SINGAPORE MANAGEMENT UNIVERSITY

School of Economics

Econ 623 Econometrics II
Assignment 2
Due: Tuesday 14 Feb, 2017

1. In this exercise you need to write a MATLAB program to examine the finite sample properties of the OLS estimates of ϕ and the bias of the OLS estimates of ϕ in the following model:

$$Y_t = \phi Y_{t-1} + \varepsilon_t, \varepsilon_t \sim iidN(0, \sigma^2), Y_0 \sim N\left(0, \frac{\sigma^2}{1-\phi^2}\right)$$

based on 5,000 replications using simulated data, each with the sample size of 20 (make sure to use the same set of random seeds for different values of ϕ when generating data).

Set the parameter value at $\sigma^2 = 1$ and
$\phi = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99.0.995$

Plot the bias of the OLS estimates obtained from the simulation. Plot the bias function ($-2\phi/n$) in the same graph. Discuss the results.

What if the initial condition is replaced by $Y_0 = 0$.

2. Repeat the same exercise as in Question 1 for the following model,

$$Y_t = \mu + \phi Y_{t-1} + \varepsilon_t, \varepsilon_t \sim iidN(0, \sigma^2), Y_0 \sim N\left(\frac{\mu}{1-\phi}, \frac{\sigma^2}{1-\phi^2}\right)$$

where you have to estimate both μ and ϕ using OLS.
Set the parameter value at $\mu = 0, \sigma^2 = 1$ and
$\phi = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99.0.995$

Note that the bias function is $-(1+3\phi)/n$ in this case.

3. Derive the bias of the OLS estimates of ϕ in the following model:

$$Y_t = \phi Y_{t-1} + \varepsilon_t, \varepsilon_t \sim iidN(0,\sigma^2), |\phi| < 1, Y_0 = 0$$

is $E(\hat{\phi}) \approx \phi - \frac{2\phi}{n}$ where n is the sample size.

4. If X and Y are two random variables with the joint density $f_{X,Y}(x,y)$, the density of a new random variable Z, defined by the ratio of these two random variables (ie $Z = X/Y$), is known to be $\int_{-\infty}^{+\infty} |y| f_{X,Y}(zy, y)dy$. Use this result to show that:

1) If X and Y are two independent $N(0,1)$, then Z follows a Cauchy distribution. Also show that if X and Y are two independent $N(0,\sigma^2)$, then Z follows the same Cauchy distribution.

2) Suppose two observations (X_1 and X_2), from the following AR(1) model, are used to estimate the AR coefficient:

$$X_t = \phi X_{t-1} + \varepsilon_t, |\phi| < 1, \varepsilon_t \sim iidN(0,\sigma^2), X_1 \sim N\left(0, \frac{\sigma^2}{1-\phi^2}\right)$$

Derive the exact distribution of the OLS estimate of ϕ. Discuss the properties of the estimator. Is this mean-unbiased or median-unbiased?