

# SINGAPORE MANAGEMENT UNIVERSITY

## School of Economics

### Econ623 Econometrics II

#### Assignment 2

*Due: Wed 17 February, 2010*

1.  $X$  and  $Y$  are two random variables. If  $E(X | Y) = 0$ , show that  $E(h(Y)X) = 0$  for any  $h$ .
2. Let  $Y = \{Y_1, Y_2, \dots, Y_n\}$  be the  $n$  random variables that represent the durations of transaction  $n$  stocks. Suppose that these variables are iid with the following pdf:

$$f(y, \theta) = \begin{cases} \theta_2 \exp\{-\theta_2(y - \theta_1)\} & \text{if } y \geq \theta_1 \\ 0 & \text{if } y < \theta_1 \end{cases}$$

This is an Exponential distribution with the shift parameter  $\theta_1$ . The support of the duration is  $[\theta_1, +\infty)$ . Let  $\{y_1, y_2, \dots, y_n\}$  be a sample from the distribution of these random variables. Answer the following questions:

- (a) Obtain the analytical expression of the MLE of the parameters  $\theta_1$  and  $\theta_2$ .
- (b) Show that the MLE estimator of  $\theta_1$  (call it  $\hat{\theta}_1$ ) is consistent (ie  $\hat{\theta}_1 \xrightarrow{p} \theta_1$ ) but not asymptotically normal.
- (c) Show that  $n(\hat{\theta}_1 - \theta_1)$  is asymptotically distributed as an Exponential random variable with parameter  $\theta_2$ .