A Comparison of QML Method and GMM for Estimating Spatial Regression Models

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Abstract

The quasi-maximum likelihood (QML) method and the generalized method of moments (GMM) are the two popular methods in the estimation and inference for spatial regression models. In this paper, a Monte Carlo comparison is made of the two methods based on several spatial regression models: the regression model with spatial lag dependence (SLD), or with spatial error dependence (SED), or with both SLD and SED.

Key Words: QML estimation; GMM; Monte Carlo; Spatial layout; Spatial lag model; spatial error model; SARAR model.

JEL Classification: C10, C12, C15, C21

1. Introduction

The maximum likelihood (ML) or quasi-ML (QML) method is popular in the estimation and inference for spatial regression models (Anselin, 1988; Anselin and Bera, 1998; Lee, 2004; Baltagi and Yang, 2013a,b; Robinson and Rossi, 2014a,b; Liu and Yang, 2015a,b; Yang, 2015a,b). Another popular method is GMM (Kelijian and Prucha, 1998,1999; Lee, 2001a,b, 2007; Lin and Lee, 2010; Liu and Lee, 2010). We consider in detail three popular spatial regression models: the linear regression model with spatial error dependence (SED), that with a spatial lag dependence (SLD), and that with both SLD and SED, also referred to as the SARAR model in the literature. See Anselin and Bera (1998) and Anselin (2001) for excellent reviews on these models.

The line-up for the paper is as follows. Section 2 outlines the QML method. In section 3 we propose a simple method of bias reduction of QML estimator for regrssion model with spatial lag dependence, regression model with spatial error dependence as well as spatial model with both SLD anf SED. Sections 4 outlines the GMM for, respectively, the SED model, the SLD model, and the SARAR model. Section 5 presents the Monte Carlo simulation results. Section 6 concludes the paper.

2. QML Estimation of Spatial Regression Models

2.1 QML estimation of SED model

Consider the following linear regression model with spatial error dependence (SED), where the SED is specified as a spatial autoregressive (SAR) process:

$$Y_n = X_n\beta + u_n, \quad u_n = \rho W_n u_n + \epsilon_n, \tag{1}$$

where Y_n is an $n \times 1$ vector of observations on the dependent variable corresponding to n spatial units, X_n is an $n \times k$ matrix containing the values of k exogenous regressors, W_n is an $n \times n$ spatial weight matrix that summarises the interactions among the spatial units, ϵ_n is an $n \times 1$ vector of independent and identically distributed (iid) disturbances with mean zero and variance σ^2, ρ is the *spatial parameter*, and β denotes the $k \times 1$ vector of regression coefficients.

QML method. The quasi Gaussian loglikelihood function of $\theta = (\beta', \sigma^2, \rho)'$ for the SED model is given by, $\ell_n(\theta) = -\frac{n}{2}\log(2\pi\sigma^2) + \log|B_n(\rho)| - \frac{1}{2\sigma^2}(Y_n - X_n\beta)'B_n'(\rho)B_n(\rho)(Y_n - X_n\beta)$, where $B_n(\rho) = I_n - \rho W_n$. Maximizing $\ell_n(\theta)$ gives the MLE, $\hat{\theta}_n$ of θ if the errors are indeed Gaussian, otherwise the QMLE. Given ρ, $\ell_n(\theta)$ is partially maximized at,

$$\tilde{\beta}_n(\rho) = [X_n'B_n'(\rho)B_n(\rho)X_n]^{-1}X_n'B_n'(\rho)B_n(\rho)Y_n, \text{ and } \tilde{\sigma}_n^2(\rho) = \tfrac{1}{n}Y_n'B_n'(\rho)M_n(\rho)B_n(\rho)Y_n, \tag{2}$$

where $M_n(\rho) = I_n - B_n(\rho)X_n[X_n'B_n'(\rho)B_n(\rho)X_n]^{-1}X_n'B_n'(\rho)$. The concentrated log-likelihood function for ρ upon substituting the constrained QMLEs $\tilde{\beta}_n(\rho)$ and $\tilde{\sigma}_n^2(\rho)$ into $\ell(\theta)$:

$$\ell_n^c(\rho) = -\frac{n}{2}[\log(2\pi) + 1] + \log|B_n(\rho)| - \frac{n}{2}\log(\tilde{\sigma}_n^2(\rho)). \tag{3}$$

Maximising $\ell_n^c(\rho)$ gives the unconstrained QMLE $\hat{\rho}_n$ of ρ, which in turn gives the unconstrained QMLEs of β and σ^2 as, $\hat{\beta}_n \equiv \tilde{\beta}_n(\hat{\rho}_n)$ and $\tilde{\sigma}_n^2 \equiv \hat{\sigma}_n^2(\hat{\rho}_n)$. Thus, $\hat{\theta}_n = (\hat{\beta}_n', \hat{\sigma}_n^2, \hat{\rho}_n)'$.

Liu and Yang (2015) show that, under regularity conditions, the QMLE $\hat{\theta}_n$ is asymptotically normal with mean θ_0, and variance-covariance (VC) matrix $\Sigma_n^{-1}\Gamma_n\Sigma_n^{-1}$, where

$$\Sigma_n = \begin{pmatrix} \frac{1}{\sigma_0^2}X_n'B_n'B_nX_n & 0 & 0 \\ 0 & \frac{n}{2\sigma_0^4} & \frac{1}{\sigma_0^2}\text{tr}(G_n) \\ 0 & \frac{1}{\sigma_0^2}\text{tr}(G_n) & \text{tr}(G_n^sG_n) \end{pmatrix},$$

$$\Gamma_n = \begin{pmatrix} \frac{1}{\sigma_0^2}X_n'B_n'B_nX_n & \frac{1}{2\sigma_0^3}\gamma X_n'B_n'\iota_n & \frac{1}{\sigma_0}\gamma X_n'B_n'g_n \\ \frac{1}{2\sigma_0^3}\gamma\iota_n'B_nX_n & \frac{n}{4\sigma_0^4}(\kappa+2) & \frac{1}{2\sigma_0^2}(\kappa+2)\text{tr}(G_n) \\ \frac{1}{\sigma_0}\gamma g_n'B_nX_n & \frac{1}{2\sigma_0^2}(\kappa+2)\text{tr}(G_n) & \kappa g_n'g_n + \text{tr}(G_n^sG_n) \end{pmatrix},$$

ι_n is an $n \times 1$ vector of ones, γ and κ are, respectively, the measures of skewness and excess kurtosis of the idiosyncratic errors $\epsilon_{n,i}$, $g_n = \text{diag}(G_n)$, $G_n = G_n(\rho_0) = W_nB_n^{-1}(\rho_0)$, and

$G_n^s = G_n + G_n'$.

2.2 QML estimation of SLD model

The regression model with spatial lag dependence (SLD) takes the form:

$$Y_n = \lambda W_n Y_n + X_n \beta + \epsilon_n, \tag{4}$$

Letting $A_n(\lambda) = I_n - \lambda W_n$, the log-likelihood function of $\theta = (\beta', \sigma^2, \lambda)'$ is $\ell_n(\theta) = -\frac{n}{2}\log(2\pi\sigma^2) + \log|A_n(\lambda)| - \frac{1}{2\sigma^2}[A_n(\lambda)Y_n - X_n\beta]'[A_n(\lambda)Y_n - X_n\beta]$. Given λ, $\ell_n(\theta)$ is maximized at

$$\tilde{\beta}_n(\lambda) = (X_n'X_n)^{-1}X_n'A_n(\lambda)Y_n \quad \text{and} \quad \tilde{\sigma}_n^2(\lambda) = \tfrac{1}{n}Y_n'A_n'(\lambda)M_nA_n(\lambda)Y_n, \tag{5}$$

where $M_n = I_n - X_n(X_n'X_n)^{-1}X_n'$. These lead to the concentrated log-likelihood of λ as,

$$\ell_n^c(\lambda) = -\frac{n}{2}[\log(2\pi) + 1] - \frac{n}{2}\log\hat{\sigma}_n^2(\lambda) + \log|A_n(\lambda)|. \tag{6}$$

Maximizing $\ell_n^c(\lambda)$ gives the unconstrained QMLE $\hat{\lambda}_n$ of λ. The unconstrained QMLEs of β and σ^2 are thus, $\hat{\beta}_n \equiv \tilde{\beta}_n(\hat{\lambda}_n)$ and $\hat{\sigma}_n^2 \equiv \tilde{\sigma}_n^2(\hat{\lambda}_n)$. Write $\hat{\theta}_n = (\hat{\beta}_n', \hat{\sigma}_n^2, \hat{\lambda}_n)'$.

Lee (2004) shows that $\hat{\theta}_n$ is asymptotically $N(\theta_0, \Sigma_n^{-1}\Gamma_n\Sigma_n^{-1})$, where

$$\Sigma_n = \begin{pmatrix} \frac{1}{\sigma_0^2}X_n'X_n & 0 & \frac{1}{\sigma_0}X_n'\eta_n \\ 0 & \frac{n}{2\sigma_0^4} & \frac{1}{\sigma_0^2}\mathrm{tr}(G_n) \\ \frac{1}{\sigma_0}\eta_n'X_n & \frac{1}{\sigma_0^2}\mathrm{tr}(G_n) & \eta_n'\eta_n + \mathrm{tr}(G_n^sG_n) \end{pmatrix},$$

$$\Gamma_n = \begin{pmatrix} 0 & \frac{1}{2\sigma_0^3}\gamma X_n'\iota_n & \frac{1}{\sigma_0}\gamma X_n'g_n \\ \frac{1}{2\sigma_0^3}\gamma\iota_n'X_n & \frac{n}{4\sigma_0^4}\kappa & \frac{1}{2\sigma_0^2}\gamma\iota_n'\eta_n + \frac{1}{2\sigma_0^2}\kappa\mathrm{tr}(G_n) \\ \frac{1}{\sigma_0}\gamma g_n'X_n & \frac{1}{2\sigma_0^2}\gamma\iota_n'\eta_n + \frac{1}{2\sigma_0^2}\kappa\mathrm{tr}(G_n) & \kappa g_n'g_n + 2\gamma g_n'\eta_n \end{pmatrix} + \Sigma_n,$$

ι_n, γ and κ are defined as in Section 3.1, $g_n = \mathrm{diag}(G_n)$, $G_n = G_n(\rho_0) = W_nA_n^{-1}(\lambda_0)$, $G_n^s = G_n + G_n'$ and $\eta_n = \sigma_0^{-1}G_nX_n\beta_0$.

Letting V_{n1} be the submatrix of $\Sigma_n^{-1}\Gamma_n\Sigma_n^{-1}$ corresponding to β and $\hat{V}_{n1}$ be its estimate, an asymptotic t-statistic for inferences for $c_0'\beta_0$ is thus,

$$t_{\texttt{SLD}} = \frac{c_0'\hat{\beta}_n - c_0'\beta_0}{\sqrt{c_0'\hat{V}_{n1}c_0}}, \tag{7}$$

which is asymptotically $N(0,1)$. Finite sample properties of $t_{\texttt{SLD}}$ is of interest.

As $\tilde{\beta}_n(\hat{\lambda}) = \beta_0 + (\lambda - \hat{\lambda})(X_n'X_n)^{-1}X_n'G_nX_n\beta_0 + o_p(1)$, we see clearly that any estimation bias of $\hat{\lambda}$ is quickly passed down to the QMLE of β_0. Thus the t-statistic computed using $\tilde{\beta}_n(\hat{\lambda})$ and the variance estimate $\hat{V}_{n1}$ can be unreliable. This fact is confirmed by the Monte Carlo results. As such it is desirable to find ways to improve $t_{\texttt{SLD}}$.

