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Asymptotic theory for rough fractional Vasicek models[☆]

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HIGHLIGHTS

- Establish asymptotic theory for the LS estimator of the persistency parameter in fVm when the Hurst parameter is less than 1/2.
- Establish asymptotic theory for the LS estimator of the drift parameter in fVm when the Hurst parameter is less than 1/2.
- Establish asymptotic theory for the ergodic-type estimator of the persistency parameter in the stationary fVm when the Hurst parameter is less than 1/2.
- Establish asymptotic theory for the ergodic-type estimator of the drift parameter in the stationary fVm when the Hurst parameter is less than 1/2.

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ABSTRACT

This paper extends the asymptotic theory for the fractional Vasicek model developed in Xiao and Yu (2018) from the case where $H \in (1/2, 1)$ to where $H \in (0, 1/2)$. It is found that the asymptotic theory of the persistence parameter (κ) critically depends on the sign of κ . Moreover, if $\kappa > 0$, the asymptotic distribution for the estimator of κ is different when $H \in (0, 1/2)$ from that when $H \in (1/2, 1)$.

1. Introduction

The fractional Vasicek model (fVm), which is a Vasicek model driven by a fractional Brownian motion (fBm), has found a considerable amount of applications in economics and finance; see Comte and Renault (1998), Comte et al. (2012), Chronopoulou and Viens (2012a), Chronopoulou and Viens (2012b), Bayer et al. (2016) and references therein. The model is given by

$$dX_t = \kappa\,(\mu - X_t)\,dt + \sigma\,dB_t^H, \tag{1.1}$$

where σ is a positive constant, $\mu, \kappa \in \mathbb{R}$, H is the Hurst parameter, and B_t^H is an fBm. In a recent study, based on a continuous record of X_t over a time period of $[0, T]$, Xiao and Yu (2018) developed the long-span asymptotic theory for alternative estimators of κ and μ when H and σ are known and H takes a value in the range of $(1/2, 1)$.

While $H \in (1/2, 1)$ is empirically relevant for some economic time series, recent findings suggest that some time series is better modelled by an fVm with $H \in (0, 1/2)$. For example, Gatheral et al. (2018) showed that the logarithm of realized variance behaves more like an fVm with H near 0.1 than that with H bigger than 0.5, regardless of timescale sampled. Hence, it is important to study Model (1.1) with $H \in (0, 1/2)$.

The present paper extends the asymptotic results of Xiao and Yu (2018) from the case where $H \in (1/2, 1)$ to the case where $H \in (0, 1/2)$. It is found that the asymptotic theory critically depends on the sign of κ , as in Xiao and Yu (2018). However, if $\kappa > 0$ the asymptotic theory for κ is different when $H \in (0, 1/2)$ from that when $H \in (1/2, 1)$.

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The rest of the paper is organized as follows. Section 2 introduces the model and discusses the least squares (LS) estimators and the ergodic-type estimators of κ and μ. Section 3 establishes strong consistency and asymptotic distributions for the LS estimators of κ and μ and those of the ergodic-type estimators of κ and μ when $\kappa > 0$. The proofs of the main results are given an online supplement.

We use the following notations throughout the paper. Let $\xrightarrow{p}$, $\xrightarrow{a.s.}$, $\xrightarrow{\mathcal{L}}$ and $\overset{a}{\sim}$ denote convergence in probability, convergence almost surely, convergence in distribution, and asymptotic equivalence, respectively, as $T \to \infty$. Let $\overset{d}{=}$ denote equivalence in distribution.

