda) A_n(\lambda) Y_n - Y_n' A_n'(\lambda) M_n \text{diag}(M_n)^{-1}\text{diag}(M_n G_n(\lambda)) A_n(\lambda) Y_n \equiv T_{1n}(\lambda) - T_{2n}(\lambda)$.

Condition (a): By $M_n X_n = 0$, $A_n(\lambda) = A_n + (\lambda_0 - \lambda) W_n$ and $G_n A_n = W_n = G_n(\lambda) A_n(\lambda)$,

$$\begin{aligned}T_{1n}(\lambda) &= Y_n' A_n'(\lambda) M_n G_n(\lambda) A_n(\lambda) Y_n\\ &= Y_n' A_n' M_n G_n A_n Y_n + (\lambda_0 - \lambda) Y_n' A_n' G_n' M_n G_n A_n Y_n\\ &= \epsilon_n' M_n G_n (X_n\beta_0 + \epsilon_n) + (\lambda_0 - \lambda)(X_n\beta_0 + \epsilon_n)' G_n' M_n G_n (X_n\beta_0 + \epsilon_n). \quad \text{(B-2)}\end{aligned}$$

Then, $\text{E}(T_{1n}(\lambda)) = (\lambda_0 - \lambda)\beta_0' X_n G_n' M_n G_n X_n \beta_0 + \sigma_0^2 \text{tr}(H_n M_n G_n) + \sigma_0^2(\lambda_0 - \lambda)\text{tr}(H_n G_n' M_n G_n)$. By Lemma A.3 and Assumptions 5 and 6, we have $\frac{1}{n}[T_{1n}(\lambda) - \text{E}(T_{1n}(\lambda))] = o_p(1)$. Now, as M_n appeared in T_{2n} is a projection matrix, by Lemma A.2, similar arguments as for $T_{1n}(\lambda)$ lead to $\frac{1}{n}[T_{2n}(\lambda) - \text{E}(T_{2n}(\lambda))] = o_p(1)$. Thus, $\frac{1}{n}\{T_n(\lambda) - \text{E}[T_n(\lambda)]\} = o_p(1)$.

Condition (b): First, we have $\text{E}[T_n(\lambda_0)] = 0$, as $\text{tr}[H_n M_n \text{diag}(M)^{-1}\text{diag}(M_n G_n)] = \text{tr}[\text{diag}(H_n M_n \text{diag}(M)^{-1})\text{diag}(M_n G_n)] = \text{tr}(H_n M_n G_n)$. Now,

$$\text{E}[T_n(\lambda)] = \beta_0' X_n' A_n'^{-1} A_n'(\lambda) M_n G_n^\circ(\lambda) A_n(\lambda) A_n^{-1} X_n \beta_0 + \sigma_0^2 \text{tr}(H_n A_n'^{-1} A_n'(\lambda) M_n G_n^\circ(\lambda) A_n(\lambda) A_n^{-1}).$$

By Assumption 6* and Lemma A.2, $\text{E}[T_n(\lambda)] \neq 0$, for any $\lambda \neq \lambda_0$. It follows that the conditions of Theorem 5.9 of van der Vaart (1998) hold, and thus the consistency of $\tilde{\lambda}_n$ follows.

To prove asymptotic normality, we have, by the mean value theorem,

$$0 = \sqrt{n}\tilde{\psi}_n^*(\tilde{\lambda}_n) = \sqrt{n}\tilde{\psi}_n^*(\lambda_0) + \tfrac{d}{d\lambda}\tilde{\psi}_n^*(\bar{\lambda}_n)\sqrt{n}(\tilde{\lambda}_n - \lambda_0), \quad \text{(B-3)}$$

where $\bar{\lambda}_n$ lies between $\tilde{\lambda}_n$ and λ_0. It suffices to show that (i) $\frac{d}{d\lambda}\tilde{\psi}_n^*(\bar{\lambda}_n) - \frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0) = o_p(1)$, (ii) $\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0) - \text{E}\big(\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0)\big) = o_p(1)$, and (iii) $\text{E}\big(\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0)\big) \neq 0$ for large enough n. Note,

$$\begin{aligned}&\tfrac{d}{d\lambda}\tilde{\psi}_n^*(\lambda) = \tfrac{1}{n\hat{\sigma}_n^2(\lambda)} Y_n' A_n'(\lambda) \dot{G}_n^{\circ\prime}(\lambda) M_n A_n(\lambda) Y_n - \tfrac{1}{n\hat{\sigma}_n^2(\lambda)} Y_n' W_n' G_n^{\circ\prime}(\lambda) M_n A_n(\lambda) Y_n\\ &-\tfrac{1}{n\hat{\sigma}_n^2(\lambda)} Y_n' A_n'(\lambda) G_n^{\circ\prime}(\lambda) M_n W_n Y_n + \tfrac{2}{n^2\hat{\sigma}_n^4(\lambda)} Y_n' A_n'(\lambda) G_n^{\circ\prime}(\lambda) M_n A_n(\lambda) Y_n \cdot Y_n' W_n' M_n A_n(\lambda) Y_n,\end{aligned}$$

where $\dot{G}_n^\circ(\lambda) = \frac{d}{d\lambda}G_n^\circ(\lambda) = G_n^2(\lambda) - \text{diag}(M_n)^{-1}\text{diag}(M_n G_n^2(\lambda))$.

Condition (i): $\frac{1}{n}Y_n' W_n' M_n A_n(\bar{\lambda}_n) Y_n = \frac{1}{n}Y_n' W_n' M_n A_n Y_n + \frac{1}{n}(\lambda_0 - \bar{\lambda}_n) Y_n' W_n' M_n W_n Y_n = \frac{1}{n}Y_n' W_n' M_n A_n Y_n + o_p(1)$. Next, by Assumptions 4 and 5 and continuous mapping theorem,

$G_n^\circ(\bar{\lambda}_n) = G_n^\circ + o_p(1)$ and $\dot{G}_n^\circ(\bar{\lambda}_n) = \dot{G}_n^\circ + o_p(1)$. These lead to $\frac{1}{n}Y_n'A_n'(\bar{\lambda}_n)G_n^{\circ\prime}(\bar{\lambda}_n)M_nA_n(\bar{\lambda}_n)Y_n = \frac{1}{n}Y_n'A_n'G_n^{\circ\prime}M_nA_nY_n + o_p(1)$, and $\frac{1}{n}Y_n'A_n'(\bar{\lambda}_n)\dot{G}_n^{\circ\prime}(\bar{\lambda}_n)M_nA_n(\bar{\lambda}_n)Y_n = \frac{1}{n}Y_n'A_n'\dot{G}_n^{\circ\prime}M_nA_nY_n + o_p(1)$, after some algebra. Similarly, $\frac{1}{n}Y_n'W_n'G_n^{\circ\prime}(\bar{\lambda}_n)M_nA_n(\bar{\lambda}_n)Y_n = \frac{1}{n}Y_n'W_n'G_n^{\circ\prime}M_nA_nY_n + o_p(1)$, and $\frac{1}{n}Y_n'A_n'(\bar{\lambda}_n)G_n^{\circ\prime}(\bar{\lambda}_n)M_nW_nY_n = \frac{1}{n}Y_n'A_n'G_n^{\circ\prime}M_nW_nY_n + o_p(1)$. Collecting these results and observing $\hat{\sigma}_n^2(\bar{\lambda}_n) = \hat{\sigma}_n^2(\lambda_0) + o_p(1)$, we have $\frac{d}{d\lambda}\tilde{\psi}_n^*(\bar{\lambda}_n) - \frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0) = o_p(1)$.

Condition *(ii)***:** Note that,

$$\begin{aligned}\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0) &= \frac{1}{n\sigma_0^2}Y_n'A_n'\dot{G}_n^{\circ\prime}M_nA_nY_n - \frac{1}{n\sigma_0^2}Y_nW_n'G_n^{\circ\prime}M_nA_nY_n - \frac{1}{n\sigma_0^2}Y_nA_n'G_n^{\circ\prime}M_nW_nY_n \\ &\quad + \frac{2}{n^2\sigma_0^4}(Y_n'A_n'G_n^{\circ\prime}M_nA_nY_n)\cdot(Y_n'W_n'M_nA_nY_n) + o_p(1) \equiv \textstyle\sum_{i=1}^4 T_{in} + o_p(1).\end{aligned}$$

Using $M_nA_nY_n = M_n\epsilon_n$ and the result $\frac{1}{n}a_n'\epsilon_n = o_p(1)$ for a vector a_n of uniformly bounded elements, we can readily verify that $T_{1n} = \frac{1}{n\sigma_0^2}\epsilon_n'\dot{G}_n^{\circ\prime}\epsilon_n + o_p(1)$, $T_{2n} = -\frac{1}{n\sigma_0^2}\epsilon_n'G_n^\circ G_n\epsilon_n + o_p(1)$, $T_{3n} = -\frac{1}{n\sigma_0^2}(c_n'\eta_n + \epsilon_n'G_n^{\circ\prime}G_n\epsilon_n) + o_p(1)$, and $T_{4n} = o_p(1)$, by Lemma A.2. It follows that

$$-\mathrm{E}\big[\tfrac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0)\big] = \tfrac{1}{n}\mathrm{tr}[H_n(G_n^\circ G_n + G_n^{\circ\prime}G_n - \dot{G}_n^\circ)] + \tfrac{1}{n\sigma_0^2}c_n'\eta_n + o(1) = \Phi_n + o(1),$$

and that $\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0) - \mathrm{E}\big[\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0)\big] = o_p(1)$.

Condition *(iii)***:** By Assumptions 3-6 and Lemmas A.2 and A.3, it is easy to see that $\Phi_n \neq 0$ for large enough n, and thus $\mathrm{E}\big(\frac{d}{d\lambda}\tilde{\psi}_n^*(\lambda_0)\big) \neq 0$ for large enough n.

With (13), and *(i)*-*(iii)* proved above, the asymptotic normality result of Theorem 2 follows.

Proof of Theorem 3: Recall $\tilde{\beta}_n = (X_n'X_n)^{-1}X_n'A_n(\tilde{\lambda}_n)Y_n$. We have,

$$\sqrt{n}(\tilde{\beta}_n - \beta_0) = (\tfrac{1}{n}X_n'X_n)^{-1}\tfrac{1}{\sqrt{n}}X_n'\epsilon_n - \sqrt{n}(\tilde{\lambda}_n - \lambda_0)(\tfrac{1}{n}X_n'X_n)^{-1}\tfrac{1}{n}X_n'\eta_n + O_p\big(\tfrac{1}{\sqrt{n}}\big). \quad \text{(B-4)}$$

The proof of the asymptotic normality of $\tilde{\lambda}_n$ in Theorem 2 and the asymptotic representation for $\sqrt{n}\tilde{\psi}_n^*$ given in (11) imply that

$$\sqrt{n}(\tilde{\lambda}_n - \lambda_0) = \Phi_n^{-1}\sqrt{n}\tilde{\psi}_n^* + o_p(1) = (\sqrt{n}\sigma_0^2\Phi_n)^{-1}(\epsilon_n'B_n\epsilon_n + c_n'\epsilon_n) + o_p(1). \quad \text{(B-5)}$$

This shows that each component of $\sqrt{n}(\tilde{\beta}_n - \beta_0)$ is a linear-quadratic form in ϵ_n. Thus, Cramèr-Wold device and the CLT for linear-quadratic form of Kelejian and Prucha (2001) lead to the asymptotic normality of $\sqrt{n}(\tilde{\beta}_n - \beta_0)$. Clearly, the asymptotic mean of $\sqrt{n}(\tilde{\beta}_n - \beta_0)$ is zero and the first-order variance of it can be easily found using (B-4) and (B-5):

$$\begin{aligned}\tau^2(\tilde{\beta}_n) &= (X_n'X_n)^{-1}X_n'\mathrm{Var}(\epsilon_n)X_n(X_n'X_n)^{-1} + \tau^2(\tilde{\lambda}_n)(X_n'X_n)^{-1}X_n'\eta_n\eta_n'X_n(X_n'X_n)^{-1} \\ &\quad -2(\sigma_0^2\Phi_n)^{-1}(X_n'X_n)^{-1}X_n'\mathrm{Cov}(\epsilon_n, \epsilon_n'B_n\epsilon_n + c_n'\epsilon_n)\eta_n'X_n(X_n'X_n)^{-1} \\ &= (X_n'X_n)^{-1}X_n'\mathbb{A}_nX_n(X_n'X_n)^{-1},\end{aligned}$$

where $\mathbb{A}_n = n\sigma_0^2H_n + \tau_n^2(\tilde{\lambda}_n)\eta_n\eta_n' - 2\Phi_n^{-1}(\sigma_0^{-2}\mathrm{diag}(B_n)s_n + H_nc_n)\eta_n'$, and $s_n = \mathrm{E}(\epsilon_n^3)$.