2.3 QML estimation of SARAR model

Combining the SED and SLD models considered above, we have the so-called spatial autoregressive model with autoregressive errors, also known as the SARAR model:

$$Y_n = \lambda W_{1n} Y_n + X_n \beta + u_n, \quad u_n = \rho W_{2n} u_n + \epsilon_n. \tag{8}$$

The Gaussian log-likelihood function of $\theta = (\beta', \sigma^2, \lambda, \rho)'$ is

$$\ell_n(\theta) = -\frac{n}{2}\log(2\pi\sigma^2) + \log|A_n(\lambda)| + \log|B_n(\rho)| - \frac{1}{2\sigma^2}\left[Y_n(\delta) - X_n(\rho)\beta\right]'\left[Y_n(\delta) - X_n(\rho)\beta\right] \tag{9}$$

where $A_n(\lambda) = I_n - \lambda W_{1n}$, $B_n(\rho) = I_n - \rho W_{2n}$, $X_n(\rho) = B_n(\rho)X_n$ and $Y_n(\delta) = B_n(\rho)A_n(\lambda)Y_n$. The constrained QMLEs of β and σ^2, given $\delta = (\lambda, \rho)'$, are

$$\tilde{\beta}_n(\delta) = [X_n'(\rho)X_n(\rho)]^{-1}X_n'(\rho)Y_n(\delta) \ \text{ and } \ \tilde{\sigma}_n^2(\delta) = \tfrac{1}{n}Y_n'(\delta)M_n(\rho)Y_n(\delta), \tag{10}$$

where $M_n(\rho) = I_n - B_n(\rho)X_n[X_n'B_n'(\rho)B_n(\rho)X_n]^{-1}X_n'B_n'(\rho)$. Then, the concentrated Gaussian loglikelihood function for δ is,

$$\ell_n^c(\delta) = -\frac{n}{2}[\ln(2\pi) + 1] - \frac{n}{2}\ln(\hat{\sigma}_n^2(\delta)) + \ln|A_n(\lambda)| + \ln|B_n(\rho)|. \tag{11}$$

Maximizing (**??**) gives the QMLE $\hat{\delta}_n$ of δ, and thus the QMLEs of β and σ^2 as $\hat{\beta}_n \equiv \tilde{\beta}_n(\hat{\delta}_n)$ and $\hat{\sigma}_n^2 \equiv \tilde{\sigma}_n^2(\hat{\delta}_n)$. Write $\hat{\theta}_n = (\hat{\beta}_n', \hat{\sigma}_n^2, \hat{\delta}_n')'$. The concentrated score function upon dividing by n is,

$$\tilde{\psi}_n(\delta) = \begin{cases} -\dfrac{1}{n}\mathrm{tr}(G_{1n}(\lambda)) + \dfrac{Y_n'(\delta)M_n(\rho)\bar{B}_n(\delta)Y_n(\delta)}{Y_n'(\delta)M_n(\rho)Y_n(\delta)}, \\ -\dfrac{1}{n}\mathrm{tr}(G_{2n}(\rho)) + \dfrac{Y_n'(\delta)M_n(\rho)G_{2n}(\rho)M_n(\rho)Y_n(\delta)}{Y_n'(\lambda)M_n(\rho)Y_n(\delta)}, \end{cases} \tag{12}$$

where $G_{1n}(\lambda) = W_{1n}A_n^{-1}(\lambda)$, $G_{2n}(\rho) = W_{2n}B_n^{-1}(\rho)$ and $\bar{B}_n(\delta) = B_n(\rho)G_{1n}(\lambda)B_n^{-1}(\rho)$.

Jin and Lee (2013) shows that under some regularity conditions, $\hat{\theta}_n$ is asymptotically normal with mean θ_0 and asymptotic VC matrix $\Sigma_n^{-1}\Gamma_n\Sigma_n^{-1}$, where

$$\Sigma_n = \begin{pmatrix} \frac{1}{\sigma_0^2}X_n'B_n'B_nX_n & 0 & \frac{1}{\sigma_0}X_n'B_n'\mu_n & 0 \\ 0 & \frac{n}{2\sigma_0^4} & \frac{1}{\sigma_0^2}\mathrm{tr}(\bar{B}_n) & \frac{1}{\sigma_0^2}\mathrm{tr}(G_{2n}) \\ \frac{1}{\sigma_0}\mu_n'B_nX_n & \frac{1}{\sigma_0^2}\mathrm{tr}(\bar{B}_n) & \mu_n'\mu_n + \mathrm{tr}(\bar{B}_n^s\bar{B}_n) & \mathrm{tr}(G_{2n}^s\bar{B}_n) \\ 0 & \frac{1}{\sigma_0^2}\mathrm{tr}(G_{2n}) & \mathrm{tr}(G_{2n}^s\bar{B}_n) & \mathrm{tr}(G_{2n}^sG_{2n}) \end{pmatrix},$$

$$\Gamma_n = \begin{pmatrix} 0 & \frac{\gamma}{2\sigma_0^3} X_n' B_n' \iota_n & \frac{\gamma}{\sigma_0} X_n' B_n' \bar{b}_n & \frac{\gamma}{\sigma_0} X_n' B_n' g_{2n} \\ \frac{\gamma}{2\sigma_0^3} \iota_n' B_n X_n & \frac{n\kappa}{4\sigma_0^4} & \frac{\kappa}{2\sigma_0^2} \mathrm{tr}(\bar{B}_n) + \frac{\gamma}{2\sigma_0^2} \iota_n' \mu_n & \frac{\kappa}{2\sigma_0^2} \mathrm{tr}(G_{2n}) \\ \frac{\gamma}{\sigma_0} \bar{b}_n' B_n X_n & \frac{\kappa}{2\sigma_0^2} \mathrm{tr}(\bar{B}_n) + \frac{\gamma}{2\sigma_0^2} \iota_n' \mu_n & \kappa \bar{b}_n' \bar{b}_n + 2\gamma \bar{b}_n' \mu_n & \kappa g_{2n}' \bar{b}_n + \gamma g_{2n}' \mu_n \\ \frac{\gamma}{\sigma_0} g_{2n}' B_n X_n & \frac{\kappa}{2\sigma_0^2} \mathrm{tr}(G_{2n}) & \kappa g_{2n}' \bar{b}_n + \gamma g_{2n}' \mu_n & \kappa g_{2n}' g_{2n} \end{pmatrix} + \Sigma_n,$$

ι_n, γ and κ are defined in the earlier sections, $\mu_n = \sigma_0^{-1} B_n G_{1n} X_n \beta_0$, $\bar{b}_n = \mathrm{diag}(\bar{B}_n)$, $g_{2n} = \mathrm{diag}(G_{2n})$, $\bar{B}_n^s = \bar{B}_n + \bar{B}_n'$ and $G_{2n}^s = G_{2n} + G_{2n}'$.

Letting V_{n1} be the submatrix of $\Sigma_n^{-1} \Gamma_n \Sigma_n^{-1}$ corresponding to β and $\hat{V}_{n1}$ be its estimate, an asymptotic t-statistic for inferences for $c_0' \beta_0$ is thus

$$t_{\mathtt{SARAR}} = \frac{c_0' \hat{\beta}_n - c_0' \beta_0}{\sqrt{c_0' \hat{V}_{n1} c_0}} \sim N(0, 1). \tag{13}$$

3. Simple Method of Bias reduction for QML Estimators

In spatial regression models, the main source of finite sample bias in model estimation and the main cause of difficulty in bias correction is the spatial parameters as it enters the model in a highly nonlinear manner. Given the spatial parameter other estimators in the model in either unbiased or can be easily bias-corrected. Many other econometric models, including dynamic regression model, dynamic panel model with fixed effect,Box-Cox regression, Weibull duration model, et cetera, share similar features as the three spatial regression models considered in this paper. Many researchers have recognised and focused on providing treatment to the bias problem arising from the estimation of the nonlinear parameter in the last two decades (among others, Kiviet, 1995; Hahn and Kuersteiner, 2002; Hahn and Newey, 2004; Bun and Carree, 2005; Bao and Ullah, 2007a,b; Bao, 2013; Yang, 2015a,b; Liu and Yang, 2015).

In this paper, we introduce a simple bias redution method for QML estimators. The proposed method is very simple and yet quite general as it is applicable to almost all the spatial econometrics models appeared in the literature.

Consider again the SARA model and $\tilde{\psi}_n(\delta)$, its concentrated score function upon dividing by n presented in previous section. Let $\theta_0(\delta_0)$ be the true value of the parameter vector $\theta(\delta)$. Denote a quantity at the true parameter value by dropping its arguments shall no confusion arise, e.g., $A_n = A_n(\lambda_0)$, $B_n = B_n(\rho_0)$, $G_{1n} = G_{1n}(\lambda_0)$, $G_{2n} = G_{2n}(\rho_0)$, and $M_n = M_n(\rho_0)$. The the concentrated score given in (**??**) at the true parameter values can be written as

$$\tilde{\psi}_n(\delta_0) = \begin{cases} \dfrac{Y_n'(\delta_0) M_n (B_n G_{1n} B_n^{-1} - \bar{G}_{1n} I_n) Y_n(\delta_0)}{Y_n'(\delta_0) M_n Y_n(\delta_0)}, \\ \dfrac{Y_n'(\delta_0) M_n (G_{2n} - \bar{G}_{2n} I_n) M_n Y_n(\delta_0)}{Y_n'(\delta_0) M_n Y_n(\delta_0)}, \end{cases} \tag{14}$$

where $\bar{G}_{rn} = \frac{1}{n} \mathrm{tr}(G_{rn}), r = 1, 2$.

Clearly, $\mathrm{E}[\tilde{\psi}_n(\delta_0)] \neq 0$ (even though the joint score function has the expectation zero). This is one of the main source of bias in estimating δ. Yang (2015b) and Liu and Yang (2015a,b) proposed general methodologies for up to thrid-order bias-correction of $\hat{\delta}_n$, which requires bootstrap to estimate the bias-correction terms. We now propose a much simpler alternative, which leads to a significant bias-reduction although may not be as effective as the general bootstrap-based methods of Yang (2015b) and Liu and Yang (2015a,b).

Consider the numerators of $\tilde{\psi}_n(\delta_0)$. Using the reduced form expression for Model (**??**): $Y(\delta_0) = X_n(\rho_0)\beta_0 + \epsilon_n$, we have

$$\begin{aligned}
Y_n'(\delta_0)M_n(B_nG_{1n}B_n^{-1} - \bar{G}_{1n}I_n)Y_n(\delta_0) &= \epsilon_n'M_n(B_nG_{1n}B_n^{-1} - \bar{G}_{1n}I_n)\epsilon_n \\
&\quad + \epsilon_n'M_n(B_nG_{1n}B_n^{-1} - \bar{G}_{1n}I_n)X_n(\rho_0)\beta_0 \equiv \epsilon_n'\Phi_1\epsilon_n + \epsilon_n'\Pi, \text{ and} \\
Y_n'(\delta_0)M_n(G_{2n} - \bar{G}_{2n}I_n)M_nY_n(\delta_0) &= \epsilon_n'M_n(G_{2n} - \bar{G}_{2n}I_n)M_n epsilon_n \equiv \epsilon_n'\Phi_2\epsilon_n,
\end{aligned}$$

where $\Pi = M_n(B_nG_{1n}B_n^{-1} - \bar{G}_{1n}I_n)X_n(\rho_0)\beta$, $\Phi_1 = M_n(B_nG_{1n}B_n^{-1} - \bar{G}_{1n}I_n)$ and $\Phi_2 = M_n(G_{2n} - \bar{G}_{2n}I_n)M_n$. It follows that

$$\begin{aligned}
&\mathrm{E}[Y_n'(\delta_0)M_n(B_nG_{1n}B_n^{-1} - \bar{G}_{1n}I_n)Y_n(\delta_0)] = \sigma_\epsilon^2\mathrm{tr}(\Phi_1), \quad \text{and} \\
&\mathrm{E}[Y_n'(\delta_0)M_n(G_{2n} - \bar{G}_{2n}I_n)M_nY_n(\delta_0)] = \sigma_\epsilon^2\mathrm{tr}(\Phi_2).
\end{aligned}$$

These suggest a set of adjusted estimating equation:

$$\tilde{\psi}_n^\circ(\delta) = \begin{cases} Y_n'(\delta)M_n(\rho)[B_n(\rho)G_{1n}(\lambda)B_n^{-1}(\rho) - \bar{G}_{1n}(\lambda)I_n]Y_n(\delta) - \frac{n}{n-k}\tilde{\sigma}_n^2(\delta)\mathrm{tr}[\Phi_1(\delta)], \\ Y_n'(\delta)M_n(\rho)[G_{2n}(\rho) - \bar{G}_{2n}(\rho)I_n)]M_n(\rho)Y_n(\delta) - \frac{n}{n-k}\tilde{\sigma}_n^2(\delta)\mathrm{tr}[\Phi_2(\delta)], \end{cases} \tag{15}$$

which is unbiased in the sense that $\mathrm{E}[\tilde{\psi}_n^\circ(\delta_0)] = 0$. Solving $\tilde{\psi}_n^\circ(\delta) = 0$ leads to a potentially finite sample bias-reduced estimator for δ, denoted as $\hat{\delta}_n^\circ$. The corresponding estimators for β and σ^2 are, respectively, $\hat{\beta}_n^\circ \equiv \tilde{\beta}_n(\hat{\delta}_n^{\mathrm{bc}})$ and $\hat{\sigma}_n^{\circ 2} \equiv \tilde{\sigma}_n^2(\hat{\delta}_n^\circ)$, which potentially also enjoy a bias-reduction.

