Reputation Building through Failure

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Revised: January 2015‡

Abstract

In China, many entrepreneurs receive strong supports each time their business fails. This contradicts existing literature and differs from rare revival elsewhere. The major explanation lies in China’s unfriendly and unstable policy environments, due to which business failure per se cannot discern competence. Therefore, entrepreneurs failing because of policy shocks have the incentive for extra efforts to build reputation of competence and trustworthiness. This mechanism prepares a pool of seasoned entrepreneurs who can help alleviate damages of not only policy shocks, but also such system shocks as business cycle and sector upgrading, and therefore makes the economy more adaptable.

JEL classification: C73, D72, D83
Keywords: Reputation Building, Entrepreneurs, Failure, Shock, Revival

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‡ We are grateful to the “CASS Senior Fellow Program,” the Hoover Institution, and the Institute for Research in Social Sciences at Stanford University. Yi Zhang is grateful to Singapore Management University research grant C244/MSS7E014 for financial support. The usual disclaimer applies.
1 Introduction

We build a theoretical model to solve an empirical puzzle: why in China many entrepreneurs, who fail almost completely several times, get strong supports each time after their business fails?

In our in-depth interviews with two successful entrepreneurs in Zhongyang County in North China in October 2007, we were surprised to notice that both of them had failed several times in their long way of entrepreneurship. At times, they lost almost all their capital accumulated. However, they managed to revive rather quickly and others were willing to help them with their businesses. Thereafter, we find out that many entrepreneurs in China have similar experiences of failures and revivals.

This phenomenon, on the one hand, contradicts existing literature. According to the basic economic wisdom, there should be little investment wasted for the ex-losers because they should be less competent than the average entrepreneurs. Compared with “fresh” entrepreneurs who have never been financed, “young” entrepreneurs whose projects were financed for the first time but failed should be deemed less competent. Therefore, the investors should turn to the pool of “fresh” entrepreneurs rather than giving the failed “young” entrepreneurs a second chance.\(^1\)

It, on the other hand, differs from the phenomena of rare revival in many other countries. Taking France and Japan for example, entrepreneurs who failed can very rarely get a second chance for supports from others. Therefore reputation can be built almost only through success.\(^2\)

The major reason for the puzzle lies in the fact that besides entrepreneur competence, social economic environmental factors also play important roles in the successes and failures of businesses. In China, due to unfriendly and unstable policy environments, business failures per se quite often cannot discern the competence of entrepreneurs.\(^3\) Therefore those entrepreneurs fail due to policy shocks have the incentive to make extra efforts beyond their obligations to minimize their supporters’ losses for a reputation of competence and trustworthiness while investors have the incentive to support such entrepreneurs for future businesses. This phenomenon of reputation building through failure helps partially alleviate the disadvantages of the unfriendly and unstable policy environments. In addition, as an unintended conse-

\(^1\)In Hörner (2002), the equilibrium requires the worst punishment – a firm should be abandoned for merely one bad outcome.

\(^2\)Robert Boyer told the authors that in France people even think failed entrepreneurs are irresponsible.

\(^3\)The correlation between success and competence in China is further twisted by the degree of personal connections, which differentiate access to government funds for revival, with powerful officials. We will not incorporate this variable in our model for two reasons besides consideration for simplicity. The first reason is that only exceptionally few privileged entrepreneurs can get insider information to avoid policy shocks that are often initiated by a tiny circle. The second reason is that connection only skews rather than changes the correlation between success and competence.
quence, entrepreneurs survive over failures caused by policy shocks can benefit from their experiences against such system shocks as business cycle and sector upgrading, and therefore make the economy more adaptable.

There are several related strands of literature. The first is on individual reputation. “Bad reputation” models (e.g., Ely and Valimaki 2003; Ely, Fudenberg, and Levine 2008) argue that to avoid falling into the non-trade zone, good type long-run players have to invest in their reputation. Foreseeing this, short-run players choose not to participate. Holmstrom (1999) investigates the dynamic incentive problem – the agent has the strongest incentive to work hard to reveal his managerial ability. As time goes by, his ability is learned, and thus the reputation effect on incentive also decreases. Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Levine (1989, 1992), Ely, Fudenberg and Levine (2004), and others investigate the settings of a single long-run player and a sequence of short-run opponents – the long-run player tries to commit to certain reputation type to achieve highest possible utility. Hörner (2002) introduces competition to keep high efforts sustainable.