The limiting distribution of $\sqrt{n}(\tilde{\sigma}_n^2 - \sigma_0^2)$ can be found in a similar manner from

$$\begin{aligned}\sqrt{n}(\tilde{\sigma}_n^2 - \sigma_0^2) &= \sqrt{n}[\tfrac{1}{n}Y_n' A_n'(\tilde{\lambda}_n)M_n A_n(\tilde{\lambda}_n)Y_n - \sigma_0^2] \\ &= \tfrac{1}{\sqrt{n}}(\epsilon_n'\epsilon_n - n\sigma_0^2) + 2\sqrt{n}(\tilde{\lambda}_n - \lambda_0)\tfrac{1}{n}\sigma_0^2 \text{tr}(H_n G_n) + o_p(1),\end{aligned}$$

which has a limiting mean of zero and first-order variance:

$$\tau_n^2(\tilde{\sigma}_n^2) = \tfrac{1}{n}\sum_{i=1}^n \text{Var}(\epsilon_{n,i}^2) + \tfrac{4}{n^2}\sigma_0^4\tau_n^2(\tilde{\lambda}_n)\text{tr}^2(H_n G_n) + \tfrac{4}{n^2}\text{Cov}(\epsilon_n'\epsilon, \epsilon_n' B_n\epsilon_n + c_n'\epsilon_n)\text{tr}(H_n G_n)\Phi_n^{-1},$$

where $\text{Cov}(\epsilon_n'\epsilon, \epsilon_n' B_n\epsilon_n + c_n'\epsilon_n)$ can be easily derived but not needed in light of Footnote 7.

Proof of Theorem 4: To prove the consistency of $\tilde{\tau}_n^2(\tilde{\lambda}_n)$ as an estimator of $\tau_n^2(\tilde{\lambda}_n)$, we need to prove (a) $\tilde{\Phi}_n - \Phi_n = o_p(1)$, and (b) $\tilde{\tau}_n^2(\tilde{\psi}_n^*) - \tau_n^2(\tilde{\psi}_n^*) = o_p(1)$. First, (a) follows from the proof of Theorem 2 (the asymptotic normality part). To prove (b), as $\tilde{\sigma}_n^2 = \sigma_0^2 + o_p(1)$ by Theorem 3, it suffices to show that, by the consistency of $\tilde{\theta}_n$ and referring to (18) and (19),

$$\tfrac{1}{n}\sum_{i=1}^n \left(\epsilon_{n,i}^2\xi_{n,i}^2 - \text{Var}(\epsilon_{n,i}\xi_{n,i})\right) = o_p(1),$$

where $\xi_{n,i} = \zeta_{n,i} + b_{n,ii}\epsilon_{n,i} + c_{n,i}$. This follows immediately by Theorem A.1 and the poof of Theorem 1 of Baltagi and Yang (2013b).

The consistency of $\tilde{\tau}_n^2(\tilde{\beta}_n)$ follows that of $\tilde{\tau}_n^2(\tilde{\lambda}_n)$ and the consistency of $\tilde{\theta}_n$.

Finally, the same procedure proves the same set of the results for the regular QMLEs $\hat{\beta}_n$ and $\hat{\sigma}_n^2$.

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Table 1: Empirical Mean(rmse)[sd]{$\hat{\text{sd}}$} of Estimators of λ for SAR Model
Cases when Regular QMLE is Consistent

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
		DGP 1: Constant Circular Neighbours (REG-1), $\beta_0 = (3, 1, 1)'$			
.50	100	.464 (.105)[.098]{.092}	.473(.117)[.114]{.099}	.469(.121)[.117]	.479(.132)[.130]
	250	.488(.061)[.060]{.064}	.492(.063)[.063]{.059}	.489(.064)[.063]	.494(.071)[.071]
	500	.494(.043)[.043]{.046}	.497(.043)[.043]{.042}	.495(.043)[.043]	.498(.048)[.048]
	1000	.497(.030)[.030]{.032}	.498(.030)[.030]{.029}	.498(.030)[.030]	.498(.033)[.033]
.25	100	.212(.133)[.127]{.115}	.230(.128)[.127]{.123}	.221(.132)[.129]	.232(.146)[.145]
	250	.233(.080)[.078]{.078}	.246(.081)[.081]{.079}	.242(.082)[.081]	.247(.090)[.090]
	500	.245(.052)[.052]{.054}	.245(.054)[.054]{.054}	.243(.054)[.054]	.244(.060)[.059]
	1000	.246(.041)[.041]{.040}	.247(.039)[.039]{.038}	.246(.039)[.039]	.247(.043)[.043]
.00	100	-.033(.153)[.149]{.142}	-.014(.150)[.149]{.142}	-.024(.156)[.154]	-.009(.172)[.172]
	250	-.017 (.090)[.089]{.089}	-.007(.091)[.091]{.089}	-.011(.092)[.092]	-.005(.102)[.102]
	500	-.006 (.063)[.063]{.062}	-.002(.061)[.061]{.064}	-.004(.061)[.061]	-.002(.069)[.069]
	1000	-.006(.046)[.046]{.046}	-.003(.043)[.043]{.045}	-.005(.043)[.043]	-.003(.047)[.047]
-.25	100	-.285(.155)[.151]{.149}	-.272(.171)[.169]{.167}	-.286(.176)[.173]	-.275(.200)[.198]
	250	-.266(.101)[.100]{.100}	-.258(.100)[.100]{.099}	-.264(.101)[.100]	-.260(.112)[.112]
	500	-.259(.070)[.070]{.072}	-.255(.070)[.070]{.070}	-.258(.070)[.070]	-.256(.077)[.076]
	1000	-.253(.050)[.050]{.050}	-.250(.050)[.050]{.049}	-.252(.050)[.050]	-.250(.055)[.055]
-.50	100	-.524(.172)[.170]{.179}	-.506(.172)[.172]{.162}	-.521(.175)[.174]	-.513(.195)[.194]
	250	-.515(.108)[.107]{.112}	-.505(.104)[.104]{.101}	-.511(.104)[.104]	-.507(.117)[.116]
	500	-.501(.075)[.075]{.080}	-.497(.075)[.075]{.073}	-.501(.075)[.075]	-.497(.084)[.084]
	1000	-.500(.054)[.054]{.058}	-.499(.051)[.051]{.051}	-.500(.051)[.051]	-.500(.057)[.057]
		DGP 2: Constant Circular Neighbours (REG-1), $\beta_0 = (3, 1, 1)'$			
.50	100	.465(.098)[.091]{.093}	.481(.107)[.105]{.099}	.475(.118)[.115]	.488(.142)[.141]
	250	.487(.062)[.061]{.063}	.494(.061)[.060]{.059}	.491(.061)[.061]	.495(.084)[.084]
	500	.494(.041)[.041]{.042}	.499(.042)[.042]{.040}	.497(.042)[.042]	.500(.059)[.059]
	1000	.498(.028)[.028]{.028}	.500(.028)[.028]{.029}	.499(.029)[.029]	.499(.041)[.041]
.25	100	.219(.129)[.126]{.124}	.238(.125)[.125]{.124}	.230(.128)[.127]	.251(.168)[.168]
	250	.236(.081)[.080]{.080}	.243(.080)[.079]{.079}	.239(.081)[.080]	.245(.108)[.108]
	500	.246(.056)[.056]{.059}	.250(.056)[.056]{.053}	.248(.056)[.056]	.251(.080)[.080]
	1000	.249(.039)[.039]{.041}	.251(.039)[.039]{.037}	.250(.039)[.039]	.250(.052)[.052]
.00	100	-.029(.146)[.143]{.139}	-.010(.143)[.143]{.139}	-.020(.150)[.148]	-.005(.209)[.209]
	250	-.011(.088)[.088]{.087}	-.003(.088)[.088]{.085}	-.008(.089)[.088]	.003(.122)[.122]
	500	-.005(.063)[.063]{.061}	-.008(.064)[.064]{.062}	-.010(.064)[.064]	-.004(.092)[.092]
	1000	-.003(.045)[.045]{.045}	-.001(.043)[.043]{.044}	-.003(.043)[.043]	.000(.060)[.060]
-.25	100	-.276(.158)[.155]{.145}	-.257(.156)[.156]{.153}	-.271(.160)[.159]	-.249(.223)[.223]
	250	-.268(.100)[.099]{.106}	-.261(.099)[.099]{.093}	-.266(.100)[.099]	-.260(.136)[.136]
	500	-.256(.073)[.073]{.077}	-.252(.073)[.073]{.069}	-.255(.074)[.073]	-.254(.102)[.102]
	1000	-.254(.050)[.050]{.050}	-.252(.049)[.049]{.048}	-.253(.050)[.049]	-.252(.068)[.068]
-.50	100	-.527(.155)[.153]{.163}	-.505(.154)[.154]{.154}	-.519(.158)[.157]	-.511(.221)[.221]
	250	-.505(.101)[.101]{.103}	-.500(.099)[.099]{.097}	-.506(.100)[.100]	-.502(.138)[.138]
	500	-.507(.075)[.075]{.077}	-.502(.072)[.072]{.072}	-.505(.072)[.072]	-.501(.103)[.103]
	1000	-.505(.050)[.049]{.049}	-.503(.050)[.049]{.050}	-.504(.050)[.050]	-.505(.071)[.071]

Table 1: Cont'd

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
DGP 3: Constant Circular Neighbours (REG-1), $\beta_0 = (3, 1, 1)'$					
.50	100	.474(.086)[.082]{.094}	.484(.096)[.095]{.089}	.476(.100)[.098]	.480(.149)[.148]
	250	.491(.057)[.056]{.054}	.497(.056)[.056]{.052}	.495(.076)[.076]	.499(.088)[.088]
	500	.493(.040)[.039]{.038}	.496(.040)[.039]{.038}	.494(.040)[.039]	.494(.067)[.067]
	1000	.496(.030)[.030]{.029}	.497(.029)[.028]{.027}	.497(.029)[.029]	.498(.045)[.045]
.25	100	.213(.124)[.119]{.110}	.231(.119)[.117]{.115}	.221(.125)[.122]	.233(.185)[.184]
	250	.240(.072)[.071]{.079}	.247(.071)[.070]{.067}	.242(.072)[.072]	.244(.116)[.116]
	500	.245(.050)[.050]{.052}	.247(.054)[.054]{.050}	.245(.055)[.054]	.245(.087)[.087]
	1000	.248(.037)[.037]{.038}	.250(.037)[.037]{.035}	.249(.037)[.037]	.250(.057)[.057]
.00	100	-.024(.124)[.122]{.116}	-.015(.140)[.140]{.143}	-.027(.148)[.145]	-.018(.221)[.220]
	250	-.010(.085)[.085]{.082}	-.002(.084)[.084]{.088}	-.007(.086)[.086]	-.002(.133)[.133]
	500	-.006(.059)[.058]{.060}	-.002(.058)[.058]{.058}	-.005(.059)[.059]	-.007(.101)[.101]
	1000	-.004(.045)[.044]{.044}	-.002(.042)[.042]{.041}	-.003(.043)[.043]	.000(.069)[.069]
-.25	100	-.276(.148)[.146]{.156}	-.258(.146)[.146]{.142}	-.272(.152)[.150]	-.261(.236)[.236]
	250	-.260(.093)[.092]{.101}	-.252(.093)[.093]{.096}	-.259(.094)[.093]	-.253(.153)[.153]
	500	-.256(.063)[.063]{.065}	-.254(.065)[.065]{.064}	-.256(.066)[.066]	-.251(.111)[.111]
	1000	-.254(.049)[.049]{.047}	-.250(.049)[.049]{.046}	-.252(.050)[.050]	-.251(.076)[.076]
-.50	100	-.514(.141)[.140]{.153}	-.508(.161)[.161]{.167}	-.526(.165)[.163]	-.513(.246)[.245]
	250	-.511(.092)[.091]{.098}	-.506(.097)[.097]{.091}	-.512(.099)[.098]	-.514(.155)[.154]
	500	-.503(.069)[.069]{.069}	-.499(.069)[.069]{.067}	-.503(.069)[.069]	-.498(.111)[.111]
	1000	-.503(.051)[.051]{.051}	-.501(.051)[.051]{.049}	-.503(.051)[.051]	-.505(.081)[.081]
DGP 1: Queen Contiguity (REG-1), $\beta_0 = (.3, .1, .1)'$					
.50	100	.447(.156)[.146]{.136}	.471(.147)[.144]{.148}	.463(.158)[.154]	.501(.207)[.207]
	250	.482(.081)[.079]{.088}	.495(.079)[.079]{.079}	.488(.081)[.080]	.499(.085)[.085]
	500	.489(.061)[.059]{.063}	.494(.056)[.056]{.056}	.491(.070)[.069]	.497(.071)[.071]
	1000	.496(.041)[.041]{.045}	.497(.042)[.042]{.040}	.495(.042)[.042]	.498(.043)[.043]
.25	100	.207(.170)[.165]{.155}	.231(.167)[.166]{.155}	.219(.172)[.169]	.240(.186)[.186]
	250	.232(.103)[.101]{.101}	.241(.102)[.102]{.099}	.234(.104)[.102]	.242(.106)[.106]
	500	.242(.072)[.072]{.072}	.249(.072)[.072]{.070}	.245(.072)[.072]	.250(.074)[.074]
	1000	.244(.050)[.050]{.052}	.247(.050)[.050]{.050}	.245(.050)[.050]	.247(.051)[.051]
.00	100	-.046(.192)[.186]{.173}	-.021(.188)[.187]{.174}	-.036(.195)[.192]	-.021(.205)[.204]
	250	-.019(.117)[.115]{.112}	-.008(.115)[.115]{.112}	-.017(.117)[.116]	-.010(.120)[.120]
	500	-.008(.080)[.080]{.079}	-.001(.080)[.080]{.080}	-.005(.080)[.080]	-.001(.082)[.082]
	1000	-.005(.058)[.058]{.057}	-.002(.058)[.058]{.057}	-.004(.058)[.058]	-.002(.059)[.059]
-.25	100	-.286(.199)[.195]{.192}	-.258(.198)[.198]{.193}	-.277(.205)[.204]	-.264(.218)[.217]
	250	-.272(.122)[.120]{.125}	-.258(.121)[.120]{.120}	-.268(.122)[.121]	-.265(.126)[.125]
	500	-.260(.089)[.088]{.089}	-.253(.089)[.089]{.086}	-.258(.089)[.089]	-.256(.090)[.090]
	1000	-.256(.063)[.063]{.064}	-.252(.063)[.063]{.061}	-.255(.063)[.063]	-.254(.064)[.064]
-.50	100	-.526(.194)[.192]{.201}	-.502(.194)[.194]{.187}	-.521(.197)[.196]	-.521(.214)[.213]
	250	-.513(.122)[.121]{.128}	-.501(.122)[.122]{.122}	-.513(.124)[.123]	-.514(.128)[.127]
	500	-.504(.087)[.087]{.088}	-.498(.088)[.088]{.087}	-.503(.088)[.088]	-.503(.089)[.089]
	1000	-.503(.063)[.063]{.061}	-.500(.063)[.063]{.063}	-.502(.063)[.063]	-.502(.064)[.064]

