SINGAPORE MANAGEMENT UNIVERSITY

School of Economics

Econ623 Econometrics II
Assignment 1
Due: Tuesday 31 January, 2017

1. If $X \sim N(\mu, \sigma^2)$, show that the moment generating function of X is $\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$.

2. If $\begin{bmatrix} X \\ Y \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix}, \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}\right)$, show that the two marginal distributions and the two conditional distributions are all normally distributed and derive the mean and variance for each distribution.

3. X and Y are two random variables. If $E(X \mid Y) = 0$, show that $E(h(Y)X) = 0$ for any h.

4. In this exercise you need to write MATLAB programs to examine the finite sample properties of the OLS estimates in the following models

$$\text{Model 1: } Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,1)$$

$$\text{Model 2: } Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} t_4$$

   Obtain the histograms of $\hat{\beta}_0$ and $\hat{\beta}_1$ based on 10,000 replications of simulated data with the sample size being 25 and 100, respectively. In all cases, simulate X_t s from $N(10,1)$ and fixed them in repeated samples. Examine how each histogram changes with the sample size and the error distribution. Testing for the normality of $\hat{\beta}_1$ in all four cases (namely Model 1 with n=25, 100 and Model 2 with n=25, 100). Write a short paragraph to summarize what you can observe from the experiment.

5. Let $Y = \{Y_1, Y_2, \ldots, Y_n\}$ be the n random variables that represent the durations of transaction n stocks. Suppose that these variables are iid with the following pdf:

$$f(y,\theta) = \begin{cases} \theta_2 \exp\{-\theta_2 (y - \theta_1)\} & if.y \geq \theta_1 \\ 0 & if.y < \theta_1 \end{cases}$$

This is an Exponential distribution with the shift parameter θ_1. The support of the duration is $[\theta_1,+\infty)$. Let $\{y_1, y_2,\ldots, y_n\}$ be a sample from the distribution of these random variables. Answer the following questions:

(a) Obtain the analytical expression of the MLE of the parameters θ_1 and θ_2.

(b) Show that the MLE estimator of θ_1 (call it $\hat{\theta}_1$) is consistent (ie $\hat{\theta}_1 \xrightarrow{p} \theta_1$) but not asymptotically normal.

(c) Show that $n(\hat{\theta}_1 - \theta_1)$ is asymptotically distributed as an Exponential random variable with parameter θ_2.