The second is on group reputation. Diamond (1989) constructs a model in debt markets. His key point is that while bad type drops out as time goes by, the reputation for the remaining agents is driven up. Wang and Zhang (2014) provide a game theoretical model of group reputation concerning reputation matching in different social contexts. They show that the regime change from acquaintance matching in small communities to anonymous matching in complex societies tends to cause rampant corruption and the effectiveness of anti-corruption policies is non-monotonic with respect to supervision efforts.

The third is on how managerial reputation affects venture finance. Shane and Cable (2002) argue that to obtain external financing, investors often rely on social ties, through which to get access to private information and social obligations regarding wealth constrained entrepreneurs. Hirshleifer (1993) provides a thorough review of managerial reputation and investment decision. But he does not address the issue of social tolerance, which is an endogenous equilibrium result of the strategic interactions among the investors and entrepreneurs, with respect to failure.

This paper is structured as follows. Section 2 analyzes the equilibrium outcome of entrepreneur survival when affected only by entrepreneurial competence. Section 3 analyzes the mechanism of reputation building through failure by introducing the effects of idiosyncratic shocks. Section 4 discusses how entrepreneurial revival systematically benefits the economy. Section 5 concludes.

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4 Our model is also related to literature on the repeated game with imperfect public monitoring. See Mailath and Samuelson (2006) for detailed analysis.
5 For the literature on reputation loss and recovery, see a recent paper by Fujiwara-Greve, Greve and Jonsson (2014).
6 See Fischer and Reuber (2007) for the reputation formation facing new firms.
2 Competence-Only Model

2.1 Basic Environment

In this section, we construct a competence-only model to analyze an investment environment, in which entrepreneurs seek financing for their investment projects from investors repeatedly. The notations of Mailath and Samuelson (2006) and Hörner (2002) are employed. Time is discrete, indexed by \( t = 0, 1, \ldots \) and the horizon is infinite. There is a continuum population of long-lived entrepreneurs with common discount factor \( \delta \in (0, 1) \), who may be the type of “competent” or “incompetent,” denoted as type “G” and “B” respectively. There is a continuum of investors with types indexed by \( m \in (0, \infty) \), which follows some distribution \( \Phi \). An investor’s type is defined by her opportunity cost of participation. Suppose investors are short-lived. That is, each generation of short-run investors play only in one period, and are replaced by another generation of short-run investors in the next period. Thus, the game is a setting of a group of long-run players and a sequence of short-run counterparts’ generations.

In each period, each entrepreneur proposes a project and investors choose which project to invest on or opt out. In the latter case, investors get reservation payoff, normalized to zero. Suppose an investor chooses to invest on an entrepreneur’s project. Then the payoff is 1 if the project succeeds and 0 if the project fails. For a “competent” type entrepreneur, the probability for his project to be successful is \( \rho_G \). For an incompetent entrepreneur, the probability for his project to be successful is \( \rho_B \). Assume \( 0 < \rho_B < \rho_G < 1 \). The payoff for the successful entrepreneur is \( V \), and the payoff for the failed entrepreneur is \( W \), with \( V > W > 0 \). If no one invests on the project of an entrepreneur, he gets zero.

