Nonlife Actuarial Models

Chapter 6 Classical Credibility

Learning Objectives

- 1. Basic framework of credibility
- 2. The limited-fluctuation (classical) credibility approach
- 3. Full credibility
- 4. Partial credibility
- 5. Prediction of claim frequency, claim severity, aggregate loss and pure premium

6.1 Framework and Notations

- We consider a block of insurance policies, referred to as a **risk group**.
- The risk group is covered over a period of time (say, one year) upon the payment of a premium.
- The premium is partially based on a rate specified in the manual, called the **manual rate** and partially on the specific risk character-istics of the group.
- Based upon the recent **claim experience** of the risk group, the premium for the next period will be revised.

• Credibility theory concerns the updating of the prediction of the claim for the next period using the recent claim experience and the manual rate. It may be applied to different measures of claim experience, such as

Claim frequency:	Let N denote the number of claims in the period.		
Aggregate loss:	We denote the amount of the <i>i</i> th claim by X_i and the aggregate loss by S , so that $S = X_1 + X_2 + \cdots + X_N$.		
Claim severity:	The average claim severity is the sample mean of $\{X_i\}$, i.e., $\bar{X} = S/N$.		
Pure premium:	Let E be the number of exposure units of the risk group, the pure premium P is defined as $P = S/E$.		

- We denote generically the predicted loss based on the manual by M, and the observed value of the loss based on recent data of the experience of the risk group by D.
- The classical credibility approach (also called the limited-fluctuation credibility approach) proposes to formulate the updated prediction of the loss measure as a weighted average of *D* and *M*.
- The weight attached to D is called the **credibility factor**, and is denoted by Z, with $0 \le Z \le 1$. Thus, the updated prediction, generically denoted by U, is given by

$$U = ZD + (1 - Z)M.$$
 (6.1)

Example 6.1: The loss per worker insured in a ship-building company was \$230 last year. If the pure premium per worker in a similar industry is \$292 and the credibility factor of the company (the risk group) is 0.46, calculate the updated predicted pure premium for the company's insurance.

Solution: We have D = 230, M = 292 and Z = 0.46, so that from equation (6.1) we obtain

$$U = (0.46)(230) + (1 - 0.46)(292) = \$263.48,$$

which will be the pure premium charged per worker of the company next year. $\hfill \Box$

- The closer Z is to 1, the closer the updated predicted value U will be to the observed measure D.
- The credibility factor Z determines the relative importance of the data in calculating the updated prediction.
- Full credibility is said to be achieved if Z = 1, in which case the prediction depends upon the data only but not the manual. When Z < 1, the data are said to have partial credibility.

6.2 Full Credibility

- The classical credibility approach determines the minimum data size required for the experience data to be given full credibility (namely, for setting Z = 1).
- The minimum data size is called the **standard for full credibility**.
- We now derive the formulas for the computation of the full-credibility standards for loss measures such as claim frequency, claim severity, aggregate loss and pure premium.
- 6.2.1 Full Credibility for Claim Frequency
 - Assume that the claim frequency random variable N has mean μ_N and variance σ_N^2 .

- For given values of k and α , full credibility is attained if the probability of observing claim frequency within 100k% of the mean is at least 1α .
- Now we have

$$\Pr(\mu_N - k\mu_N \le N \le \mu_N + k\mu_N) = \Pr\left(-\frac{k\mu_N}{\sigma_N} \le \frac{N - \mu_N}{\sigma_N} \le \frac{k\mu_N}{\sigma_N}\right).$$
(6.2)

• If we further assume that N is normally distributed, then $(N - \mu_N)/\sigma_N$ follows a standard normal distribution. Thus, denoting $\Phi(\cdot)$ as the df of the standard normal random variable, expression (6.2) becomes

$$\Pr\left(-\frac{k\mu_N}{\sigma_N} \le \frac{N-\mu_N}{\sigma_N} \le \frac{k\mu_N}{\sigma_N}\right) = \Phi\left(\frac{k\mu_N}{\sigma_N}\right) - \Phi\left(-\frac{k\mu_N}{\sigma_N}\right)$$

$$= \Phi\left(\frac{k\mu_N}{\sigma_N}\right) - \left[1 - \Phi\left(\frac{k\mu_N}{\sigma_N}\right)\right]$$
$$= 2\Phi\left(\frac{k\mu_N}{\sigma_N}\right) - 1. \quad (6.3)$$

• If

$$\frac{k\mu_N}{\sigma_N} = z_{1-\frac{\alpha}{2}},\tag{6.4}$$

where z_{β} is the 100 β th percentile of the standard normal, i.e., $\Phi(z_{\beta}) = \beta$, then the probability in (6.3) is given by $2(1 - \alpha/2) - 1 = 1 - \alpha$. Thus, there is a probability of $1 - \alpha$ that an observed claim frequency is within 100k% of the true mean.