This bias reduction method can be applied to spatial autoregressive (SAR) or spatial lag dependence (SLD) model simply by setting $\rho = 0$ and deleting the second equation in (15) . The adjusted estimating equation for SLD model is thus:

$$\tilde{\psi}_n^\circ(\delta) = Y_n'(\lambda)M_n[G_{1n}(\lambda) - \bar{G}_{1n}(\lambda)I_n]Y_n(\lambda) - \frac{n}{n-k}\tilde{\sigma}_n^2(\lambda)\mathrm{tr}[\Phi_1(\lambda)] \tag{16}$$

where $\Phi_1 = M_n(G_{1n} - \bar{G}_{1n}I_n)$. Similarly, setting $\lambda = 0$ and deleting the first equation in (15) gives the method for the spatial error dependence (SED) model. The adjusted estimating equation for SED model is the following:

$$Y_n'(\rho)M_n(\rho)[G_{2n}(\rho) - \bar{G}_{2n}(\rho)I_n)]M_n(\rho)Y_n(\rho) - \frac{n}{n-k}\tilde{\sigma}_n^2(\rho)\mathrm{tr}[\Phi_2(\rho)] \tag{17}$$

where $\Phi_2 = M_n(\rho)(G_{2n} - \bar{G}_{2n}I_n)M_n(\rho)$.

Mote Carlo experiments have been done to investigate the finite smmple performance of the bias reduction method proposed. The result suggested stable performance of the estimator and more importantly significant bias reduction of regular QML estimators in all three models considered.

It would be interesting to present some theories to show that $\hat{\delta}_n^{\circ}$ has a smaller finite sample bias than the QMLE $\hat{\delta}_n$, so do the $\hat{\beta}_n^{\circ}$ and $\hat{\sigma}_n^{\circ 2}$ compared with the QMLEs $\hat{\beta}_n$ and $\hat{\sigma}_n^2$.

To make the comparison complete. We also examined the performance of the general method of bias correction based on stochastic expansion and bootstrap proposed by Yang (2015), where stochastic expansion provides tractable approximations to the bias and variance of the nonlinear estimatiors and the bootstrap makes these expansions practically implementable.

4. GMM Estimation of Spatial Regression Models

As argued by many researchers, the ML method can be computationally damanding for some spatial weights matrices. Alternative methods have subsequently been proposed. The 2SLS method has been noted for spatial lag model with exogenous regressors in Anselin (1988,1990), Kelejian and Prucha (1997,1998) and Lee (2003), among others. For spatial error dependent model, the generalized least square can be easily adopted. Morover, a method of moments (MOM) approach has been introduced in Kelejian and Prucha (2001). Finally for the SARA model, Kelejian and Prucha (1998) introduced a method of generalized 2SLS in combination with the MOM approach. Lee (2010) proposed distribution-free best generalized method of moments estimators for all three spatial regression models aforementioned. Lee (2010) argued that these BGMMEs, within the class of GMMEs based on linear and quadratic moment conditions, have the merit of computational simplicity and asymptotic efficiency. Is it argued that the BGMMEs are asymptotically as efficient as the MLE when the disturbances are normally distributed and asymptotically more efficient than the Gaussian QMLE otherwise.

4.1 GMM Estimation of SARAR Model

Let us first consider the SARA model. For the estimation of model (8). We consider the transformed equation $B_nY_n = B_nX_n\beta_n + \lambda B_nW_nY_n + \epsilon_n$. Let Q_n be an $n \times q$ matrix of instuments variables constructed as functions of the regressors and spatial weight matrices. Denote $\epsilon_n(\theta) = B_n(\rho)\left[A_n(\lambda)Y_n - X_n\beta\right]$, where $\theta = (\rho, \lambda, \beta',)'$. The moment functions corresponding to the orthognality conditions of X_n and $\epsilon_n = \epsilon_n(\theta_0)$ are $Q_n'\epsilon_n(\theta)$. In addition to the linear moment functions $Q_n'\epsilon_n(\theta)$, Lee (2001b, 2007) suggests the use of the quadratic moment functions $\epsilon_n'(\theta)P_{jn}\epsilon_n(\theta)$ where P_{jn} are $n \times n$ constant matices such that $\mathrm{tr}(P_{jn}) = 0 \ for \ j = 1, ..., m$. With the selected P_{jn} and Q_n, the GMM uses the empirical moments:

$$g_n(\theta) = (Q_n,\ P_{1n}\epsilon_n(\theta), ..., P_{mn}\epsilon_n(\theta))'\,\epsilon_n(\theta). \quad (18)$$

At θ_0, $g_n(\theta_0) = (Q_n,\ P_{1n}\epsilon_n, ..., P_{mn}\epsilon_n)'\ \epsilon_n$ has zero mean because $E(Q_n'\epsilon_n) = Q_n'E(\epsilon_n) = 0$ and $E(\epsilon_n' P_{jn}'\epsilon_n) = \sigma_0^2 tr(P_{jn}) = 0\ for\ j = 1, ..., m$. The variance of the moment functions takes the form:

$$\Omega_n = var(g_n(\theta_0)) = \begin{bmatrix} \sigma_0^2 Q_n' Q_n & \mu_3 Q_n' \omega_{mn} \\ \mu_3 \omega_{mn}' Q_n & (\mu_4 - 3\sigma_0^4)\omega_{mn}'\omega_{mn} + \sigma_0^4 \Delta_m \end{bmatrix}, \tag{19}$$

with $\omega_{mn} = [diag(P_{1n}), ..., diag(P_{mn})]$, $\Delta_m = \left[vec(P_{1n}^{(s)}), ..., vec(P_{mn}^{(s)})\right]$, where$P^{(s)} = P + P'$.

Under some regularity conditions, Lee and Liu (2010) carefully picked the IV matrices to be $Q_{1n}^* = \bar{X}^*{}_n,\ Q_{2n}^* = \bar{G}_{1n}\bar{X}_n\beta_0,\ Q_{3n}^* = l_n,\ Q_{4n}^* = diag(\bar{G}_{1n}^{(t)}),\ and\ Q_{5n}^* = diag(G_{2n}^{(t)})$, and the weighting matrices of the quadratic moments to be $P_{1n}^* = \bar{G}_{1n}^{(t)}, P_{2n}^* = D(\bar{G}_{1n}^{(t)}), P_{3n}^* = D(\bar{G}_{1n}\bar{X}_n\beta_0)^{(t)}, P_{4n}^* = G_{2n}^{(t)}, P_{5n}^* = D(G_{2n}^{(t)})\ and\ P_{l+5,n}^* = D(\bar{X}_{nl}^*)^{(t)},\ for\ l = 1, ..., k^*$. Where $\bar{X}_n = B_n X_n$, $\bar{X}_n^*$ is the submatrix of $\bar{X}_n$with the intercept column deleted, and k^* is the number of columns in $\bar{X}_n^*$. For an $n \times n$ matrix$A$, let $A^{(t)} = A - (1/n)tr(A)I_n$. Let $D(A)$ be a diagnal matrix with diagnal elements being A if A is a vector or diagnal elements of A if A is a square matrix.

With these selection of moment functions, Lee and Liu (2010) showed that

$$\hat{\theta}_b = arg\ min_{\theta\in\Theta} g_n^{'*}(\theta)\Omega_n^{*-1} g_n^*(\theta) \tag{20}$$

is the BGMME with the class of optimal GMMEs derived from $min_{\theta\in\Theta} g_n'(\theta)\Omega_n^{-1}g_n(\theta)$, where $g_n(\theta)$ is given by (18). And it has the asymptotic distribution that $\sqrt{n}(\hat{\theta}_B - \theta_0 \rightarrow N(0, \Sigma_B^{-1})$, where $\Sigma_B = lim_{n\rightarrow\infty}\frac{1}{n}D_n'^*\Omega_n^{*-1}D_n^*$ and

$$D_n^* = E(\frac{\partial}{\partial\theta'}g_n^*(\theta_0)) = -\begin{bmatrix} 0 & Q_n'^*\bar{G}_{1n}\bar{X}_n\beta_0 & Q_n'^*\bar{X}_n \\ \sigma_0^2 tr(P_{1n}^{*(s)}G_{2n}) & \sigma_0^2 tr(P_{1n}^{*(s)}\bar{G}_{1n}) & 0 \\ \vdots & \vdots & \vdots \\ \sigma_0^2 tr(P_{k^*+5,n}^{*(s)}H_n) & \sigma_0^2 tr(P_{k^*+5,n}^{*(s)}\bar{G}_{2n}) & 0 \end{bmatrix}$$

With the initial $\sqrt{n}$ consistent estimators $\beta_0, \sigma_0^2, \mu_3, \mu_4,\ P_{jn}^*$ and $Q*_n$ can be estimated. And the feasible BGMM estimator has the same limiting distribution as the BGMM estimator defined in (20).

Another approach by Kelejian and Prucha (1998) is also examined in this papar in contrast with BGMM proposed in Lee and Liu (2010). Kelejian and Prucha (1998) first estimated $\hat{\beta}$ and $\hat{\lambda}$ by 2SLS. With the initial esimates and the residuals, they then estimated $\hat{\rho}$ using method of moment. With $\hat{\rho}$, they again used 2SLS to estimate $\hat{\beta}$ and $\hat{\lambda}$. Results of this method is also presented in the section of Monte Carlo experiment for a more complete comparison.

4.2 GMM Estimation of SED Model

By setting $\lambda_0 = 0$ in SARA model, we go back spatial error dependence model. Everything follows from SARA model. Let $P^{\dagger}_{1n} = G^{(t)}_{2n}, P^{\dagger}_{2n} = D(G^{(t)}_{2n})$ and $P^{\dagger}_{j+2,n} = D(\bar{X}^{*}_{nj})^{(t)} forj = 1, ..., k^{*}$ be the weighting matrices of the quadratic moments. Let $Q^{\dagger}_{1n} = \bar{X}^{*}_{n}, Q^{\dagger}_{2n} = l_n, Q^{\dagger}_{3n} = diag(G^{(t)}_{2n})$ be the IV matrices. Let $g^{\dagger}_{\rho n}(\rho, \beta) = (Q^{\dagger}_{n}, P^{\dagger}_{1n}\epsilon_{\rho n}(\rho, \beta), ..., P^{\dagger}_{k^{*}+2,n}\epsilon_{\rho n}(\rho, \beta))'\epsilon_{\rho n}(\rho, \beta)$ and $\Omega^{\dagger}_{\rho n} = var(g^{\dagger}_{\rho n}(\rho_0, \beta_0))$, where $Q^{\dagger}_{n} = (Q^{\dagger}_{1n}, Q^{\dagger}_{2n}, Q^{\dagger}_{3n})$. Then, $\hat{\rho}_B$ and $\hat{\beta}_B$ derived from $min_{\rho,\beta}\ g^{\dagger}_{\rho n}(\rho, \beta)'(\Omega^{\dagger}_{\rho n})^{-1} g^{\dagger}_{\rho n}(\rho, \beta)$ is the BGMM estimator.