2. The model and estimation methods

Before introducing the model, we first state some basic facts about the fBm (see Taqqu (1977), Cheridito et al. (2003) and Nualart (2006) for more details). An fBm $B^H = \{B_t^H, t \in \mathbb{R}\}$ with the Hurst parameter $H \in (0,1)$ is a zero mean Gaussian process, defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with the following covariance function

$$\mathbb{E}(B_t^H B_s^H) = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right). \tag{2.1}$$

This covariance function implies that the fBm is self-similar with the self-similarity parameter H, that is, $B_{\lambda t}^H \overset{d}{=} \lambda^H B_t^H$. A direct consequence of (2.1) is that $B_n^H - B_{n-1}^H$ is a discrete-time Gaussian process with a covariance function

$$\begin{aligned} r(n) &= \mathbb{E}\left[\left(B_{k+n}^H - B_{k+n-1}^H\right)\left(B_k^H - B_{k-1}^H\right)\right] \\ &= \frac{1}{2}\left[(n+1)^{2H} + (n-1)^{2H} - 2n^{2H}\right] \overset{a}{\sim} H(2H-1)n^{2H-2}. \end{aligned}$$

By the convexity of $g(n) = n^{2H}$, the increments, $B_{k+n}^H - B_{k+n-1}^H$ and $B_k^H - B_{k-1}^H$, are positively correlated if $1/2 < H < 1$. However, the increments are negatively correlated if $0 < H < 1/2$, generating the feature of roughness in the sample path. Thus, B^H is persistent when $1/2 < H < 1$ and antipersistent when $0 < H < 1/2$, If $H = 1/2$, B_t^H becomes a standard Brownian motion W_t. Moreover, if $H \in (1/2, 1)$, $\sum_{n=1}^{\infty} r(n) = \infty$, suggesting that the process exhibits long-range dependence. However, if $H \in (0, 1/2)$, $\sum_{n=1}^{\infty} r(n) < \infty$. While Gatheral et al. (2018) showed the empirical relevance of the fVm with H near 0.1, it did not estimate any parameter in fVm nor provide any asymptotic theory for making statistical inference.

The model concerned in the present paper is given by (1.1). It is worth to emphasize that, with a continuous record, both σ^2 and H can be recovered. For example, for any $\epsilon \neq \xi$,

$$H = \lim_{\epsilon\downarrow 0, \xi\downarrow 0} \frac{1}{2}\log\left(\frac{\epsilon}{\xi}\right)\log\left(\frac{\int_0^T \left(X_{t+\xi} - X_t\right)^2 dt}{\int_0^T \left(X_{t+\epsilon} - X_t\right)^2 dt}\right),$$

$$\sigma^2 = \frac{\lim_{\epsilon\downarrow 0}\int_0^T \left(X_{t+\epsilon} - X_t\right)^2 dt}{\epsilon^{2H} T}.$$

Consequently, for further statistical analysis, we assume that both σ and H are known. Xiao and Yu (2018) developed the long-span asymptotic theory for κ and μ when $H \in (1/2, 1)$. The goal of the present paper is to extend the results in Xiao and Yu (2018) to the case where $H \in (0, 1/2)$. This extension is important in light of the empirical results in Gatheral et al. (2018). Following Xiao and Yu (2018), we assume the whole trajectory of X_t for $t \in [0, T]$ is available. The asymptotic theory is developed by requiring $T \to \infty$.

The LS estimators of κ and μ are,

$$\hat{\kappa}_{LS} = \frac{(X_T - X_0)\int_0^T X_t dt - T\int_0^T X_t dX_t}{T\int_0^T X_t^2 dt - \left(\int_0^T X_t dt\right)^2}, \tag{2.2}$$

$$\hat{\mu}_{LS} = \frac{(X_T - X_0)\int_0^T X_t^2 dt - \int_0^T X_t dX_t \int_0^T X_t dt}{(X_T - X_0)\int_0^T X_t dt - T\int_0^T X_t dX_t}. \tag{2.3}$$

When $H \in (0, 1/2)$, X_t is no longer a semimartingale. In this case, for $\hat{\kappa}_{LS}$ and $\hat{\mu}_{LS}$ to consistently estimate κ and μ, we have to interpret the stochastic integral $\int_0^T X_t dX_t$ carefully. In fact, we interpret it differently when the sign of κ is different. If $\kappa > 0$, following the idea of Hu et al. (2018), we interpret it as a divergence integral; if $\kappa < 0$, borrowing an idea from El Machkouri et al. (2016), we interpret it as a Young integral; if $\kappa = 0$, we can interpret it as either a divergence integral or a Young integral. The asymptotic distribution of $\hat{\kappa}_{LS}$ is different across these three cases.