The common prior proportion of competent entrepreneur among the population is \( \mu_0 \). For simplicity, assume all players are uninformed regarding the types of entrepreneurs. The only public history observed is whether or not an entrepreneur is financed, and if financed, whether the project succeeds or fails. Till this point, entrepreneurs are passive. Investors are competing with other investors in the same generation after observing the previous records of entrepreneurs. Further, entrepreneurs alive in period \( t \) remain in the economy in period \( t + 1 \) with probability \( \lambda \in (0, 1) \). Each quit is offset by the arrival of a new entrepreneur with the common prior proportion \( \mu_0 \) of competence. Therefore, the population size of entrepreneurs remains constant.
2.2 Stage Game

Consider the stage game in period $t$. The historical record of an entrepreneur at the beginning of period $t$ is denoted by $(i,j)_t$: $i$ successes and $j$ failures, where $0 \leq i \leq t, 0 \leq j \leq t, i + j \leq t$. Let $\mu^{(i,j)}_t$ represent investors’ belief at the beginning of period $t$ for the entrepreneurs with history $(i,j)_t$ to be competent. By Bayes’ rule, the following remark says that if there are more successes, then the posterior belief to be competent is higher; if there are more failures, then the posterior belief to be competent is lower.

Remark 1

$$\forall k > i : \mu^{(i,j)}_t < \mu^{(k,j)}_t$$
$$\forall l > j : \mu^{(i,j)}_t > \mu^{(i,j)}_t$$

As an inference of the remark above, $\mu^{(0,1)}_t < \mu^{(0,0)}_t$. This implies that at the beginning of any period $t$, investors believe that the “young” entrepreneurs whose projects were financed for the first time but failed are less competent than the “fresh” entrepreneurs who have never been financed.

An equilibrium of this game refers to a Perfect Bayesian Equilibrium in symmetric, pure, Markovian strategies. Markovian strategies are strategies in which investors’ decisions depend only on their own beliefs about the entrepreneurs’ types. At the beginning of period $t$, the expected return investing on entrepreneurs with history $(i,j)_t$ is $r(\mu^{(i,j)}_t) = \mu^{(i,j)}_t \rho_G + (1-\mu^{(i,j)}_t) \rho_B$, and the net expected return is $r(\mu^{(i,j)}_t) - m$ for investor $m$. The total amount of investment in period $t$, denoted by $x_t$, depends on the competition among investors and entrepreneurs. Denote the mass of entrepreneurs with history $(i,j)_t$ who are financed in period $t$ as $N^{(i,j)}_t$. Clearly, $\sum_{i,j} N^{(i,j)}_t = x_t$.

Note, $\mu^{(i,j)}_t$ has the property of Markovian: the order of successes and failures does not matter; only the number of successes and failures counts.\(^7\)

At the beginning of period $t$, investors have an ordering of $\mu^{(i,j)}_t$. Suppose after the competition among investors and entrepreneurs, eventually there exists a marginal type of entrepreneurs with history $(i,j)_t$, such that entrepreneurs with history $(i,j)_t$, where $\mu^{(i,j)}_t > \mu^{(i,j)}_t$, will be financed; all others with history $(i,j)_t$, where $\mu^{(i,j)}_t < \mu^{(i,j)}_t$, will not be financed. The expected return investing on the marginal type $(i,j)_t$ is $r(\mu^{(i,j)}_t) = \mu^{(i,j)}_t \rho_G + (1-\mu^{(i,j)}_t) \rho_B$, and the net expected return is $r(\mu^{(i,j)}_t) - m$ for investor $m$. Thus, the total amount of investment in period $t$, $x_t = \Phi \left( r(\mu^{(i,j)}_t) \right)$.

Due to the competition among investors, they must pay some premium $d^{(i,j)}_t$ to the entrepreneurs with history $(i,j)_t$, where $\mu^{(i,j)}_t > \mu^{(i,j)}_t$. Market clearing requires

\(^7\)To avoid triviality, assume the population size of entrepreneurs and the mass of investors are sufficiently large to ensure the market has an interior equilibrium as in Mailath and Samuelson (2006).
For the marginal type \((i,j)_t\), at least some will be financed while others will not, depending upon the availability of investors.\(^8\) For the transition from period \(t\) to period \(t+1\), at the beginning of period \(t+1\), the historical record for those who were financed in period \(t\), with \(\mu^{(i,j)_t} \geq \mu^{(i,j)_t}\), will be either \((i+1,j)_t\) or \((i,j+1)_t\); at the beginning of period \(t+1\), the historical record for those who were not financed in period \(t\), with \(\mu^{(i,j)_t} \leq \mu^{(i,j)_t}\), will be \((i,j)_{t+1}\).