4.3 GMM Estimation of SLD Model

By setting $\rho_0 = 0$ in SARA model, we obtain spatial lag dependence model. Let $P^{*}_{1n} = G^{(t)}_{1n}, P^{*}_{2n} = D(G^{(t)}_{2n}), P^{*}_{3n} = D(G_{1n}X_n\beta_0)^{(t)}$, and $P^{*}_{j+3,n} = D(X^{*}_{jn})^{(t)} forj = 1, ..., k^{*}$ be the weighting matriced of the quadratic moments. And $Q^{*}_{1n} = X^{*}_{jn}, Q^{*}_{2n} = G_{1n}X_n\beta_0, Q^{*}_{3n} = l_n$ and $Q^{*}_{4n} = diag(G^{(t)}_{n})$ be the IV matrices. Let

$$g^{*}_{\lambda n}(\lambda, \beta) = (Q^{*}_{n}, P^{*}_{1n}\epsilon_{\lambda n}(\lambda, \beta), ..., P^{*}_{k^{*}+3,n}\epsilon_{\lambda n}(\lambda, \beta))'\epsilon_{\lambda n}(\lambda, \beta)$$

and $\Omega^{*}_{\lambda n} = var(g^{*}_{\lambda n}(\lambda_0, \beta_0))$, where $Q^{*}_{n} = (Q^{*}_{1n}, Q^{*}_{2n}, Q^{*}_{3n}, Q^{*}_{4n})$. Then $\hat{\lambda}_B$ and $\hat{\beta}_B$ derived from $min_{\lambda,\beta}\ g^{*}_{\lambda n}(\lambda, \beta)'(\Omega^{*}_{\lambda n})^{-1} g^{*}_{\lambda n}(\lambda, \beta)$ is the BGMM estimator.

5. Monte Carlo results

In the Monte Carlo experiments, the model is specified as

Regresion 1:

$$Y_n = \lambda W_{1n}Y_n + \iota_n\beta_0 + X_{1n}\beta_1 + X_{2n}\beta_2 + u_n, \quad u_n = \rho W_{2n}u_n + \epsilon_n,$$

Regression 2:

$$Y_n = \lambda W_{1n}Y_n + X_{1n}\beta_0 + X_{2n}\beta_1 + X_{3n}\beta_2 + u_n, \quad u_n = \rho W_{2n}u_n + \epsilon_n,$$

The regressors X_{1n}, X_{2n} and X_{3n} are mutually independent vectors of independent standard normal random variables. The error term, $epsilon_{ni}$ are independently generated from the following two distributions: DGP1: $epsilon_{ni}$ are iid standard normal , DGP2: $epsilon_{ni}$ iid standardized log-normal with parameters 0 and 1, whose distribution is both skewed and leptokurtic. The number of repetition is 1000 for each case in the Monte Carlo experiments. The regressors are fixed across replications. In each case, we report the mean and standard deviation (SD) of the empirical distribution of the estimates. We set $\beta_0 = 2, \beta_1 = 1$ and $\beta_2 = -1$ in the data generating process. The sample sizes considerd are $n = 50, 100, 200, 500$. The weight matrices W_{1n} and W_{2n} are set to be the same and are generated by Queen Contiguity.

The first case considered is SLD model where $\rho_0 = 0$ and λ take values from $\{-0.5, -0.250.25, 0.5\}$. The estimators considered are (i) 2SLS estimator with IV set to be $Q_n = (X_n, W_n X_n, W_n^2 X_n)$, (ii) the BGMME presented above, (iii) the Gaussian QMLE, (iv) bias reducted QMLE using the simple method proposed in section 2, (v) second order bias corrected QMLE by Yang (2015). We use initial estimates from 2SLS to implement the BGMM.

Table 1.1 reports the estimation results of SLD model in REG1. First we observe that SD and bias reduce as sample size get larger for all estimators. Looking at the SDs, the 2SLS estimator of λ_0 have the largest SDs than other estimators for all sample size considered. When the disturbances are normally distributed, BGMM estimator has larger SD than QMLE for small sample size $n = 50, 100$. When sample size is $200, 500$, BGMME performs as good as QMLE in terms of SD. When the disturbances are not normally distributed, BGMME improves upon QMLE interms of SD for both λ and β under all sample sizes. BRQMLE and BCQMLE have almost the same SDs as QMLE is both cases when errors are normally distributed and follows log-normal distribution under all sample sizes. Looking at the bias, 2SLS estimator of λ tend to be biased upward while QMLE estimator of λ tend to be biased downward. BGMM estimator has smaller bias than QMLE in both spatial parameter λ and linear parameter β under all sample sizes in both DGP. BRQMLE significantly reduces the bias of QMLE and has almost the same level of bias as BGMM. BCQMLE has the smallest bias among all the estimators under all sample sizes and both DGP. Table 1.2 reports the estimation results of SLD model in REG2. The pattern of relative performance of the estimators maintains but the overall performance of every estimator inproved in terms of both smaller SDs and smaller bias under all sample sizes. Improvment in QMLE is most significant, and is it now performing almost as good as BRQMLE in terms of the bias, where the latter only improved a little after the change of regressor.

The second case we consider is SED model, where $\lambda_0 = 0$ and ρ_0 take values from $\{-0.5, -0.250.25, 0.5\}$. The estimators considered are (i) fesible GLS estimator (ii) BGMME discussed in section 3.2 (iii) the Gaussian QMLE (iv) bias reducted QMLE proposed in section 2 (v) second order bias corrected QMLE by Yang (2015). We use initial estimates from FGLS to implement BGMM.

Table 2.1 reports the estimation results of SED model. First we observe that SD and bias reduce as sample size get larger for all estimators. BGMM estimators gives strange result under all sample sizes. We found that some of the estimates during the Monte Carlo replications exploded, therefore the mean and SD of BGMM exploded. The objective function seems to be unbounded for some replications. Performace of other estimators are as expected. GLS has the largest SD. QMLE is biased downward. BRQMLE again significantly reduced the bias of

QMLE and the SD of BRQMLE is even slightly smaller than that of QMLE under all sample sizes. BCQMLE still gives smallest bias among all the estimators.

Table 2.2 reports the estimation results of SED model in REG2. In this specification, the abnormal performace occurs only when sample size is 50. When BGMM is well-behaved, it has slightly bigger SD and slightly bigger bias than QMLE. Similar to SLD model, performace of QMLE improved significantly after the exclusion of constant term. BRQMLE no longer improves upon QMLE, as it seems not to be sensitive to the change in regressor. The BRQMLE is almost the same as QMLE in terms of SD and bias under all sample sizes.

Lastly,we consider SARA model, where both λ_0 and ρ_0 take values from $\{-0.3, 0.3\}$. The estimators considered are (i) G2sls estimator in Kelejian and Prucha (1998), (ii) BGMME in section 3.1, (iii) he Gaussian QMLE (iv) bias reducted QMLE proposed in section 2 (v) second order bias corrected QMLE by Yang (2015). We use preliminary estimates from G2SLS to implement feasible BGMM.

The estimation results of SARA model in REG 1 are are given in table 3.1. Except for the the case where $n = 500$, BGMM performs abnormally in all other cases. When sample size is 500, BGMME has the same level of bias as QMLE and give smaller SD when error term in not normal. QMLE give downward bias in both estimates of λ_0 and ρ_0. BRQMLE improves upon QMLE in terms of biasness while giving a larger SD. BCQMLE again has the smallest bias while maintaining the SDs at same level with QMLE. In REG2 of SARA model presented in Table 3.2, we see again the abnormal performance of BGMM disapears. BGMM tend to have upward bias in ρ and a small downward bias in λ while QMLE tend to have downward bias in both λ and ρ. BGMME has smaller SD than QMLE when error term in non-normal under sample size 100,200,500. Again, we see that BRQMLE does not significantly improve upon QMLE when constant term is excluded, especially when sample size is small, as QMLE has fairly small bias after the exclusion of constant term. BCQMLE in this case performs only slightly better than the regular QMLE.

6. Conclusions

This paper compares QML method with GMM when used in estimating spatial regression models and proposed a new method to reduce to bias of QML estimator in estimation of spatial regression models. We found that, for SED and SARA model, BGMM estimator can be unstable to the inclusion and exclusion of the constant term as well as specification of the parameters, such that the estimates can be explosive in some cases, especially when sample size is small. QML estimator is also sensitive to the inclusion and exclusion of the constant term. It produces fairly small bias when constant term is excluded. Non-iid regressors always lead to worse estimations, and inclusion of constant term certainly make the regressors non-iid. We think this might be the reason that QMLE performs significantly better when there is no intercept in our cases. The proposed bias reducted QML estimator is stable to the inclusion or exclusion of the constant term and it always improves QMLE when constant term is included in the regressors. Because constant term is always included in the model in practice, we think that this method is valuable for its simplicity and effectiveness.

Appendix A: Monte Carlo Result Table

Appendix B: Settings of Monte Carlo Experiments

Spatial Weight Matrix: We use three different methods for generating the spatial weight matrix W_n: (*i*) `Rook contiguity`, (*ii*) `Queen contiguity`, and (*iii*) `Group Interaction`, with details given in Yang (2015b). The degree of spatial dependence specified by layouts (*i*) and (*ii*) are fixed while in (*iii*) it grows with the increase in sample size. This is attained by allowing for the number of groups, k, for each sample to be directly related to n. We have considered $k = n^{0.5}$ and $k = n^{0.65}$, where k is the number of groups for each n and hence the degree of spatial dependence indicated by the average group size is $m = n/k$. The actual sizes of the groups are generated from a discrete uniform distribution from $.5m$ to $1.5m$.

Regressors: The fixed regressors are generated by `REG1:` $\{x_{1i}, x_{2i}\} \stackrel{iid}{\sim} N(0,1)/\sqrt{2}$ when Rook or Queen contiguity is followed; and according to either `REG1` or `REG2:` $\{x_{1,ir}, x_{2,ir}\} \stackrel{iid}{\sim} (2z_r + z_{ir})/\sqrt{10}$, where, $(z_r, z_{ir}) \stackrel{iid}{\sim} N(0,1)$ when group interaction scheme is followed. The `REG2` scheme gives non-iid regressors where the group means of the regressors' values are different, see Lee (2004). Note that both schemes give a signal-to-noise ratio of 1 when $\beta_1 = \beta_2 = \sigma = 1$.

Error Distribution: To generate $\epsilon_n = \sigma e_n$, three DGPs are considered: `DGP1:` $\{e_{n,i}\}$ are iid standard normal, `DGP2:` $\{e_{n,i}\}$ are iid standardized normal mixture with 10% of values from $N(0,4)$ and the remaining from $N(0,1)$, and `DGP3:` $\{e_{n,i}\}$ iid standardized log-normal with parameters 0 and 1. Thus, the error distribution from `DGP2` is leptokurtic, and that of `DGP3` is both skewed and leptokurtic.

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Table1.1 **Regression Model with Spatial Lag Dependence REG1**

DGP1		n=50				n=100			
		lambda=0.5	beta_0=2.0	beta_1=1.0	beta_2=-1.0	lambda=0.5	beta_0=2.0	beta_1=1.0	beta_2=-1.0
2SLS	mean	0.5181	1.9232	0.9813	-0.9770	0.5027	1.9927	0.9971	-0.9984
	sd	0.1498	0.5764	0.1473	0.1509	0.1243	0.5173	0.0946	0.1027
GMM	mean	0.4907	2.0314	0.9866	-0.9822	0.4884	2.0499	0.9983	-0.9987
	sd	0.1483	0.5811	0.1593	0.1631	0.1003	0.4246	0.1002	0.1070
QMLE	mean	0.4535	2.1722	1.0056	-0.9900	0.4666	2.1400	1.0021	-1.0012
	sd	0.1264	0.4998	0.1438	0.1495	0.0927	0.3934	0.0946	0.1031
BRQMLE	mean	0.4812	2.0662	0.9965	-0.9854	0.4827	2.0744	1.0007	-1.0005
	sd	0.1259	0.4969	0.1443	0.1497	0.0920	0.3897	0.0945	0.1029
BCQMLE	mean	0.4994	1.9966	0.9904	-0.9822	0.4957	2.0211	0.9995	-0.9999
	sd	0.1258	0.4951	0.1446	0.1499	0.0920	0.3891	0.0943	0.1028
		n=200				n=500			
2SLS	mean	0.5106	1.9596	0.9947	-0.9979	0.5049	1.9801	0.9995	-0.9976
	sd	0.1070	0.4541	0.0686	0.0700	0.0669	0.2720	0.0475	0.0426
GMM	mean	0.4943	2.0279	0.9946	-0.9984	0.4955	2.0175	0.9999	-0.9976
	sd	0.0713	0.3088	0.0701	0.0717	0.0445	0.1843	0.0480	0.0428
QMLE	mean	0.4843	2.0697	0.9972	-1.0004	0.4920	2.0315	1.0011	-0.9987
	sd	0.0683	0.2966	0.0686	0.0698	0.0442	0.1834	0.0474	0.0425
BRQMLE	mean	0.4930	2.0331	0.9967	-1.0001	0.4955	2.0176	1.0008	-0.9985
	sd	0.0682	0.2959	0.0686	0.0698	0.0441	0.1832	0.0474	0.0425
BCQMLE	mean	0.4999	2.0043	0.9963	-0.9998	0.4987	2.0048	1.0006	-0.9984
	sd	0.0686	0.2970	0.0686	0.0698	0.0442	0.1833	0.0474	0.0425