When $\kappa > 0$, one may use the ergodic-type estimator of Hu and Nualart (2010) to estimate κ and μ, which is given by

$$\hat{\kappa}_{HN} = \left(\frac{T\int_0^T X_t^2 dt - \left(\int_0^T X_t dt\right)^2}{T^2\sigma^2 H\Gamma(2H)}\right)^{-\frac{1}{2H}}, \tag{2.4}$$

$$\hat{\mu}_{HN} = \frac{1}{T}\int_0^T X_t dt. \tag{2.5}$$

Compared with (2.2) and (2.3) which involve the stochastic integral $\int_0^T X_t dX_t$, the ergodic-type estimators in (2.4) and (2.5) do not contain any stochastic integral.

3. Asymptotic theory

Let us first consider the case when $\kappa > 0$. Using Lemma 2.1 of Kloeden and Neuenkirch (2007), we have the following results.

Lemma 3.1. *Let $H \in (0, 1/2)$, $X_0/\sqrt{T} = o_{a.s.}(1)$, $\kappa > 0$ in Model (1.1). As $T \to \infty$,*

$$\frac{1}{T}\int_0^T X_t dt \xrightarrow{a.s.} \mu, \tag{3.1}$$

$$\frac{1}{T}\int_0^T X_t^2 dt \xrightarrow{a.s.} \sigma^2\kappa^{-2H} H\Gamma(2H) + \mu^2, \tag{3.2}$$

$$\frac{1}{T}\int_0^T X_t dX_t \xrightarrow{a.s.} -\sigma^2\kappa^{1-2H} H\Gamma(2H). \tag{3.3}$$

Theorem 3.1. *Let $H \in (0, 1/2)$, $X_0/\sqrt{T} = o_{a.s.}(1)$, $\kappa > 0$ in Model (1.1). Then, as $T \to \infty$, $\hat{\kappa}_{LS} \xrightarrow{a.s.} \kappa$ and $\hat{\mu}_{LS} \xrightarrow{a.s.} \mu$. Moreover, let $H \in (0, 1/2)$, $X_0/\sqrt{T} = o_p(1)$, $\kappa > 0$ in Model (1.1). Then, as $T \to \infty$,*

$$\sqrt{T}\left(\hat{\kappa}_{LS} - \kappa\right) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, \kappa\delta_{LS}^2\right), \tag{3.4}$$

$$T^{1-H}\left(\hat{\mu}_{LS} - \mu\right) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, \frac{\sigma^2}{\kappa^2}\right), \tag{3.5}$$

where $\delta_{LS}^2 = (4H-1) + \frac{2\Gamma(2-4H)\Gamma(4H)}{\Gamma(2H)\Gamma(1-2H)}$.

Theorem 3.2. *Let $H \in (0, 1/2)$, $X_0/\sqrt{T} = o_{a.s.}(1)$, $\kappa > 0$ in Model (1.1). Then, as $T \to \infty$, $\hat{\kappa}_{HN} \xrightarrow{a.s.} \kappa$ and $\hat{\mu}_{HN} \xrightarrow{a.s.} \mu$. Moreover, let $H \in (0, 1/2)$, $X_0/\sqrt{T} = o_p(1)$, $\kappa > 0$ in Model (1.1). Then, as $T \to \infty$,*

$$\sqrt{T}\left(\hat{\kappa}_{HN} - \kappa\right) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, \kappa\delta_{HN}^2\right), \tag{3.6}$$

$$T^{1-H}\left(\hat{\mu}_{HN}-\mu\right)\xrightarrow{\mathcal{L}}\mathcal{N}\left(0,\frac{\sigma^2}{\kappa^2}\right),\tag{3.7}$$

where $\delta_{HN}^2=\frac{1}{4H^2}\left[(4H-1)+\frac{2\Gamma(2-4H)\Gamma(4H)}{\Gamma(2H)\Gamma(1-2H)}\right]$.

Remark 3.1. Theorems 3.1 and 3.2 extend Theorems 3.2 and 3.3 of Xiao and Yu (2018) and provide the full coverage of asymptotic laws of the LS and the ergodic-type estimators for κ and μ in fVm for $H\in(0,1)$.

