

is allowed to possess a general structure of the form

$$u_{it} = \mu_i + \varepsilon_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T, \quad (3.2)$$

where μ_i denotes the unobservable space-specific effect, due to aspects of regional structure, firm's specific feature, etc. Spatial units may be dependent. To allow for such a possibility, Anselin (1988) introduced a SAR process into the disturbance vector $\varepsilon_t = \{\varepsilon_{1t}, \dots, \varepsilon_{Nt}\}'$,

$$\varepsilon_t = \lambda W \varepsilon_t + v_t, \quad t = 1, \dots, T, \quad (3.3)$$

where the spatial weight matrix W is defined similarly to that in Model (2.2), and v_t is an $N \times 1$ vector of iid remainder disturbances with mean zero and variance σ_v^2 .

We are interested in testing the hypothesis $H_0 : \lambda = 0$. We consider the scenario where the time dimension T is small and the 'space' dimension N is large. This is the typical feature for many micro-level panel data sets. The space-specific effects μ_i can be random or fixed. As T is small, the time-specific effects can be directly built into the model.

3.1. Panel linear regression with random effects

Let $B = I_N - \lambda W$. Stacking the vectors (Y_t, u_t, v_t) and the matrix X_t , the model can be written in matrix form:

$$Y = X\beta + u, \quad u = (\iota_T \otimes I_N)\mu + (I_T \otimes B^{-1})v, \quad (3.4)$$

where ι_m represents an $m \times 1$ vector of ones, I_m represents an $m \times m$ identity matrix.

Assuming (i) the elements of μ are iid with mean zero and variance σ_μ^2 , (ii) the elements of v are iid with mean zero and variance σ_v^2 , and (iii) μ and v are independent. The log-likelihood function, assuming μ and v are both normally distributed, is given by:

$$\ell(\beta, \sigma_v^2, \sigma_\mu^2, \lambda) = -\frac{NT}{2} \log(2\pi\sigma_v^2) - \frac{1}{2} \log |\Sigma| - \frac{1}{2\sigma_v^2} u' \Sigma^{-1} u, \quad (3.5)$$

where $\Sigma = \frac{1}{\sigma_v^2} E(uu') = \phi(J_T \otimes I_N) + I_T \otimes (B' B)^{-1}$, $\Sigma^{-1} = \bar{J}_T \otimes (T\phi I_N + (B' B)^{-1})^{-1} + E_T \otimes (B' B)$, $\phi = \sigma_\mu^2 / \sigma_v^2$, $J_T = \iota_T \iota_T'$, $\bar{J}_T = \frac{1}{T} J_T$, and $E_T = I_T - \bar{J}_T$. See Anselin (1988) and Baltagi et al. (2003) for details. Maximizing (3.3) gives the maximum likelihood estimator (MLE) of the model parameters if the error components are normally distributed, otherwise it gives a quasi-maximum likelihood estimator (QMLE).

Anselin (1988, p. 155) presents an LM test of $H_0 : \lambda = 0$ for Model (3.4) in the presence of random space-specific effects, which can be written in the form

$$\text{LM}_{\text{RE}} = \frac{\tilde{u}' [\tilde{\rho}^2 (\bar{J}_T \otimes W) + E_T \otimes W] \tilde{u}}{\tilde{\sigma}_v^2 [(T - 1 + \tilde{\rho}^2) S_0]^{\frac{1}{2}}}, \quad (3.6)$$

where $S_0 = \text{tr}(W'W) + W^2$, $\tilde{\rho}$ and $\tilde{\sigma}_v^2$ are the constrained QMLEs of $\rho = \sigma_\mu^2 / (T\sigma_\mu^2 + \sigma_v^2)$ and σ_v^2 under H_0 , and \tilde{u} is the vector of constrained QMLE residuals.⁴

A nice feature of the LM test is that it requires only the estimates of the model under

⁴Baltagi et al. (2003) considered the joint, marginal and conditional LM tests for λ and/or σ_μ^2 , which includes (3.6) as a special case, and presented Monte Carlo results under spatial layouts with a fixed number of neighbors. Apparently, the LM test given in (3.6) does not fit into the framework of Robinson (2008), but it does if the test concerns $H_0 : \lambda = 0, \sigma_\mu = 0$. We note that our approach is applicable to all scenarios similar to (3.6), i.e., testing spatial effect allowing other type of effects (such as random effects, heteroskedasticity, etc.) to exist in the model.

H_0 . However, even under H_0 , the constrained QMLE of ρ (or ϕ) does not possess an explicit expression, meaning that $\tilde{\rho}$ has to be obtained via numerical optimization. In fact, under H_0 , the partially maximized log-likelihood (with respect to β and σ_v^2) is given by:

$$\ell_{\max}(\rho) = \text{constant} - \frac{NT}{2} \log \tilde{\sigma}_v^2(\rho) + \frac{N}{2} \log \rho, \quad (3.7)$$

where $\tilde{\sigma}_v^2(\rho) = \frac{1}{NT} \tilde{u}'(\rho) \Sigma_0^{-1} \tilde{u}(\rho)$, $\tilde{u}(\rho) = Y - X\tilde{\beta}(\rho)$, $\tilde{\beta}(\rho) = (X'\Sigma_0^{-1}X)^{-1}X'\Sigma_0^{-1}Y$, and $\Sigma_0^{-1} = \Sigma^{-1}|_{\lambda=0} = \rho\bar{J}_T \otimes I_N + E_T \otimes I_N$. Maximizing (3.7) gives the constrained QMLE (under H_0) $\tilde{\rho}$ of ρ , which in turn gives the constrained QMLEs $\tilde{\beta} \equiv \tilde{\beta}(\tilde{\rho})$, $\tilde{\sigma}_v^2 \equiv \tilde{\sigma}_v^2(\tilde{\rho})$, $\tilde{\Sigma}_0^{-1} \equiv \tilde{\rho}\bar{J}_T \otimes I_N + E_T \otimes I_N$, and $\tilde{u} \equiv \tilde{u}(\tilde{\rho})$, for β , σ_v^2 , Σ_0^{-1} and $u(\rho)$, respectively.

Similar to the LM test in the linear regression model, the numerator of LM_{RE} given in (3.6) is again a quadratic form in the disturbance vector u , but now u contains two independent components. The large sample mean of this quadratic form is zero, but its finite sample mean is not necessarily zero. This may distort the finite sample distribution of the test statistic, in particular the tail probability. We now present a standardized version of the LM_{RE} test, which corrects both the mean and the variance and has a better finite sample performance in the situation where each spatial unit has ‘many’ neighbors. Lemma A.3 given in the Appendix is essential in deriving the modified test statistics. Some basic regularity conditions are listed below.

ASSUMPTION 3.1. *The random effects $\{\mu_i\}$ are iid with mean zero, variance σ_μ^2 , and excess kurtosis κ_μ . The idiosyncratic errors $\{v_{it}\}$ are iid with mean zero, variance σ_v^2 , and excess kurtosis κ_v . Also, the moments $E|\mu_i|^{4+\eta_1}$ and $E|v_{it}|^{4+\eta_2}$ exist for some $\eta_1, \eta_2 > 0$.*

ASSUMPTION 3.2. *The elements $\{w_{ij}\}$ of W are at most of order h_N^{-1} uniformly for all i, j , with the rate sequence $\{h_N\}$, bounded or divergent, satisfying $h_N/N \rightarrow 0$ as N goes to infinity. The $N \times N$ matrices $\{W\}$ are uniformly bounded in both row and column sums with $w_{ii} = 0$ and $\sum_j w_{ij} = 1$ for all i .*

ASSUMPTION 3.3. *The elements of the $NT \times k$ matrix X are uniformly bounded for all N and $\lim_{N \rightarrow \infty} \frac{1}{N} X'X$ exists and is nonsingular.*

Now, define $A(\rho) = \rho^2(\bar{J}_T \otimes W) + E_T \otimes W$, $M(\rho) = I_{NT} - X(X'\Sigma_0^{-1}X)^{-1}X'\Sigma_0^{-1}$, $C(\rho) = M'(\rho)[A(\rho) - a_0(\rho)\Sigma_0^{-1}]M(\rho)$, and $a_0(\rho) = \frac{1}{NT-k} \text{tr}[\Sigma_0 M'(\rho)A(\rho)M(\rho)]$. Let $\text{diagv}(A)$ be a column vector formed by the diagonal elements of a square matrix A . We have the following theorem.

THEOREM 3.1. *Assume that the constrained QMLE $\tilde{\rho}$ under H_0 is a consistent estimator of ρ . Under Assumptions 3.1-3.3, for testing $H_0; \lambda = 0$, the standardized LM test which corrects both the mean and variance takes the form:*

$$\text{SLM}_{\text{RE}} = \frac{\tilde{u}'(\tilde{A} - \tilde{a}_0\tilde{\Sigma}_0^{-1})\tilde{u}}{[\tilde{\phi}^2\tilde{\kappa}_\mu\tilde{a}'_1\tilde{a}_1 + \tilde{\kappa}_v\tilde{a}'_2\tilde{a}_2 + \text{tr}(\tilde{\Sigma}(\tilde{C}' + \tilde{C})\tilde{\Sigma}\tilde{C})]^{1/2}\tilde{\sigma}_v^2}, \quad (3.8)$$

where $\tilde{A} = A(\tilde{\rho})$, $\tilde{C} = C(\tilde{\rho})$, $\tilde{a}_0 = a_0(\tilde{\rho})$, $\tilde{\kappa}_\mu$ is the sample excess kurtosis of $\tilde{\mu} = (\bar{J}_T \otimes I_N)\tilde{u}$, $\tilde{\kappa}_v$ is the sample excess kurtosis of $\tilde{v} = \tilde{u} - (\iota \otimes I_N)\tilde{\mu}$, $\tilde{a}_1 = \text{diagv}[(\iota_T \otimes I_N)\tilde{C}(\iota_T \otimes I_N)]$, and $\tilde{a}_2 = \text{diagv}(\tilde{C})$. Under H_0 , we have (i) $\text{SLM}_{\text{RE}} \xrightarrow{D} N(0, 1)$, and (ii) $\text{SLM}_{\text{RE}} \sim \text{LM}_{\text{RE}}$.

Similar to the results of Theorem 2.1, the results of Theorem 3.1 show that the mean correction factor for the standardized LM test is also of the order $O_p((h_N/N)^{\frac{1}{2}})$. Thus, the LM_{RE} test can have large mean bias when h_N is large.⁵

3.2. Panel linear regression with fixed effects

When the space-specific effects $\{\mu_i\}$ are treated as fixed, the incidental parameters problem occurs. The standard practice is to remove these fixed effects by some kind of transformation. Recently, Lee and Yu (2010) studied the asymptotic properties of QML estimation of spatial panel models with fixed effects, which contain the above model as a special case. They used an orthogonal transformation to the model specified by (3.1)-(3.3) to obtain

$$Y_t^* = X_t^* \beta + \varepsilon_t^*, \quad \varepsilon_t^* = \lambda W \varepsilon_t^* + v_t^*, \quad t = 1, \dots, T-1,$$

where $(Y_1^*, Y_2^*, \dots, Y_{T-1}^*) = (Y_1, Y_2, \dots, Y_T) F_{T, T-1}$, $F_{T, T-1}$ is a $T \times (T-1)$ matrix whose columns are the eigenvectors of $I_T - \frac{1}{T} \iota_T \iota_T'$ corresponding to the eigenvalues of one, and similarly ε_t^* , v_t^* , and the columns of X_t^* are defined.

Debarsy and Ertur (2010) followed up with LM tests for spatial dependence. In case of a spatial error panel model with fixed space-specific effects, the LM test takes the form:

$$\text{LM}_{\text{FE}} = \frac{N(T-1)}{\sqrt{\mathbb{S}_0}} \frac{\tilde{\varepsilon}^{*'} \mathbb{W} \tilde{\varepsilon}^*}{\tilde{\varepsilon}^{*'} \tilde{\varepsilon}^*}, \quad (3.9)$$

where $\tilde{\varepsilon}^*$ is OLS residuals from regressing Y^* on X^* with Y^* being the stacked $\{Y_t^*\}$ and X^* the stacked $\{X_t^*\}$, $\mathbb{S}_0 = (T-1)\mathbb{S}_0$, and $\mathbb{W} = I_{T-1} \otimes W$. With the fixed effects specification, the model wipes out time-invariant regressors.