		n=50				n=100			
		lambda=0.25	beta_0=2.0	beta_1=1.0	beta_2=-1.0	lambda=0.25	beta_0=2.0	beta_1=1.0	beta_2=-1.0
2SLS	mean	0.2694	1.9449	0.9820	-0.9901	0.2571	1.9871	0.9912	-1.0011
	sd	0.1856	0.4864	0.1499	0.1530	0.1578	0.4476	0.0953	0.1010
GMM	mean	0.2356	2.0315	0.9858	-0.9917	0.2354	2.0453	0.9894	-0.9988
	sd	0.1786	0.4772	0.1604	0.1662	0.1236	0.3598	0.0994	0.1062
QMLE	mean	0.2021	2.1176	0.9999	-0.9982	0.2174	2.0946	0.9945	-1.0027
	sd	0.1501	0.4044	0.1464	0.1515	0.1125	0.3296	0.0953	0.1013
BR_QMLE	mean	0.2312	2.0432	0.9938	-0.9959	0.2345	2.0480	0.9939	-1.0028
	sd	0.1500	0.4024	0.1467	0.1518	0.1122	0.3278	0.0952	0.1012
BC_QMLE	mean	0.2483	1.9995	0.9901	-0.9944	0.2468	2.0145	0.9935	-1.0028
	sd	0.1519	0.4061	0.1470	0.1519	0.1130	0.3292	0.0951	0.1011
		n=200				n=500			
2SLS	mean	0.2608	1.9689	0.9988	-0.9952	0.2541	1.9883	1.0019	-0.9984
	sd	0.1230	0.3475	0.0683	0.0679	0.0828	0.2283	0.0478	0.0442
GMM	mean	0.2455	2.0117	0.9991	-0.9937	0.2472	2.0068	1.0018	-0.9984
	sd	0.0872	0.2541	0.0702	0.0699	0.0569	0.1617	0.0484	0.0447
QMLE	mean	0.2361	2.0384	1.0004	-0.9963	0.2447	2.0135	1.0027	-0.9990
	sd	0.0846	0.2468	0.0683	0.0677	0.0559	0.1596	0.0477	0.0442
BR_QMLE	mean	0.2448	2.0141	1.0002	-0.9963	0.2483	2.0039	1.0026	-0.9991
	sd	0.0846	0.2467	0.0683	0.0677	0.0559	0.1595	0.0477	0.0442
BC_QMLE	mean	0.2505	1.9982	1.0000	-0.9964	0.2511	1.9962	1.0025	-0.9991
	sd	0.0851	0.2478	0.0683	0.0677	0.0560	0.1595	0.0477	0.0442

DGP 2		n=50				n=100			
		lambda=0.5	beta_0=2.0	beta_1=1.0	beta_2=-1.0	lambda=0.5	beta_0=2.0	beta_1=1.0	beta_2=-1.0
2SLS	mean	0.5141	1.9367	0.9847	-0.9970	0.4977	2.0054	0.9998	-0.9976
	sd	0.1953	0.8190	0.1423	0.1593	0.1348	0.5930	0.0961	0.1082
GMM	mean	0.5061	2.0297	0.9878	-1.0079	0.4936	2.0445	1.0000	-0.9976
	sd	0.1012	0.4389	0.0792	0.0894	0.0634	0.2915	0.0519	0.0563
QMLE	mean	0.4665	2.1404	0.9899	-1.0017	0.4701	2.1252	1.0021	-1.0040
	sd	0.1223	0.5353	0.1412	0.1587	0.0895	0.4052	0.0949	0.1081
BR_QMLE	mean	0.4908	2.0385	0.9893	-1.0008	0.4837	2.0663	1.0016	-1.0023
	sd	0.1222	0.5242	0.1414	0.1588	0.0887	0.3980	0.0949	0.1080
BC_QMLE	mean	0.5013	1.9947	0.9891	-1.0003	0.4933	2.0248	1.0013	-1.0010
	sd	0.1230	0.5237	0.1415	0.1589	0.0888	0.3958	0.0949	0.1079
		n=200				n=500			
2SLS	mean	0.5094	1.9606	0.9964	-0.9977	0.5061	1.9772	0.9998	-0.9994
	sd	0.1074	0.4595	0.0652	0.0687	0.0675	0.2756	0.0480	0.0450
GMM	mean	0.4984	2.0141	0.9996	-0.9997	5.0001	2.0036	0.9987	-1.0014
	sd	0.0517	0.2321	0.0369	0.0368	0.0333	0.1424	0.0294	0.0270
QMLE	mean	0.4844	2.0660	0.9989	-1.0001	0.4968	2.0147	1.0011	-1.0005
	sd	0.0672	0.2953	0.0648	0.0684	0.0429	0.1797	0.0479	0.0447
BR_QMLE	mean	0.4927	2.0312	0.9984	-0.9997	0.5002	2.0012	1.0008	-1.0003
	sd	0.0671	0.2931	0.0648	0.0684	0.0428	0.1790	0.0479	0.0447
BC_QMLE	mean	0.4980	2.0090	0.9981	-0.9995	0.5030	1.9898	1.0006	-1.0002
	sd	0.0673	0.2931	0.0648	0.0683	0.0428	0.1787	0.0479	0.0447

		n=50				n=100			
		lambda=0.25	beta_0=2.0	beta_1=1.0	beta_2=-1.0	lambda=0.25	beta_0=2.0	beta_1=1.0	beta_2=-1.0
2SLS	mean	0.2594	1.9701	0.9903	-0.9965	0.2700	1.9370	0.9988	-0.9898
	sd	0.2056	0.5964	0.1315	0.1576	0.1542	0.4569	0.0984	0.1094
GMM	mean	0.2453	2.0588	0.9909	-0.9988	0.2476	2.0218	0.9993	-1.0001
	sd	0.1078	0.3386	0.0709	0.0810	0.0766	0.2534	0.0518	0.0578
QMLE	mean	0.2144	2.0975	0.9915	-0.9990	0.2272	2.0607	1.0002	-0.9947
	sd	0.1345	0.4121	0.1294	0.1546	0.1059	0.3294	0.0978	0.1090
BR_QMLE	mean	0.2367	2.0354	0.9925	-0.9991	0.2414	2.0198	1.0002	-0.9939
	sd	0.1348	0.4035	0.1294	0.1550	0.1057	0.3249	0.0979	0.1089
BC_QMLE	mean	0.2454	2.0112	0.9928	-0.9991	0.2505	1.9937	1.0002	-0.9933
	sd	0.1367	0.4061	0.1295	0.1552	0.1062	0.3243	0.0979	0.1089

		n=200				n=500			
2SLS	mean	0.2627	1.9622	0.9984	-0.9993	0.2581	1.9776	0.9974	-0.9971
	sd	0.1214	0.3453	0.0705	0.0691	0.0814	0.2215	0.0506	0.0459
GMM	mean	0.2502	2.0042	0.9986	-0.9985	0.2499	2.0017	0.9982	-0.9986
	sd	0.0580	0.1758	0.0385	0.0389	0.0384	0.1111	0.028	0.0264
QMLE	mean	0.2385	2.0304	1.0000	-1.0004	0.245	2.0122	0.9984	-0.9977
	sd	0.0776	0.2295	0.0713	0.0686	0.0506	0.1403	0.0504	0.0458
BR_QMLE	mean	0.2467	2.0075	0.9998	-1.0004	0.2486	2.003	0.9983	-0.9977
	sd	0.0776	0.2281	0.0712	0.0686	0.0506	0.14	0.0504	0.0458
BC_QMLE	mean	0.2507	1.9962	0.9997	-1.0005	0.2512	1.9961	0.9982	-0.9977
	sd	0.0781	0.2288	0.0712	0.0686	0.0507	0.14	0.0504	0.0458

Table 1.2 **Regression Model with Spatial Lag Dependence REG2**

DGP1		n=50				n=100			
		lambda=0.5	beta0=2	beta1=1	beta2=-1	lambda=0.5	beta0=2	beta1=1	beta2=-1
2SLS	mean	0.51004	1.98123	0.99495	-1.00301	0.5080	1.9986	0.9927	-0.9983
	sd	0.06756	0.14550	0.13504	0.13415	0.0803	0.1013	0.0838	0.0939
GMM	mean	0.50677	1.98009	0.99513	-1.00275	0.5086	1.9961	0.9906	-0.9957
	sd	0.07474	0.15894	0.14885	0.14541	0.0750	0.1044	0.0892	0.1006
QMLE	mean	0.49228	2.00036	0.99634	-0.99667	0.4920	2.0007	0.9937	-0.9984
	sd	0.06549	0.14311	0.13570	0.13409	0.0679	0.1011	0.0840	0.0940
BR_QMLE	mean	0.49498	1.99755	0.99613	-0.99761	0.4911	2.0008	0.9937	-0.9984
	sd	0.06537	0.14322	0.13560	0.13409	0.0679	0.1011	0.0840	0.0940
BC_QMLE	mean	0.50267	1.98947	0.99551	-1.00031	0.4979	2.0002	0.9935	-0.9984
	sd	0.06537	0.14357	0.13534	0.13407	0.0683	0.1011	0.0840	0.0940

		n=200			
2SLS	mean	0.5063	1.9990	0.9968	-1.0011
	sd	0.0534	0.0668	0.0745	0.0715
GMM	mean	0.5059	1.9970	0.9962	-0.9998
	sd	0.0491	0.0695	0.0769	0.0737
QMLE	mean	0.4974	2.0009	0.9987	-1.0029
	sd	0.0465	0.0668	0.0744	0.0712
BR_QMLE	mean	0.4972	2.0010	0.9987	-1.0029
	sd	0.0465	0.0668	0.0744	0.0712
BC_QMLE	mean	0.5007	2.0003	0.9981	-1.0023
	sd	0.0465	0.0668	0.0743	0.0712

		n=50				n=100			
		lambda=0.25	beta0=2	beta1=1	beta2=-1	lambda=0.25	beta0=2	beta1=1	beta2=-1
2SLS	mean	0.2597	1.9872	1.0091	-1.0033	0.2592	2.0017	0.9979	-0.9997
	sd	0.0914	0.1367	0.1339	0.1359	0.0939	0.0967	0.0871	0.0938
GMM	mean	0.2529	1.9907	1.0086	-1.0015	0.2588	1.9989	0.9987	-0.9993
	sd	0.0975	0.1515	0.1471	0.1466	0.0887	0.1023	0.0918	0.0984
QMLE	mean	0.2380	2.0024	1.0102	-0.9973	0.2428	2.0024	0.9980	-0.9993
	sd	0.0872	0.1349	0.1343	0.1354	0.0825	0.0967	0.0871	0.0936
BR_QMLE	mean	0.2415	2.0001	1.0101	-0.9983	0.2417	2.0024	0.9980	-0.9992
	sd	0.0871	0.1350	0.1342	0.1354	0.0825	0.0967	0.0871	0.0936
BC_QMLE	mean	0.2494	1.9948	1.0096	-1.0004	0.2473	2.0024	0.9981	-0.9994
	sd	0.0876	0.1354	0.1341	0.1354	0.0829	0.0967	0.0871	0.0936
		n=200							
2SLS	mean	0.2588	1.9987	0.9984	-0.9963				
	sd	0.0623	0.0662	0.0754	0.0727				
GMM	mean	0.2575	1.9969	0.9974	-0.9956				
	sd	0.0585	0.0687	0.0773	0.0766				
QMLE	mean	0.2496	1.9997	0.9996	-0.9975				
	sd	0.0559	0.0663	0.0751	0.0725				
BR_QMLE	mean	0.2494	1.9997	0.9996	-0.9975				
	sd	0.0559	0.0663	0.0751	0.0725				
BC_QMLE	mean	0.2526	1.9995	0.9993	-0.9972				
	sd	0.0561	0.0663	0.0752	0.0725				