Remark 3.2. The rate of convergence for $\hat{\kappa}_{HN}$ and $\hat{\kappa}_{LS}$ is the same (i.e. $\sqrt{T}$) and independent of H, but their asymptotic variances depend on H. Since $\lim_{z\to 0} z\Gamma(z)=1$, $\lim_{H\to 1/2}\delta_{LS}^2=\lim_{H\to 1/2}\delta_{HN}^2=2$, suggesting that, when $H\to 1/2$, $\hat{\kappa}_{LS}$ and $\hat{\kappa}_{HN}$ have the same asymptotic variance of 2κ. In this case, the asymptotic distribution is identical to that in Feigin (1976). When $0<H<1/2$, $4H^2<1$ and the asymptotic variance of $\hat{\kappa}_{LS}$ is smaller than that of $\hat{\kappa}_{HN}$, suggesting $\hat{\kappa}_{LS}$ is asymptotically more efficient than $\hat{\kappa}_{HN}$. Fig. 1 plots δ_{LS}^2 and δ_{HN}^2 as a function of H. It can be seen that as H increases, δ_{LS}^2 monotonically increases while δ_{HN}^2 monotonically decreases. They both converge to 2 when H approaches 1/2. The relative asymptotic efficiency increases as H decreases. When $H=0.1$ which is an empirically realistic value for H according to Gatheral et al. (2018), the relative asymptotic efficiency is 25, favouring $\hat{\kappa}_{LS}$. This difference is very significant. The direction of relative asymptotic efficiency is different from that in Xiao and Yu (2018) where $\hat{\kappa}_{LS}$ is found to be asymptotically less efficient than $\hat{\kappa}_{HN}$ when $H>1/2$.

Remark 3.3. Unlike κ, the asymptotic distribution for $\hat{\mu}_{LS}$ is identical to that of $\hat{\mu}_{HN}$, which is also the same as those obtained in Xiao and Yu (2018) when $H>1/2$.

Remark 3.4. When $0<H<1/2$, paths generated from an fBm are irregular. In this case, the stochastic integration with respect to fBm should be interpreted as a divergence integral introduced by Cheridito and Nualart (2005). If we interpret the integral $\int_0^T X_t dX_t$ in (2.2) as a Young integral, then, as $T\to\infty$,

$$\hat{\kappa}_{LS}=\frac{\frac{X_T-X_0}{T}\frac{\int_0^T X_t dt}{T}-\frac{1}{2}\frac{X_T^2-X_0^2}{T}}{\frac{1}{T}\int_0^T X_t^2dt-\left(\frac{1}{T}\int_0^T X_t dt\right)^2}\xrightarrow{a.s.}0,\tag{3.8}$$

by (3.1), (3.2) and Lemma 18 of Hu et al. (2018), implying that $\hat{\kappa}_{LS}$ will be inconsistent.

Now, we consider the case where $\kappa<0$. Applying the Young integral to (2.2) and (2.3), we can rewrite $\hat{\kappa}_{LS}$ and $\hat{\mu}_{LS}$ as

$$\begin{aligned}\hat{\kappa}_{LS}&=\frac{(X_T-X_0)\int_0^T X_t dt-\frac{T}{2}\left(X_T^2-X_0^2\right)}{T\int_0^T X_t^2dt-\left(\int_0^T X_t dt\right)^2}\\&=\frac{\frac{X_T}{T}e^{\kappa T}e^{\kappa T}\int_0^T X_t dt-\frac{X_0}{T}e^{\kappa T}e^{\kappa T}\int_0^T X_t dt-\frac{1}{2}X_T^2e^{2\kappa T}+\frac{1}{2}X_0^2e^{2\kappa T}}{e^{2\kappa T}\int_0^T X_t^2dt-e^{2\kappa T}\frac{1}{T}\left(\int_0^T X_t dt\right)^2},\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{LS}&=\frac{(X_T-X_0)\int_0^T X_t^2dt-\frac{X_T^2-X_0^2}{2}\int_0^T X_t dt}{(X_T-X_0)\int_0^T X_t dt-T\frac{X_T^2-X_0^2}{2}}\\&=\frac{\frac{e^{\kappa T}}{T}\int_0^T X_t^2dt-\frac{X_T+X_0}{2T}e^{\kappa T}\int_0^T X_t dt}{\frac{e^{\kappa T}}{T}\int_0^T X_t dt-\frac{X_T+X_0}{2}e^{\kappa T}}.\end{aligned}$$