Table 1: Cont'd

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
DGP 2: Queen Contiguity (REG-1), $\beta_0 = (.3, .1, .1)'$					
.50	100	.455(.136)[.129]{.137}	.481(.129)[.128]{.123}	.470(.135)[.132]	.581(.354)[.345]
	250	.480(.087)[.083]{.100}	.493(.078)[.078]{.076}	.487(.080)[.079]	.533(.160)[.157]
	500	.490(.057)[.056]{.057}	.497(.056)[.056]{.054}	.495(.068)[.068]	.518(.088)[.086]
	1000	.496(.042)[.042]{.047}	.499(.042)[.042]{.039}	.498(.042)[.042]	.510(.053)[.052]
.25	100	.206(.171)[.166]{.155}	.233(.166)[.165]{.161}	.224(.180)[.178]	.308(.366)[.361]
	250	.222(.108)[.104]{.105}	.240(.097)[.096]{.094}	.232(.099)[.098]	.272(.139)[.137]
	500	.239(.072)[.071]{.076}	.246(.071)[.071]{.068}	.242(.072)[.071]	.259(.089)[.089]
	1000	.246(.050)[.050]{.050}	.245(.052)[.052]{.050}	.244(.053)[.052]	.257(.070)[.070]
.00	100	-.035(.177)[.174]{.165}	-.023(.184)[.182]{.188}	-.039(.191)[.187]	.002(.243)[.243]
	250	-.019(.116)[.115]{.109}	-.005(.115)[.114]{.106}	-.014(.117)[.116]	.016(.153)[.152]
	500	-.009(.081)[.080]{.078}	-.004(.081)[.081]{.077}	-.008(.082)[.081]	.012(.105)[.105]
	1000	-.004(.057)[.057]{.057}	-.002(.057)[.057]{.056}	-.005(.057)[.057]	.007(.069)[.069]
-.25	100	-.283(.185)[.182]{.190}	-.268(.186)[.185]{.186}	-.285(.192)[.189]	-.254(.251)[.251]
	250	-.270(.122)[.120]{.125}	-.256(.121)[.120]{.114}	-.267(.123)[.122]	-.253(.161)[.161]
	500	-.256(.085)[.084]{.085}	-.250(.085)[.085]{.082}	-.254(.085)[.085]	-.242(.106)[.106]
	1000	-.252(.063)[.063]{.060}	-.249(.063)[.063]{.060}	-.251(.063)[.063]	-.245(.078)[.078]
-.50	100	-.518(.195)[.194]{.204}	-.506(.188)[.187]{.180}	-.529(.193)[.190]	-.523(.255)[.254]
	250	-.513(.127)[.126]{.128}	-.501(.127)[.127]{.125}	-.512(.128)[.128]	-.513(.168)[.167]
	500	-.505(.088)[.088]{.084}	-.500(.089)[.089]{.085}	-.505(.089)[.088]	-.500(.110)[.110]
	1000	-.503(.063)[.063]{.060}	-.500(.063)[.063]{.061}	-.503(.063)[.063]	-.501(.077)[.077]
DGP 3: Queen Contiguity (REG-1), $\beta_0 = (.3, .1, .1)'$					
.50	100	.453(.128)[.119]{.126}	.479(.120)[.118]{.109}	.470(.144)[.141]	.631(.463)[.444]
	250	.479(.079)[.076]{.072}	.492(.076)[.075]{.069}	.487(.079)[.077]	.583(.287)[.275]
	500	.486(.056)[.054]{.057}	.492(.054)[.054]{.049}	.489(.055)[.054]	.554(.206)[.198]
	1000	.494(.039)[.038]{.031}	.497(.039)[.038]{.037}	.496(.039)[.039]	.530(.107)[.103]
.25	100	.205(.151)[.144]{.146}	.232(.145)[.144]{.148}	.220(.154)[.151]	.354(.469)[.458]
	250	.231(.100)[.098]{.100}	.245(.098)[.098]{.095}	.237(.100)[.099]	.307(.277)[.271]
	500	.237(.071)[.070]{.072}	.244(.070)[.070]{.069}	.240(.071)[.070]	.306(.250)[.244]
	1000	.246(.049)[.049]{.055}	.248(.050)[.050]{.049}	.246(.051)[.050]	.271(.126)[.124]
.00	100	-.048(.164)[.157]{.159}	-.015(.169)[.168]{.164}	-.029(.175)[.172]	.057(.327)[.321]
	250	-.018(.106)[.104]{.104}	-.004(.104)[.104]{.099}	-.013(.107)[.106]	.038(.214)[.210]
	500	-.011(.077)[.076]{.075}	-.003(.077)[.076]{.071}	-.008(.077)[.077]	.032(.169)[.166]
	1000	-.004(.055)[.055]{.055}	-.001(.055)[.055]{.053}	-.003(.055)[.055]	.028(.132)[.129]
-.25	100	-.284(.170)[.167]{.179}	-.263(.169)[.168]{.163}	-.284(.175)[.172]	-.245(.283)[.283]
	250	-.268(.119)[.117]{.110}	-.254(.118)[.117]{.115}	-.265(.120)[.119]	-.220(.214)[.211]
	500	-.258(.081)[.081]{.083}	-.252(.081)[.081]{.079}	-.257(.081)[.081]	-.221(.176)[.174]
	1000	-.252(.059)[.059]{.054}	-.254(.059)[.059]{.056}	-.256(.059)[.059]	-.224(.151)[.148]
-.50	100	-.523(.176)[.175]{.189}	-.516(.182)[.182]{.187}	-.539(.192)[.188]	-.528(.312)[.311]
	250	-.514(.120)[.119]{.113}	-.501(.119)[.119]{.118}	-.513(.120)[.119]	-.501(.215)[.215]
	500	-.503(.085)[.085]{.084}	-.500(.085)[.085]{.088}	-.505(.085)[.085]	-.491(.172)[.172]
	1000	-.503(.063)[.063]{.061}	-.500(.063)[.063]{.059}	-.502(.063)[.063]	-.496(.150)[.150]

Table 2: Empirical Mean(rmse)[sd]{$\hat{\text{sd}}$} of Estimators of λ for SAR Model
Case I of Inconsistent QMLE: Circular Neighbours (REG-1)

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
		DGP 1: $\beta_0 = (3,1,1)'$			
.50	100	.434(.119)[.100]	.481(.103)[.101]{.093}	.477(.107)[.104]	.483(.113)[.112]
	250	.458(.071)[.057]	.491(.059)[.059]{.057}	.489(.058)[.057]	.492(.061)[.060]
	500	.463(.056)[.043]	.496(.044)[.044]{.043}	.495(.043)[.043]	.496(.046)[.046]
	1000	.472(.040)[.028]	.500(.029)[.029]{.028}	.499(.028)[.028]	.500(.030)[.030]
.25	100	.197(.120)[.107]	.233(.116)[.115]{.115}	.226(.117)[.115]	.232(.127)[.125]
	250	.218(.077)[.070]	.242(.075)[.074]{.070}	.239(.073)[.072]	.242(.075)[.075]
	500	.222(.060)[.053]	.246(.057)[.057]{.054}	.245(.057)[.057]	.247(.061)[.060]
	1000	.225(.042)[.034]	.246(.037)[.036]{.035}	.245(.036)[.036]	.246(.038)[.038]
.00	100	-.023(.114)[.111]	-.009(.127)[.126]{.127}	-.015(.127)[.127]	-.006(.136)[.136]
	250	-.012(.073)[.072]	-.007(.081)[.080]{.078}	-.009(.080)[.079]	-.005(.084)[.084]
	500	-.005(.054)[.053]	-.002(.060)[.060]{.060}	-.003(.060)[.060]	-.001(.064)[.064]
	1000	-.002(.036)[.036]	-.001(.040)[.040]{.039}	-.002(.039)[.039]	-.001(.042)[.042]
-.25	100	-.249(.110)[.110]	-.271(.137)[.135]{.139}	-.271(.132)[.131]	-.270(.155)[.154]
	250	-.226(.072)[.068]	-.250(.082)[.081]{.080}	-.251(.076)[.076]	-.250(.081)[.081]
	500	-.224(.058)[.052]	-.252(.063)[.063]{.062}	-.252(.060)[.060]	-.251(.064)[.064]
	1000	-.225(.043)[.034]	-.252(.040)[.040]{.040}	-.252(.039)[.039]	-.252(.042)[.042]
-.50	100	-.449(.105)[.092]	-.494(.114)[.114]{.119}	-.492(.105)[.104]	-.498(.112)[.112]
	250	-.448(.079)[.059]	-.503(.076)[.076]{.076}	-.498(.065)[.065]	-.500(.070)[.070]
	500	-.444(.073)[.046]	-.506(.061)[.061]{.059}	-.505(.054)[.054]	-.506(.057)[.056]
	1000	-.444(.064)[.030]	-.501(.037)[.037]{.037}	-.500(.034)[.034]	-.501(.035)[.035]
		DGP 2: $\beta_0 = (3,1,1)'$			
.50	100	.438(.114)[.096]	.483(.098)[.097]{.089}	.477(.105)[.102]	.485(.130)[.129]
	250	.462(.066)[.054]	.495(.055)[.055]{.055}	.492(.053)[.053]	.496(.067)[.067]
	500	.467(.054)[.043]	.500(.044)[.044]{.042}	.498(.043)[.043]	.499(.057)[.057]
	1000	.473(.039)[.027]	.501(.028)[.028]{.028}	.500(.027)[.027]	.501(.034)[.034]
.25	100	.201(.123)[.113]	.236(.120)[.119]{.109}	.228(.122)[.120]	.235(.147)[.146]
	250	.219(.072)[.066]	.244(.070)[.070]{.069}	.242(.070)[.069]	.245(.087)[.087]
	500	.220(.059)[.051]	.244(.055)[.054]{.053}	.243(.054)[.054]	.247(.071)[.071]
	1000	.228(.040)[.033]	.248(.035)[.035]{.035}	.248(.035)[.034]	.249(.043)[.043]
.00	100	-.022(.116)[.114]	-.010(.131)[.131]{.129}	-.016(.129)[.128]	-.005(.159)[.158]
	250	-.010(.073)[.072]	-.005(.081)[.081]{.079}	-.008(.080)[.079]	-.004(.097)[.096]
	500	-.004(.051)[.051]	-.001(.058)[.058]{.058}	-.002(.057)[.057]	.001(.075)[.075]
	1000	-.003(.036)[.036]	-.002(.040)[.040]{.039}	-.002(.039)[.039]	-.001(.048)[.048]
-.25	100	-.239(.109)[.108]	-.257(.131)[.131]{.129}	-.256(.122)[.122]	-.248(.150)[.150]
	250	-.232(.071)[.069]	-.257(.083)[.082]{.079}	-.257(.077)[.077]	-.253(.093)[.093]
	500	-.223(.059)[.052]	-.251(.062)[.062]{.060}	-.251(.060)[.060]	-.247(.078)[.078]
	1000	-.222(.045)[.036]	-.249(.041)[.041]{.040}	-.249(.040)[.040]	-.249(.048)[.048]
-.50	100	-.452(.105)[.093]	-.499(.114)[.114]{.116}	-.495(.110)[.110]	-.496(.123)[.123]
	250	-.448(.080)[.061]	-.501(.073)[.073]{.073}	-.499(.066)[.066]	-.499(.079)[.079]
	500	-.438(.077)[.046]	-.500(.059)[.059]{.058}	-.498(.052)[.052]	-.497(.065)[.065]
	1000	-.444(.064)[.031]	-.501(.037)[.037]{.037}	-.502(.034)[.034]	-.502(.041)[.041]