DGP2		n=50				100			
		lambda=0.5	beta0=2	beta1=1	beta2=-1	lambda=0.5	beta0=2	beta1=1	beta2=-1
2SLS	mean	0.5254	1.9844	0.9842	-0.9857	0.5085	1.9893	0.9975	-0.9956
	sd	0.1161	0.1250	0.1758	0.1673	0.0797	0.1168	0.1061	0.0985
GMM	mean	0.5033	1.9831	1.0053	-0.9907	0.5040	1.9924	0.9963	-0.9968
	sd	0.0850	0.0793	0.1210	0.1095	0.0425	0.0608	0.0528	0.0537
QMLE	mean	0.4832	2.0039	0.9989	-0.9984	0.4947	1.9955	0.9979	-0.9996
	sd	0.0973	0.1197	0.1828	0.1636	0.0646	0.1142	0.1069	0.0985
BR_QMLE	mean	0.4875	2.0020	0.9977	-0.9972	0.4948	1.9955	0.9979	-0.9996
	sd	0.0970	0.1198	0.1821	0.1636	0.0646	0.1142	0.1069	0.0984
BC_QMLE	mean	0.4987	1.9971	0.9941	-0.9937	0.5046	1.9918	0.9980	-0.9969
	sd	0.0978	0.1200	0.1808	0.1641	0.0639	0.1142	0.1069	0.0980
		n=200							
2SLS	mean	0.5025	1.9985	0.9953	-0.9980				
	sd	0.0422	0.0672	0.0746	0.0672				
GMM	mean	0.5024	1.9971	0.9973	-0.9979				
	sd	0.0242	0.0383	0.0412	0.0407				
QMLE	mean	0.4956	1.9999	0.9987	-0.9997				
	sd	0.0378	0.0670	0.0743	0.0672				
BR_QMLE	mean	0.4959	1.9998	0.9985	-0.9996				
	sd	0.0378	0.0670	0.0743	0.0672				
BC_QMLE	mean	0.4988	1.9994	0.9972	-0.9990				
	sd	0.0379	0.0671	0.0743	0.0672				

		n=50				100			
		lambda=0.25	beta0=2	beta1=1	beta2=-1	lambda=0.25	beta0=2	beta1=1	beta2=-1
2SLS	mean	0.2780	1.9830	0.9942	-0.9828	0.2572	1.9957	0.9936	-0.9952
	sd	0.1468	0.1355	0.2001	0.1769	0.0923	0.1146	0.0980	0.1015
GMM	mean	0.2496	1.9797	1.0056	-0.9981	0.2525	1.9967	0.9960	-0.9995
	sd	0.1027	0.0851	0.1300	0.1084	0.0525	0.0604	0.0555	0.0583
QMLE	mean	0.2336	1.9983	1.0062	-0.9929	0.2424	1.9997	0.9932	-0.9985
	sd	0.1172	0.1282	0.2043	0.1755	0.0773	0.1129	0.0979	0.1015
BR_QMLE	mean	0.2388	1.9966	1.0051	-0.9919	0.2424	1.9997	0.9932	-0.9985
	sd	0.1167	0.1283	0.2036	0.1754	0.0773	0.1129	0.0979	0.1015
BC_QMLE	mean	0.2496	1.9932	1.0026	-0.9897	0.2517	1.9976	0.9937	-0.9966
	sd	0.1179	0.1287	0.2024	0.1754	0.0768	0.1131	0.0980	0.1015

		n=200			
2SLS	mean	0.2554	2.0011	1.0013	-0.9985
	sd	0.0555	0.0662	0.0743	0.0700
GMM	mean	0.2519	2.0014	1.0013	-0.9994
	sd	0.0312	0.0381	0.0418	0.0376
QMLE	mean	0.2480	2.0017	1.0037	-0.9997
	sd	0.0484	0.0661	0.0742	0.0699
BR_QMLE	mean	0.2484	2.0016	1.0036	-0.9996
	sd	0.0484	0.0661	0.0742	0.0699
BC_QMLE	mean	0.2515	2.0015	1.0027	-0.9992
	sd	0.0486	0.0661	0.0742	0.0699

Table 2.1 Regression Model with Spatial Error Dependence REG1

DGP1		n=50				n=100			
		rho=0.5	beta0=2	beta1=1	beta2=-1	**rho=0.5**	beta0=2	beta1=1	beta2=-1
GLS	mean	0.3823	2.0084	1.0055	-0.9958	0.4270	2.0062	1.0041	-0.9968
	sd	0.2330	0.3555	0.1287	0.1705	0.1488	0.2142	0.1004	0.1079
GMM	mean	0.103984465	-4.107E+14	-9.79E+14	4.424E+14	0.301265392	-9.4107E+13	-2.1124E+14	1.4462E+14
	sd	0.577312593	1.16E+16	2.70E+16	1.25E+16	0.35099118	2.64E+15	6.23E+15	4.38E+15
QMLE	mean	0.3887	2.0074	1.0055	-0.9962	0.4360	2.0052	1.0038	-0.9969
	sd	0.2187	0.2860	0.1283	0.1697	0.1464	0.2038	0.0999	0.1081
BR_QMLE	mean	0.4380	2.0069	1.0055	-0.9962	0.4646	2.0053	1.0036	-0.9970
	sd	0.2159	0.2862	0.1280	0.1689	0.1435	0.2039	0.0997	0.1080
BC_QMLE	mean	0.4860	2.0064	1.0055	-0.9961	0.4903	2.0053	1.0035	-0.9970
	sd	0.2170	0.2870	0.1279	0.1686	0.1437	0.2041	0.0997	0.1081
		n=200				n=500			
GLS	mean	0.4644	1.9964	1.0025	-1.0041	0.4885	2.0032	0.9996	-1.0012
	sd	0.1005	0.1459	0.0735	0.0725	0.0562	0.0900	0.0435	0.0444
GMM	mean	0.079041693	-3.718E+13	-6.874E+13	7.618E+13	0.176075218	-5.8733E+10	-1.322E+12	1.2723E+12
	sd	0.438284518	1.11E+15	2.30E+15	2.48E+15	0.298796885	1.4247E+13	3.4609E+13	3.5703E+13
QMLE	mean	0.4669	1.9965	1.0024	-1.0041	0.4897	2.0040	0.9995	-1.0013
	sd	0.0986	0.1397	0.0735	0.0725	0.0557	0.0886	0.0434	0.0442
BR_QMLE	mean	0.4799	1.9965	1.0024	-1.0041	0.4946	2.0040	0.9995	-1.0013
	sd	0.0979	0.1397	0.0734	0.0724	0.0556	0.0886	0.0434	0.0442
BC_QMLE	mean	0.4927	1.9965	1.0023	-1.0041	0.4997	2.0040	0.9995	-1.0013
	sd	0.0982	0.1398	0.0734	0.0724	0.0556	0.0886	0.0434	0.0442

		n=50				n=100			
		lambda=0.25	beta0=2	beta1=1	beta2=-1	**lambda=0.25**	beta0=2	beta1=1	beta2=-1
GLS	mean	0.1357	2.0027	0.9938	-0.9901	0.1890	2.0018	0.9995	-0.9991
	sd	0.2584	0.1913	0.1303	0.1696	0.1764	0.1367	0.1014	0.1049
GMM	mean	0.170443396	-7.495E+12	-1.403E+13	4.803E+12	0.184904024	-3.8229E+13	-7.7695E+13	5.4111E+13
	sd	0.490206319	1.23E+14	2.503E+14	1.135E+14	0.376119155	7.3697E+14	1.62E+15	1.21E+15
QMLE	mean	0.1502	2.0028	0.9942	-0.9904	0.1994	2.0021	0.9994	-0.9992
	sd	0.2466	0.1869	0.1299	0.1694	0.1747	0.1354	0.1012	0.1047
BR_QMLE	mean	0.2027	2.0026	0.9943	-0.9900	0.2331	2.0021	0.9993	-0.9991
	sd	0.2426	0.1870	0.1295	0.1691	0.1709	0.1354	0.1012	0.1047
BC_QMLE	mean	0.2485	2.0024	0.9943	-0.9896	0.2574	2.0020	0.9993	-0.9991
	sd	0.2493	0.1873	0.1293	0.1693	0.1735	0.1354	0.1013	0.1047
		n=200				n=500			
GLS	mean	0.2221	2.0004	1.0018	-1.0003	0.2345	1.9985	1.0012	-0.9996
	sd	0.1159	0.0917	0.0743	0.0716	0.0714	0.0592	0.0455	0.0427
GMM	mean	0.078993544	-4.544E+14	-1.84E+15	1.99E+15	0.179792891	-3.7209E+13	-2.3352E+14	2.1751E+14
	sd	0.464724207	6.94E+15	2.94E+16	3.17E+16	0.353560038	1.13E+15	7.13E+15	6.58E+15
QMLE	mean	0.2263	2.0001	1.0018	-1.0004	0.2362	1.9984	1.0012	-0.9996
	sd	0.1144	0.0909	0.0743	0.0716	0.0711	0.0590	0.0454	0.0426
BR_QMLE	mean	0.2410	2.0001	1.0018	-1.0004	0.2419	1.9984	1.0012	-0.9996
	sd	0.1135	0.0910	0.0743	0.0716	0.0709	0.0590	0.0454	0.0426
BC_QMLE	mean	0.2529	2.0001	1.0018	-1.0004	0.2466	1.9984	1.0012	-0.9996
	sd	0.1142	0.0910	0.0743	0.0715	0.0710	0.0590	0.0454	0.0426

DGP2		n=50				n=100			
		rho=0.5	beta0=2	beta1=1	beta2=-1	**rho=0.5**	beta0=2	beta1=1	beta2=-1
GLS	mean	0.4093	2.0017	0.9945	-0.9988	0.4370	1.9978	0.9997	-1.0051
	sd	0.1880	0.3110	0.1285	0.1638	0.1313	0.2112	0.1064	0.1091
GMM	mean	0.241184283	1.699E+13	-9.447E+13	1.154E+14	0.425379467	8.1614E+10	-1.0271E+11	-7480820504
	sd	0.502515748	1.09E+15	1.77E+15	3.38E+15	0.269367538	1.6419E+12	2.4234E+12	1.1711E+12
QMLE	mean	0.4140	2.0057	0.9940	-0.9986	0.4465	2.0004	0.9995	-1.0046
	sd	0.1741	0.2897	0.1281	0.1631	0.1257	0.2029	0.1059	0.1086
BR_QMLE	mean	0.4632	2.0063	0.9939	-0.9985	0.4750	2.0005	0.9995	-1.0044
	sd	0.1713	0.2904	0.1281	0.1628	0.1229	0.2031	0.1058	0.1084
BC_QMLE	mean	0.5045	2.0062	0.9940	-0.9986	0.4975	2.0003	0.9995	-1.0043
	sd	0.1722	0.2909	0.1283	0.1628	0.1235	0.2031	0.1058	0.1083
		n=200				n=500			
GLS	mean	0.4706	1.9965	1.0024	-1.0002	0.4879	2.0027	0.9976	-1.0004
	sd	0.0887	0.1445	0.0714	0.0681	0.0550	0.0933	0.0449	0.0421
GMM	mean	0.399240243	-1.855E+12	-1E+13	9.717E+12	0.485726254	-132857.311	17017.6017	-20496.9479
	sd	0.268312245	5.963E+13	3.138E+14	3.082E+14	0.077134113	3873859.95	455066.388	815038.656
QMLE	mean	0.4732	1.9968	1.0025	-1.0003	0.4887	2.0033	0.9976	-1.0004
	sd	0.0862	0.1365	0.0713	0.0680	0.0537	0.0922	0.0448	0.0420
BR_QMLE	mean	0.4862	1.9968	1.0025	-1.0003	0.4937	2.0033	0.9976	-1.0004
	sd	0.0855	0.1366	0.0713	0.0680	0.0535	0.0922	0.0448	0.0420
BC_QMLE	mean	0.4976	1.9968	1.0026	-1.0003	0.4985	2.0033	0.9976	-1.0004
	sd	0.0857	0.1366	0.0713	0.0680	0.0536	0.0922	0.0448	0.0420