Using similar arguments as those in Xiao and Yu (2018), we can obtain asymptotic properties of $\hat{\kappa}_{LS}$ and $\hat{\mu}_{LS}$. In particular, let $H\in(0,1/2)$, $X_0=O_p(1)$ and $\kappa<0$ in Model (1.1). Then, as $T\to\infty$, $\hat{\kappa}_{LS}\xrightarrow{a.s.}\kappa$, $\hat{\mu}_{LS}\xrightarrow{a.s.}\mu$ and

$$T^{1-H}\left(\hat{\mu}_{LS}-\mu\right)\xrightarrow{\mathcal{L}}\mathcal{N}\left(0,\frac{\sigma^2}{\kappa^2}\right),$$

$$\frac{e^{-\kappa T}}{2\kappa}\left(\hat{\kappa}_{LS}-\kappa\right)\xrightarrow{\mathcal{L}}\frac{\sigma\frac{\sqrt{H\Gamma(2H)}}{|\kappa|^H}\nu}{X_0-\mu+\sigma\frac{\sqrt{H\Gamma(2H)}}{|\kappa|^H}\omega},$$

where ν and ω are two independent standard normal variables. The asymptotic law for $\hat{\kappa}_{LS}$ is a Cauchy-type and is similar to that developed in the explosive discrete-time and continuous-time models in Phillips and Magdalinos (2007), Magdalinos (2012), Wang and Yu (2015), Wang and Yu (2016) and Arvanitis and Magdalinos (2018) . It is the same as that in Xiao and Yu (2018) for the fVm when $H\in(1/2,1)$.

Finally, we consider the case when $\kappa=0$. In this case, μ vanishes and the fVm reduces to an fBm without drift. In this case $X_t=X_0+\sigma B_t^H$. Using the relationship between the divergence integral and the Stratonovich integral and applying the divergence integral to (2.2), we can rewrite the LS estimator of κ as

$$\hat{\kappa}_{1,LS}=\frac{B_T^H\int_0^T B_t^H dt-\frac{T}{2}\left(\left(B_T^H\right)^2-T^{2H}\right)}{T\int_0^T (B_t^H)^2dt-\left(\int_0^T B_t^H dt\right)^2},$$

where the equality $\int_0^T B_t^H dB_t^H=\int_0^T B_t^H\circ dB_t^H-\mathbb{E}\left[\int_0^T B_t^H\circ dB_t^H\right]=(B_T^H)^2/2-\mathbb{E}\left[(B_T^H)^2/2\right]=(B_T^H)^2/2-T^{2H}/2$ is used. If we interpret $\int_0^T B_t^H dB_t^H$ in (2.2) as a Young integral, then the LS estimator of κ can be rewritten as

$$\hat{\kappa}_{2,LS}=\frac{B_T^H\int_0^T B_t^H dt-\frac{T}{2}\left(B_T^H\right)^2}{T\int_0^T (B_t^H)^2dt-\left(\int_0^T B_t^H dt\right)^2}.$$

Let $H\in(0,1/2)$, $X_0=O_p(1)$, $\kappa=0$ in (1.1). Then, as $T\to\infty$, using similar arguments as in Theorem 3.6 of Xiao and Yu (2018), we have $\hat{\kappa}_{1,LS}\xrightarrow{a.s.}0$ and $\hat{\kappa}_{2,LS}\xrightarrow{a.s.}0$. Moreover, for any T and $i=1,2$,

$$T\hat{\kappa}_{i,LS}\overset{d}{=}-\frac{\int_0^1\overline{B}_u^H dB_u^H}{\int_0^1\left(\overline{B}_u^H\right)^2du},$$

where $\overline{B}_u^H=B_u^H-\int_0^1 B_t^H dt$. This is the Dickey–Fuller–Phillips type of distribution of Phillips (1987) and the same as that in Xiao and Yu (2018) for fVm when $H\in(1/2,1)$.