• We further assume that N is Poisson with mean λ_N so that

$$\frac{k\mu_N}{\sigma_N} = k\sqrt{\lambda_N}.$$

• Now define

$$\lambda_F \equiv \left(\frac{z_{1-\frac{\alpha}{2}}}{k}\right)^2. \tag{6.7}$$

• Full credibility is attained if $\lambda_N \geq \lambda_F$.

Example 6.4: If an insurance company requires a coverage probability of 99% for the number of claims to be within 5% of the true expected claim frequency, how many claims in the recent period are required for full credibility? If the insurance company receives 2,890 claims this year from the risk group and the manual list of expected claim is 3,000, what is the updated expected number of claims next year? Assume the claim-frequency distribution is Poisson and the normal approximation applies.

Solution: We compute λ_F using equation (6.7) to obtain

$$\lambda_F = \left(\frac{z_{0.995}}{0.05}\right)^2 = \left(\frac{2.576}{0.05}\right)^2 = 2,654.31.$$

Hence, 2,655 claims are required for full credibility. As the observed claim frequency of 2,890 is larger than 2,655, full credibility is attributed to the data, i.e., Z = 1. Thus, 1 - Z = 0, and from equation (6.1) the updated estimate of the expected number of claims in the next period is 2,890. Note that as full credibility is attained, the updated prediction does not depend on the manual value of M = 3,000.

• Table 6.1 presents the values of λ_F for selected values of α and k.

Table 6.1:Selected values of standard for full
credibility λ_F for claim frequency

	Coverage		k	
lpha	Prob –	10%	5%	1%
0.20	80%	164	657	16,424
0.10	90%	271	$1,\!082$	$27,\!055$
0.05	95%	384	$1,\!537$	$38,\!415$
0.01	99%	664	$2,\!654$	$66,\!349$

• Given the accuracy parameter k, the standard for full credibility λ_F increases with the required coverage probability $1 - \alpha$. Likewise, given the required coverage probability $1 - \alpha$, λ_F increases with the required accuracy (i.e., decreases with k).

6.2.2 Full Credibility for Claim Severity

- We now consider the standard for full credibility when the loss measure of interest is the claim severity.
- Suppose there is a sample of N claims of amounts X_1, X_2, \dots, X_N .
- We assume $\{X_i\}$ are independently and identically distributed with mean μ_X and variance σ_X^2 , and use the sample mean \overline{X} to estimate μ_X .
- Full credibility is attributed to \overline{X} if the probability of \overline{X} being within 100k% of the true mean of claim loss μ_X is at least 1α , for given values of k and α .
- We assume that the sample size N is sufficiently large so that \overline{X} is approximately normally distributed with mean μ_X and variance

 σ_X^2/N . Hence, the coverage probability is

$$\Pr(\mu_X - k\mu_X \le \bar{X} \le \mu_X + k\mu_X) = \Pr\left(-\frac{k\mu_X}{\frac{\sigma_X}{\sqrt{N}}} \le \frac{\bar{X} - \mu_X}{\frac{\sigma_X}{\sqrt{N}}} \le \frac{k\mu_X}{\frac{\sigma_X}{\sqrt{N}}}\right)$$
$$\simeq 2\Phi\left(\frac{k\mu_X}{\frac{\sigma_X}{\sqrt{N}}}\right) - 1. \tag{6.8}$$

• For the coverage probability to be larger than $1 - \alpha$, we must have

$$\frac{k\mu_X}{\frac{\sigma_X}{\sqrt{N}}} \ge z_{1-\frac{\alpha}{2}},\tag{6.9}$$

so that

$$N \ge \left(\frac{z_{1-\frac{\alpha}{2}}}{k}\right)^2 \left(\frac{\sigma_X}{\mu_X}\right)^2,\tag{6.10}$$

which is the standard for full credibility for severity.