ASSUMPTION 3.4. *The idiosyncratic errors $\{v_{it}\}$ are iid with mean zero, variance σ_v^2 , and excess kurtosis κ_v . Also, the moment $E|v_{it}|^{4+\eta}$ exists for some $\eta > 0$.*

ASSUMPTION 3.5. *The elements of the $NT \times k$ matrix X are uniformly bounded for all N and $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^T (X_t - \bar{X})'(X_t - \bar{X})$ exists and is nonsingular, where $\bar{X} = \frac{1}{T} \sum_{t=1}^T X_t$.*

Define $\mathbb{M} = I_{N(T-1)} - X^*(X^{*'} X^*)^{-1} X^{*'}$, $\mathbb{A} = (F_{T, T-1} \otimes I_N)(\mathbb{M} \mathbb{W} \mathbb{M} - \mathbb{S}_1 \mathbb{M})(F'_{T, T-1} \otimes I_N)$, and \mathbf{a}_{ii} as the diagonal elements of \mathbb{A} . We have the following theorem:

THEOREM 3.2. *Under Assumptions 3.2, 3.4 and 3.5, for testing $H_0; \lambda = 0$, the standardized LM test which corrects both the mean and variance takes the form:*

$$\text{SLM}_{\text{FE}} = \frac{N(T-1)}{\sqrt{\tilde{\kappa}_v \mathbb{S}_2 + \mathbb{S}_3}} \frac{\tilde{\varepsilon}^{*'} (\mathbb{W} - \mathbb{S}_1 I_{N(T-1)}) \tilde{\varepsilon}^*}{\tilde{\varepsilon}^{*'} \tilde{\varepsilon}^*}, \quad (3.10)$$

where $\mathbb{S}_1 = \frac{1}{N(T-1)-k} \text{tr}(\mathbb{W} \mathbb{M})$, $\mathbb{S}_2 = \sum_{i=1}^{N(T-1)} \mathbf{a}_{ii}^2$, $\mathbb{S}_3 = \text{tr}(\mathbb{A} \mathbb{A}' + \mathbb{A}^2)$, and $\tilde{\kappa}_v$ is a consistent estimator of κ_v . Under H_0 , we have (i) $\text{SLM}_{\text{FE}} \xrightarrow{D} N(0, 1)$, and (ii) $\text{SLM}_{\text{FE}} \sim \text{LM}_{\text{FE}}$.

For practical applications of the above theorem, one needs a consistent estimator of κ_v . While the elements of ε^* are uncorrelated, they may not be independent and thus

⁵The condition in Theorem 3.1 may be relaxed to allow $\bar{\rho}$ to be an arbitrary consistent estimator of ρ .

the sample kurtosis of $\tilde{\varepsilon}^*$ may not provide a consistent estimator for κ_v as in the case of a linear regression model. The following corollary provides the needed result.

COROLLARY 3.1. *Under the assumptions of Theorem 3.2, a method of moments type estimator for κ_v that is consistent under H_0 takes the form:*

$$\tilde{\kappa}_v = \frac{(T-1)^2}{\sum_{t=1}^T c_t^4} \left(\frac{\sum_{i=1}^N (1'_{T-1} \tilde{\varepsilon}_i^*)^4}{N^{-1} (\tilde{\varepsilon}^{*'} \tilde{\varepsilon}^*)^2} - 3 \right), \quad (3.11)$$

where c_t is the t th element of $F_{T,T-1} 1_{T-1}$, and $\tilde{\varepsilon}_i^{*'}$ is the i th row of $(\tilde{\varepsilon}_1^*, \tilde{\varepsilon}_2^*, \dots, \tilde{\varepsilon}_{T-1}^*)$.

Tests based on bootstrap p -values. Again, for each of the LM tests presented above, one may construct a bootstrap counterpart by extending the procedure given at the end of Section 2. This is typically the case for the fixed effects model as seen below, but for the random effects model there are two complications: one is the existence of error components that makes it unclear on the way of resampling, and the other is that the parameter σ_μ^2 has to be estimated in each bootstrap sample, making the bootstrap procedure computationally more demanding. We thus present a bootstrap version only for LM_{FE} , denoted as BLM_{FE} . Noting that $LM_{FE} = \frac{N(T-1)}{\sqrt{S_0}} \frac{\tilde{\varepsilon}^{*'} M W M \tilde{\varepsilon}^*}{\tilde{\varepsilon}^{*'} M \tilde{\varepsilon}^*}$, a bootstrap procedure similar to that for LM_{EI} can be obtained as follows:

- (a) Draw a bootstrap sample $\tilde{\varepsilon}^{*b}$ from the OLS residuals $\tilde{\varepsilon}^*$;
- (b) Compute the bootstrap value of LM_{FE} as $BLM_{FE}^b = \frac{N(T-1)}{\sqrt{S_0}} \frac{\tilde{\varepsilon}^{*b'} M W M \tilde{\varepsilon}^{*b}}{\tilde{\varepsilon}^{*b'} M \tilde{\varepsilon}^{*b}}$;
- (c) Repeat (a)-(b) B times to give $\{BLM_{FE}^b\}_{b=1}^B$, and thus the bootstrap p -value.

The suggested BLM_{FE} test can be used as a benchmark for the finite sample performance of the SLM tests. Its validity can again be inferred from Hall and Horowitz (1996).

4. MONTE CARLO RESULTS

The finite sample performance of the test statistics introduced in this paper are evaluated based on a series of Monte Carlo experiments. These experiments involve a number of different error distributions and a number of different spatial layouts. Comparisons are made between the standardized tests and their non-standardized counterparts to see the effects of the error distributions, the spatial layouts, and the design of the regression model. In cases of a linear regression and panel linear regression with fixed effects, LM tests referring to bootstrap p -values are also implemented to serve as benchmarks for the comparison.

4.1. Spatial layouts and error distributions

Three general spatial layouts are considered in the Monte Carlo experiments and they are applied to all the test statistics involved in the experiments. The first is based on the Rook contiguity, the second is based on Queen contiguity and the third is based on the notion of group or social interactions with the number of groups $G = N^d$ where $0 < d < 1$. In the first two cases, the number of neighbors for each spatial unit stays the same (2-4 for Rook and 3-8 for Queen) and does not change when sample size N increases.

However, in the last case, the number of neighbors for each spatial unit increases with the sample size but at a slower rate, and changes from group to group.

The details for generating the W matrix under Rook contiguity is as follows: (i) index the N spatial units by $\{1, 2, \dots, N\}$, randomly permute these indices and then allocate them into a lattice of $r \times m (\geq N)$ squares, (ii) let $W_{ij} = 1$ if the index j is in a square which is on the immediate left, or right, or above, or below the square which contains the index i , otherwise $W_{ij} = 0$, and (iii) divide each element of W by its row sum. The W matrix under Queen contiguity is generated in a similar way, but with additional neighbors which share a common vertex with the unit of interest.

To generate the W matrix according to the group interaction scheme, (i) calculate the number of groups according to $G = \text{Round}(N^d)$, and the approximate average group size $m = N/G$, (ii) generate the group sizes (n_1, n_2, \dots, n_G) according to a discrete uniform distribution from $m/2$ to $3m/2$, (iii) adjust the group sizes so that $\sum_{i=1}^G n_i = N$, and (iv) define $W = \text{diag}\{W_i/(n_i - 1), i = 1, \dots, G\}$, a matrix formed by placing the submatrices W_i along the diagonal direction, where W_i is an $n_i \times n_i$ matrix with ones on the off-diagonal positions and zeros on the diagonal positions. In our Monte Carlo experiments, we choose $d = 0.2, 0.5$, and 0.8 , representing respectively the situations where (i) there are few groups and many spatial units in a group, (ii) the number of groups and the sizes of the groups are of the same magnitude, and (iii) there are many groups with few elements in each. Under Rook or Queen contiguity, h_N defined in the theorems is bounded, whereas under group interaction h_N is divergent with rate N^{1-d} . This spatial layout covers the scenario considered in Case (1991). Lee (2007) shows that the group size variation plays an important role in the identification and estimation of econometric models with group interactions, contextual factors and fixed effects. Yang (2010) shows that it also plays an important role in the robustness of the LM test of spatial error components.

The reported Monte Carlo results correspond to the following three error distributions: (i) standard normal, (ii) mixture normal, standardized to have mean zero and variance 1, and (iii) log-normal, also standardized to have mean zero and variance one. The standardized normal-mixture variates are generated according to

$$u_i = ((1 - \xi_i)Z_i + \xi_i\tau Z_i)/(1 - p + p * \tau^2)^{0.5},$$

where ξ is a Bernoulli random variable with probability of success p and Z_i is standard normal independent of ξ . The parameter p in this case also represents the proportion of mixing the two normal populations. In our experiments, we choose $p = 0.05$, meaning that 95% of the random variates are from standard normal and the remaining 5% are from another normal population with standard deviation τ . We choose $\tau = 10$ to simulate the situation where there are gross errors in the data. The standardized lognormal random variates are generated according to

$$u_i = [\exp(Z_i) - \exp(0.5)]/[\exp(2) - \exp(1)]^{0.5}.$$

This gives an error distribution that is both skewed and leptokurtic. The normal mixture gives an error distribution that is still symmetric like normal but leptokurtic. Other non-normal distributions, such as normal-gamma mixture and chi-squared, are also considered and the results are available from the authors upon request. All the Monte Carlo experiments are based on 10,000 replications.

4.2. Performance of the tests for the linear regression model

The finite sample performance of seven LM-type test statistics are investigated and compared: (1) LM_{EI} , (2) SLM_{EI} , (3) BLM_{EI} which is LM_{EI} referring to the bootstrap p -values (Lin et al., 2007), (4) I^o , (5) I^* , (6) LM_{OPG} , and (7) SLM_{OPG} . The Monte Carlo experiments are carried out based on the following data generating process:

$$Y_i = \beta_0 + X_{1i}\beta_1 + X_{2i}\beta_2 + u_i.$$

The design of the experiment, or the way the regressors are generated also matters. We thus consider two scenarios: (i) IID scheme: $X_{1i} \stackrel{iid}{\sim} \sqrt{6}U(0, 1)$ and $X_{2i} \stackrel{iid}{\sim} N(0, 1)/\sqrt{2}$; and (ii) Non-IID scheme: the i th pair of X values in the g th group are generated according to $X_{1,ig} = (2z_g + z_{ig})/\sqrt{7}$ and $X_{2,ig} = (v_g + v_{ig})/\sqrt{7}$, where $\{z_g, z_{ig}, v_g, v_{ig}\}$ are iid $N(0, 1)$ across all i and g (see Lee, 2004a). Both X_1 and X_2 are treated as fixed in the experiments. The parameters $\beta = \{5, 1, 1\}'$ and $\sigma = 1$, resulting in a signal-to-noise ratio of 1. Five different sample sizes are considered, i.e., $N = 50, 100, 200, 500$, and 1000.

Null behavior of the tests. Table 1(a-c) reports the (null) empirical mean, standard deviation, and the tail probabilities (10%, 5% and 1%) for the seven test statistics. From the results (reported and unreported), the general observations are as follows: (i) in terms of closeness to $N(0, 1)$, the standardized tests (SLM_{EI} , I^* and SLM_{OPG}) improve significantly over their non-standardized counterparts (LM_{EI} , I^o and LM_{OPG}); (ii) the finite sample null distributions of LM_{EI} , I^o and LM_{OPG} can be altered greatly by the spatial layout, and they can also be affected by the error distributions and the way the regressors are generated; and (iii) in general, SLM_{EI} and in particular SLM_{OPG} , perform comparably with BLM_{EI} .

Some details are as follows: All tests including BLM_{EI} perform better under (i) light spatial dependence compared with heavy spatial dependence, (ii) normal errors rather than non-normal errors, (iii) IID regressors rather than Non-IID regressors. The tests LM_{EI} , I^o and LM_{OPG} have a downward mean shift, which can be sizable even when N is quite large. Besides the mean shift, LM_{EI} also has a downward SD shift, which can be sizable as well when N is not large, but goes to zero as N increases. In contrast, SLM_{EI} , I^* and SLM_{OPG} have mean close to zero and SD close to 1 which explain why they have better size in all experiments. Recalling that LM_{EI} corrects neither mean nor SD, and that I^o and LM_{OPG} correct only for SD, it is clear now why I^o and LM_{OPG} have size distortions, and why LM_{EI} is more severely undersized than I^o . Thus, the LM tests of spatial dependence need to be *both* mean- and variance-adjusted for good finite sample performance.