4. Concluding remarks

Based on a continuous record of observations with an increasing time span from an fVm with $H<1/2$, this paper develops asymptotic theory for the two parameters in the drift function, κ and μ. When $\kappa>0$, two type estimators are considered, the LS estimators and the ergodic-type estimators. When $\kappa=0$ or <0, the LS estimators are considered. It is shown that when $\kappa>0$, the two estimators of κ and μ are asymptotically normally distributed. However, the LS estimator of κ is asymptotically more efficient than that of the ergodic-type estimator of κ. The relative efficiency is especially large when H takes a value close to zero. When $\kappa<0$, the LS estimator follows a Cauchy-type distribution asymptotically. When $\kappa=0$, the LS estimator follows the Dickey–Fuller–Phillips type of distribution.

It is assumed that a continuous record of an increasing time span is available for the development of asymptotic theory. In practice, data is typically discretely sampled at, say $(0,h,2h,\ldots,Nh(:=T))$ where h is the sampling interval and T is the time span. When

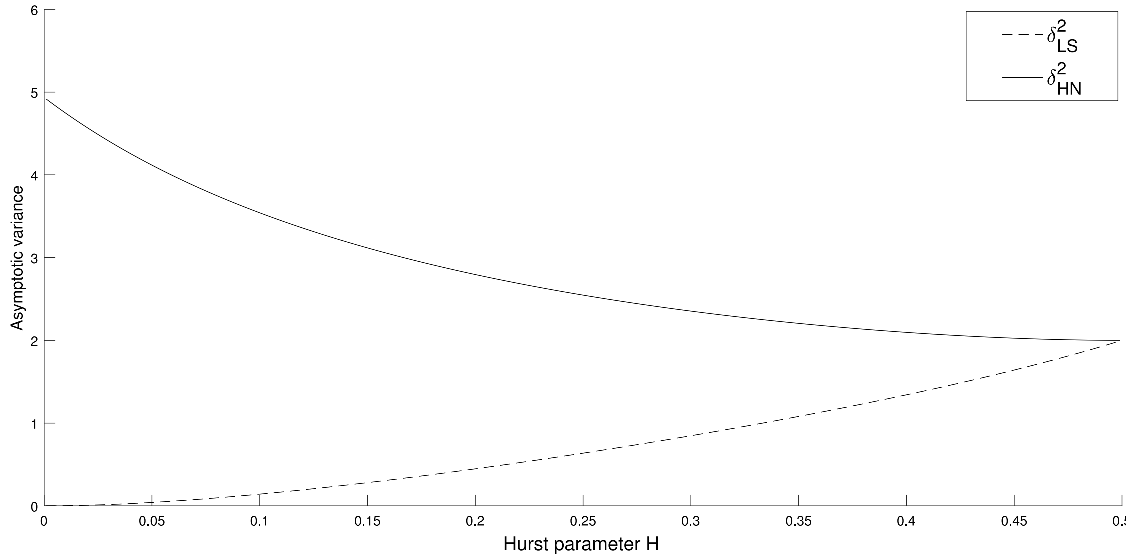

Fig. 1. Plots of δ_{LS}^2 and δ_{HN}^2 as functions of H.

high frequency data over a long span of time period is available, one may consider using a double asymptotic scheme by assuming $h \to 0$ and $T \to \infty$. The discretized model corresponding to (1.1) is given by

$$y_{th} = \mu + \exp(-\kappa h)(y_{(t-1)h} - \mu) + u_t,\ (1-L)^d u_t = \varepsilon_t,\ t = 1, \ldots, N,$$

where L is the lag operator, $d = H - 1/2$. As shown in Wang and Yu (2016), under the double asymptotic scheme, $\exp(-\kappa h) = \exp\{-\kappa/k_N\} = 1 - \kappa/k_N + O(k_N^{-2}) \to 1$ where $k_N := 1/h \to \infty$ as $h \to 0$ and $k_N/N = 1/T \to 0$ as $T \to \infty$. This implies an autoregressive (AR) model with an AR root being moderately deviated from unity and with a fractionally integrated error term with $d \in (-1/2, 0)$. This model is closely related to a model considered in Magdalinos (2012) where it is assumed that $d \in (0, 1/2)$. Developing double asymptotic theory based on discretely sampled data will allow one to extend the results of Magdalinos (2012) to the case where $d \in (-1/2, 0)$. This analysis will be reported in later work.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2019.01.020.