• Note that the coefficient of variation of X is $C_X = \sigma_X/\mu_X$. Using equation (6.7), expression (6.10) can be written as

$$N \ge \lambda_F C_X^2. \tag{6.11}$$

• Hence, $\lambda_F C_X^2$ is the standard for full credibility for claim severity. If the experience claim frequency exceeds $\lambda_F C_X^2$, \bar{X} will be the predictor for the average severity of the next period (i.e., the manual rate will not be relevant). **Example 6.7:** What is the standard for full credibility for claim severity with $\alpha = 0.01$ and k = 0.05, given that the mean and variance estimates of the severity are 1,000 and 2,000,000, respectively?

Solution: From Table 1, we have $\lambda_F = 2,654$. Thus, using equation (6.11), the standard for full credibility for severity is

$$2,654\left[\frac{2,000,000}{(1,000)}\right] = 5,308.$$

6.2.3 Full Credibility for Aggregate Loss

• To derive the standard for full credibility for aggregate loss, we determine the minimum (expected) claim frequency such that the probability of the observed aggregate loss S being within 100k% of the expected aggregate loss is at least $1 - \alpha$.

• Denoting μ_S and σ_S^2 as the mean and variance of S, respectively, we need to evaluate

$$\Pr(\mu_S - k\mu_S \le S \le \mu_S + k\mu_S) = \Pr\left(-\frac{k\mu_S}{\sigma_S} \le \frac{S - \mu_S}{\sigma_S} \le \frac{k\mu_S}{\sigma_S}\right).$$
(6.12)

- To compute μ_S and σ_S^2 , we use the compound distribution formulas.
- If N and X_1, X_2, \dots, X_N are mutually independent, we have $\mu_S = \mu_N \mu_X$ and $\sigma_S^2 = \mu_N \sigma_X^2 + \mu_X^2 \sigma_N^2$.
- If we further assume that N is distributed as a Poisson variable with mean λ_N , then $\mu_N = \sigma_N^2 = \lambda_N$, and we have $\mu_S = \lambda_N \mu_X$ and $\sigma_S^2 = \lambda_N (\mu_X^2 + \sigma_X^2)$.

• Thus,

$$\frac{\mu_S}{\sigma_S} = \frac{\lambda_N \mu_X}{\sqrt{\lambda_N (\mu_X^2 + \sigma_X^2)}} = \frac{\mu_X \sqrt{\lambda_N}}{\sqrt{\mu_X^2 + \sigma_X^2}}.$$
(6.13)

• Applying normal approximation to the distribution of the aggregate loss S, equation (6.12) can be written as

$$\Pr(\mu_S - k\mu_S \le S \le \mu_S + k\mu_S) \simeq 2\Phi\left(\frac{k\mu_S}{\sigma_S}\right) - 1$$
$$= 2\Phi\left(\frac{k\mu_X\sqrt{\lambda_N}}{\sqrt{\mu_X^2 + \sigma_X^2}}\right) - 1.$$

• For the above probability to be at least $1 - \alpha$, we must have

$$\frac{k\mu_X\sqrt{\lambda_N}}{\sqrt{\mu_X^2 + \sigma_X^2}} \ge z_{1-\frac{\alpha}{2}},\tag{6.15}$$

so that

$$\lambda_N \ge \left(\frac{z_{1-\frac{\alpha}{2}}}{k}\right)^2 \left(\frac{\mu_X^2 + \sigma_X^2}{\mu_X^2}\right). \tag{6.16}$$

• Thus, the standard for full credibility for aggregate loss is

$$\left(\frac{z_{1-\frac{\alpha}{2}}}{k}\right)^2 \left(\frac{\mu_X^2 + \sigma_X^2}{\mu_X^2}\right) = \left(\frac{z_{1-\frac{\alpha}{2}}}{k}\right)^2 \left(1 + \frac{\sigma_X^2}{\mu_X^2}\right)$$
$$= \lambda_F \left(1 + C_X^2\right). \quad (6.17)$$

• We note that

$$\lambda_F \left(1 + C_X^2 \right) = \lambda_F + \lambda_F C_X^2, \tag{6.18}$$

so that

Standard for full credibility for aggregate loss = Standard for full credibility for claim frequency + Standard for full credibility for claim severity **Example 6.8:** A block of health insurance policies has estimated mean severity of 25 and variance of severity of 800. For $\alpha = 0.15$ and k = 0.08, calculate the standard for full credibility for claim frequency and aggregate loss. Assume the claim frequency follows a Poisson distribution and normal approximation can be used for the claim-frequency and aggregate-loss distributions. If the block has an expected number of claims of 400 for the next period, is full credibility attained?