### 2.3 Dynamic Game

The following lemma says that the marginal type will be the “fresh” entrepreneurs who have never been financed.

**Lemma 1** \((i,j)_t = (0,0)_t\)

**Proof.** See the Appendix. ■

Intuitively, at the beginning of period 0, there is no historical record, \(\mu^{(0,0)}_0 = \mu_0\). The marginal type of entrepreneurs \((i,j)_0 = (0,0)_0\). The corresponding expected return is \(r(\mu^{(0,0)}_0) = r(\mu_0) = \mu_0\rho_G + (1 - \mu_0)\rho_B\). Investors with the opportunity cost of participation \(m \leq \mu_0\rho_G + (1 - \mu_0)\rho_B\) will have incentive to invest. The total amount of investment in period 0, \(x_0 = \Phi(r(\mu_0)) = N^{(0,0)}_0\).

By induction, assume at the beginning period of \(t-1\) the marginal type of entrepreneurs \((i,j)_{t-1} = (0,0)_{t-1}\). Let’s check if it is true that at the beginning period of \(t\) the marginal type of entrepreneurs \((i,j)_t = (0,0)_t\). As \((i,j)_{t-1} = (0,0)_{t-1}, x_{t-1} = x_0\). As assumed, entrepreneurs alive in period \(t-1\) remain in the economy in period \(t\) with probability \(\lambda \in (0,1)\). By Bayesian updating, the mass of entrepreneurs with \((i,j)_t\) such that \(\mu^{(i,j)_t} > \mu_0\) is less than \(\lambda x_0\). Further, as the population size of entrepreneurs are sufficiently large, entrepreneurs with \((0,1)_t\) drop out. Investors will turn to the pool of “fresh” entrepreneurs rather than giving a second chance to the failed “young” entrepreneurs. One failure will cause the “young” entrepreneurs “drop-out” of the race. Consequently, the marginal type of entrepreneurs \((i,j)_t = (0,0)_t\).

Similar to period 0, investors with the opportunity cost of participation \(m \leq r(\mu_0) = \mu_0\rho_G + (1 - \mu_0)\rho_B\) will have incentive to invest on the entrepreneurs with history \((i,j)_t\) such that \(\mu^{(i,j)_t} > \mu^{(0,0)_t} = \mu_0\). Since \(\sum_{i,j} N^{(i,j)_t}_{\mu^{(i,j)_t} > \mu_0} < \lambda x_0 < x_0\), there are some “fresh” entrepreneurs being financed in period \(t\). Still, \(x_t = \Phi(r(\mu^{(0,0)_t})) = \Phi(r(\mu_0)) = x_0\). Consequently, we have the following proposition. The level of investment over time is constant at \(x_0 = \Phi(r(\mu_0))\).

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\(^8\)For simplicity, assume investors will invest on entrepreneurs with higher \(\mu^{(i,j)_t}\), even though the net expected return is the same from all entrepreneurs with \(\mu^{(i,j)_t} > \mu^{(i,j)_t}\) after paying the premium \(d^{(i,j)_t}\).
Proposition 1 \( \forall \ t : \ x_t = x_0 = \Phi (r(\mu_0)) \)

2.3.1 Asymptotic Property

The following proposition says that beyond “natural death,” only the competent entrepreneurs will survive while the incompetent entrepreneurs will drop out in the long run. Eventually, we get into a “mature” period, in which investors believe that all “old” active entrepreneurs (who get their projects financed for a long time) are competent. Due to “natural death,” the “old” active entrepreneurs will be replaced by the “fresh” entrepreneurs at the rate \( \lambda \) in each period.

Proposition 2 As \( t \to \infty \),

\[
\begin{align*}
p_t^G & \to p^G \geq \rho_G - \rho_B \\
p_t^B & \to 0
\end{align*}
\]

where \( p_t^G \) is the ex ante probability of competent entrepreneurs surviving after \( t \) periods “trial” beyond “natural death”; \( p_t^B \) is the ex ante probability of incompetent entrepreneurs surviving after \( t \) periods “trial” beyond “natural death.”