The results in Table 1 show that one of the major factors affecting the null distribution of LM_{EI} , I^o and LM_{OPG} is the spatial layout, or rather the degree of spatial dependence. In situations of a large group interaction, e.g., $G = \text{Round}(N^{0.2})$ (results not reported to conserve space), the number of groups ranges from 2 to 4 for N ranging from 50 to 1000. Thus, there are only a few groups, each containing many spatial units which are all neighbors of each other. This ‘heavy’ spatial dependence distorts severely the null distributions of LM_{EI} , I^o and LM_{OPG} , and combined with Non-IID regressors these tests fail completely. In a sharp contrast, SLM_{EI} still performs reasonably under these extreme situations. In contrast, in situations of small group interaction, e.g., $G = \text{Round}(N^{0.8})$ (results not reported to conserve space), the number of groups ranges from 23 to 251 for N ranging from 50 to 1000. In this case, there are many groups each having only 2 to 4 units, giving a spatial layout with very weak spatial dependence. As a result, the null distributions of LM_{EI} , I^o and LM_{OPG} are much closer to $N(0, 1)$ though still not as

close as those of the null distributions of SLM_{EI} , I^* , and SLM_{OPG} . These observations are consistent with the discussion following Theorem 2.1. Another factor affecting the null distribution of LM_{EI} , I^0 and LM_{OPG} is the way the regressors were generated (or the design of the model). Under the group interaction spatial layout, the null distributions of LM_{EI} , I^0 and LM_{OPG} are much closer to $N(0, 1)$ when the regressors are generated under the IID

Table 1a. Empirical Means, SDs and Tail Probabilities: Linear Regression, Normal Errors

Test	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
<u>$N = 50$</u>										
1	-0.5270	0.8408	.0615	.0144	.0038	-0.2268	0.9312	.0784	.0363	.0058
2	-0.0047	1.0367	.0836	.0507	.0214	-0.0132	1.0336	.1034	.0545	.0143
3	-0.5270	0.8408	.0977	.0485	.0113	-0.2268	0.9312	.0946	.0493	.0110
4	-0.6235	0.9948	.1583	.0583	.0085	-0.2416	0.9919	.0994	.0493	.0092
5	-0.0045	0.9948	.0743	.0445	.0184	-0.0127	0.9919	.0901	.0441	.0115
6	-0.7146	0.9806	.1843	.0972	.0163	-0.2946	0.9950	.1116	.0518	.0071
7	-0.1840	1.0669	.1315	.0641	.0094	-0.0748	1.0320	.1100	.0531	.0070
<u>$N = 100$</u>										
1	-0.5027	0.8892	.0859	.0244	.0048	-0.1811	0.9585	.0906	.0396	.0070
2	-0.0035	1.0334	.0884	.0477	.0193	0.0091	1.0154	.1010	.0515	.0130
3	-0.5027	0.8892	.1018	.0519	.0107	-0.1811	0.9585	.0988	.0498	.0097
4	-0.5725	1.0126	.1553	.0677	.0086	-0.1880	0.9950	.1030	.0476	.0095
5	-0.0034	1.0126	.0828	.0453	.0174	0.0089	0.9950	.0936	.0459	.0109
6	-0.6701	1.0085	.1825	.1001	.0221	-0.2271	0.9920	.1080	.0492	.0077
7	-0.1576	1.0667	.1280	.0659	.0119	-0.0361	1.0124	.1044	.0508	.0084
<u>$N = 200$</u>										
1	-0.4032	0.9200	.0920	.0323	.0045	-0.1246	0.9781	.0962	.0446	.0088
2	0.0168	1.0199	.0924	.0498	.0177	-0.0057	1.0040	.1031	.0515	.0100
3	-0.4032	0.9200	.1074	.0557	.0110	-0.1246	0.9781	.1029	.0493	.0111
4	-0.4425	1.0097	.1362	.0591	.0076	-0.1266	0.9939	.1017	.0478	.0095
5	0.0167	1.0097	.0887	.0475	.0172	-0.0056	0.9939	.0991	.0490	.0094
6	-0.5403	1.0171	.1597	.0878	.0223	-0.1571	0.9962	.1050	.0496	.0088
7	-0.1133	1.0485	.1192	.0638	.0141	-0.0371	1.0052	.1052	.0491	.0079
<u>$N = 500$</u>										
1	-0.3315	0.9401	.0865	.0368	.0059	-0.0844	0.9869	.0961	.0462	.0078
2	0.0045	1.0010	.0888	.0434	.0139	-0.0017	0.9975	.0988	.0480	.0087
3	-0.3315	0.9401	.0925	.0501	.0117	-0.0844	0.9869	.0998	.0482	.0088
4	-0.3516	0.9970	.1123	.0516	.0086	-0.0850	0.9935	.0986	.0480	.0082
5	0.0045	0.9970	.0876	.0426	.0137	-0.0017	0.9935	.0971	.0469	.0084
6	-0.4395	1.0082	.1380	.0703	.0187	-0.1064	0.9947	.0999	.0484	.0088
7	-0.0976	1.0235	.1029	.0527	.0120	-0.0236	0.9980	.1010	.0468	.0085
<u>$N = 1000$</u>										
1	-0.2929	0.9654	.0956	.0427	.0076	-0.0591	0.9884	.0946	.0454	.0101
2	-0.0060	1.0089	.0965	.0473	.0145	-0.0011	0.9937	.0954	.0467	.0108
3	-0.2929	0.9654	.1035	.0525	.0109	-0.0591	0.9884	.0969	.0487	.0119
4	-0.3055	1.0069	.1118	.0540	.0103	-0.0593	0.9918	.0959	.0462	.0103
5	-0.0059	1.0069	.0958	.0470	.0145	-0.0011	0.9918	.0950	.0460	.0106
6	-0.3829	1.0143	.1256	.0715	.0166	-0.0743	0.9931	.0978	.0478	.0108
7	-0.0923	1.0234	.1112	.0575	.0121	-0.0162	0.9947	.0969	.0477	.0111

Note: (1) Test: 1= LM_{EI} , 2= SLM_{EI} , 3=Bootstrap LM_{EI} , 4= I^0 , 5= I^* , 6= LM_{OPG} , and 7= SLM_{OPG} .
 (2) X-Value: Non-IID for group interaction scheme, and IID for queen contiguity.
 (3) True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma = 1$.

Table 1b. Empirical Means, SDs and Tail Probabilities: Linear Regression, Normal Mixtures

Test	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
$N = 50$										
1	-0.5280	0.7350	.0414	.0112	.0031	-0.2253	0.8103	.0510	.0230	.0036
2	-0.0060	0.8916	.0559	.0324	.0112	-0.0114	0.8927	.0669	.0328	.0076
3	-0.5280	0.7350	.0838	.0407	.0098	-0.2253	0.8103	.0825	.0410	.0088
4	-0.6247	0.8696	.1115	.0395	.0058	-0.2400	0.8631	.0659	.0315	.0059
5	-0.0057	0.8696	.0520	.0288	.0099	-0.0111	0.8631	.0588	.0283	.0059
6	-0.6874	0.8907	.1288	.0543	.0052	-0.3028	0.9863	.0967	.0392	.0025
7	-0.0920	1.0212	.1017	.0415	.0044	-0.0456	1.0136	.0954	.0363	.0028
$N = 100$										
1	-0.4969	0.7844	.0552	.0227	.0057	-0.1875	0.8417	.0588	.0283	.0062
2	0.0032	0.8946	.0601	.0341	.0139	0.0023	0.8882	.0667	.0342	.0094
3	-0.4969	0.7844	.0873	.0437	.0114	-0.1875	0.8417	.0875	.0433	.0100
4	-0.5658	0.8932	.1000	.0436	.0105	-0.1946	0.8738	.0669	.0331	.0074
5	0.0033	0.8932	.0597	.0346	.0137	0.0023	0.8738	.0635	.0314	.0087
6	-0.6550	0.9357	.1408	.0624	.0094	-0.2491	0.9764	.0870	.0327	.0039
7	-0.0598	1.0185	.1028	.0421	.0059	-0.0091	0.9913	.0815	.0308	.0033
$N = 200$										
1	-0.4273	0.8505	.0722	.0259	.0040	-0.0994	0.9181	.0754	.0412	.0111
2	-0.0098	0.9295	.0693	.0335	.0097	0.0201	0.9413	.0811	.0442	.0139
3	-0.4273	0.8505	.1001	.0499	.0108	-0.0994	0.9181	.1008	.0522	.0139
4	-0.4690	0.9334	.1092	.0451	.0074	-0.1010	0.9330	.0799	.0438	.0127
5	-0.0098	0.9334	.0703	.0343	.0097	0.0200	0.9330	.0786	.0425	.0134
6	-0.5547	0.9754	.1338	.0609	.0104	-0.1472	0.9982	.0904	.0368	.0038
7	-0.0783	1.0256	.1014	.0434	.0052	0.0012	1.0033	.0871	.0351	.0043
$N = 500$										
1	-0.3249	0.9106	.0791	.0340	.0065	-0.0890	0.9471	.0840	.0459	.0116
2	0.0115	0.9678	.0826	.0409	.0117	-0.0063	0.9570	.0856	.0471	.0118
3	-0.3249	0.9106	.0995	.0517	.0113	-0.0890	0.9471	.0987	.0514	.0103
4	-0.3446	0.9657	.1024	.0475	.0085	-0.0896	0.9534	.0863	.0474	.0123
5	0.0115	0.9657	.0822	.0407	.0116	-0.0063	0.9534	.0830	.0466	.0115
6	-0.4241	0.9924	.1273	.0594	.0079	-0.1127	0.9897	.0919	.0373	.0045
7	-0.0582	1.0187	.1068	.0471	.0043	-0.0145	0.9917	.0874	.0371	.0043
$N = 1000$										
1	-0.2814	0.9555	.0901	.0404	.0079	-0.0640	0.9819	.0948	.0494	.0115
2	0.0061	0.9973	.0921	.0467	.0133	-0.0060	0.9872	.0957	.0486	.0122
3	-0.2814	0.9555	.1042	.0522	.0120	-0.0640	0.9819	.1031	.0507	.0107
4	-0.2935	0.9966	.1069	.0495	.0105	-0.0642	0.9853	.0958	.0500	.0116
5	0.0061	0.9966	.0918	.0465	.0132	-0.0060	0.9853	.0951	.0483	.0120
6	-0.3642	1.0036	.1268	.0597	.0089	-0.0847	1.0027	.1010	.0387	.0039
7	-0.0619	1.0189	.1058	.0499	.0079	-0.0192	1.0035	.0963	.0393	.0038

Note: (1) Test: 1=LM_{EI}, 2=SLM_{EI}, 3=Bootstrap LM_{EI}, 4= I° , 5= I^* , 6=LM_{OPG}, and 7=SLM_{OPG}.
 (2) X-Value: Non-IID for group interaction scheme, and IID for queen contiguity.
 (3) True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma = 1$. For normal mixture, $p = .1$ and $\tau = 5$.

scheme than under the Non-IID scheme.

Table 1c. Empirical Means, SDs and Tail Probabilities: Linear Regression, Lognormal Errors

Test	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
<u>$N = 50$</u>										
1	-0.5198	0.7849	.0441	.0137	.0042	-0.2105	0.8586	.0563	.0241	.0064
2	0.0041	0.9573	.0674	.0394	.0178	0.0050	0.9483	.0789	.0430	.0137
3	-0.5198	0.7849	.0861	.0446	.0120	-0.2105	0.8586	.0898	.0438	.0085
4	-0.6151	0.9287	.1222	.0424	.0082	-0.2242	0.9145	.0745	.0328	.0091
5	0.0040	0.9287	.0610	.0366	.0162	0.0047	0.9145	.0691	.0359	.0115
6	-0.7387	0.9345	.1623	.0842	.0170	-0.3630	0.9778	.1083	.0461	.0057
7	-0.1662	1.0361	.1116	.0532	.0070	-0.1177	1.0033	.0963	.0392	.0036
<u>$N = 100$</u>										
1	-0.5076	0.8186	.0631	.0223	.0050	-0.2058	0.8854	.0653	.0283	.0056
2	-0.0093	0.9403	.0643	.0392	.0164	-0.0171	0.9358	.0752	.0382	.0095
3	-0.5076	0.8186	.0908	.0450	.0112	-0.2058	0.8854	.0926	.0453	.0094
4	-0.5780	0.9321	.1151	.0486	.0101	-0.2137	0.9192	.0751	.0358	.0074
5	-0.0090	0.9321	.0630	.0386	.0158	-0.0168	0.9192	.0695	.0354	.0082
6	-0.7197	0.9460	.1667	.0865	.0169	-0.3548	0.9971	.1176	.0580	.0099
7	-0.1581	1.0032	.0996	.0465	.0059	-0.1436	1.0117	.1027	.0486	.0054
<u>$N = 200$</u>										
1	-0.3979	0.8718	.0660	.0242	.0055	-0.1305	0.9350	.0754	.0337	.0096
2	0.0225	0.9573	.0718	.0367	.0138	-0.0117	0.9590	.0783	.0406	.0136
3	-0.3979	0.8718	.0947	.0487	.0116	-0.1305	0.9350	.1007	.0451	.0108
4	-0.4367	0.9568	.1024	.0421	.0084	-0.1326	0.9502	.0796	.0360	.0105
5	0.0225	0.9568	.0715	.0371	.0137	-0.0116	0.9502	.0748	.0387	.0128
6	-0.5879	0.9876	.1496	.0811	.0199	-0.2868	1.0210	.1188	.0579	.0102
7	-0.1343	1.0185	.1031	.0507	.0096	-0.1552	1.0237	.1082	.0515	.0077
<u>$N = 500$</u>										
1	-0.3326	0.9001	.0710	.0276	.0061	-0.0895	0.9524	.0803	.0386	.0078
2	0.0034	0.9569	.0736	.0369	.0136	-0.0068	0.9624	.0840	.0408	.0093
3	-0.3326	0.9001	.0905	.0426	.0090	-0.0895	0.9524	.0968	.0480	.0089
4	-0.3527	0.9545	.0937	.0380	.0084	-0.0901	0.9588	.0828	.0399	.0078
5	0.0034	0.9545	.0730	.0365	.0135	-0.0068	0.9588	.0825	.0400	.0089
6	-0.4923	0.9882	.1385	.0760	.0132	-0.2348	1.0151	.1129	.0560	.0118
7	-0.1320	1.0019	.1010	.0471	.0086	-0.1448	1.0122	.1050	.0500	.0094
<u>$N = 1000$</u>										
1	-0.2946	0.9392	.0877	.0367	.0069	-0.0712	0.9479	.0827	.0399	.0077
2	-0.0077	0.9801	.0830	.0414	.0131	-0.0132	0.9530	.0835	.0417	.0086
3	-0.2946	0.9392	.1022	.0497	.0114	-0.0712	0.9479	.0950	.0447	.0073
4	-0.3073	0.9796	.1020	.0466	.0084	-0.0714	0.9512	.0839	.0405	.0080
5	-0.0077	0.9796	.0827	.0414	.0131	-0.0132	0.9512	.0830	.0411	.0086
6	-0.4311	1.0054	.1336	.0718	.0162	-0.1974	1.0057	.1076	.0528	.0129
7	-0.1305	1.0110	.1057	.0504	.0087	-0.1348	1.0025	.1023	.0494	.0105

Note: (1) Test: 1=LM_{EI}, 2=SLM_{EI}, 3=Bootstrap LM_{EI}, 4= I^0 , 5= I^* , 6=LM_{OPG}, and 7=SLM_{OPG}.
 (2) X-Value: Non-IID for group interaction scheme, and IID for queen contiguity.
 (3) True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma = 1$.