Table 2: Cont'd

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
			DGP 3: $\beta_0 = (3, 1, 1)'$		
.50	100	.445(.107)[.092]	.486(.087)[.086]{.079}	.482(.092)[.090]	.493(.144)[.144]
	250	.464(.066)[.055]	.495(.054)[.054]{.049}	.493(.054)[.053]	.497(.073)[.073]
	500	.467(.055)[.044]	.497(.041)[.041]{.039}	.496(.042)[.041]	.497(.060)[.060]
	1000	.473(.040)[.030]	.499(.027)[.027]{.026}	.499(.027)[.027]	.500(.037)[.037]
.25	100	.199(.116)[.105]	.230(.110)[.108]{.099}	.241(.068)[.067]	.245(.090)[.089]
	250	.219(.071)[.064]	.243(.069)[.068]{.063}	.241(.068)[.067]	.245(.090)[.089]
	500	.222(.058)[.050]	.244(.054)[.053]{.049}	.243(.053)[.053]	.242(.078)[.078]
	1000	.228(.040)[.033]	.248(.035)[.034]{.033}	.248(.034)[.034]	.250(.045)[.045]
.00	100	-.019(.107)[.105]	-.008(.120)[.120]{.119}	-.013(.119)[.119]	-.005(.164)[.164]
	250	-.008(.065)[.065]	-.003(.072)[.072]{.069}	-.006(.072)[.072]	-.003(.101)[.101]
	500	-.006(.051)[.050]	-.004(.057)[.057]{.054}	-.006(.058)[.058]	-.007(.089)[.089]
	1000	-.003(.035)[.034]	-.002(.038)[.038]{.037}	-.003(.038)[.038]	-.003(.053)[.053]
-.25	100	-.243(.102)[.102]	-.260(.123)[.123]{.120}	-.262(.118)[.117]	-.257(.157)[.156]
	250	-.230(.072)[.069]	-.250(.077)[.077]{.072}	-.251(.074)[.074]	-.248(.098)[.098]
	500	-.228(.055)[.050]	-.255(.058)[.058]{.056}	-.256(.058)[.057]	-.255(.083)[.083]
	1000	-.223(.044)[.035]	-.250(.039)[.039]{.038}	-.250(.039)[.039]	-.249(.052)[.052]
-.50	100	-.450(.107)[.095]	-.486(.110)[.109]{.112}	-.485(.105)[.104]	-.484(.125)[.123]
	250	-.450(.081)[.063]	-.502(.074)[.074]{.070}	-.498(.064)[.064]	-.496(.085)[.085]
	500	-.439(.081)[.053]	-.499(.061)[.061]{.059}	-.497(.051)[.051]	-.499(.069)[.069]
	1000	-.445(.066)[.037]	-.500(.038)[.038]{.036}	-.500(.034)[.034]	-.501(.044)[.044]
			DGP 1: $\beta_0 = (.3, .1, .1)'$		
.50	100	.407(.154)[.123]	.474(.129)[.127]{.119}	.467(.148)[.144]	.499(.189)[.189]
	250	.437(.100)[.078]	.489(.080)[.079]{.075}	.485(.082)[.080]	.494(.083)[.083]
	500	.445(.076)[.053]	.494(.054)[.054]{.053}	.493(.069)[.069]	.497(.066)[.066]
	1000	.453(.060)[.037]	.499(.037)[.037]{.038}	.498(.037)[.037]	.500(.038)[.038]
.25	100	.174(.156)[.136]	.226(.155)[.153]{.149}	.213(.165)[.161]	.235(.195)[.194]
	250	.199(.101)[.087]	.238(.097)[.097]{.096}	.233(.100)[.098]	.241(.102)[.102]
	500	.208(.076)[.063]	.243(.069)[.069]{.068}	.240(.070)[.069]	.243(.070)[.070]
	1000	.213(.058)[.045]	.246(.049)[.049]{.048}	.245(.050)[.049]	.246(.050)[.050]
.00	100	-.041(.146)[.140]	-.023(.170)[.168]{.165}	-.040(.179)[.174]	-.026(.184)[.182]
	250	-.016(.096)[.095]	-.009(.114)[.113]{.117}	-.015(.115)[.114]	-.009(.116)[.116]
	500	-.008(.066)[.066]	-.004(.078)[.078]{.077}	-.008(.079)[.079]	-.005(.080)[.080]
	1000	-.004(.044)[.044]	-.002(.052)[.052]{.054}	-.003(.052)[.052]	-.002(.053)[.053]
-.25	100	-.240(.136)[.136]	-.270(.176)[.175]{.172}	-.292(.185)[.180]	-.291(.201)[.197]
	250	-.213(.095)[.087]	-.251(.110)[.110]{.111}	-.259(.111)[.111]	-.256(.114)[.114]
	500	-.210(.074)[.062]	-.252(.079)[.079]{.079}	-.256(.079)[.079]	-.255(.080)[080]
	1000	-.209(.060)[.044]	-.252(.055)[.055]{.056}	-.254(.055)[.055]	-.254(.056)[.056]
-.50	100	-.417(.149)[.124]	-.496(.164)[.164]{.159}	-.531(.202)[.199]	-.535(.213)[.210]
	250	-.413(.117)[.078]	-.504(.103)[.103]{.102}	-.512(.103)[.102]	-.516(.107)[.106]
	500	-.409(.107)[.056]	-.501(.073)[.073]{.073}	-.506(.073)[.073]	-.507(.074)[.074]
	1000	-.405(.103)[.039]	-.498(.051)[.051]{.052}	-.501(.051)[.051]	-.501(.051)[.051]

Table 2: Cont'd

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
			DGP 2: $\beta_0 = (.3, .1, .1)'$		
.50	100	.416(.147)[.121]	.482(.123)[.121]{.119}	.475(.138)[.136]	.592(.342)[.329]
	250	.438(.101)[.080]	.490(.081)[.080]{.079}	.487(.090)[.089]	.528(.157)[.154]
	500	.448(.074)[.053]	.496(.053)[.053]{.052}	.494(.054)[.053]	.511(.068)[.067]
	1000	.452(.061)[.038]	.499(.038)[.038]{.037}	.498(.038)[.038]	.508(.047)[.047]
.25	100	.184(.152)[.137]	.236(.154)[.154]{.157}	.224(.165)[.163]	.304(.305)[.301]
	250	.203(.100)[.088]	.242(.097)[.097]{.091}	.236(.099)[.098]	.271(.149)[.147]
	500	.211(.073)[.062]	.246(.067)[.067]{.066}	.243(.068)[.068]	.264(.109)[.109]
	1000	.217(.055)[.044]	.250(.048)[.048]{.047}	.249(.048)[.048]	.258(.058)[.058]
.00	100	-.040(.144)[.139]	-.021(.171)[.169]{.164}	-.039(.180)[.176]	.014(.262)[.262]
	250	-.016(.091)[.089]	-.010(.107)[.107]{.104}	-.016(.109)[.108]	.008(.134)[.134]
	500	-.007(.063)[.063]	-.003(.075)[.075]{.074}	-.006(.075)[.075]	.008(.090)[.090]
	1000	-.003(.046)[.046]	-.001(.054)[.054]{.053}	-.003(.054)[.054]	.006(.066)[.066]
-.25	100	-.232(.133)[.131]	-.259(.169)[.169]{.159}	-.281(.180)[.177]	-.254(.266)[.266]
	250	-.216(.090)[.083]	-.254(.106)[.106]{.107}	-.262(.108)[.107]	-.249(.138)[.138]
	500	-.210(.073)[.061]	-.251(.077)[.077]{.077}	-.255(.077)[.077]	-.246(.088)[.088]
	1000	-.207(.063)[.046]	-.249(.057)[.057]{.055}	-.251(.057)[.057]	-.247(.067)[.067]
-.50	100	-.424(.148)[.127]	-.503(.163)[.163]{.160}	-.535(.191)[.187]	-.549(.246)[.241]
	250	-.410(.123)[.084]	-.499(.105)[.105]{.099}	-.507(.106)[.105]	-.513(.151)[.151]
	500	-.409(.108)[.058]	-.500(.071)[.071]{.072}	-.504(.071)[.071]	-.507(.086)[.086]
	1000	-.409(.100)[.041]	-.503(.050)[.050]{.051}	-.506(.051)[.050]	-.509(.063)[.062]
			DGP 3: $\beta_0 = (.3, .1, .1)'$		
.50	100	.416(.147)[.120]	.480(.118)[.116]{.099}	.473(.130)[.128]	.652(.453)[.426]
	250	.439(.096)[.074]	.490(.071)[.070]{.065}	.486(.073)[.071]	.572(.247)[.236]
	500	.449(.074)[.054]	.497(.050)[.050]{.048}	.495(.051)[.051]	.547(.189)[.184]
	1000	.453(.060)[.037]	.498(.034)[.034]{.035}	.497(.035)[.034]	.523(.104)[.101]
.25	100	.174(.153)[.133]	.224(.147)[.144]{.137}	.212(.156)[.152]	.335(.387)[.378]
	250	.210(.089)[.080]	.249(.087)[.087]{.083}	.243(.087)[.087]	.310(.245)[.237]
	500	.211(.072)[.061]	.244(.065)[.065]{.061}	.242(.066)[.065]	.283(.198)[.195]
	1000	.214(.057)[.044]	.247(.046)[.046]{.044}	.246(.047)[.046]	.266(.116)[.115]
.00	100	-.027(.135)[.133]	-.008(.161)[.160]{.153}	-.026(.172)[.170]	.077(.422)[.414]
	250	-.014(.087)[.086]	-.006(.103)[.103]{.099}	-.013(.105)[.104]	.052(.263)[.258]
	500	-.008(.059)[.058]	-.004(.070)[.070]{.069}	-.008(.071)[.070]	.026(.151)[.149]
	1000	-.003(.042)[.042]	-.001(.050)[.050]{.050}	-.003(.050)[.050]	.025(.116)[.114]
-.25	100	-.234(.131)[.130]	-.262(.172)[.172]{.179}	-.288(.184)[.180]	-.238(.295)[.295]
	250	-.218(.090)[.084]	-.254(.105)[.105]{.099}	-.262(.107)[.106]	-.223(.239)[.238]
	500	-.213(.073)[.063]	-.252(.076)[.076]{.071}	-.256(.077)[.076]	-.233(.161)[.160]
	1000	-.208(.062)[.046]	-.250(.055)[.055]{.053}	-.252(.055)[.055]	-.238(.128)[.127]
-.50	100	-.418(.151)[.127]	-.495(.158)[.158]{.151}	-.526(.178)[.176]	-.544(.304)[.301]
	250	-.411(.126)[.089]	-.503(.105)[.105]{.099}	-.511(.105)[.104]	-.508(.199)[.198]
	500	-.408(.113)[.066]	-.500(.073)[.073]{.069}	-.504(.072)[.072]	-.501(.156)[.156]
	1000	-.403(.109)[.049]	-.496(.051)[.051]{.049}	-.498(.051)[.051]	-.502(.129)[.129]

Table 3: Empirical Mean(rmse)[sd]{$\hat{\text{sd}}$} of Estimators of λ for SAR Model
Case II of Inconsistent QMLE: Group Interaction (REG-2)