Proof. See the Appendix.

Intuitively, from lemma 1, the marginal type is always the “fresh” entrepreneurs. Failed entrepreneurs will drop out if investors’ belief for their competence is smaller than \( \mu_0 \). Further, the share of successes in the amount of trials must be greater than \( \rho_B \) to avoid dropping out in the long run. By the Law of Large Numbers, for the incompetent entrepreneurs, the share of successes in the amount of trials must converge to \( \rho_B \). Therefore, eventually the incompetent entrepreneurs will drop out even if there is no “natural death.”

In contrast, for the competent entrepreneurs, ex ante the probability of dropping out beyond “natural death” in any period is non-negative, which is smaller than that for the incompetent entrepreneurs. Thus, \( p_t^G \) is a decreasing function of \( t \) and bounded below by \( \rho_G - \rho_B \), which is the difference between the probabilities of dropping out in period 1 for the competent and incompetent entrepreneurs.

3 Plus-Shock Model

From section 2, we show that one failure is sufficient to induce the investors to believe that the failed “young” entrepreneurs are less competent than the “fresh” entrepreneurs. Therefore, investors will naturally turn to the latter rather than giving a second chance to the former.

But what if business failure per se cannot discern competence?
3.1 Idiosyncratic Shock

Indeed, besides competence of entrepreneurs, social economic environment factors can also play important roles on the success or failure of projects. Sometimes, it is the unfriendly and unstable policy environments that cause failures. In this case, entrepreneurs may complain that they are not their faults.

Consider a variant from the basic model. Suppose in a society with idiosyncratic shocks, even though the existence and possible impacts of such shocks are common knowledge, the exact impact of an idiosyncratic shock on a specific project can be observed only by the entrepreneur himself. With probability $\pi \in (0,1)$ the project will fail, on top of the competence of the entrepreneurs. Specifically, for a competent entrepreneur, the probability for his project to be successful reduces to $(1-\pi)\rho_G$. For the remaining probability $1-(1-\pi)\rho_G$, the project will fail, in which the project fails due to the idiosyncratic shock with probability $\pi\rho_G$. Similarly, for an incompetent entrepreneur, the probability for his project to be successful reduces to $(1-\pi)\rho_B$. For the remaining probability $1-(1-\pi)\rho_B$, the project will fail, in which the project fails due to the idiosyncratic shock with probability $\pi\rho_B$.

As investors cannot observe the exact impact of an idiosyncratic shock on specific projects, one-failure-out is still prevalent. We are at an even worse situation. The level of investment over time is down to $\Phi((1-\pi)r(\mu_0)) < \Phi(r(\mu_0)) = x_0$.

3.2 Reputation Building through Failure

It looks as if we were stuck in the trap forever. However, an entrepreneur fails due to the unfriendly and unstable policy environments may argue that if it were not for the idiosyncratic shock, he would have been successful. Subsequently, a signalling device may be spontaneously generated, such that entrepreneurs failing for idiosyncratic shocks have the incentive to send out costly signals to separate themselves from those failing for their own incompetence. For instance, after each failure, the failed entrepreneur may make a transfer to the investor, which is public information, to indicate that his failure is due to the idiosyncratic shock and he is trustworthy for future business relationships.

Consider the stage game in period $t$. The public record of an entrepreneur at the beginning of period $t$ is denoted by $(i,j,k)_t$: $i$ successes, $j$ failures, and $k$ “transfer” signals, where $0 \leq i \leq t, 0 \leq j \leq t, i + j \leq t, k \leq j$. Suppose the entrepreneur is financed for his project in period $t$. If the project is successful, he will be continually financed in period $t+1$, as his public record is improved. Instead, if the project fails, he is in jeopardy of dropping out if his public record $(i,j+1,k)_{t+1}$ is worse than the marginal type in period $t+1$. In this case, he may send out the costly signal after the failure, such that his public record becomes $(i,j+1,k+1)_{t+1}$ at the beginning of period $t+1$, to indicate that his failure is due to the idiosyncratic shock and he is
trustworthy for future business relationships.\textsuperscript{9}

Meanwhile, as the exact impact of an idiosyncratic shock on a specific project can be observed only by the entrepreneur himself, after the failure, his private record becomes \((i, j + 1, k, l)_t\), where \(l\) is the number of failures due to idiosyncratic shocks till period \(t\). The following proposition shows that there exists a separating equilibrium, in which only entrepreneurs failing for idiosyncratic shocks \((l > k)\), who are at the brink of dropping out, have the incentive to send out costly signals, while those failing for their own incompetence do not.