Solution: As $z_{0.925} = \Phi^{-1}(0.925) = 1.4395$, from equation (6.7) we have

$$\lambda_F = \left(\frac{1.4395}{0.08}\right)^2 = 323.78,$$

which is the standard for full credibility for claim frequency. The coefficient of variation of the claim severity is

$$\frac{\sqrt{800}}{25} = 1.1314.$$

Thus, using equation (6.17) the standard for full credibility for aggregate loss is

$$(323.78) \left[1 + (1.1314)^2 \right] = (323.78)(2.28) = 738.24,$$

which is 2.28 times that of the standard for full credibility for claim frequency. The expected number of claims for the next period, 400, is larger than 323.78 but smaller than 728.22. Thus, full credibility is attained for the risk group for claim frequency but not for aggregate loss. \Box

6.3 Partial Credibility

- When the risk group is not sufficiently large, full credibility cannot be attained.
- In this case, a value of Z < 1 has to be determined.
- Denoting generically the loss measure of interest by W, the basic assumption in deriving Z is that the probability of ZW lying within the interval $[Z\mu_W - k\mu_W, Z\mu_W + k\mu_W]$ is $1 - \alpha$ for a given value of k.
- For the case when the loss measure of interest is the claim frequency N, we require

$$\Pr\left(Z\mu_N - k\mu_N \le ZN \le Z\mu_N + k\mu_N\right) = 1 - \alpha, \qquad (6.20)$$

which upon standardization becomes

$$\Pr\left(\frac{-k\mu_N}{Z\sigma_N} \le \frac{N-\mu_N}{\sigma_N} \le \frac{k\mu_N}{Z\sigma_N}\right) = 1 - \alpha.$$
(6.21)

• Assuming Poisson claim-frequency distribution with mean λ_N and applying the normal approximation, the left-hand side of the above equation reduces to

$$2\Phi\left(\frac{k\sqrt{\lambda_N}}{Z}\right) - 1. \tag{6.22}$$

• Thus, we have

$$\frac{k\sqrt{\lambda_N}}{Z} = z_{1-\frac{\alpha}{2}},\tag{6.23}$$

so that

$$Z = \left(\frac{k}{z_{1-\frac{\alpha}{2}}}\right)\sqrt{\lambda_N} = \sqrt{\frac{\lambda_N}{\lambda_F}}.$$
 (6.24)

- Equation (6.24) is called the square-root rule for partial credibility.
- For predicting claim frequency the rule states that the **partialcredibility factor** Z is the square root of the ratio of the expected claim frequency to the standard for full credibility for claim frequency.
- The partial credibility factors for claim severity, aggregate loss and pure premium are summarized below:

Claim severity:

$$Z = \sqrt{\frac{N}{\lambda_F C_X^2}}$$

Aggregate loss/Pure premium:

$$Z = \sqrt{\frac{\lambda_N}{\lambda_F (1 + C_X^2)}}$$

Example 6.11: A block of insurance policies had 896 claims this period with mean loss of 45 and variance of loss of 5,067. Full credibility is based on a coverage probability of 98% for a range of within 10% deviation from the true mean. The mean frequency of claim is 0.09 per policy and the block has 18,600 policies. Calculate Z for the claim frequency, claim severity and aggregate loss of the next period.

Solution: The expected claim frequency is $\lambda_N = 18,600(0.09) = 1,674$. We have $z_{0.99} = \Phi^{-1}(0.99) = 2.3263$, so that the full-credibility standard for claim frequency is

$$\lambda_F = \left(\frac{2.3263}{0.1}\right)^2 = 541.17 < 1,674 = \lambda_N.$$

Thus, for claim frequency there is full credibility and Z = 1. The esti-

mated coefficient of variation of claim severity is

$$C_X = \frac{\sqrt{5,067}}{45} = 1.5818,$$

so that the standard for full credibility for claim severity is

$$\lambda_F C_X^2 = (541.17)(1.5818)^2 = 1,354.13,$$

which is larger than the sample size 896. Hence, full credibility is not attained for claim severity. The partial credibility factor is

$$Z = \sqrt{\frac{896}{1,354.13}} = 0.8134$$

For aggregate loss, the standard for full credibility is

$$\lambda_F(1+C_X^2) = 1,895.23 > 1,674 = \lambda_N.$$

Thus, full credibility is not attained for aggregate loss, and the partial credibility factor is

$$Z = \sqrt{\frac{1,674}{1,895.23}} = 0.9398.$$