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
		DGP 1: $\beta_0 = (3,1,1)'$			
.50	100	.422(.124)[.096]	.478(.102)[.099]{.093}	.469(.109)[.105]	.470(.112)[.108]
	250	.461(.069)[.057]	.493(.059)[.059]{.056}	.488(.061)[.060]	.491(.065)[.064]
	500	.472(.047)[.037]	.497(.039)[.038]{.038}	.494(.039)[.039]	.496(.041)[.041]
	1000	.476(.037)[.028]	.499(.029)[.029]{.028}	.497(.029)[.029]	.498(.031)[.030]
.25	100	.159(.161)[.132]	.224(.142)[.140]{.139}	.210(.156)[.150]	.215(.162)[.158]
	250	.210(.087)[.078]	.244(.082)[.081]{.080}	.237(.085)[.084]	.242(.090)[.090]
	500	.223(.060)[.053]	.247(.056)[.056]{.055}	.243(.057)[.057]	.246(.061)[.061]
	1000	.232(.042)[.037]	.251(.039)[.039]{.040}	.249(.040)[.040]	.251(.043)[.043]
.00	100	-.079(.179)[.160]	-.023(.183)[.181]{.183}	-.035(.194)[.191]	-.026(.203)[.201]
	250	-.034(.100)[.094]	-.011(.103)[.103]{.102}	-.020(.107)[.105]	-.014(.112)[.111]
	500	-.018(.067)[.065]	-.006(.071)[.070]{.070}	-.013(.072)[.071]	-.009(.075)[.075]
	1000	-.011(.049)[.048]	-.005(.052)[.052]{.051}	-.009(.054)[.053]	-.007(.057)[.057]
-.25	100	-.317(.184)[.171]	-.285(.210)[.207]{.213}	-.300(.222)[.216]	-.291(.234)[.231]
	250	-.264(.109)[.108]	-.266(.126)[.124]{.123}	-.276(.128)[.125]	-.271(.134)[.132]
	500	-.247(.074)[.074]	-.258(.085)[.085]{.084}	-.265(.086)[.085]	-.262(.091)[.090]
	1000	-.235(.056)[.054]	-.254(.061)[.060]{.060}	-.257(.062)[.061]	-.255(.065)[.065]
-.50	100	-.532(.181)[.178]	-.534(.226)[.224]{.219}	-.546(.231)[.226]	-.543(.245)[.241]
	250	-.468(.120)[.116]	-.505(.146)[.146]{.144}	-.515(.143)[.142]	-.511(.151)[.150]
	500	-.460(.090)[.080]	-.507(.101)[.100]{.097}	-.511(.096)[.095]	-.509(.101)[.101]
	1000	-.448(.078)[.057]	-.501(.070)[.070]{.069}	-.505(.069)[.069]	-.503(.073)[.073]
		DGP 2: $\beta_0 = (3,1,1)'$			
.50	100	.437(.117)[.098]	.492(.099)[.098]{.089}	.487(.110)[.110]	.497(.126)[.126]
	250	.465(.066)[.056]	.499(.057)[.057]{.054}	.494(.060)[.059]	.504(.074)[.074]
	500	.471(.047)[.037]	.497(.038)[.038]{.038}	.494(.039)[.038]	.499(.050)[.050]
	1000	.477(.035)[.027]	.500(.028)[.028]{.028}	.498(.028)[.028]	.500(.036)[.036]
.25	100	.167(.155)[.130]	.230(.137)[.135]{.129}	.220(.151)[.148]	.235(.172)[.171]
	250	.211(.085)[.076]	.245(.079)[.079]{.077}	.236(.082)[.081]	.245(.100)[.100]
	500	.219(.060)[.051]	.243(.054)[.053]{.054}	.238(.055)[.054]	.245(.067)[.067]
	1000	.231(.042)[.038]	.251(.040)[.040]{.039}	.248(.040)[.040]	.250(.052)[.052]
.00	100	-.084(.181)[.160]	-.028(.179)[.176]{.169}	-.044(.195)[.190]	-.019(.228)[.227]
	250	-.031(.098)[.093]	-.008(.101)[.101]{.098}	-.018(.107)[.105]	-.005(.134)[.134]
	500	-.015(.068)[.067]	-.003(.073)[.073]{.069}	-.009(.074)[.074]	.001(.095)[.095]
	1000	-.008(.050)[.049]	-.002(.053)[.053]{.050}	-.005(.054)[.054]	.000(.069)[.069]
-.25	100	-.313(.178)[.167]	-.283(.206)[.203]{.211}	-.296(.215)[.210]	-.268(.259)[.258]
	250	-.262(.109)[.108]	-.263(.126)[.126]{.119}	-.272(.128)[.126]	-.256(.159)[.159]
	500	-.243(.072)[.072]	-.254(.082)[.082]{.082}	-.260(.081)[.081]	-.252(.101)[.101]
	1000	-.235(.055)[.053]	-.253(.060)[.060]{.060}	-.256(.061)[.061]	-.252(.080)[.080]
-.50	100	-.523(.182)[.181]	-.531(.241)[.239]{.230}	-.541(.237)[.233]	-.510(.284)[.283]
	250	-.471(.118)[.114]	-.510(.142)[.142]{.140}	-.517(.138)[.137]	-.497(.174)[.174]
	500	-.458(.092)[.082]	-.503(.101)[.101]{.095}	-.509(.098)[.097]	-.498(.121)[.121]
	1000	-.445(.079)[.057]	-.497(.068)[.068]{.069}	-.500(.068)[.068]	-.493(.090)[.089]

Table 3: Cont'd

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
		DGP 3: $\beta_0 = (3, 1, 1)'$			
.50	100	.433(.115)[.094]	.484(.090)[.089]{.081}	.476(.110)[.107]	.485(.138)[.138]
	250	.469(.062)[.054]	.500(.053)[.053]{.050}	.495(.055)[.055]	.503(.076)[.076]
	500	.473(.046)[.037]	.497(.036)[.036]{.035}	.494(.037)[.037]	.496(.051)[.051]
	1000	.478(.035)[.027]	.500(.026)[.026]{.026}	.498(.027)[.027]	.502(.038)[.038]
.25	100	.173(.145)[.123]	.232(.125)[.124]{.114}	.221(.150)[.147]	.236(.187)[.186]
	250	.211(.086)[.077]	.243(.079)[.079]{.071}	.236(.084)[.083]	.247(.115)[.115]
	500	.225(.056)[.051]	.248(.052)[.052]{.051}	.244(.054)[.053]	.250(.078)[.078]
	1000	.228(.044)[.038]	.246(.039)[.039]{.038}	.244(.040)[.039]	.248(.056)[.056]
.00	100	-.078(.169)[.150]	-.026(.174)[.172]{.164}	-.044(.188)[.183]	-.019(.229)[.228]
	250	-.030(.098)[.093]	-.008(.102)[.102]{.099}	-.018(.107)[.106]	-.002(.145)[.145]
	500	-.017(.066)[.063]	-.005(.069)[.069]{.066}	-.012(.071)[.070]	-.005(.097)[.097]
	1000	-.007(.047)[.046]	-.001(.050)[.050]{.048}	-.005(.051)[.051]	-.003(.073)[.073]
-.25	100	-.305(.178)[.170]	-.270(.197)[.196]{.199}	-.291(.218)[.214]	-.262(.280)[.280]
	250	-.262(.104)[.103]	-.264(.123)[.122]{.119}	-.272(.124)[.122]	-.256(.173)[.173]
	500	-.248(.071)[.071]	-.259(.081)[.080]{.078}	-.265(.082)[.081]	-.256(.115)[.115]
	1000	-.234(.055)[.053]	-.251(.059)[.059]{.057}	-.255(.060)[.060]	-.249(.090)[.090]
-.50	100	-.535(.181)[.177]	-.530(.218)[.216]{.223}	-.555(.236)[.229]	-.528(.304)[.303]
	250	-.474(.118)[.115]	-.515(.148)[.147]{.139}	-.523(.142)[.141]	-.505(.195)[.195]
	500	-.457(.091)[.080]	-.504(.094)[.093]{.092}	-.509(.091)[.090]	-.500(.125)[.125]
	1000	-.449(.081)[.063]	-.502(.069)[.069]{.067}	-.505(.069)[.069]	-.498(.101)[.101]
		DGP 1: $\beta_0 = (.3, .1, .1)'$			
.50	100	.364(.203)[.150]	.456(.148)[.141]{.129}	.419(.219)[.204]	.423(.234)[.220]
	250	.433(.105)[.080]	.487(.079)[.078]{.073}	.468(.095)[.090]	.469(.095)[.090]
	500	.450(.073)[.053]	.494(.053)[.053]{.051}	.482(.057)[.054]	.483(.057)[.054]
	1000	.460(.054)[.036]	.497(.036)[.036]{.036}	.491(.038)[.037]	.491(.038)[.037]
.25	100	.092(.246)[.188]	.193(.206)[.197]{.185}	.126(.269)[.239]	.127(.289)[.261]
	250	.178(.129)[.107]	.232(.114)[.112]{.109}	.203(.126)[.116]	.202(.127)[.117]
	500	.202(.084)[.069]	.242(.074)[.073]{.073}	.225(.079)[.075]	.225(.079)[.075]
	1000	.215(.059)[.048]	.246(.051)[.051]{.051}	.238(.053)[.051]	.238(.053)[.051]
.00	100	-.150(.258)[.211]	-.070(.257)[.247]{.233}	-.161(.331)[.289]	-.159(.346)[.307]
	250	-.060(.141)[.127]	-.028(.148)[.146]{.133}	-.066(.164)[.150]	-.066(.165)[.151]
	500	-.030(.090)[.085]	-.011(.097)[.097]{.093}	-.033(.104)[.099]	-.032(.104)[.099]
	1000	-.016(.059)[.057]	-.007(.065)[.065]{.066}	-.018(.068)[.066]	-.018(.069)[.066]
-.25	100	-.365(.241)[.212]	-.328(.294)[.283]{.272}	-.441(.381)[.330]	-.432(.409)[.366]
	250	-.260(.127)[.126]	-.264(.159)[.158]{.156}	-.308(.172)[.162]	-.309(.173)[.162]
	500	-.243(.093)[.093]	-.263(.116)[.115]{.110}	-.289(.123)[.117]	-.289(.123)[.117]
	1000	-.228(.071)[.068]	-.258(.084)[.084]{.088}	-.271(.087)[.085]	-.272(.088)[.085]
-.50	100	-.556(.216)[.209]	-.581(.312)[.301]{.299}	-.712(.409)[.350]	-.706(.404)[.347]
	250	-.464(.137)[.132]	-.526(.185)[.183]{.179}	-.576(.202)[.186]	-.579(.204)[.188]
	500	-.439(.113)[.095]	-.514(.129)[.128]{.124}	-.543(.137)[.130]	-.544(.138)[.131]
	1000	-.423(.101)[.066]	-.506(.089)[.089]{.088}	-.520(.092)[.090]	-.521(.092)[.090]

Table 3: Cont'd

λ_0	n	QMLE	MQMLE	RGMM	ORGMM
DGP 2: $\beta_0 = (.3, .1, .1)'$					
.50	100	.361(.206)[.152]	.453(.150)[.143]{.137}	.426(.251)[.240]	.518(.396)[.396]
	250	.435(.103)[.080]	.489(.078)[.077]{.070}	.469(.085)[.079]	.510(.185)[.185]
	500	.453(.070)[.052]	.496(.050)[.050]{.049}	.485(.053)[.051]	.502(.113)[.113]
	1000	.460(.054)[.037]	.497(.036)[.036]{.035}	.492(.038)[.037]	.494(.042)[.042]
.25	100	.098(.241)[.187]	.197(.202)[.194]{.186}	.134(.269)[.242]	.230(.459)[.459]
	250	.176(.131)[.108]	.229(.116)[.114]{.109}	.199(.128)[.117]	.231(.219)[.218]
	500	.200(.086)[.070]	.239(.075)[.074]{.071}	.222(.080)[.075]	.234(.113)[.112]
	1000	.215(.062)[.052]	.246(.055)[.055]{.051}	.238(.057)[.055]	.239(.062)[.061]
.00	100	-.144(.254)[.209]	-.064(.257)[.249]{.241}	-.154(.314)[.273]	-.029(.573)[.573]
	250	-.052(.127)[.116]	-.017(.132)[.131]{.129}	-.054(.146)[.136]	-.015(.267)[.266]
	500	-.032(.091)[.085]	-.014(.098)[.097]{.090}	-.036(.105)[.099]	-.024(.119)[.116]
	1000	-.018(.063)[.060]	-.009(.069)[.069]{.065}	-.020(.072)[.069]	-.014(.082)[.081]
-.25	100	-.354(.235)[.211]	-.311(.283)[.276]{.265}	-.423(.348)[.302]	-.320(.534)[.529]
	250	-.264(.131)[.130]	-.268(.164)[.163]{.159}	-.312(.180)[.168]	-.278(.271)[.269]
	500	-.241(.090)[.089]	-.260(.110)[.109]{.106}	-.286(.117)[.111]	-.269(.136)[.135]
	1000	-.228(.067)[.064]	-.257(.078)[.078]{.077}	-.270(.081)[.078]	-.266(.092)[.091]
-.50	100	-.543(.218)[.214]	-.559(.308)[.302]{.296}	-.696(.424)[.376]	-.621(.616)[.604]
	250	-.468(.138)[.135]	-.532(.186)[.183]{.179}	-.583(.203)[.186]	-.563(.248)[.240]
	500	-.444(.113)[.098]	-.520(.129)[.128]{.122}	-.549(.138)[.129]	-.538(.161)[.156]
	1000	-.420(.104)[.066]	-.503(.086)[.086]{.087}	-.517(.088)[.086]	-.512(.101)[.100]
DGP 3: $\beta_0 = (.3, .1, .1)'$					
.50	100	.378(.186)[.140]	.470(.131)[.127]{.114}	.439(.225)[.217]	.575(.428)[.421]
	250	.434(.100)[.076]	.487(.071)[.070]{.074}	.467(.080)[.073]	.536(.255)[.253]
	500	.450(.074)[.055]	.492(.052)[.051]{.049}	.481(.056)[.052]	.539(.229)[.226]
	1000	.460(.055)[.037]	.497(.035)[.035]{.033}	.491(.036)[.035]	.523(.168)[.167]
.25	100	.109(.217)[.165]	.210(.173)[.168]{.160}	.151(.252)[.232]	.286(.518)[.517]
	250	.183(.120)[.099]	.235(.103)[.102]{.099}	.207(.114)[.106]	.310(.398)[.394]
	500	.205(.081)[.067]	.243(.069)[.069]{.066}	.227(.074)[.070]	.287(.286)[.284]
	1000	.215(.058)[.046]	.246(.048)[.048]{.047}	.237(.050)[.048]	.265(.179)[.179]
.00	100	-.144(.241)[.194]	-.063(.235)[.227]{.199}	-.144(.329)[.296]	.056(.696)[.694]
	250	-.051(.123)[.112]	-.018(.130)[.129]{.119}	-.054(.144)[.133]	.094(.551)[.543]
	500	-.027(.084)[.079]	-.008(.091)[.090]{.089}	-.030(.098)[.093]	.032(.337)[.336]
	1000	-.015(.058)[.056]	-.006(.065)[.064]{.061}	-.017(.067)[.065]	.020(.210)[.209]
-.25	100	-.355(.231)[.205]	-.313(.273)[.265]{.250}	-.432(.357)[.307]	-.193(.780)[.778]
	250	-.267(.129)[.128]	-.272(.162)[.160]{.151}	-.317(.180)[.167]	-.183(.540)[.536]
	500	-.240(.087)[.086]	-.259(.106)[.106]{.100}	-.285(.114)[.108]	-.202(.376)[.373]
	1000	-.224(.068)[.063]	-.254(.075)[.075]{.073}	-.267(.078)[.076]	-.213(.253)[.251]
-.50	100	-.544(.209)[.204]	-.557(.290)[.284]{.279}	-.684(.447)[.407]	-.442(.904)[.903]
	250	-.467(.139)[.135]	-.526(.179)[.177]{.168}	-.577(.196)[.180]	-.464(.523)[.522]
	500	-.433(.119)[.099]	-.506(.123)[.123]{.119}	-.535(.130)[.125]	-.412(.483)[.475]
	1000	-.423(.107)[.074]	-.504(.086)[.086]{.083}	-.519(.088)[.086]	-.466(.257)[.255]