Power of the tests. Empirical frequencies of rejection of the seven tests are plotted in Figures 1a-1c against the values of λ (horizontal line). Simulated critical values for each test are used, which means that the reported powers of the tests are size-adjusted. In each plot of Figures 1a-1c, the power lines for LM_{EI}, SLM_{EI}, I^0 and I^* (tests 1, 2, 4 and 5) overlap. This means that once size-adjusted, these four tests have almost identical power.

This is not surprising as all four tests share the same term $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$. The four tests differ mainly in their locations and scales, and thus have different sizes or null behaviors in general when referred to the standard normal distribution. If, however, the exact critical values are used, they become essentially the same test. However, in empirical applications, asymptotic critical values are often used. In this case, it is important to do the mean and variance corrections to the test statistics so that the asymptotic critical values give a better approximation. An alternative way is to bootstrap. The power of BLM_{EI} is generally very close to that of the four tests, but the power of the two OPG-based LM tests can be noticeably different from that of the four tests.

Figure 1 further reveals that the spatial layout and the sample size are the two important factors affecting the power of these tests. With less neighbors (plots on the right) or with a larger sample, the tests become more powerful. It is interesting to note that when the spatial dependence is strong, it is harder to detect the spatial dependence when the spatial parameter is negative than when it is positive (see the plots on the left). Another factor affecting the power of the tests is the way that the regressors are generated. The results (not reported to conserve space) show that the tests under IID regressors are more powerful than tests under Non-IID regressors, although the signal-to-noise ratios are the same. The error distribution also affects the power of the tests, but to a lesser degree.

4.3. Performance of the tests for the random effects panel model

The LM and SLM tests (LM_{RE} and SLM_{RE}) introduced in Section 3.1 are compared by Monte Carlo simulation based on the following DGP

$$Y_t = \beta_0 + X_{1t}\beta_1 + X_{2t}\beta_2 + u_t, \text{ with } u_t = \mu + \varepsilon_t, \quad t = 1, \dots, T,$$

where the error components μ and ε_t can be drawn from any of the three distributions used in the previous two subsections, or the combination of any two distributions. For example, μ and ε_t can both be drawn from the normal mixture, or μ from the normal mixture but ε_t from the normal or log-normal distribution. The beta parameters are set at the same values as before, $\sigma_\mu = \sigma_v = 1$. For sample sizes, $T = 3, 10$; and $N = 20, 50, 100, 200, 500$. The same spatial layouts are used as described above. The two regressors follow either the IID scheme where $\{X_{1,it}\} \stackrel{iid}{\sim} \sqrt{12}(U(0, 1) - 0.5)$, and $\{X_{2,it}\} \stackrel{iid}{\sim} N(0, 1)$, or the Non-IID scheme for group interaction layout: $X_{1,itg} = (2z_{tg} + z_{itg})/\sqrt{7}$ and $X_{2,itg} = (v_{tg} + v_{itg})/\sqrt{7}$, with $\{z_{tg}, z_{itg}, v_{tg}, v_{itg}\}$ being iid $N(0, 1)$ across all i, t and g . For this model, we are unable to implement the bootstrap method due to the extra complication in the error structure.

Null behavior of the tests. The results presented in Table 2 correspond to cases where both μ and v_t are normal, both are normal mixture, and both are log-normal. Essentially, the same conclusions hold as in the case of the spatial linear regression model. The SLM test outperforms its LM counterpart in all the experiments considered. Increasing the value of T from 3 to 10 significantly improves both tests. Another interesting phenomenon is that the null behavior of LM_{RE} also depends upon the relative magnitude of the variance components σ_μ^2 and σ_v^2 . The larger the ratio σ_v^2/σ_μ^2 , the worse is the performance of the LM_{RE} test. In contrast, the performance of SLM_{RE} is very robust.

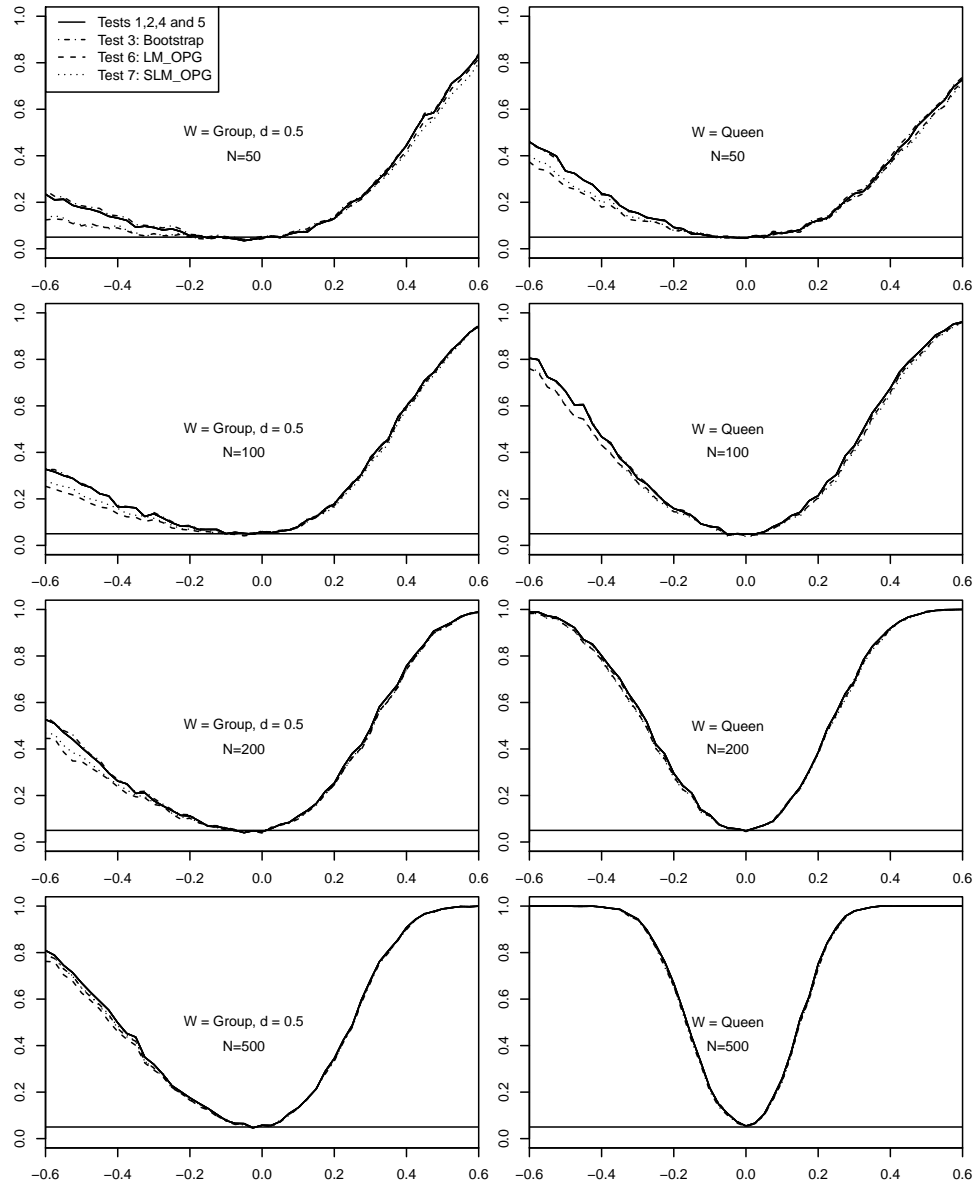


Figure 1a. Size-Adjusted Empirical Power of Tests 1-7, Normal Errors.

Power of the tests. Empirical frequencies of rejection, based on the simulated critical values, of the two tests are plotted in Figure 2 against the values of λ (horizontal line). Now each line we see from each plot of Figure 2 is in fact an overlap of two lines, one for LM_{RE} and the other for SLM_{RE} . Similar to the case of the linear regression model,

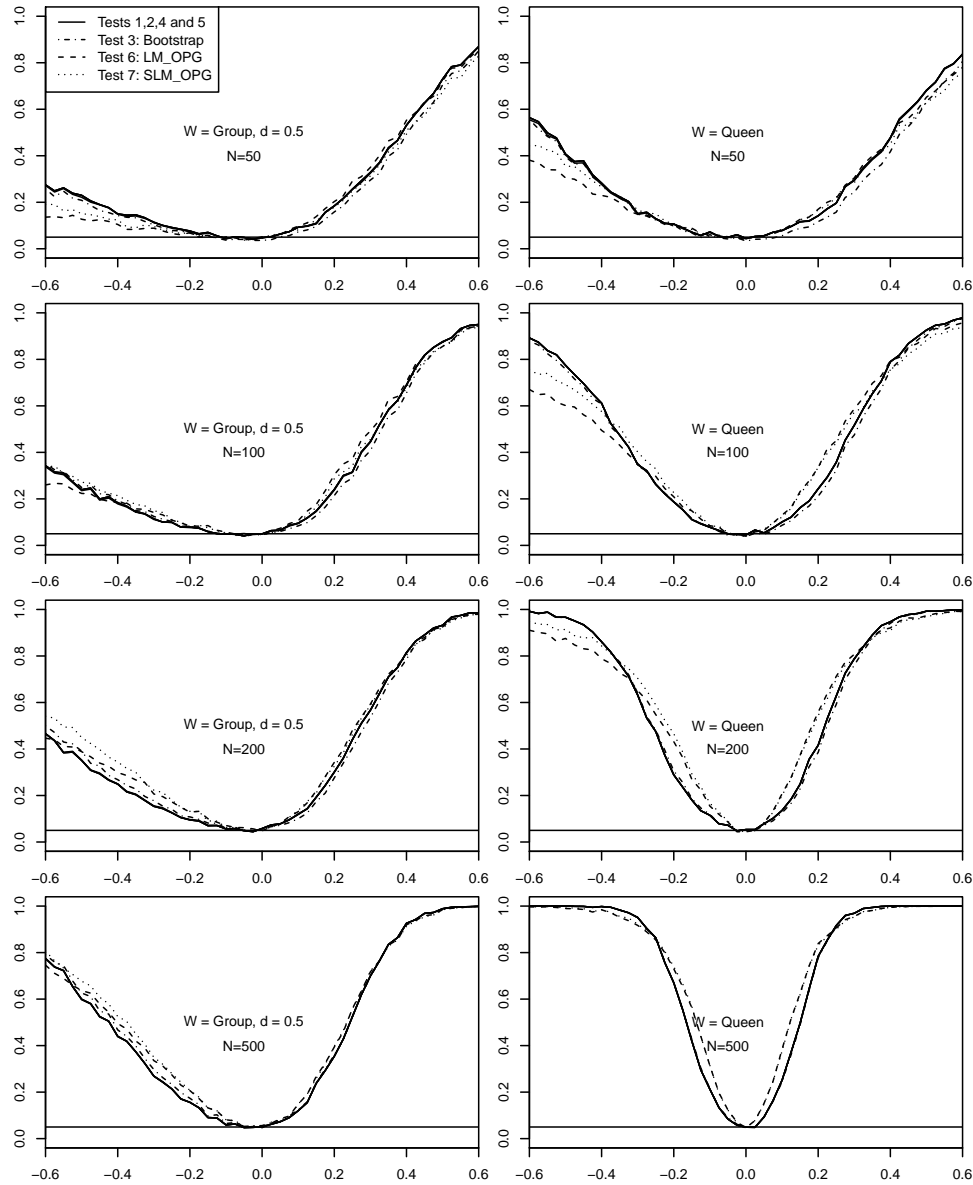


Figure 1b. Size-Adjusted Empirical Power of Tests 1-7, Normal Mixture Errors.

the two tests have almost identical power once they are size-adjusted. The power of the tests depend heavily on the degree of spatial dependence and on the sample size. It also depends on the error distributions and the type of regressors, though to a lesser degree.