		n=50				n=100			
		lambda=0.25	beta0=2	beta1=1	beta2=-1	lambda=0.25	beta0=2	beta1=1	beta2=-1
GLS	mean	0.1467	-875.8152	1.0032	-0.9976	0.1754	1.9964	0.9995	-1.0002
	sd	0.2286	27744.9241	0.1263	0.1728	0.1551	0.1339	0.1097	0.0990
GMM	mean	0.179511735	-1.79E+13	-5.045E+13	1.143E+13	0.212293002	-2.7183E+10	-5.3483E+10	5.9635E+10
	sd	0.389078	3.622E+14	9.481E+14	2.426E+14	0.20518617	9.0093E+11	1.6758E+12	1.8536E+12
QMLE	mean	0.1586	1.9966	1.0027	-0.9978	0.1862	1.9969	0.9995	-1.0004
	sd	0.2200	0.1891	0.1260	0.1723	0.1512	0.1329	0.1100	0.0986
BR_QMLE	mean	0.2108	1.9972	1.0025	-0.9977	0.2203	1.9971	0.9994	-1.0005
	sd	0.2160	0.1898	0.1257	0.1719	0.1478	0.1331	0.1100	0.0984
BC_QMLE	mean	0.2499	1.9974	1.0027	-0.9977	0.2412	1.9971	0.9993	-1.0005
	sd	0.2210	0.1905	0.1258	0.1719	0.1501	0.1332	0.1101	0.0984

		n=200				n=500			
GLS	mean	0.2235	1.9983	0.9998	-1.0008	0.2377	2.0007	1.0012	-0.9985
	sd	0.1089	0.0918	0.0719	0.0680	0.0709	0.0599	0.0453	0.0432
GMM	mean	0.232997904	1.51E+10	-1.049E+10	1.286E+10	0.242072501	2.00308744	1.0014416	-0.99979688
	sd	0.136833557	3.379E+11	2.483E+11	2.964E+11	0.070192151	0.06007642	0.03006871	0.02884611
QMLE	mean	0.2277	1.9987	0.9996	-1.0008	0.2394	2.0010	1.0013	-0.9986
	sd	0.1074	0.0921	0.0716	0.0679	0.0696	0.0597	0.0453	0.0431
BR_QMLE	mean	0.2424	1.9987	0.9997	-1.0008	0.2450	2.0010	1.0013	-0.9986
	sd	0.1065	0.0920	0.0716	0.0679	0.0694	0.0597	0.0453	0.0431
BC_QMLE	mean	0.2531	1.9987	0.9997	-1.0007	0.2495	2.0010	1.0013	-0.9986
	sd	0.1073	0.0920	0.0717	0.0679	0.0695	0.0597	0.0452	0.0431

Table 2.2 **Regression Model with Spatial Error Dependence REG2**

DGP1		n=50				n=100			
		rho=0.5	beta0=2	beta1=1	beta2=-1	rho=0.5	beta0=2	beta1=1	beta2=-1
GLS	mean	0.4335	1.9989	1.0063	-1.0028	0.4722	2.0001	1.0015	-0.9969
	sd	0.1950	0.1369	0.1321	0.1365	0.1280	0.0982	0.0957	0.1136
GMM	mean	0.5082	-33.1141	-13.8978	28.5464	0.5033	1.9922	0.9998	-0.9975
	sd	0.2224	1108.8966	470.9113	934.1790	0.1417	0.3366	0.1026	0.1188
QMLE	mean	0.4635	1.9988	1.0062	-1.0029	0.4855	2.0001	1.0014	-0.9971
	sd	0.1960	0.1369	0.1322	0.1358	0.1264	0.0981	0.0955	0.1133
BR_QMLE	mean	0.4464	1.9987	1.0063	-1.0029	0.4773	2.0000	1.0014	-0.9971
	sd	0.1937	0.1370	0.1322	0.1359	0.1258	0.0981	0.0955	0.1133
BC_QMLE	mean	0.4987	1.9990	1.0062	-1.0029	0.5034	2.0001	1.0014	-0.9970
	sd	0.1940	0.1367	0.1322	0.1356	0.1260	0.0981	0.0954	0.1133
		n=200				n=500			
GLS	mean	0.4875	2.0007	1.0007	-1.0007	0.4924	1.9983	1.0025	-0.9999
	sd	0.0872	0.0648	0.0644	0.0610	0.0563	0.0462	0.0460	0.0388
GMM	mean	0.4758	1.9846	1.0025	-1.0001	0.4925	2.0051	1.0002	-1.0006
	sd	0.1039	0.3027	0.0682	0.0625	0.0749	0.3265	0.0463	0.0403
QMLE	mean	0.4942	2.0008	1.0007	-1.0007	0.4950	1.9984	1.0025	-0.9998
	sd	0.0865	0.0645	0.0644	0.0608	0.0558	0.0462	0.0458	0.0386
BR_QMLE	mean	0.4901	2.0009	1.0007	-1.0006	0.4931	1.9984	1.0025	-0.9998
	sd	0.0864	0.0645	0.0644	0.0608	0.0557	0.0462	0.0458	0.0386
BC_QMLE	mean	0.5029	2.0008	1.0007	-1.0007	0.4983	1.9984	1.0025	-0.9998
	sd	0.0864	0.0645	0.0644	0.0607	0.0558	0.0462	0.0458	0.0386

		n=50				n=100			
		lambda=0.25	beta0=2	beta1=1	beta2=-1	lambda=0.25	beta0=2	beta1=1	beta2=-1
GLS	mean	0.19142	1.99593	1.00135	-0.99859	0.21150	1.99533	0.99985	-1.00582
	sd	0.23272	0.13973	0.13749	0.13907	0.16986	0.10315	0.09583	0.11775
GMM	mean	0.26932	1.94895	0.99319	-1.01327	0.23340	2.00826	1.00116	-1.00123
	sd	0.26440	0.33063	0.15145	0.18113	0.18602	0.36611	0.10564	0.12495
QMLE	mean	0.20672	1.99590	1.00092	-0.99862	0.22184	1.99528	0.99994	-1.00567
	sd	0.24365	0.14001	0.13739	0.13890	0.17258	0.10338	0.09601	0.11743
BR_QMLE	mean	0.19485	1.99595	1.00096	-0.99855	0.21677	1.99529	0.99993	-1.00569
	sd	0.23400	0.13993	0.13731	0.13882	0.16908	0.10333	0.09599	0.11743
BC_QMLE	mean	0.24476	1.99600	1.00102	-0.99879	0.24083	1.99519	0.99996	-1.00559
	sd	0.24094	0.13976	0.13716	0.13875	0.17167	0.10336	0.09596	0.11739
		n=200				n=500			
GLS	mean	0.23911	1.99745	1.00007	-1.00265	0.24498	2.00148	1.00100	-1.00147
	sd	0.11089	0.06641	0.06548	0.06255	0.07232	0.04592	0.04676	0.04202
GMM	mean	0.24880	1.98899	0.99933	-1.00590	0.24921	1.98624	0.99967	-1.00293
	sd	0.11729	0.29834	0.06909	0.06474	0.08102	0.32210	0.04718	0.04309
QMLE	mean	0.24430	1.99748	0.99996	-1.00264	0.24721	2.00149	1.00103	-1.00147
	sd	0.11232	0.06642	0.06539	0.06254	0.07254	0.04592	0.04674	0.04202
BR_QMLE	mean	0.24145	1.99748	0.99996	-1.00264	0.24581	2.00150	1.00103	-1.00147
	sd	0.11127	0.06642	0.06538	0.06255	0.07231	0.04592	0.04674	0.04202
BC_QMLE	mean	0.25351	1.99748	0.99995	-1.00264	0.25053	2.00149	1.00104	-1.00146
	sd	0.11192	0.06642	0.06538	0.06253	0.07249	0.04592	0.04674	0.04203

DGP2		n=50				n=100			
		rho=0.5	beta0=2	beta1=1	beta2=-1	rho=0.5	beta0=2	beta1=1	beta2=-1
GLS	mean	0.4567	2.0011	0.9950	-0.9957	0.4736	2.0040	0.9979	-1.0042
	sd	0.1773	0.1429	0.1406	0.1446	0.1239	0.1002	0.0919	0.1156
GMM	mean	0.513011867	-5.9974E+10	-1.953E+11	2.2833E+11	0.4537	2.0519	1.0014	-0.9994
	sd	0.289346262	1.13637E+12	3.7491E+12	4.4533E+12	0.1649	0.6003	0.0816	0.0922
QMLE	mean	0.4876	2.0007	0.9949	-0.9962	0.4882	2.0043	0.9981	-1.0045
	sd	0.1777	0.1408	0.1408	0.1420	0.1207	0.0990	0.0914	0.1147
BR_QMLE	mean	0.4701	2.0006	0.9948	-0.9961	0.4800	2.0043	0.9981	-1.0045
	sd	0.1772	0.1410	0.1408	0.1421	0.1204	0.0990	0.0915	0.1147
BC_QMLE	mean	0.5169	2.0004	0.9948	-0.9967	0.5002	2.0044	0.9981	-1.0045
	sd	0.1810	0.1411	0.1407	0.1409	0.1215	0.0991	0.0914	0.1146
		n=200				n=500			
GLS	mean	0.4899	1.9997	0.9992	-1.0001	0.4961	1.9978	1.0022	-0.9983
	sd	0.0855	0.0639	0.0664	0.0611	0.0540	0.0442	0.0445	0.0410
GMM	mean	0.4905	1.9688	0.9969	-0.9982	0.4854	2.0134	1.0025	-0.9868
	sd	0.1081	0.3490	0.0446	0.0399	0.0995	0.5396	0.0281	0.0300
QMLE	mean	0.4969	1.9998	0.9992	-0.9999	0.4986	1.9977	1.0021	-0.9984
	sd	0.0841	0.0635	0.0662	0.0610	0.0530	0.0441	0.0445	0.0410
BR_QMLE	mean	0.4927	1.9998	0.9992	-0.9999	0.4968	1.9977	1.0021	-0.9984
	sd	0.0840	0.0635	0.0662	0.0610	0.0530	0.0441	0.0445	0.0410
BC_QMLE	mean	0.5028	1.9998	0.9992	-1.0000	0.5012	1.9977	1.0021	-0.9984
	sd	0.0844	0.0636	0.0662	0.0609	0.0532	0.0441	0.0445	0.0410

		n=50				n=100			
		lambda=0.25	beta0=2	beta1=1	beta2=-1	lambda=0.25	beta0=2	beta1=1	beta2=-1
GLS	mean	0.2125	2.0026	1.0016	-0.9951	0.2276	2.0027	1.0033	-0.9956
	sd	0.2176	0.1489	0.1503	0.1443	0.1478	0.1063	0.0908	0.1226
GMM	mean	0.267511565	-2979564793	-2.46E+09	-156106021	0.2112	2.0055	0.9988	-1.0010
	sd	0.284783999	87074714212	7.9018E+10	5160487882	0.1863	0.6658	0.0840	0.0982
QMLE	mean	0.2340	2.0024	1.0017	-0.9950	0.2379	2.0028	1.0034	-0.9958
	sd	0.2266	0.1487	0.1505	0.1442	0.1486	0.1059	0.0910	0.1222
BR_QMLE	mean	0.2211	2.0023	1.0016	-0.9950	0.2324	2.0028	1.0033	-0.9958
	sd	0.2185	0.1485	0.1505	0.1441	0.1459	0.1059	0.0910	0.1221
BC_QMLE	mean	0.2585	2.0023	1.0018	-0.9950	0.2486	2.0028	1.0034	-0.9958
	sd	0.2292	0.1488	0.1508	0.1444	0.1489	0.1060	0.0910	0.1222

		n=200				n=500			
GLS	mean	0.2356	2.0012	1.0012	-1.0002	0.2419	1.9992	0.9983	-0.9989
	sd	0.1029	0.0660	0.0650	0.0629	0.0702	0.0480	0.0459	0.0415
GMM	mean	0.2393	1.9743	0.9979	-0.9985	0.2403	2.0063	1.0057	-0.9811
	sd	0.1168	0.3120	0.0416	0.0385	0.0904	0.4674	0.0295	0.0287
QMLE	mean	0.2413	2.0011	1.0011	-1.0002	0.2449	1.9992	0.9983	-0.9989
	sd	0.1026	0.0660	0.0648	0.0628	0.0698	0.0479	0.0458	0.0414
BR_QMLE	mean	0.2384	2.0011	1.0011	-1.0002	0.2435	1.9992	0.9983	-0.9989
	sd	0.1017	0.0660	0.0648	0.0628	0.0695	0.0479	0.0458	0.0414
BC_QMLE	mean	0.2472	2.0012	1.0011	-1.0002	0.2474	1.9992	0.9983	-0.9989
	sd	0.1025	0.0659	0.0648	0.0628	0.0698	0.0479	0.0458	0.0414