Table 4: Empirical Mean(rmse)[sd]{$\hat{\text{sd}}$} of Estimators of β for SAR Model
Cases of Consistent QMLEs

λ_0	n	β_0	QMLE	MQMLE	RGMM	ORGMM
			DGP 1: Constant Circular Neighbours (REG-1), $\beta_0 = (3,1,1)'$			
.5	100	3	3.220(.644)[.606]{.592}	3.166(.708)[.688]{.691}	3.192(.733)[.707]	3.129(.797)[.786]
		1	1.006(.131)[.131]{.123}	0.992(.153)[.152]{.143}	0.989(.152)[.152]	0.988(.152)[.152]
		1	1.003(.201)[.201]{.203}	0.990(.229)[.228]{.222}	0.983(.228)[.228]	0.981(.229)[.229]
	250	3	3.089(.396)[.386]{.392}	3.051(.388)[.385]{.369}	3.069(.395)[.389]	3.040(.437)[.435]
		1	0.999(.096)[.096]{.093}	0.999(.096)[.096]{.093}	0.996(.096)[.096]	0.996(.096)[.096]
		1	1.003(.138)[.138]{.134}	1.004(.149)[.149]{.144}	1.002(.149)[.149]	1.002(.149)[.149]
	500	3	3.039(.264)[.261]{.276}	3.019(.261)[.260]{.253}	3.030(.264)[.263]	3.013(.290)[.290]
		1	1.000(.068)[.068]{.068}	0.996(.070)[.070]{.070}	0.995(.070)[.070]	0.995(.070)[.070]
		1	0.999(.106)[.106]{.104}	0.998(.106)[.106]{.104}	0.997(.106)[.106]	0.997(.106)[.106]
-.5	100	3	3.047(.357)[.353]{.360}	3.011(.356)[.355]{.339}	3.041(.362)[.360]	3.024(.400)[.399]
		1	0.994(.130)[.130]{.123}	0.994(.157)[.157]{.149}	0.988(.157)[.157]	0.988(.158)[.158]
		1	0.995(.226)[.226]{.222}	0.996(.227)[.227]{.222}	0.988(.226)[.226]	0.987(.227)[.227]
	250	3	3.026(.221)[.220]{.230}	3.011(.220)[.220]{.214}	3.024(.221)[.220]	3.016(.246)[.245]
		1	0.999(.098)[.098]{.100}	0.995(.093)[.093]{.094}	0.992(.094)[.093]	0.992(.094)[.093]
		1	1.002(.130)[.130]{.135}	0.992(.143)[.143]{.144}	0.989(.143)[.143]	0.990(.144)[.143]
	500	3	3.001(.157)[.157]{.166}	2.993(.158)[.157]{.152}	3.000(.158)[.158]	2.993(.174)[.174]
		1	0.998(.067)[.067]{.068}	0.998(.067)[.067]{.070}	0.997(.067)[.067]	0.997(.067)[.067]
		1	0.999(.104)[.104]{.103}	0.999(.104)[.104]{.103}	0.997(.104)[.104]	0.998(.104)[.104]
			DGP 2: Constant Circular Neighbours (REG-1), $\beta_0 = (3,1,1)'$			
.5	100	3	3.207(.597)[.560]{.570}	3.117(.641)[.631]{.645}	3.150(.706)[.690]	3.071(.843)[.840]
		1	1.007(.154)[.154]{.148}	1.007(.154)[.154]{.148}	1.003(.154)[.154]	1.003(.151)[.151]
		1	1.000(.207)[.207]{.198}	0.999(.220)[.220]{.211}	0.993(.220)[.220]	0.991(.217)[.217]
	250	3	3.078(.380)[.372]{.345}	3.041(.372)[.370]{.345}	3.057(.377)[.372]	3.029(.512)[.512]
		1	1.004(.096)[.096]{.092}	1.004(.096)[.096]{.092}	1.001(.095)[.095]	1.001(.095)[.095]
		1	0.993(.141)[.141]{.132}	1.010(.146)[.146]{.141}	1.007(.146)[.146]	1.007(.145)[.145]
	500	3	3.028(.254)[.253]{.229}	3.009(.252)[.252]{.245}	3.020(.254)[.253]	2.998(.357)[.357]
		1	1.001(.067)[.067]{.068}	0.996(.071)[.070]{.069}	0.995(.071)[.070]	0.995(.070)[.070]
		1	0.999(.100)[.100]{.097}	1.002(.108)[.108]{.103}	1.001(.108)[.108]	1.000(.108)[.108]
-.5	100	3	3.044(.326)[.323]{.310}	3.010(.324)[.324]{.316}	3.039(.331)[.329]	3.021(.450)[.449]
		1	0.997(.154)[.154]{.141}	0.999(.154)[.154]{.140}	0.992(.154)[.153]	0.993(.153)[.153]
		1	0.999(.235)[.235]{.217}	1.000(.235)[.235]{.218}	0.992(.234)[.234]	0.990(.231)[.231]
	250	3	3.012(.205)[.205]{.201}	2.997(.205)[.205]{.206}	3.010(.206)[.206]	3.002(.281)[.281]
		1	1.000(.097)[.097]{.093}	1.001(.097)[.097]{.093}	0.998(.097)[.097]	0.999(.097)[.097]
		1	0.997(.147)[.147]{.141}	0.998(.147)[.147]{.142}	0.994(.147)[.147]	0.995(.146)[.145]
	500	3	3.010(.148)[.148]{.101}	3.002(.148)[.148]{.150}	3.009(.148)[.148]	3.002(.207)[.207]
		1	1.001(.070)[.070]{.067}	0.995(.069)[.069]{.069}	0.994(.069)[.069]	0.994(.069)[.069]
		1	1.000(.104)[.104]{.103}	1.000(.104)[.104]{.103}	0.998(.104)[.104]	0.999(.103)[.103]

Table 4: Cont'd

λ_0	n	β_0	QMLE	MQMLE	RGMM	ORGMM
DGP 1: Queen Contiguity (REG-1), $\beta_0 = (.3, .1, .1)'$						
.5	100	.3	.338(.154)[.149]{.139}	.323(.146)[.145]{.137}	.328(.154)[.151]	.306(.167)[.167]
		.1	.094(.163)[.163]{.159}	.094(.163)[.163]{.169}	.093(.162)[.162]	.092(.165)[.165]
		.1	.100(.204)[.204]{.195}	.100(.204)[.204]{.195}	.099(.202)[.202]	.100(.202)[.202]
	250	.3	.310(.082)[.081]{.082}	.303(.080)[.080]{.079}	.307(.081)[.081]	.300(.081)[.081]
		.1	.109(.096)[.096]{.096}	.109(.096)[.096]{.096}	.109(.096)[.095]	.108(.096)[.096]
		.1	.101(.139)[.139]{.134}	.096(.141)[.141]{.139}	.096(.141)[.141]	.096(.140)[.140]
	500	.3	.308(.060)[.059]{.059}	.304(.059)[.058]{.056}	.306(.064)[.064]	.302(.064)[.064]
		.1	.101(.067)[.067]{.068}	.101(.067)[.067]{.068}	.101(.067)[.067]	.100(.067)[.067]
		.1	.102(.100)[.100]{.098}	.102(.100)[.100]{.098}	.102(.100)[.100]	.101(.100)[.100]
-.5	100	.3	.306(.109)[.109]{.106}	.301(.108)[.108]{.104}	.305(.110)[.109]	.304(.110)[.110]
		.1	.100(.167)[.167]{.157}	.100(.168)[.168]{.159}	.099(.166)[.166]	.097(.168)[.168]
		.1	.087(.195)[.194]{.185}	.084(.199)[.198]{.189}	.082(.196)[.195]	.082(.196)[.195]
	250	.3	.303(.069)[.069]{.069}	.303(.069)[.069]{.068}	.305(.069)[.069]	.306(.069)[.069]
		.1	.097(.099)[.098]{.095}	.107(.100)[.100]{.095}	.106(.100)[.100]	.106(.100)[.099]
		.1	.096(.138)[.138]{.134}	.106(.138)[.138]{.133}	.105(.138)[.138]	.105(.138)[.138]
	500	.3	.301(.048)[.048]{.048}	.297(.049)[.049]{.048}	.298(.049)[.049]	.298(.049)[.049]
		.1	.100(.069)[.069]{.067}	.101(.069)[.069]{.067}	.101(.069)[.069]	.101(.069)[.069]
		.1	.100(.097)[.097]{.098}	.100(.097)[.097]{.098}	.100(.097)[.097]	.100(.097)[.097]
DGP 2: Queen Contiguity (REG-1), $\beta_0 = (.3, .1, .1)'$						
.5	100	.3	.327(.136)[.133]{.128}	.311(.129)[.129]{.120}	.318(.134)[.133]	.251(.234)[.229]
		.1	.103(.161)[.161]{.153}	.103(.161)[.161]{.152}	.103(.161)[.161]	.102(.161)[.161]
		.1	.103(.194)[.194]{.189}	.094(.194)[.194]{.180}	.092(.193)[.193]	.093(.192)[.192]
	250	.3	.311(.080)[.079]{.087}	.304(.078)[.078]{.078}	.308(.079)[.079]	.280(.111)[.110]
		.1	.104(.095)[.095]{.093}	.108(.097)[.097]{.093}	.107(.097)[.097]	.106(.095)[.095]
		.1	.096(.130)[.130]{.132}	.096(.130)[.130]{.132}	.096(.129)[.129]	.096(.129)[.129]
	500	.3	.307(.057)[.057]{.064}	.305(.058)[.058]{.056}	.306(.064)[.063]	.292(.070)[.069]
		.1	.101(.069)[.069]{.067}	.101(.069)[.069]{.067}	.101(.069)[.069]	.100(.068)[.068]
		.1	.104(.102)[.102]{.098}	.094(.101)[.101]{.098}	.094(.101)[.101]	.092(.100)[.099]
-.5	100	.3	.306(.109)[.109]{.110}	.301(.108)[.108]{.103}	.306(.109)[.109]	.304(.111)[.111]
		.1	.104(.171)[.171]{.162}	.104(.172)[.172]{.164}	.103(.170)[.170]	.103(.159)[.159]
		.1	.101(.194)[.194]{.181}	.089(.194)[.194]{.181}	.088(.192)[.191]	.084(.181)[.180]
	250	.3	.300(.069)[.069]{.072}	.302(.067)[.067]{.066}	.304(.067)[.067]	.303(.070)[.070]
		.1	.103(.095)[.095]{.093}	.103(.095)[.095]{.093}	.102(.095)[.094]	.101(.092)[.092]
		.1	.101(.133)[.133]{.132}	.095(.138)[.138]{.130}	.094(.138)[.138]	.093(.133)[.133]
	500	.3	.299(.048)[.048]{.051}	.298(.048)[.048]{.048}	.299(.048)[.048]	.299(.049)[.049]
		.1	.102(.067)[.067]{.068}	.102(.067)[.067]{.068}	.101(.067)[.067]	.100(.066)[.066]
		.1	.099(.099)[.099]{.096}	.103(.103)[.103]{.098}	.103(.102)[.102]	.103(.101)[.101]