Some interesting details are as follows. The two plots in the first row of Figure 2 show

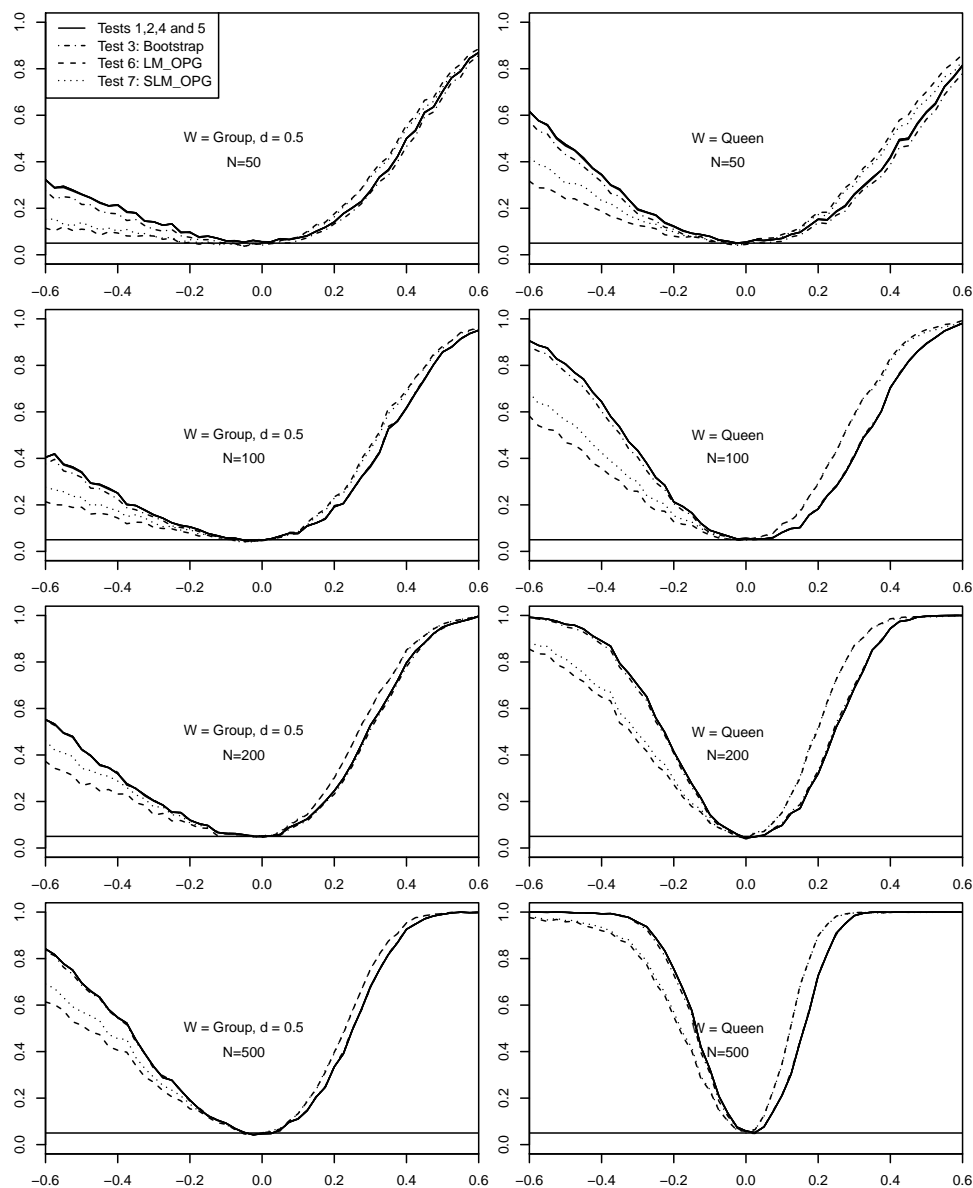


Figure 1c. Size-Adjusted Empirical Power of Tests 1-7, Log-Normal Errors.

that the two tests possess very low power and that the power does not seem to increase as N increase from 20 to 50 (with T fixed at 3). This is because the underlying spatial layout generates very strong spatial dependence. When N is increased from 20 to 50, the number of groups stays at $G = \text{Round}(N^{0.2}) = 2$. This means that under this spatial

Table 2a. Empirical Means, SDs and Tail Probabilities: Panel with Random Effects, $T = 3$

N	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
Normal Errors										
20	-0.4242	0.9192	.0915	.0308	.0036	-0.0776	0.9854	.0889	.0434	.0090
	0.0040	1.0275	.0960	.0514	.0184	0.0054	1.0635	.1143	.0602	.0154
50	-0.2383	0.9491	.0873	.0334	.0063	-0.0382	0.9904	.0965	.0490	.0102
	0.0044	1.0042	.0905	.0493	.0135	-0.0090	1.0272	.1083	.0562	.0131
100	-0.2456	0.9627	.0919	.0414	.0071	-0.0211	0.9878	.0943	.0459	.0105
	-0.0049	1.0045	.0943	.0485	.0131	0.0211	1.0152	.1032	.0537	.0136
200	-0.2132	0.9866	.0959	.0433	.0091	-0.0278	0.9938	.0970	.0474	.0097
	0.0045	1.0187	.0971	.0520	.0143	-0.0042	1.0122	.1037	.0518	.0108
500	-0.1808	0.9789	.0910	.0415	.0073	-0.0139	1.0018	.1025	.0510	.0105
	0.0004	0.9993	.0932	.0463	.0102	0.0001	1.0160	.1063	.0547	.0118
Normal Mixture Error										
20	-0.4533	0.8550	.0715	.0272	.0046	-0.1006	0.9109	.0698	.0305	.0061
	-0.0193	0.9520	.0726	.0398	.0125	-0.0075	0.9832	.0927	.0463	.0100
50	-0.2483	0.9255	.0780	.0335	.0070	-0.0473	0.9579	.0848	.0423	.0080
	-0.0020	0.9774	.0823	.0416	.0131	-0.0131	0.9932	.0959	.0508	.0104
100	-0.2501	0.9399	.0815	.0357	.0075	-0.0361	0.9764	.0931	.0443	.0086
	-0.0076	0.9800	.0826	.0432	.0117	0.0074	1.0035	.1033	.0511	.0109
200	-0.2293	0.9515	.0885	.0385	.0067	-0.0335	0.9867	.0950	.0474	.0092
	-0.0113	0.9822	.0879	.0440	.0107	-0.0093	1.0048	.1013	.0508	.0103
500	-0.1779	0.9830	.0947	.0461	.0087	-0.0231	0.9973	.1004	.0484	.0096
	0.0035	1.0033	.0947	.0478	.0110	-0.0090	1.0114	.1050	.0517	.0103
Lognormal Errors										
20	-0.4311	0.8532	.0691	.0270	.0041	-0.0997	0.8820	.0583	.0249	.0062
	0.0079	0.9501	.0786	.0418	.0147	-0.0008	0.9510	.0782	.0373	.0098
50	-0.2535	0.9076	.0682	.0311	.0071	-0.0329	0.9338	.0746	.0366	.0090
	-0.0034	0.9572	.0735	.0380	.0125	0.0063	0.9681	.0846	.0426	.0120
100	-0.2377	0.9272	.0761	.0347	.0080	-0.0443	0.9555	.0826	.0430	.0090
	0.0077	0.9663	.0809	.0435	.0126	0.0011	0.9818	.0898	.0481	.0114
200	-0.2014	0.9424	.0798	.0343	.0070	-0.0293	0.9641	.0864	.0448	.0094
	0.0186	0.9724	.0814	.0409	.0112	-0.0039	0.9819	.0917	.0491	.0105
500	-0.1779	0.9741	.0936	.0426	.0089	-0.0039	0.9776	.0888	.0449	.0117
	0.0042	0.9939	.0944	.0473	.0134	0.0110	0.9914	.0931	.0478	.0125

Note: (1) under each N , the first row corresponds to LM_{RE} and the second corresponds to SLM_{RE} .
 (2) X-Value: Non-IID for group interaction scheme, and IID for queen contiguity.
 (3) True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma_\mu = \sigma_v = 1$.

layout, the degree of spatial dependence at $N = 50$ is bigger than that at $N = 20$. As a result, the power does not go up, and might even go down slightly.

Table 2b. Empirical Means, SDs and Tail Probabilities: Panel with Random Effects, $T = 10$

N	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
Normal Errors										
20	-0.1770	0.9675	.0926	.0441	.0062	0.0045	1.0036	.1011	.0516	.0126
	-0.0032	0.9902	.0956	.0477	.0093	0.0097	1.0431	.1136	.0606	.0155
50	-0.1474	0.9849	.1017	.0448	.0074	-0.0062	0.9968	.0982	.0495	.0111
	-0.0127	0.9988	.1000	.0463	.0087	0.0035	1.0210	.1082	.0555	.0128
100	-0.1255	0.9903	.0943	.0461	.0089	0.0020	1.0096	.1013	.0501	.0119
	-0.0058	1.0003	.0956	.0453	.0108	0.0076	1.0276	.1075	.0554	.0125
200	-0.0942	0.9967	.1005	.0491	.0089	0.0016	0.9987	.1006	.0467	.0089
	0.0063	1.0041	.1010	.0509	.0102	0.0050	1.0134	.1053	.0513	.0096
500	-0.0685	0.9882	.0983	.0464	.0085	0.0048	1.0048	.0981	.0492	.0094
	0.0131	0.9928	.0955	.0472	.0099	0.0072	1.0176	.1029	.0523	.0103
Normal Mixture Error										
20	-0.1698	0.9571	.0849	.0410	.0096	-0.0079	0.9645	.0896	.0432	.0089
	0.0087	0.9788	.0852	.0443	.0123	0.0035	1.0023	.1030	.0518	.0117
50	-0.1247	0.9790	.0905	.0442	.0102	-0.0132	0.9922	.0965	.0481	.0098
	0.0119	0.9925	.0927	.0460	.0128	-0.0019	1.0162	.1051	.0534	.0111
100	-0.1203	0.9924	.0960	.0478	.0102	0.0076	0.9932	.0997	.0496	.0095
	0.0001	1.0022	.0972	.0498	.0113	0.0140	1.0110	.1046	.0544	.0113
200	-0.1027	0.9919	.0966	.0454	.0090	-0.0027	0.9973	.0980	.0509	.0100
	-0.0020	0.9992	.0974	.0482	.0102	0.0008	1.0119	.1034	.0548	.0111
500	-0.0652	0.9984	.1017	.0490	.0104	0.0024	0.9974	.0985	.0505	.0100
	0.0165	1.0031	.1001	.0509	.0097	0.0048	1.0100	.1024	.0531	.0106
Lognormal Errors										
20	-0.1893	0.9216	.0711	.0334	.0081	-0.0204	0.9433	.0753	.0380	.0098
	-0.0081	0.9417	.0721	.0395	.0118	-0.0061	0.9801	.0859	.0455	.0127
50	-0.1255	0.9560	.0800	.0382	.0107	-0.0200	0.9553	.0805	.0393	.0090
	0.0128	0.9688	.0789	.0439	.0135	-0.0068	0.9783	.0869	.0449	.0107
100	-0.1267	0.9615	.0876	.0397	.0098	0.0004	0.9760	.0865	.0452	.0115
	-0.0054	0.9709	.0864	.0429	.0116	0.0076	0.9934	.0937	.0481	.0128
200	-0.0987	0.9755	.0919	.0444	.0101	0.0023	0.9845	.0922	.0467	.0109
	0.0027	0.9825	.0907	.0472	.0108	0.0064	0.9989	.0961	.0495	.0123
500	-0.0750	0.9778	.0948	.0446	.0074	0.0048	0.9928	.0954	.0467	.0113
	0.0068	0.9823	.0946	.0466	.0090	0.0074	1.0054	.0991	.0496	.0125

Note: (1) under each N , the first row corresponds to LM_{RE} and the second corresponds to SLM_{RE} .
 (2) X-Value: Non-IID for group interaction scheme, and IID for queen contiguity.
 (3) True parameter values: $\beta = \{5, 1, 1\}'$, and $\sigma_\mu = \sigma_v = 1$.