Table 3.1 SARAR Model RGE1

DGP1		n=50					n=100				
		lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1	lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1
G2SLS	mean	0.3460	0.0572	1.8850	0.9871	-0.9762	0.3260	0.1693	1.9378	1.0004	-0.9924
	sd	0.2148	0.3479	0.5763	0.1654	0.1424	0.1910	0.2541	0.5534	0.1131	0.1143
GMM	mean	-0.1761	0.7266	114.6236	0.8776	-0.8680	0.0548	0.5159	102.0239	0.9743	-0.8984
	sd	0.5104	0.3777	1110.2722	0.2084	0.2893	0.3853	0.4048	649.9452	0.1319	0.3505
QMLE	mean	0.2850	0.1267	2.0400	0.9874	-0.9853	0.2865	0.2167	2.0470	1.0063	-0.9981
	sd	0.2019	0.3501	0.5455	0.1654	0.1407	0.1832	0.2451	0.5331	0.1118	0.1130
BR_QMLE	mean	0.2747	0.2036	2.0614	0.9761	-0.9750	0.2672	0.2729	2.0976	0.9989	-0.9896
	sd	0.2554	0.4180	0.6670	0.1746	0.1453	0.2574	0.3602	0.7295	0.1229	0.1368
BC_QMLE	mean	0.3000	0.2740	2.0014	0.9841	-0.9820	0.2965	0.2874	2.0193	1.0036	-0.9961
	sd	0.2197	0.3650	0.5908	0.1646	0.1408	0.1945	0.2526	0.5634	0.1124	0.1134
		n=200					n=500				
G2SLS	mean	0.3092	0.2425	1.9778	0.9956	-0.9981	0.3045	0.2833	2.0018	0.9990	-0.9952
	sd	0.1197	0.1600	0.3385	0.0631	0.0697	0.0483	0.0841	0.0435	0.0509	0.0481
GMM	mean	0.2350	0.3511	6.1440	0.9894	-0.9621	0.2976	0.3101	2.0021	0.9988	-0.9948
	sd	0.2246	0.2449	167.8520	0.0709	0.2799	0.0504	0.0865	0.0447	0.0517	0.0489
QMLE	mean	0.2861	0.2703	2.0403	0.9951	-0.9981	0.2982	0.2943	2.0015	0.9983	-0.9948
	sd	0.1201	0.1575	0.3424	0.0633	0.0695	0.0481	0.0842	0.0434	0.0508	0.0481
BR_QMLE	mean	0.2898	0.2304	2.0251	0.9696	-0.9686	0.2982	0.2927	2.0015	0.9983	-0.9948
	sd	0.2751	0.5363	0.7590	0.1163	0.1442	0.0481	0.0839	0.0434	0.0508	0.0481
BC_QMLE	mean	0.2918	0.3043	2.0249	0.9951	-0.9979	0.2995	0.3006	2.0016	0.9985	-0.9949
	sd	0.1237	0.1604	0.3514	0.0635	0.0694	0.0483	0.0842	0.0434	0.0508	0.0481

DGP2		n=50					n=100				
		lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1	lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1
G2SLS	mean	0.3339	0.0816	1.9025	0.9790	-0.9758	0.3153	0.1987	1.9628	0.9920	-0.9876
	sd	0.2220	0.3153	0.6064	0.1491	0.1375	0.2175	0.2319	0.6548	0.1067	0.1151
GMM	mean	-0.0538	0.6689	215.2459	0.9197	-0.8616	0.1270	0.4399	85.9188	0.9844	-0.9546
	sd	0.4453	0.3764	884.9968	0.1411	0.3792	0.3681	0.3722	536.9220	0.1032	0.5611
QMLE	mean	0.2799	0.1481	2.0537	0.9812	-0.9843	0.2544	0.2643	2.1435	0.9994	-0.9934
	sd	0.2004	0.2980	0.5829	0.1471	0.1373	0.1944	0.2191	0.6087	0.1052	0.1099
BR_QMLE	mean	0.2720	0.2271	2.0740	0.9741	-0.9780	0.2544	0.2990	2.1401	0.9913	-0.9845
	sd	0.2343	0.3568	0.6739	0.1526	0.1405	0.2532	0.3191	0.7600	0.1222	0.1272
BC_QMLE	mean	0.2905	0.2949	2.0278	0.9806	-0.9817	0.2594	0.3346	2.1313	0.9979	-0.9920
	sd	0.2144	0.3026	0.6252	0.1468	0.1399	0.2079	0.2242	0.6520	0.1053	0.1100

		lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1	lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1
G2SLS	mean	0.3107	0.2482	1.9799	0.9962	-0.9937	0.3071	0.2817	1.9984	1.0035	-1.0014
	sd	0.1190	0.1609	0.3319	0.0680	0.0719	0.0460	0.0802	0.0433	0.0487	0.0465
GMM	mean	0.2592	0.3268	0.6230	0.9984	-0.9023	0.3015	0.3065	1.9975	1.0004	-1.0016
	sd	0.1962	0.2401	34.6243	0.0531	0.3986	0.0284	0.0713	0.0258	0.0287	0.0274
QMLE	mean	0.2907	0.2707	2.0354	0.9957	-0.9938	0.3019	0.2889	1.9981	1.0030	-1.0012
	sd	0.1211	0.1590	0.3439	0.0683	0.0718	0.0463	0.0782	0.0434	0.0487	0.0465
BR_QMLE	mean	0.2635	0.2875	2.1106	0.9740	-0.9673	0.3003	0.2892	1.9975	1.0026	-1.0009
	sd	0.2957	0.4822	0.8479	0.1137	0.1324	0.0616	0.0897	0.0454	0.0491	0.0475
BC_QMLE	mean	0.2952	0.3039	2.0230	0.9957	-0.9935	0.3034	0.2942	1.9982	1.0031	-1.0012
	sd	0.1251	0.1617	0.3542	0.0683	0.0718	0.0464	0.0783	0.0434	0.0487	0.0465

Table 3.2 SARAR Model RGE2

DGP1		n=50					n=100				
		lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1	lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1
G2SLS	mean	0.3533	0.1562	1.9970	0.9987	-1.0003	0.3170	0.2185	1.9977	0.9957	-0.9946
	sd	0.1623	0.2603	0.1454	0.1539	0.1299	0.1056	0.1811	0.1127	0.1000	0.1137
GMM	mean	0.2681	0.3859	1.9721	0.9903	-1.0005	0.2638	0.3607	1.9783	0.9931	-0.9889
	sd	0.2304	0.3960	0.1723	0.1725	0.1422	0.1578	0.2588	0.1195	0.1099	0.1199
QMLE	mean	0.2867	0.2385	1.9827	0.9980	-0.9985	0.2878	0.2623	1.9916	0.9971	-0.9976
	sd	0.1708	0.3031	0.1466	0.1549	0.1301	0.1084	0.1926	0.1129	0.0999	0.1130
BR_QMLE	mean	0.2936	0.1324	1.9636	0.9869	-0.9950	0.2873	0.2597	1.9908	0.9965	-0.9966
	sd	0.1977	0.6388	0.1574	0.1634	0.1328	0.1194	0.2206	0.1138	0.1001	0.1138
BC_QMLE	mean	0.2957	0.3108	1.9839	0.9972	-0.9983	0.2935	0.3000	1.9927	0.9970	-0.9970
	sd	0.1785	0.3013	0.1469	0.1547	0.1304	0.1094	0.1910	0.1127	0.0997	0.1128
		n=200					n=500				
G2SLS	mean	0.3071	0.2748	1.9983	1.0022	-0.9978	0.3045	0.2833	2.0018	0.9990	-0.9952
	sd	0.0732	0.1268	0.0763	0.0722	0.0678	0.0483	0.0841	0.0435	0.0509	0.0481
GMM	mean	0.2931	0.3351	1.9949	0.9998	-0.9957	0.2976	0.3101	2.0021	0.9988	-0.9948
	sd	0.0992	0.1504	0.0837	0.0756	0.0714	0.0504	0.0865	0.0447	0.0517	0.0489
QMLE	mean	0.2963	0.2916	1.9982	1.0000	-0.9974	0.2982	0.2943	2.0015	0.9983	-0.9948
	sd	0.0748	0.1275	0.0762	0.0722	0.0678	0.0481	0.0842	0.0434	0.0508	0.0481
BR_QMLE	mean	0.2963	0.2888	1.9982	1.0000	-0.9974	0.2982	0.2927	2.0015	0.9983	-0.9948
	sd	0.0747	0.1264	0.0762	0.0722	0.0677	0.0481	0.0839	0.0434	0.0508	0.0481
BC_QMLE	mean	0.2996	0.3093	1.9983	1.0006	-0.9975	0.2995	0.3006	2.0016	0.9985	-0.9949
	sd	0.0750	0.1269	0.0763	0.0722	0.0678	0.0483	0.0842	0.0434	0.0508	0.0481

DGP2		n=50					n=100				
		lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1	lambda=0.3	rho=0.3	beta0=2	beta1=1	beta2=-1
G2SLS	mean	0.3471	0.1743	2.0067	0.9821	-1.0018	0.3164	0.2416	1.9923	1.0023	-0.9922
	sd	0.1587	0.2693	0.1369	0.1566	0.1343	0.1120	0.1870	0.1031	0.1024	0.1055
GMM	mean	0.2676	0.4120	1.9846	0.9821	-0.9975	0.2767	0.3886	1.9891	1.0007	-0.9940
	sd	0.2181	0.4003	0.1169	0.1125	0.0860	0.1078	0.2263	0.0717	0.0546	0.0590
QMLE	mean	0.2887	0.2479	1.9969	0.9841	-0.9998	0.2891	0.2827	1.9876	1.0040	-0.9952
	sd	0.1614	0.2853	0.1386	0.1605	0.1344	0.1107	0.1864	0.1035	0.1025	0.1026
BR_QMLE	mean	0.3000	0.1979	1.9919	0.9817	-1.0008	0.2895	0.2628	1.9807	0.9994	-0.9893
	sd	0.1617	0.4117	0.1435	0.1643	0.1337	0.1355	0.2998	0.1303	0.1115	0.1239
BC_QMLE	mean	0.2839	0.3211	1.9933	0.9822	-0.9991	0.2915	0.3137	1.9879	1.0038	-0.9945
	sd	0.1779	0.2914	0.1397	0.1601	0.1343	0.1144	0.1902	0.1035	0.1024	0.1039

		n=200					n=500				
G2SLS	mean	0.3073	0.2711	1.9973	1.0000	-1.0037	0.3071	0.2817	1.9984	1.0035	-1.0014
	sd	0.0660	0.1155	0.0703	0.0715	0.0674	0.0460	0.0802	0.0433	0.0487	0.0465
GMM	mean	0.2852	0.3519	1.9927	0.9970	-1.0024	0.3015	0.3065	1.9975	1.0004	-1.0016
	sd	0.0659	0.1634	0.0529	0.0429	0.0412	0.0284	0.0713	0.0258	0.0287	0.0274
QMLE	mean	0.2966	0.2884	1.9972	0.9978	-1.0034	0.3019	0.2889	1.9981	1.0030	-1.0012
	sd	0.0668	0.1173	0.0701	0.0718	0.0674	0.0463	0.0782	0.0434	0.0487	0.0465
BR_QMLE	mean	0.2941	0.2884	1.9961	0.9973	-1.0030	0.3003	0.2892	1.9975	1.0026	-1.0009
	sd	0.0822	0.1296	0.0724	0.0731	0.0675	0.0616	0.0897	0.0454	0.0491	0.0475
BC_QMLE	mean	0.2990	0.3032	1.9972	0.9983	-1.0035	0.3034	0.2942	1.9982	1.0031	-1.0012
	sd	0.0671	0.1178	0.0701	0.0719	0.0674	0.0464	0.0783	0.0434	0.0487	0.0465