Table 5: Empirical Mean(rmse)[sd]{$\hat{\text{sd}}$} of Estimators of β for SAR Model
Case I of Inconsistent QMLEs: Circular Neighbours (REG-1)

λ_0	n	β_0	QMLE	MQMLE	RGMM	ORGMM
			DGP 1: $\beta_0 = (3, 1, 1)'$			
.5	100	3	3.398(.598)[.719]	3.116(.596)[.607]{.594}	3.145(.641)[.624]	3.104(.679)[.671]
		1	1.001(.125)[.125]	0.997(.125)[.125]{.118}	0.993(.125)[.125]	0.993(.126)[.126]
		1	0.999(.190)[.190]	0.992(.189)[.189]{.188}	0.986(.188)[.187]	0.987(.187)[.187]
	250	3	3.254(.346)[.429]	3.055(.350)[.355]{.349}	3.067(.351)[.345]	3.048(.370)[.367]
		1	1.001(.076)[.076]	0.998(.076)[.076]{.073}	0.997(.076)[.076]	0.997(.076)[.076]
		1	1.011(.125)[.125]	1.004(.124)[.124]{.119}	1.002(.124)[.124]	1.002(.124)[.124]
	500	3	3.219(.263)[.342]	3.024(.265)[.266]{.262}	3.030(.266)[.264]	3.021(.281)[.280]
		1	1.006(.054)[.055]	1.000(.054)[.054]{.056}	0.999(.054)[.054]	0.999(.055)[.055]
		1	1.008(.090)[.090]	1.002(.089)[.089]{.089}	1.001(.089)[.089]	1.001(.089)[.089]
-.5	100	3	2.897(.206)[.231]	2.986(.259)[.259]{.270}	2.981(.232)[.231]	2.993(.245)[.245]
		1	1.003(.127)[.127]	0.999(.127)[.127]{.120}	0.996(.127)[.127]	0.995(.127)[.127]
		1	1.014(.191)[.191]	1.003(.192)[.192]{.194}	0.996(.192)[.192]	0.993(.192)[.192]
	250	3	2.898(.134)[.169]	3.010(.177)[.177]{.166}	3.000(.146)[.146]	3.003(.154)[.154]
		1	1.005(.072)[.073]	0.996(.072)[.072]{.074}	0.995(.073)[.073]	0.995(.073)[.073]
		1	1.001(.122)[.122]	0.996(.121)[.121]{.119}	0.995(.121)[.121]	0.995(.121)[.121]
	500	3	2.887(.101)[.152]	3.011(.136)[.137]{.135}	3.009(.115)[.115]	3.011(.121)[.120]
		1	1.003(.055)[.055]	1.000(.055)[.055]{.055}	0.999(.055)[.055]	0.999(.055)[.055]
		1	1.002(.089)[.089]	0.995(.089)[.089]{.088}	0.994(.089)[.089]	0.993(.089)[.089]
			DGP 2: $\beta_0 = (3, 1, 1)'$			
.5	100	3	3.374(.572)[.683]	3.104(.568)[.578]{.563}	3.136(.623)[.607]	3.092(.767)[.762]
		1	1.009(.122)[.122]	1.005(.122)[.122]{.115}	1.001(.122)[.122]	1.001(.121)[.121]
		1	0.995(.193)[.193]	0.988(.192)[.193]{.182}	0.982(.192)[.191]	0.983(.192)[.191]
	250	3	3.229(.325)[.397]	3.030(.327)[.328]{.330}	3.045(.321)[.318]	3.021(.399)[.399]
		1	1.000(.073)[.073]	0.997(.073)[.073]{.072}	0.995(.074)[.073]	0.995(.074)[.074]
		1	1.013(.118)[.118]	1.006(.117)[.118]{.117}	1.004(.118)[.118]	1.005(.118)[.118]
	500	3	3.200(.261)[.329]	3.003(.262)[.262]{.259}	3.013(.265)[.264]	3.005(.343)[.343]
		1	1.007(.054)[.055]	1.001(.054)[.054]{.055}	1.000(.054)[.054]	1.000(.054)[.054]
		1	1.006(.089)[.089]	1.001(.088)[.088]{.087}	1.000(.088)[.088]	1.000(.088)[.088]
-.5	100	3	2.907(.209)[.229]	3.002(.260)[.260]{.273}	2.992(.239)[.239]	2.994(.265)[.265]
		1	0.997(.125)[.125]	0.993(.124)[.124]{.119}	0.990(.125)[.124]	0.991(.124)[.124]
		1	1.016(.198)[.199]	1.003(.199)[.199]{.195}	0.997(.200)[.200]	0.998(.199)[.199]
	250	3	2.892(.135)[.173]	3.000(.168)[.168]{.161}	2.995(.145)[.145]	2.996(.169)[.168]
		1	1.010(.075)[.076]	1.001(.075)[.075]{.072}	1.000(.076)[.076]	1.000(.076)[.076]
		1	0.996(.122)[.122]	0.991(.121)[.121]{.116}	0.989(.121)[.121]	0.990(.121)[.121]
	500	3	2.875(.101)[.161]	2.997(.133)[.133]{.129}	2.994(.113)[.113]	2.991(.137)[.137]
		1	1.007(.056)[.057]	1.004(.056)[.056]{.055}	1.003(.056)[.056]	1.003(.056)[.056]
		1	1.010(.090)[.090]	1.002(.090)[.090]{.088}	1.001(.090)[.090]	1.001(.090)[.090]

Table 6: Empirical Mean(rmse)[sd]{$\hat{\text{sd}}$} of Estimators of β for SAR Model
Case II of Inconsistent QMLEs: Group Interaction (REG-2)

λ_0	n	β_0	QMLE	MQMLE	RGMM	ORGMM
			DGP 1: $\beta_0 = (3, 1, 1)'$			
.5	100	3	3.493(.795)[.623]	3.146(.645)[.628]{.599}	3.207(.698)[.667]	3.196(.714)[.687]
		1	1.131(.253)[.217]	1.036(.221)[.218]{.205}	1.043(.237)[.233]	1.043(.239)[.235]
		1	1.096(.272)[.254]	1.015(.245)[.244]{.247}	1.019(.260)[.260]	1.019(.262)[.261]
	250	3	3.239(.423)[.348]	3.041(.358)[.355]{.349}	3.074(.375)[.367]	3.054(.397)[.394]
		1	1.059(.160)[.149]	1.008(.149)[.149]{.142}	1.012(.151)[.151]	1.008(.155)[.155]
		1	1.058(.160)[.149]	1.007(.149)[.149]{.139}	1.011(.152)[.151]	1.008(.155)[.155]
	500	3	3.173(.291)[.234]	3.017(.237)[.236]{.239}	3.038(.245)[.242]	3.027(.258)[.256]
		1	1.045(.101)[.090]	1.002(.090)[.090]{.091}	1.006(.091)[.091]	1.003(.093)[.093]
		1	1.045(.106)[.096]	1.004(.096)[.096]{.099}	1.008(.097)[.096]	1.005(.099)[.099]
-.5	100	3	3.070(.388)[.382]	3.075(.489)[.483]{.480}	3.104(.493)[.482]	3.097(.521)[.512]
		1	1.011(.168)[.168]	1.011(.183)[.182]{.202}	1.009(.190)[.190]	1.009(.194)[.194]
		1	1.019(.230)[.229]	1.020(.247)[.246]{.245}	1.016(.243)[.242]	1.015(.245)[.245]
	250	3	2.938(.251)[.243]	3.015(.308)[.307]{.301}	3.033(.296)[.294]	3.025(.312)[.310]
		1	0.980(.129)[.127]	0.997(.135)[.135]{.134}	0.998(.136)[.136]	0.997(.139)[.139]
		1	0.982(.127)[.125]	1.000(.134)[.134]{.131}	1.001(.134)[.134]	1.001(.136)[.136]
	500	3	2.918(.189)[.170]	3.013(.216)[.215]{.204}	3.023(.202)[.200]	3.017(.212)[.212]
		1	0.976(.082)[.078]	1.001(.087)[.087]{.083}	1.002(.083)[.083]	1.001(.085)[.085]
		1	0.976(.086)[.083]	1.000(.088)[.088]{.092}	1.001(.087)[.087]	0.999(.089)[.089]
			DGP 2: $\beta_0 = (3, 1, 1)'$			
.5	100	3	3.397(.746)[.631]	3.057(.622)[.620]{.654}	3.088(.693)[.688]	3.027(.786)[.786]
		1	1.106(.239)[.214]	1.012(.213)[.213]{.198}	1.009(.234)[.234]	0.998(.255)[.255]
		1	1.084(.277)[.264]	1.003(.252)[.252]{.239}	0.999(.275)[.275]	0.989(.285)[.285]
	250	3	3.211(.408)[.349]	3.006(.349)[.349]{.333}	3.036(.366)[.364]	2.979(.450)[.449]
		1	1.045(.152)[.146]	0.993(.145)[.144]{.141}	0.996(.148)[.148]	0.984(.165)[.165]
		1	1.046(.153)[.145]	0.993(.144)[.144]{.138}	0.997(.148)[.148]	0.984(.163)[.162]
	500	3	3.172(.287)[.229]	3.016(.230)[.230]{.235}	3.036(.238)[.235]	3.005(.303)[.303]
		1	1.049(.102)[.090]	1.005(.090)[.090]{.091}	1.009(.091)[.091]	1.001(.105)[.105]
		1	1.046(.110)[.100]	1.005(.101)[.101]{.099}	1.008(.101)[.101]	1.001(.112)[.112]
-.5	100	3	3.055(.397)[.394]	3.073(.520)[.515]{.508}	3.096(.508)[.499]	3.031(.598)[.597]
		1	1.016(.174)[.173]	1.020(.197)[.196]{.218}	1.019(.197)[.196]	1.004(.214)[.214]
		1	1.004(.225)[.225]	1.009(.246)[.246]{.260}	1.001(.241)[.241]	0.991(.248)[.248]
	250	3	2.939(.247)[.239]	3.018(.301)[.300]{.392}	3.031(.286)[.284]	2.992(.357)[.357]
		1	0.986(.128)[.127]	1.006(.136)[.136]{.133}	1.005(.137)[.137]	0.997(.149)[.148]
		1	0.986(.123)[.122]	1.005(.132)[.131]{.130}	1.006(.130)[.130]	0.997(.140)[.140]
	500	3	2.912(.195)[.174]	3.003(.216)[.216]{.200}	3.015(.206)[.205]	2.993(.253)[.253]
		1	0.976(.081)[.078]	1.000(.085)[.085]{.083}	1.002(.083)[.083]	0.996(.091)[.091]
		1	0.982(.090)[.088]	1.005(.093)[.093]{.092}	1.007(.094)[.093]	1.002(.100)[.100]

Table 7: Empirical Mean(rmse)[sd] of Estimators of λ and ρ for SARAR(1,1) Model
Case I of Inconsistent QMLEs: Circular Neighbours (REG-1)