4.4. Performance of the tests for the fixed effects panel model

The LM and SLM tests (LM_{FE} and SLM_{FE}) introduced in Section 3.2 are compared by Monte Carlo simulation based on the following DGP:

$$Y_t = X_{1t}\beta_1 + X_{2t}\beta_2 + X_{3t}\beta_3 + u_t, \text{ with } u_t = \mu + \varepsilon_t, \quad t = 1, \dots, T.$$

As this model (after the transformation) and the corresponding test LM_{FE} are quite similar to the model and the test LM_{EI} given in Section 2, a bootstrap version of LM_{FE} , denoted

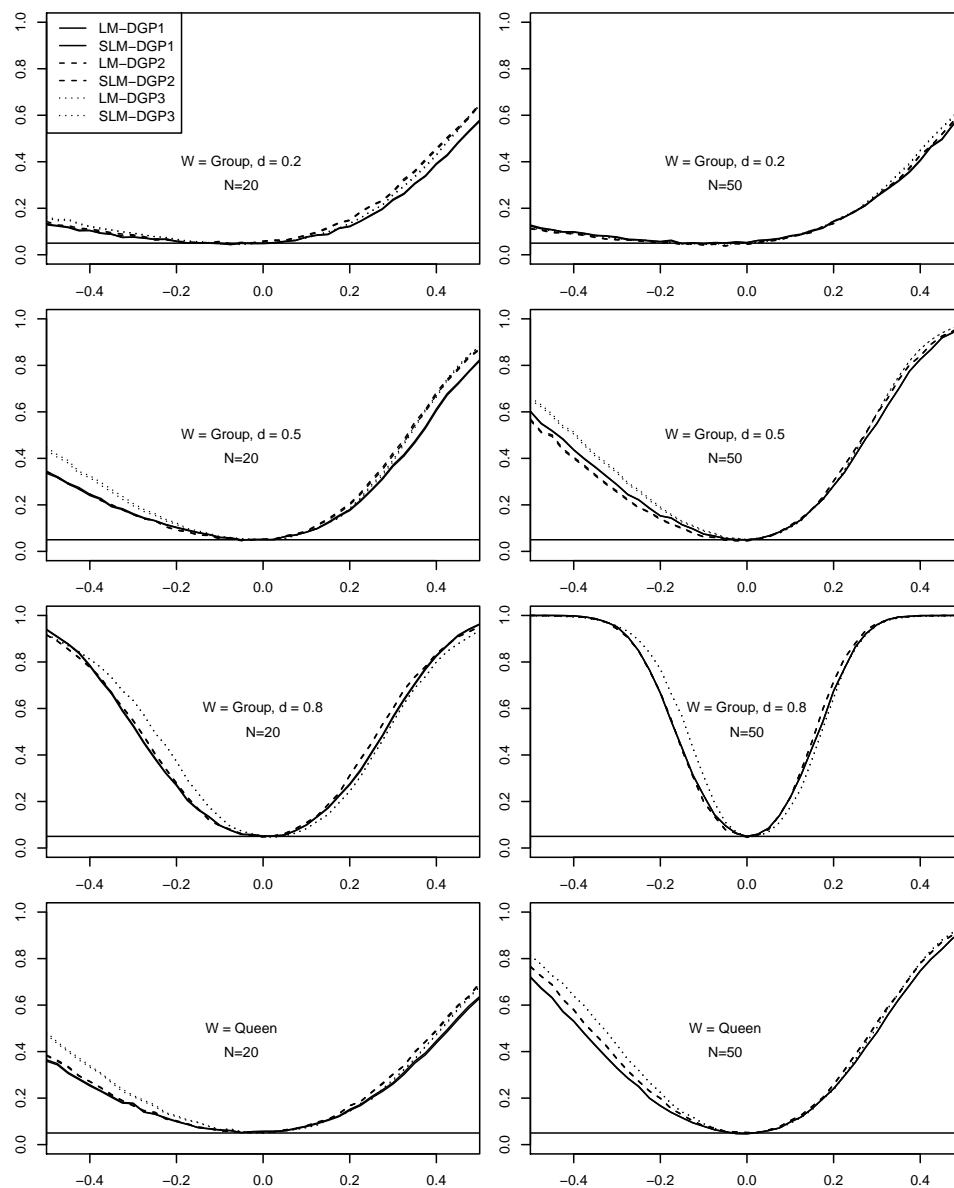


Figure 2. Size-Adjusted Empirical Power of LM and SLM for Random Effects Panel Model.

as BLM_{FE} , is also implemented to serve as a benchmark for the finite sample performance of the proposed test SLM_{FE} . The fixed effects are generated either according to $\mu = \frac{1}{T} \sum_{t=1}^T X_{3t}$, or as a vector of iid $N(0, 1)$ random numbers independent of the X -values. The regressors are generated according to either the IID scheme: $X_{1,it} \stackrel{iid}{\sim} 2U(0, 1)$,

$X_{2,it} \stackrel{iid}{\sim} N(0, 1)/\sqrt{3}$, and $X_{3,it} \stackrel{iid}{\sim} [\exp(N(0, 1) - \exp(0.5))]/(3\exp(2) - 3\exp(1))^{0.5}$, or the Non-IID scheme for group interaction layout: $X_{1,itg} = (2z_{tg} + z_{itg})/\sqrt{15}$, $X_{2,itg} = (2v_{tg} + v_{itg})/\sqrt{15}$, and $X_{3,itg} = (2e_{tg} + e_{itg})/\sqrt{15}$ with $\{z_{tg}, z_{itg}, v_{tg}, v_{itg}\}$ being iid $N(0, 1)$ and $\{e_{tg}, e_{itg}\}$ iid $[\exp(N(0, 1) - \exp(0.5))]/(3\exp(2) - 3\exp(1))^{0.5}$ across all i, t and g .

Table 3a. Empirical Means, SDs and Tail Probabilities: Panel with Fixed Effects, $T = 3$

N	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
Normal Errors										
20	-0.5108	0.8690	.0831	.0254	.0026	-0.0393	0.9691	.0830	.0394	.0105
	-0.0124	1.0487	.0988	.0549	.0193	-0.0061	1.0405	.1060	.0577	.0150
	-0.5108	0.8690	.1031	.0531	.0113	-0.0393	0.9691	.0929	.0470	.0101
100	-0.2865	0.9491	.0916	.0392	.0063	0.0098	0.9947	.0976	.0465	.0102
	-0.0013	1.0110	.0977	.0504	.0142	0.0051	1.0089	.1016	.0498	.0111
	-0.2865	0.9491	.1035	.0521	.0126	0.0098	0.9947	.0982	.0503	.0119
200	-0.2776	0.9483	.0896	.0378	.0058	-0.0102	1.0048	.1024	.0530	.0117
	0.0020	0.9970	.0917	.0455	.0119	-0.0101	1.0122	.1054	.0547	.0121
	-0.2776	0.9483	.0983	.0495	.0112	-0.0102	1.0048	.1061	.0556	.0119
500	-0.2439	0.9751	.0980	.0440	.0078	0.0160	0.9914	.0970	.0477	.0084
	0.0027	1.0076	.0973	.0473	.0122	0.0134	0.9943	.0980	.0481	.0086
	-0.2439	0.9751	.1024	.0507	.0118	0.0160	0.9914	.0992	.0485	.0092
Normal Mixture Errors										
20	-0.5117	0.8234	.0645	.0224	.0041	-0.0215	0.9317	.0745	.0358	.0068
	-0.0129	0.9825	.0804	.0467	.0171	0.0128	0.9936	.0938	.0492	.0120
	-0.5117	0.8234	.0866	.0434	.0109	-0.0215	0.9317	.0923	.0447	.0095
100	-0.2592	0.9455	.0869	.0358	.0078	-0.0032	0.9657	.0891	.0422	.0070
	0.0278	1.0049	.0908	.0484	.0152	-0.0080	0.9789	.0929	.0447	.0075
	-0.2592	0.9455	.1030	.0514	.0130	-0.0032	0.9657	.0957	.0462	.0092
200	-0.2816	0.9314	.0842	.0359	.0050	0.0100	0.9920	.0964	.0498	.0099
	-0.0022	0.9782	.0853	.0435	.0108	0.0103	0.9991	.0990	.0518	.0105
	-0.2816	0.9314	.0939	.0473	.0098	0.0100	0.9920	.1004	.0527	.0119
500	-0.2533	0.9644	.0938	.0415	.0087	-0.0020	0.9972	.0990	.0482	.0101
	-0.0070	0.9960	.0920	.0459	.0129	-0.0047	1.0001	.0992	.0489	.0101
	-0.2533	0.9644	.0971	.0497	.0118	-0.0020	0.9972	.1016	.0505	.0105
Lognormal Errors										
20	-0.4935	0.8121	.0583	.0217	.0046	-0.0300	0.8875	.0617	.0273	.0063
	0.0095	0.9678	.0782	.0448	.0178	0.0033	0.9449	.0784	.0360	.0097
	-0.4935	0.8121	.0822	.0422	.0095	-0.0300	0.8875	.0728	.0341	.0050
100	-0.2813	0.9135	.0761	.0341	.0077	-0.0053	0.9501	.0789	.0405	.0105
	0.0045	0.9686	.0816	.0434	.0135	-0.0101	0.9626	.0820	.0429	.0113
	-0.2813	0.9135	.0923	.0495	.0115	-0.0053	0.9501	.0917	.0459	.0103
200	-0.2842	0.9061	.0705	.0317	.0049	-0.0129	0.9654	.0883	.0431	.0083
	-0.0049	0.9498	.0744	.0369	.0105	-0.0128	0.9722	.0899	.0448	.0087
	-0.2842	0.9061	.0832	.0420	.0087	-0.0129	0.9654	.0955	.0470	.0088
500	-0.2550	0.9530	.0902	.0399	.0077	-0.0043	0.9905	.0964	.0481	.0088
	-0.0088	0.9830	.0884	.0439	.0112	-0.0070	0.9933	.0975	.0484	.0089
	-0.2550	0.9530	.0962	.0497	.0109	-0.0043	0.9905	.1008	.0500	.0089

Note: (1) under each N, 1st row: LM_{FE}, 2nd row: SLM_{FE}, and 3rd row: Bootstrap LM_{FE}.
 (2) X-Value: Non-IID for group interaction, IID for queen contiguity; $\beta = \{1, 1, 1\}'$, $\sigma_v = 1$.

Table 3b. Empirical Means, SDs and Tail Probabilities: Panel with Fixed Effects, $T = 10$

N	Group: $G = N^{0.5}$					Queen Contiguity				
	Mean	SD	10%	5%	1%	Mean	SD	10%	5%	1%
Normal Errors										
20	-0.2167	0.9780	.0972	.0508	.0064	0.0193	1.0151	.1036	.0538	.0126
	0.0048	1.0133	.1034	.0540	.0116	0.0197	1.0322	.1084	.0594	.0146
	-0.2167	0.9780	.1048	.0574	.0114	0.0193	1.0151	.1104	.0580	.0134
50	-0.1363	1.0044	.1044	.0472	.0090	-0.0243	0.9919	.1006	.0462	.0084
	0.0189	1.0230	.1058	.0532	.0122	-0.0279	0.9983	.1026	.0472	.0084
	-0.1363	1.0044	.1110	.0532	.0116	-0.0243	0.9919	.1038	.0510	.0106
100	-0.1658	0.9835	.0994	.0450	.0070	-0.0349	0.9819	.0896	.0460	.0088
	-0.0057	0.9982	.1000	.0514	.0088	-0.0329	0.9852	.0908	.0464	.0088
	-0.1658	0.9835	.1014	.0498	.0100	-0.0349	0.9819	.0934	.0466	.0114
200	-0.1345	0.9909	.1026	.0470	.0074	-0.0054	1.0029	.1026	.0470	.0086
	-0.0016	1.0014	.0980	.0494	.0094	-0.0068	1.0046	.1032	.0472	.0088
	-0.1345	0.9909	.1036	.0512	.0088	-0.0054	1.0029	.1042	.0486	.0104
Normal Mixture Errors										
20	-0.2345	0.9639	.0944	.0440	.0096	0.0282	0.9654	.0850	.0434	.0090
	-0.0136	0.9953	.0930	.0514	.0134	0.0288	0.9810	.0896	.0468	.0100
	-0.2345	0.9639	.0970	.0520	.0140	0.0282	0.9654	.0904	.0510	.0120
50	-0.1518	0.9726	.0904	.0422	.0112	0.0080	0.9845	.0944	.0478	.0088
	0.0031	0.9898	.0948	.0458	.0108	0.0047	0.9907	.0968	.0494	.0088
	-0.1518	0.9726	.0980	.0474	.0150	0.0080	0.9845	.1028	.0508	.0134
100	-0.1409	0.9941	.0986	.0510	.0090	-0.0362	0.9922	.0992	.0460	.0086
	0.0196	1.0077	.1054	.0536	.0120	-0.0342	0.9955	.0996	.0466	.0088
	-0.1409	0.9941	.1062	.0562	.0122	-0.0362	0.9922	.1028	.0464	.0108
200	-0.1529	0.9825	.0966	.0498	.0096	-0.0067	0.9767	.0896	.0422	.0072
	-0.0202	0.9929	.0958	.0472	.0100	-0.0081	0.9783	.0900	.0424	.0072
	-0.1529	0.9825	.1000	.0532	.0112	-0.0067	0.9767	.0940	.0452	.0102
Lognormal Errors										
20	-0.2386	0.9202	.0780	.0348	.0088	-0.0008	0.9355	.0702	.0348	.0096
	-0.0181	0.9481	.0764	.0432	.0128	-0.0007	0.9500	.0752	.0362	.0100
	-0.2386	0.9202	.0818	.0424	.0108	-0.0008	0.9355	.0808	.0398	.0104
50	-0.1520	0.9623	.0806	.0396	.0108	0.0065	0.9628	.0840	.0438	.0118
	0.0030	0.9777	.0772	.0424	.0138	0.0031	0.9687	.0854	.0442	.0118
	-0.1520	0.9623	.0870	.0452	.0134	0.0065	0.9628	.0912	.0464	.0128
100	-0.1487	0.9747	.0868	.0428	.0122	-0.0028	0.9622	.0816	.0426	.0094
	0.0117	0.9853	.0868	.0450	.0156	-0.0006	0.9653	.0834	.0434	.0096
	-0.1487	0.9747	.0936	.0486	.0140	-0.0028	0.9622	.0858	.0424	.0102
200	-0.1288	0.9845	.0938	.0470	.0090	-0.0021	0.9953	.0922	.0522	.0154
	0.0042	0.9944	.0984	.0502	.0126	-0.0035	0.9969	.0924	.0528	.0154
	-0.1288	0.9845	.1004	.0524	.0112	-0.0021	0.9953	.0982	.0534	.0158

Note: (1) under each N , 1st row: LM_{FE} , 2nd row: SLM_{FE} , and 3rd row: Bootstrap LM_{FE} .
 (2) X-Value: Non-IID for group interaction, IID for queen contiguity; $\beta = \{1, 1, 1\}'$, $\sigma_v = 1$.