References

Arvanitis, S., Magdalinos, T., 2018. Mildly explosive autoregression under stationary conditional heteroskedasticity. J. Time. Ser. Anal. (forthcoming).
Bayer, C., Friz, P., Gatheral, J., 2016. Pricing under rough volatility. Quant. Finance 16 (6), 887–904.
Cheridito, P., Kawaguchi, H., Maejima, M., 2003. Fractional ornstein–uhlenbeck processes. Electron. J. Probab. 8 (3), 1–14.
Cheridito, P., Nualart, D., 2005. Stochastic integral of divergence type with respect to fractional brownian motion with hurst parameter $H \in (0, 1/2)$. Ann. I. H. Poincaré-PR. 41 (6), 1049–1081.
Chronopoulou, A., Viens, F.G., 2012a. Estimation and pricing under long-memory stochastic volatility. Ann. Finance 8 (2–3), 379–403.
Chronopoulou, A., Viens, F.G., 2012b. Stochastic volatility and option pricing with long-memory in discrete and continuous time. Quant. Finance 12 (4), 635–649.
Comte, F., Coutin, L., É, Renault., 2012. Affine fractional stochastic volatility models. Ann. Finance 8 (2–3), 337–378.
Comte, F., Renault, É., 1998. Long memory in continuous-time stochastic volatility models. Math. Finance 8 (4), 291–323.
El Machkouri, M., Es-Sebaiy, K., Ouknine, Y., 2016. Least squares estimator for non-ergodic ornstein–uhlenbeck processes driven by gaussian processes. J. Korean. Stat. Soc. 45 (3), 329–341.
Feigin, P.D., 1976. Maximum likelihood estimation for continuous-time stochastic processes. Adv. Appl. Probab. 8 (4), 712–736.
Gatheral, J., Jaisson, T., Rosenbaum, M., 2018. Volatility is rough. Quant. Finance 18 (6), 933–949.
Hu, Y., Nualart, D., 2010. Parameter estimation for fractional ornstein–uhlenbeck processes. Statist. Probab. Lett. 80 (11), 1030–1038.
Hu, Y., Nualart, D., Zhou, H., 2018. Parameter estimation for fractional ornstein–uhlenbeck processes of general hurst parameter. Stat. Infer. Stoch. Proc. (forthcoming).
Kloeden, P., Neuenkirch, A., 2007. The pathwise convergence of approximation schemes for stochastic differential equations. LMS J. Comput. Math. 10, 235–253.
Magdalinos, T., 2012. Mildly explosive autoregression under weak and strong dependence. J. Econometrics 169 (2), 179–187.
Nualart, D., 2006. The Malliavin Calculus and Related Topics, second ed. Springer.
Phillips, P.C.B., 1987. Time series regression with a unit root. Econometrica 55 (2), 277–301.
Phillips, P.C.B., Magdalinos, T., 2007. Limit theory for moderate deviations from a unit root. J. Econometrics 136 (1), 115–130.
Taqqu, M.S., 1977. Law of the iterated logarithm for sums of non-linear functions of gaussian variables that exhibit a long range dependence. Probab. Theory. Rel. 40 (3), 203–238.
Wang, X., Yu, J., 2015. Limit theory for an explosive autoregressive process. Econom. Lett. 126, 176–180.
Wang, X., Yu, J., 2016. Double asymptotics for explosive continuous time models. J. Econometrics 193 (1), 35–53.
Xiao, W., Yu, J., 2018. Asymptotic theory for estimating drift parameters in the fractional vasicek model. Econometric Theory (forthcoming).