Par	QMLE-λ	MQMLE-λ	KP-λ	QMLE-ρ	MQMLE-ρ	KP-ρ
			DGP 1: $\beta_0 = (3,1,1)'$			
1-1	.470(.141)[.138]	.472(.197)[.195]	.578(.219)[.204]	.409(.195)[.172]	.446(.237)[.231]	.335(.341)[.299]
	.484(.080)[.078]	.482(.118)[.117]	.528(.109)[.105]	.445(.116)[.102]	.488(.140)[.139]	.479(.180)[.179]
	.487(.065)[.064]	.489(.097)[.097]	.515(.093)[.092]	.454(.088)[.075]	.491(.110)[.109]	.512(.156)[.156]
	.490(.043)[.042]	.495(.060)[.059]	.505(.057)[.057]	.458(.066)[.051]	.497(.070)[.070]	.533(.103)[.097]
1-2	.372(.173)[.116]	.418(.233)[.218]	.494(.143)[.143]	-.307(.249)[.158]	-.505(.252)[.239]	-.507(.244)[.244]
	.411(.109)[.063]	.488(.095)[.094]	.501(.072)[.072]	-.324(.202)[.100]	-.502(.153)[.153]	-.492(.150)[.150]
	.400(.112)[.050]	.498(.071)[.071]	.498(.060)[.060]	-.305(.208)[.072]	-.504(.126)[.125]	-.476(.121)[.119]
	.421(.084)[.030]	.502(.047)[.047]	.499(.035)[.035]	-.321(.186)[.051]	-.506(.109)[.108]	-.470(.083)[.078]
2-1	.280(.144)[.141]	.250(.200)[.200]	.333(.239)[.224]	.374(.208)[.165]	.441(.225)[.217]	.358(.307)[.272]
	.292(.095)[.086]	.253(.128)[.127]	.297(.133)[.124]	.399(.140)[.097]	.470(.135)[.131]	.464(.176)[.172]
	.293(.080)[.067]	.252(.106)[.106]	.276(.105)[.101]	.408(.119)[.075]	.491(.109)[.107]	.499(.146)[.146]
	.287(.057)[.043]	.250(.064)[.064]	.259(.064)[.064]	.421(.093)[.049]	.494(.065)[.065]	.524(.092)[.089]
2-2	.113(.189)[.130]	.233(.188)[.163]	.235(.186)[.186]	-.330(.231)[.156]	-.582(.269)[.249]	-.507(.259)[.259]
	.156(.120)[.074]	.239(.131)[.131]	.248(.092)[.092]	-.337(.188)[.095]	-.503(.209)[.209]	-.484(.151)[.150]
	.140(.125)[.059]	.248(.099)[.099]	.247(.079)[.079]	-.319(.193)[.069]	-.510(.115)[.114]	-.484(.117)[.116]
	.164(.093)[.036]	.250(.052)[.052]	.250(.045)[.045]	-.332(.175)[.047]	-.501(.102)[.101]	-.475(.080)[.076]
3-1	.082(.168)[.147]	.015(.210)[.209]	.080(.236)[.222]	.335(.239)[.172]	.428(.241)[.230]	.367(.292)[.260]
	.090(.124)[.086]	.012(.126)[.125]	.047(.131)[.123]	.373(.160)[.097]	.472(.127)[.124]	.467(.165)[.161]
	.094(.116)[.067]	.006(.099)[.099]	.025(.103)[.100]	.380(.140)[.072]	.495(.093)[.091]	.502(.131)[.131]
	.082(.093)[.043]	.001(.062)[.062]	.009(.064)[.063]	.397(.114)[.048]	.496(.059)[.059]	.526(.088)[.083]
3-2	-.104(.163)[.125]	-.027(.171)[.148]	-.027(.196)[.194]	-.353(.208)[.147]	-.488(.208)[.187]	-.485(.250)[.250]
	-.078(.109)[.076]	-.023(.152)[.150]	-.006(.108)[.108]	-.356(.170)[.091]	-.489(.117)[.116]	-.481(.148)[.147]
	-.086(.106)[.062]	.000(.123)[.123]	-.001(.096)[.096]	-.343(.170)[.065]	-.501(.102)[.102]	-.478(.120)[.117]
	-.071(.081)[.040]	.001(.060)[.059]	-.001(.055)[.055]	-.350(.156)[.045]	-.502(.106)[.104]	-.473(.081)[.077]
4-1	-.126(.183)[.135]	-.219(.194)[.192]	-.189(.210)[.201]	.323(.246)[.170]	.430(.239)[.228]	.395(.258)[.235]
	-.132(.144)[.082]	-.240(.106)[.105]	-.224(.117)[.114]	.363(.169)[.099]	.478(.112)[.109]	.485(.150)[.149]
	-.119(.148)[.068]	-.247(.090)[.090]	-.236(.095)[.093]	.365(.155)[.075]	.490(.085)[.085]	.510(.118)[.118]
	-.131(.127)[.043]	-.247(.057)[.057]	-.242(.055)[.055]	.376(.134)[.049]	.492(.056)[.055]	.520(.079)[.076]
4-2	-.303(.130)[.119]	-.300(.217)[.215]	-.279(.208)[.206]	-.395(.168)[.131]	-.484(.224)[.228]	-.488(.221)[.221]
	-.288(.084)[.075]	-.272(.155)[.154]	-.260(.122)[.121]	-.384(.145)[.086]	-.487(.200)[.199]	-.475(.152)[.150]
	-.289(.069)[.057]	-.249(.106)[.106]	-.255(.098)[.098]	-.378(.137)[.061]	-.508(.101)[.100]	-.472(.115)[.112]
	-.284(.050)[.037]	-.244(.056)[.056]	-.253(.058)[.058]	-.381(.126)[.043]	-.506(.104)[.103]	-.471(.081)[.076]
5-1	-.357(.192)[.128]	-.458(.165)[.160]	-.449(.169)[.161]	.320(.244)[.164]	.438(.201)[.191]	.413(.215)[.197]
	-.373(.146)[.071]	-.491(.082)[.082]	-.481(.088)[.086]	.362(.169)[.097]	.483(.101)[.100]	.488(.126)[.126]
	-.352(.159)[.057]	-.496(.068)[.068]	-.493(.073)[.073]	.357(.160)[.072]	.491(.074)[.074]	.509(.097)[.097]
	-.374(.131)[.037]	-.499(.041)[.041]	-.498(.045)[.045]	.377(.132)[.047]	.497(.047)[.047]	.526(.069)[.064]
5-2	-.478(.104)[.101]	-.523(.180)[.179]	-.518(.189)[.188]	-.437(.140)[.125]	-.490(.215)[.214]	-.491(.217)[.217]
	-.480(.069)[.066]	-.513(.128)[.126]	-.511(.111)[.111]	-.424(.113)[.084]	-.491(.146)[.145]	-.474(.142)[.140]
	-.472(.057)[.050]	-.498(.109)[.109]	-.501(.093)[.093]	-.429(.092)[.059]	-.507(.107)[.106]	-.474(.113)[.110]
	-.478(.040)[.033]	-.499(.054)[.053]	-.502(.056)[.056]	-.424(.086)[.042]	-.500(.077)[.076]	-.470(.078)[.073]

Note: (i) The DGP used: $Y_n = \lambda W_n Y_n + \iota_n \beta_0 + X_{1n}\beta_1 + X_{2n}\beta_2 + \epsilon_n, \quad \epsilon_n = \rho W_n \epsilon_n + v_n$.

(ii) Par = i-j, where '$i = 1, 2, 3, 4, 5$' represents '$\lambda = .5, .25, 0, -.25, -.5$'; '$j = 1, 2$' represents '$\rho = .5, -.5$'. Under each Par setting, $n = 100, 250, 500, 1000$, corresponding to the four rows.

(iii) KP denotes Kelejian and Prucha's (2010) three-step estimator.

Table 7: Cont'd

Par	λ_{QML}	λ_{MQML}	λ_{GS}	ρ_{QML}	ρ_{MQML}	ρ_{GS}
			DGP 2: $\beta_0 = (3, 1, 1)'$			
1-1	.471(.138)[.135]	.473(.188)[.186]	.576(.216)[.202]	.404(.195)[.170]	.443(.235)[.228]	.329(.340)[.294]
	.487(.085)[.084]	.489(.123)[.122]	.529(.114)[.110]	.440(.119)[.103]	.477(.144)[.142]	.466(.180)[.177]
	.490(.064)[.063]	.494(.096)[.096]	.521(.092)[.089]	.454(.090)[.077]	.489(.113)[.113]	.513(.158)[.157]
	.493(.042)[.041]	.499(.059)[.059]	.508(.056)[.056]	.455(.068)[.051]	.493(.070)[.070]	.526(.099)[.095]
1-2	.380(.173)[.124]	.474(.153)[.138]	.495(.141)[.141]	-.318(.253)[.175]	-.498(.266)[.252]	-.509(.246)[.246]
	.408(.114)[.067]	.483(.101)[.100]	.495(.072)[.072]	-.320(.212)[.113]	-.492(.200)[.199]	-.483(.156)[.155]
	.399(.116)[.056]	.497(.063)[.063]	.497(.060)[.060]	-.303(.215)[.085]	-.504(.124)[.123]	-.476(.125)[.123]
	.422(.085)[.035]	.502(.038)[.038]	.500(.035)[.035]	-.324(.185)[.056]	-.506(.100)[.100]	-.473(.079)[.074]
2-1	.285(.143)[.139]	.262(.194)[.194]	.349(.241)[.220]	.370(.214)[.170]	.432(.232)[.222]	.345(.317)[.276]
	.280(.092)[.087]	.244(.126)[.126]	.282(.127)[.123]	.407(.136)[.099]	.478(.134)[.132]	.478(.177)[.176]
	.292(.081)[.069]	.253(.105)[.105]	.275(.106)[.103]	.411(.116)[.074]	.483(.103)[.102]	.502(.145)[.145]
	.285(.056)[.043]	.247(.064)[.064]	.257(.063)[.062]	.424(.091)[.049]	.497(.065)[.064]	.529(.096)[.092]
2-2	.120(.186)[.134]	.235(.198)[.175]	.227(.184)[.182]	-.337(.235)[.169]	-.490(.271)[.254]	-.504(.263)[.263]
	.157(.121)[.078]	.247(.109)[.108]	.249(.094)[.094]	-.336(.194)[.104]	-.492(.125)[.124]	-.485(.153)[.152]
	.141(.125)[.062]	.251(.101)[.101]	.247(.081)[.081]	-.318(.197)[.076]	-.502(.129)[.127]	-.479(.121)[.119]
	.163(.096)[.039]	.252(.043)[.043]	.248(.047)[.046]	-.334(.174)[.053]	-.507(.101)[.109]	-.474(.083)[.079]
3-1	.086(.171)[.148]	.033(.210)[.207]	.085(.234)[.218]	.339(.231)[.166]	.419(.235)[.221]	.371(.282)[.250]
	.082(.121)[.089]	.007(.124)[.124]	.037(.130)[.125]	.376(.161)[.103]	.472(.128)[.125]	.476(.166)[.164]
	.092(.114)[.068]	.005(.096)[.096]	.022(.102)[.100]	.382(.138)[.071]	.490(.088)[.087]	.502(.124)[.124]
	.081(.092)[.044]	.001(.061)[.061]	.009(.061)[.060]	.395(.116)[.050]	.493(.058)[.058]	.525(.088)[.085]
3-2	-.087(.156)[.129]	-.026(.205)[.201]	-.012(.199)[.199]	-.367(.196)[.144]	-.479(.211)[.197]	-.492(.238)[.238]
	-.078(.109)[.077]	-.021(.138)[.137]	-.007(.109)[.109]	-.356(.173)[.096]	-.487(.195)[.195]	-.479(.152)[.150]
	-.088(.108)[.062]	-.005(.106)[.106]	-.009(.092)[.091]	-.344(.171)[.071]	-.508(.115)[.115]	-.474(.120)[.117]
	-.070(.080)[.039]	.008(.057)[.057]	.000(.054)[.054]	-.352(.156)[.049]	-.508(.107)[.105]	-.472(.080)[.075]
4-1	-.132(.186)[.144]	-.214(.201)[.198]	-.185(.215)[.205]	.329(.238)[.165]	.428(.227)[.215]	.393(.245)[.221]
	-.132(.148)[.090]	-.237(.119)[.118]	-.219(.120)[.116]	.366(.170)[.105]	.477(.128)[.126]	.481(.150)[.149]
	-.119(.149)[.072]	-.246(.088)[.088]	-.236(.096)[.095]	.368(.153)[.076]	.491(.082)[.082]	.508(.114)[.114]
	-.133(.125)[.045]	-.248(.055)[.055]	-.244(.057)[.056]	.382(.129)[.051]	.497(.054)[.054]	.527(.079)[.074]
4-2	-.297(.132)[.123]	-.287(.206)[.205]	-.280(.207)[.205]	-.401(.175)[.144]	-.462(.232)[.235]	-.492(.233)[.233]
	-.288(.083)[.074]	-.278(.107)[.104]	-.261(.115)[.115]	-.387(.146)[.092]	-.479(.157)[.156]	-.480(.151)[.150]
	-.289(.071)[.060]	-.253(.100)[.100]	-.253(.102)[.102]	-.381(.135)[.064]	-.508(.117)[.117]	-.480(.118)[.116]
	-.283(.050)[.038]	-.252(.053)[.053]	-.254(.058)[.058]	-.381(.127)[.045]	-.509(.107)[.106]	-.470(.082)[.076]
5-1	-.373(.186)[.135]	-.472(.154)[.151]	-.459(.163)[.158]	.327(.249)[.180]	.445(.205)[.198]	.414(.223)[.206]
	-.380(.144)[.080]	-.492(.082)[.082]	-.485(.090)[.089]	.369(.167)[.103]	.484(.104)[.102]	.492(.127)[.127]
	-.355(.158)[.063]	-.498(.066)[.066]	-.491(.068)[.067]	.362(.156)[.074]	.495(.070)[.070]	.510(.093)[.092]
	-.375(.132)[.042]	-.498(.043)[.043]	-.498(.044)[.044]	.377(.133)[.051]	.495(.048)[.048]	.524(.068)[.063]
5-2	-.473(.120)[.117]	-.520(.191)[.190]	-.518(.198)[.198]	-.436(.156)[.143]	-.483(.226)[.226]	-.483(.229)[.229]
	-.476(.072)[.068]	-.505(.109)[.108]	-.507(.112)[.112]	-.427(.113)[.086]	-.485(.146)[.146]	-.477(.146)[.144]
	-.472(.059)[.052]	-.502(.107)[.107]	-.500(.096)[.096]	-.431(.093)[.062]	-.503(.107)[.107]	-.481(.109)[.107]
	-.480(.040)[.034]	-.498(.063)[.063]	-.507(.057)[.057]	-.422(.089)[.042]	-.505(.075)[.074]	-.469(.081)[.074]