Null behavior of the tests. The results reported in Table 3 provide even stronger evidence for the effectiveness of centering and rescaling in improving the finite sample performance of an LM test, compared with the case of the random effects model. General observations made from the Monte Carlo results for the earlier two models still hold. Our SLM test is generally comparable to the BLM test in terms of tail probabilities.

Power of the tests. Selected results on size-adjusted power of the tests under the nominal size 5% are plotted in Figure 3. Again, LM_{FE} and SLM_{FE} have almost identical size-adjusted power. The power of BLM_{FE} (based on bootstrap size) can be lower than the other two tests when the error distribution is skewed.

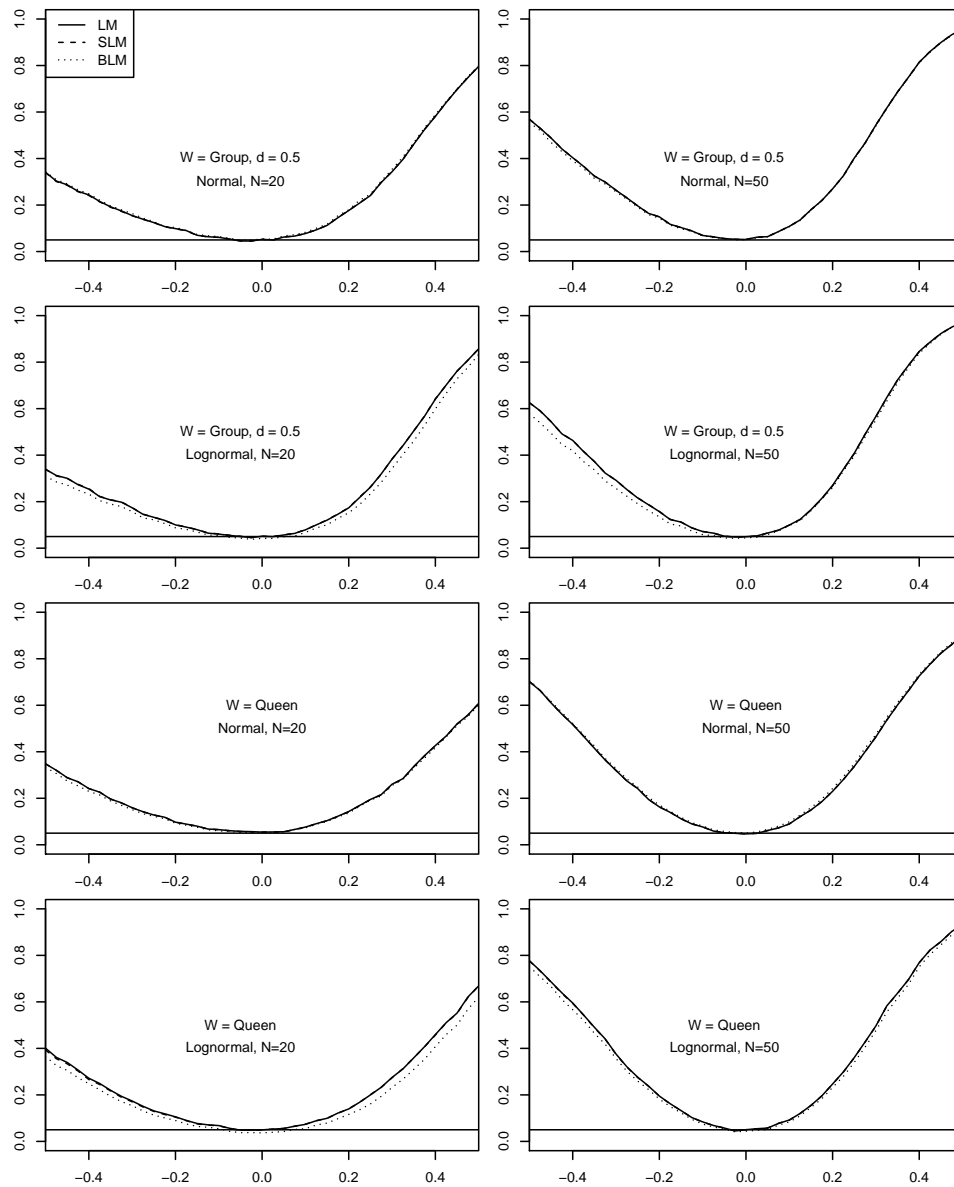


Figure 3. Size-Adjusted Empirical Power of LM, SLM and Bootstrap LM for Fixed Effects Panel Model.

5. CONCLUSION AND DISCUSSION

This paper recommends *standardized* LM tests of spatial error dependence for the linear as well as the panel regression model. We showed that when standardizing the LM tests for spatial effects, it is important to adjust for *both* the mean and variance of the LM statistics. The mean adjustment is, however, often neglected in the literature. One important reason for the mean adjustment of the LM tests for spatial effects is that the degree of spatial dependence may grow with the sample size. This slows down the convergence speed of the maximum likelihood estimators (Lee, 2004a), making the concentrated score function (the key element of the LM test) more biased.

There are other LM tests for other spatial models that are derived under normal assumptions such as Baltagi et al. (2003), and the LM test for spatial lag effect in the linear spatial autoregressive models (Anselin, 1988) and panel linear spatial autoregressive models (Debarys and Ertur, 2010), which can be studied in a similar manner. This paper recommends the standardized version of these LM tests because it offers improvements in their finite sample performance, in addition to preserving the simplicity of the original LM tests so that they can be easily adopted by applied researchers.

Two related and important issues: *bootstrap and heteroskedasticity*, deserve some further discussions as both are of potential interest for future research.⁶ The two bootstrap tests and the corresponding Monte Carlo results presented in this paper are rather encouraging. The questions are whether similar results can be obtained for more complicated models, and whether a formal justification for the validity of the bootstrap methods can be given in a more general framework. A detailed study of these issues is beyond the scope of this paper. We plan to pursue them in future research. Another important issue in testing spatial error dependence is the possible existence of heteroskedasticity. Our tests are developed under the assumption that the idiosyncratic errors are homoskedastic. By extending the idea of Born and Breitung (2011), we have successfully obtained SLM_{OPG} , which greatly improves upon their LM_{OPG} in case of homoskedasticity. However, the finite sample mean correction in SLM_{OPG} is still subject to the homoskedasticity assumption. Nevertheless, the derivation of SLM_{OPG} sheds much light on a possible solution to the general issue of standardizing spatial LM tests so that they are robust against unknown heteroskedasticity in both large and finite samples.

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⁶We are very grateful to an anonymous referee for raising these two important issues.

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APPENDIX: PROOFS OF RESULTS

To prove the theorems, we need the following lemmas.

LEMMA A.1. (Lee, 2004a): Let v be an $N \times 1$ random vector of iid elements with mean zero, variance σ^2 , and finite excess kurtosis κ . Let A be an $N \times N$ matrix with its elements denoted by $\{a_{ij}\}$. Then $E(v'Av) = \sigma^2 \text{tr}(A)$ and $\text{Var}(v'Av) = \sigma^4 \kappa \sum_{i=1}^N a_{ii}^2 + \sigma^4 \text{tr}(AA' + A^2)$.

LEMMA A.2. (Lemma A.9, Lee, 2004b): Suppose that A represents a sequence of $N \times N$ matrices that are uniformly bounded in both row and column sums. Elements of the $N \times k$ matrix X are uniformly bounded; and $\lim_{n \rightarrow \infty} \frac{1}{N} X'X$ exists and is nonsingular. Let $M = I_N - X(X'X)^{-1}X'$. Then

- (i) $\text{tr}(MA) = \text{tr}(A) + O(1)$
- (ii) $\text{tr}(A'MA) = \text{tr}(A'A) + O(1)$
- (iii) $\text{tr}[(MA)^2] = \text{tr}(A^2) + O(1)$, and
- (iv) $\text{tr}[(A'MA)^2] = \text{tr}[(MA'A)^2] = \text{tr}[A'A^2] + O(1)$

Furthermore, if the elements of A are such that $a_{ij} = O(h_N^{-1})$ for all i and j , then

- (vi) $\text{tr}^2(MA) = \text{tr}^2(A) + O(\frac{N}{h_N})$, and
- (vii) $\sum_{i=1}^N [(MA)_{ii}]^2 = \sum_{i=1}^N (a_{ii})^2 + O(h_N^{-1})$,

where $(MA)_{ii}$ are the diagonal elements of MA , and a_{ii} are the diagonal elements of A .

LEMMA A.3. Let $u = G_1\mu + G_2v$, where u and v are two independent random vectors not necessarily of the same length containing, respectively, iid elements of means zero, variances σ_μ^2 and σ_v^2 , skewness α_μ and α_v , and excess kurtosis κ_μ and κ_v ; and G_1 and G_2 are two conformable non-stochastic matrices. Let A be a conformable square matrix. Then,

- (i) $E(u'Au) = \sigma_v^2 \text{tr}(\Sigma A)$,
- (ii) $\text{Var}(u'Au) = \sigma_\mu^4 \kappa_\mu a_1' a_1 + \sigma_v^4 \kappa_v a_2' a_2 + \sigma_v^4 \text{tr}[\Sigma(A' + A)\Sigma A]$,

where $\Sigma = \sigma_v^{-2} E(uu') = \frac{\sigma_\mu^2}{\sigma_v^2} G_1 G_1' + G_2 G_2'$, $a_1 = \text{diagv}(G_1' A G_1)$, and $a_2 = \text{diagv}(G_2' A G_2)$.

Proof of Lemma A.3: The result (i) is trivial. For (ii), we have,

$$u'Au = \mu'G_1'AG_1\mu + v'G_2'AG_2v + \mu'G_1'(A + A')G_2v.$$

It is easy to see that the three terms are uncorrelated. Thus,

$$\text{Var}(u'Au) = \text{Var}(\mu'G_1'AG_1\mu) + \text{Var}(v'G_2'AG_2v) + \text{Var}[\mu'G_1'(A + A')G_2v].$$

From Lemma A.1, we obtain $\text{Var}(\mu'G_1'AG_1\mu) = \sigma_\mu^4 \kappa_\mu a_1' a_1 + \sigma_\mu^4 \text{tr}[AG_1G_1'(A + A)G_1G_1']$, and $\text{Var}(v'G_2'AG_2v) = \sigma_v^4 \kappa_v a_2' a_2 + \sigma_v^4 \text{tr}[AG_2G_2'(A + A)G_2G_2']$. It is easy to show that $\text{Var}(\mu'G_1'(A + A')G_2v) = \sigma_\mu^2 \sigma_v^2 \text{tr}[(A' + A)G_2G_2'(A + A)G_1G_1']$. Putting these three expressions together leads to (ii). ■

Proof of Theorem 2.1: First, we note that

$$\tilde{u}'W\tilde{u} - S_1\tilde{u}'\tilde{u} = \tilde{u}'(W - S_1I_N)\tilde{u} = u'M(W - S_1I_N)Mu = u'Au.$$

Under H_0 and Assumption 2.1, Lemma A.1 is applicable to $u' Au$, which gives $\text{E}u' Au = \sigma_\varepsilon^2 \text{tr} A = 0$ and $\text{Var}(u' Au) = \sigma_\varepsilon^4 \kappa_\varepsilon \sum_{i=1}^n a_{ii}^2 + \sigma_\varepsilon^4 [\text{tr}(AA') + \text{tr}(A^2)]$. Letting $W^* = W - S_1 I_N$, we have $A = MW^*M$. By Lemma A.2(i) and Assumption 2.2, $\text{tr}(WM) = O(1)$ which gives $S_1 = O(N^{-1})$. Hence, the elements of W^* are of uniform order $O(h_N^{-1})$. Under Assumption 2.3, M is uniformly bounded in both row and column sums (Lee, 2004a, Appendix A). It follows that the elements of A are of uniform order $O(1/h_N)$, and that the row and column sums of the matrix A are uniformly bounded. Thus, the generalized central limit theorem for linear-quadratic forms of Lee (2004a, Appendix A) is applicable,⁷ which shows that $u' Au$ is asymptotically normal, or equivalently,

$$\frac{u' Au}{\sigma_\varepsilon^2(\kappa_\varepsilon S_2 + S_3)^{\frac{1}{2}}} = \frac{\tilde{u}' W \tilde{u} - S_1 \tilde{u}' \tilde{u}}{\sigma_\varepsilon^2(\kappa_\varepsilon S_2 + S_3)^{\frac{1}{2}}} \xrightarrow{D} N(0, 1).$$

Now, it is easy to show that under H_0 $\tilde{\sigma}_\varepsilon^2 \equiv \tilde{u}' \tilde{u} / N \xrightarrow{P} \sigma_\varepsilon^2$ and $\tilde{\kappa}_\varepsilon \equiv \frac{1}{n \tilde{\sigma}_\varepsilon^4} \sum_{i=1}^n \tilde{u}_i^4 - 3 \xrightarrow{P} \kappa_\varepsilon$ (see Yang, 2010) for the proof of a similar result). The result (i) thus follows from Slutsky's theorem by replacing σ_ε by $\tilde{\sigma}_\varepsilon$ and κ_ε by $\tilde{\kappa}_\varepsilon$.

To prove the asymptotic equivalence of LM_{EI} and SLM_{EI} , we note that

$$\text{SLM}_{\text{EI}} = \left(\frac{S_0}{\tilde{\kappa}_\varepsilon S_2 + S_3} \right)^{\frac{1}{2}} \text{LM}_{\text{EI}} - \frac{N S_1}{(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}}}. \quad (\text{A.1})$$

Thus, it is sufficient to show that the factor in front of LM_{EI} is $O_p(1)$ and the second term is $o_p(1)$. As the elements $\{w_{ij}^*\}$ of W^* are uniformly $O(h_N^{-1})$, Lemma A.2(vi) and Assumption 2.2 ($w_{ii} = 0$) lead to $S_2 = \sum_{i=1}^n a_{ii}^2 = \sum_{i=1}^N (w_{ii}^*)^2 + O(h_N^{-1}) = O(h_N^{-1})$. Lemma A.2(ii) and (iii) lead to $S_3 = S_0 + O(1)$. Since the elements of W are uniformly $O(h_N^{-1})$ and the row sums of W are uniformly bounded, it follows that the elements of WW' and W^2 are uniformly $O(h_N^{-1})$. Hence, S_0 is $O(N/h_N)$, and so is S_3 . Furthermore, $\tilde{\kappa}_\varepsilon = O_p(1)$. These lead to $(S_0/(\tilde{\kappa}_\varepsilon S_2 + S_3))^{\frac{1}{2}} = O_p(1)$ and $N S_1/(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}} = O_p((h_N/N)^{\frac{1}{2}}) = o_p(1)$, showing that $\text{LM}_B \sim \text{LM}_B^*$. Similarly, one can show that $\text{Var}(I) \sim S_0$, and hence $\text{LM}_B \sim I^*$. Finally, it is evident that $I^o \sim I^*$. ■

Proof of Theorem 2.2: To show (i), we have, $\tilde{u}'(W - S_1 I_N)\tilde{u} = u' Au = u'(A_1 + A_u)u + u' A_d u = u' \zeta + u' A_d u$. It can be shown that (a) $u' \zeta = \sum_{i=1}^n u_i \zeta_i$ and $u' A_d u = \sum_{i=1}^n a_{ii} u_i^2$ are uncorrelated, and (b) $u_i \zeta_i$ and $u_j \zeta_j, i \neq j$, are uncorrelated, where $\{a_{ii}\}$ are the diagonal elements of A . These lead to a natural estimator of $\text{Var}(u' Au)$:

$$\sum_{i=1}^n \tilde{u}_i^2 \tilde{\zeta}_i^2 + \sum_{i=1}^n a_{ii}^2 \tilde{u}_i^4.$$

The result (i) thus follows from $\frac{1}{N} (\sum_{i=1}^N \tilde{u}_i^2 \tilde{\zeta}_i^2 + \sum_{i=1}^n a_{ii}^2 \tilde{u}_i^4) - \frac{1}{N} \sigma_\varepsilon^4 (\kappa_\varepsilon S_2 + S_3) \xrightarrow{P} 0$, and the result (ii) follows from (a) $\frac{1}{\sqrt{N}} \tilde{u}'(W - S_1 I_N)\tilde{u} - \frac{1}{\sqrt{N}} \tilde{u}' W \tilde{u} \xrightarrow{P} 0$, and (b) $\frac{1}{N} (\sum_{i=1}^N \tilde{u}_i^2 \tilde{\zeta}_i^2 + \sum_{i=1}^n a_{ii}^2 \tilde{u}_i^4) - \frac{1}{N} \sum_{i=1}^N \tilde{u}_i^2 \tilde{\zeta}_i^2 \xrightarrow{P} 0$, which are all trivial. ■

Proof of Theorem 3.1: We have $\tilde{u} = Y - X\tilde{\beta} = Y - X(X'\tilde{\Sigma}^{-1}X)^{-1}X'\tilde{\Sigma}^{-1}Y \equiv M(\tilde{\rho})Y$. The numerator of LM_{RE} becomes $\tilde{u}' A(\tilde{\rho})\tilde{u} = Y' M'(\tilde{\rho}) A(\tilde{\rho}) M(\tilde{\rho}) Y = u' M'(\tilde{\rho}) A(\tilde{\rho}) M(\tilde{\rho}) u \equiv$

⁷Lee (2004a) generalized the results of Kelejian and Prucha (2001) to cover the case where h_N is unbounded. Lee's results require the matrix A to be symmetric. If it is not, it can be replaced by $\frac{1}{2}(A + A')$.

$u' C_0(\tilde{\rho})u$. By the mean value theorem,

$$u' C_0(\tilde{\rho})u = u' C_0(\rho)u + u' \dot{C}_0(\tilde{\rho})u (\tilde{\rho} - \rho),$$

where $\tilde{\rho}$ lies between $\tilde{\rho}$ and ρ , $\dot{C}_0(\rho) = \frac{\partial C_0(\rho)}{\partial \rho} = 2M'(\rho)[\rho(\bar{J}_T \otimes W) - (\bar{J}_T \otimes I_N)P(\rho)A(\rho)]M(\rho)$, and $P(\rho) = X(X'\Sigma_0^{-1}X)^{-1}X'$. From the results of Lee (2004a, Appendix), it is easy to see the elements of $C_0(\rho)$ are of uniform order $O(1/h_N)$ uniformly in ρ , and so are the elements of $\dot{C}_0(\tilde{\rho})$. As $\tilde{\rho}$ is consistent, it follows that $E[u' C_0(\tilde{\rho})u] \sim E[u' C_0(\rho)u] = \sigma_v^2 \text{tr}[\Sigma_0 C_0(\rho)]$. This leads to a centered quantity $\tilde{u}' A(\tilde{\rho})\tilde{u} - \sigma_v^2 \text{tr}[\Sigma_0 C_0(\rho)]$, or its feasible version:

$$\tilde{u}' A(\tilde{\rho})\tilde{u} - \frac{1}{NT - k} \text{tr}[\tilde{\Sigma}_0 C_0(\tilde{\rho})] \tilde{u}' \tilde{\Sigma}_0^{-1} \tilde{u} = \tilde{u}' (\tilde{A} - \tilde{s}_0 \tilde{\Sigma}_0^{-1}) \tilde{u},$$

which gives the numerator of SLM_{SE} .

As $\tilde{u}' (\tilde{A} - \tilde{s}_0 \tilde{\Sigma}_0^{-1}) \tilde{u} = u' M(\tilde{\rho})' (\tilde{A} - \tilde{s}_0 \tilde{\Sigma}_0^{-1}) M(\tilde{\rho})u = u' C(\tilde{\rho})u$, applying the mean value theorem again leads to $u' C(\tilde{\rho})u \sim u' C(\rho)u$. It follows that $\text{Var}[u' C(\tilde{\rho})u] \sim \text{Var}[u' C(\rho)u]$. Now, $u' C(\rho)u$ can be decomposed into the following three terms,

$$\mu' (\iota_T' \otimes I_N) C(\rho) (\iota_T \otimes I_N) \mu + v' C(\rho) v + \mu' (\iota_T' \otimes I_N) C(\rho) v,$$

which are either independent or asymptotically independent. Thus, the asymptotic normality of the first two terms on the right hand side of the above equation follow from the generalized central limit theorem for linear-quadratic forms of Lee (2004a, Appendix A). The asymptotic normality of the last term follows from the fact that the two random vectors involved are independent. The mean and variance of $u' C(\rho)u$ can be easily obtained from Lemma A.3 in the Appendix. In fact, $E(u' C(\rho)u) = 0$, and

$$\text{Var}(u' C(\rho)u) = \sigma_v^4 [\phi^2 \kappa_\mu a_1' a_1 + \kappa_v a_2' a_2 + \text{tr}[\Sigma_0 (C(\rho)' + C(\rho)) \Sigma_0 C(\rho)]].$$

Thus the result in (i) follows and $\text{SLM}_{\text{RE}} \xrightarrow{D} N(0, 1)$.

To prove the result in (ii), let $X(\rho) = \Sigma_0^{-\frac{1}{2}} X$ and $M^*(\rho) = I_{NT} - X(\rho)[X'(\rho)X(\rho)]^{-1} X'(\rho)$. Assumption 3.3 and the structure of $\Sigma_0^{-\frac{1}{2}}$ guarantee that the elements of $X(\rho)$ are bounded uniformly in both N and ρ . Thus, Lemma A.2 in the Appendix is applicable on $M^*(\rho)$ for each ρ . We have $C_0(\rho) = M'(\rho)A(\rho)M(\rho) = \Sigma_0^{-\frac{1}{2}} M^*(\rho)A(\rho)M^*(\rho)\Sigma_0^{-\frac{1}{2}}$. Thus,

$$\begin{aligned} \text{tr}[\Sigma_0 C_0(\rho)] &= \text{tr}[M^*(\rho)A(\rho)M^*(\rho)\Sigma_0] \\ &= \text{tr}[A(\rho)M^*(\rho)\Sigma_0] + O(1) \quad (\text{by Lemma A.2}) \\ &= \text{tr}[M^*(\rho)\Sigma_0 A(\rho)] + O(1) \\ &= \text{tr}[\Sigma_0 A(\rho)] + O(1) \quad (\text{by Lemma A.2}) \\ &= O(1). \end{aligned}$$

Thus, $a_0(\rho) = \frac{1}{NT-k} \text{tr}(\Sigma_0 C_0(\rho)) = O(\frac{1}{N})$. Similarly, by successively applying Lemma A.2, one shows that

$$\begin{aligned} \text{tr}[\Sigma_0 (C_0(\rho)' + C_0(\rho)) \Sigma_0 C_0(\rho)] &= \text{tr}[M^*(\rho)(A(\rho)' + A(\rho))M^*(\rho)\Sigma_0 M^*(\rho)A(\rho)M^*(\rho)\Sigma_0] \\ &= \text{tr}[(A(\rho)' + A(\rho))\Sigma_0 A(\rho)\Sigma_0] + O(1) \\ &= (T - 1 + \rho^2)S_0 + O(1). \end{aligned}$$

It follows that $\text{tr}[\Sigma_0 (C(\rho)' + C(\rho)) \Sigma_0 C(\rho)] = (T - 1 + \rho^2)S_0 + O(1)$ as $C(\rho) = C_0(\rho) - a_0(\rho)\Sigma_0^{-1}M(\rho)$. Under Assumption 3.2, the elements of W^2 and WW' are of uniform

order $O(1/h_N)$. It follows that S_0 , the quantity in LM_{RE} , is $O(N/h_N)$. Hence,

$$\text{tr}[\Sigma_0 C(\rho) \Sigma_0 C(\rho)] \sim (T - 1 + \rho^2) S_0 = O(N/h_N).$$

Finally, Lemma A.2(vii) in the Appendix leads to $a'_1 a_1 = O(1/h_N)$ and $a'_2 a_2 = O(1/h_N)$. The result in (ii) thus follows and the two LM tests given in (3.6) and (3.8) are asymptotically equivalent. ■

Proof of Theorem 3.2: The proof of this theorem parallels that of Theorem 2.1. ■

Proof of Corollary 3.1: Note that $(\varepsilon_1^*, \varepsilon_2^*, \dots, \varepsilon_{T-1}^*) = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T) F_{T,T-1}$. With $\varepsilon_{i.}^{*'} denoting the i th row of the $N \times (T-1)$ matrix $(\varepsilon_1^*, \varepsilon_2^*, \dots, \varepsilon_{T-1}^*)$ and $\varepsilon_{i.}'$ the i th row of the $N \times T$ matrix $(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T)$, we have$

$$\text{Var}(1'_{T-1} \varepsilon_{i.}^*) = \text{Var}(1'_{T-1} F'_{T,T-1} \varepsilon_{i.}) = 1'_{T-1} F'_{T,T-1} \text{Var}(\varepsilon_{i.}) F_{T,T-1} 1_{T-1} = (T-1) \sigma_\varepsilon^2.$$

Denoting $c = F_{T,T-1} 1_{T-1}$, and applying Lemma A.1 in Appendix, we have

$$\text{Var}[(1'_{T-1} \varepsilon_{i.}^*)^2] = \text{Var}[(c' \varepsilon_{i.})^2] = \text{Var}[\varepsilon_{i.}' (cc') \varepsilon_{i.}] = \sigma_v^4 \kappa_v \sum_{t=1}^T c_t^4 + 2(T-1)^2 \sigma_v^4.$$

It follows that $E[(1'_{T-1} \varepsilon_{i.}^*)^4] = E[(c' \varepsilon_{i.})^4] = \sigma_v^4 \kappa_v \sum_{t=1}^T c_t^4 + 3(T-1)^2 \sigma_v^4$. As $c' \varepsilon_{i.}$ are iid, Kolmogorov's law of large numbers ensures that

$$\frac{1}{N} \sum_{i=1}^N (1'_{T-1} \varepsilon_{i.}^*)^4 \rightarrow \sigma_v^4 \kappa_v \sum_{t=1}^T c_t^4 + 3(T-1)^2 \sigma_v^4.$$

The result follows by moving the terms other than κ_v to the left and then replacing $\varepsilon_{i.}^*$ by $\tilde{\varepsilon}_{i.}^*$, and σ_v^2 by $\frac{1}{N(T-1)} \tilde{\varepsilon}^{*'} \tilde{\varepsilon}^*$. ■