**Proposition 3** Consider an entrepreneur with a public record \((i, j, k)_t\) at the beginning of period \(t\). Suppose his project was financed but failed in period \(t\). His private record becomes \((i, j + 1, k, l)_t\), where \(l\) is the number of failures due to idiosyncratic shocks till period \(t\). There exists a \(C^{(i, j + 1, k)_t}\) such that we have a separating equilibrium, in which the failed entrepreneur, who is at the brink of dropping out with \(l > k\), has the incentive to transfer \(C^{(i, j + 1, k)_t}\) to the investor if and only if \(l > k\).

**Proof.** Let us consider the incentive constraint for the entrepreneur whose project failed in period \(t\) and who is at the brink of dropping out in period \(t + 1\) with \(l > k\).

\[-C^{(i, j + 1, k)}_t + \delta \mu^{(i, j + 1, k + 1, l)}_{t+1} > \delta \lambda \mathcal{U}^{(i, j + 1, k, l)}_{t+1} \quad (1)\]

where \(\mathcal{U}^{(i, j + 1, k + 1, l)}_{t+1}\) is the continuation payoff in period \(t + 1\) for the entrepreneur who transfers \(C^{(i, j + 1, k)_t}\) to the investor after period \(t\) failure; \(\mathcal{U}^{(i, j + 1, k, l)}_{t+1}\) is the continuation payoff in period \(t + 1\) for the entrepreneur who does not transfer after period \(t\) failure. According to lemma 1, the marginal type is always the “fresh” entrepreneurs. For the entrepreneur at the brink of dropping out in period \(t + 1\), \(\mu^{(i, j + 1, k + 1, l)}_{t+1} < \mu_0\), and hence \(\mathcal{U}^{(i, j + 1, k, l)}_{t+1} = 0\).

Further, the incentive constraint for the entrepreneur whose project failed in period \(t\) and who is at the brink of dropping out in period \(t + 1\) with \(l \leq k\) is as follows.

\[-C^{(i, j + 1, k)}_t + \delta \lambda \mathcal{U}^{(i, j + 1, k + 1, l)}_{t+1} \leq \delta \lambda \mathcal{U}^{(i, j + 1, k, l)}_{t+1} \quad (2)\]

Similar to remark 1, \(\mathcal{U}^{(i, j + 1, k + 1, l)}_{t+1}\) is (strictly) increasing in \(l\). It is easy to see

\textsuperscript{9}Since investors are short-lived, suppose payoffs for the investors are public information. That is, the subsequent generations of investors can observe the historical payoff records. The costly “responsible” signal only serves the role of signalling and therefore are purely opportunistic behavior. Only in the situation that they believe that the failure will cause them to drop out (no future financing opportunity), do they have the incentive to send out such costly signals.
that there exists a $C^{(i,j+1,k)} = \delta \mathcal{U}_{(i,j+1,k+1,k)}$, such that\footnote{By the intuitive criterion, it is not necessary to set the amount of transfer more than $\delta \mathcal{U}_{(i,j+1,k+1,k)}$.}
\[
\begin{align*}
\forall \ l > k : & \quad -C^{(i,j+1,k)} + \delta \mathcal{U}_{(i,j+1,k+1,l)} > 0 \\
\forall \ l \leq k : & \quad -C^{(i,j+1,k)} + \delta \mathcal{U}_{(i,j+1,k+1,l)} \leq 0
\end{align*}
\]

For the entrepreneur whose project failed in period $t$ and who is at the brink of dropping out in period $t + 1$, inequalities 1 and 2 are satisfied.

Similar to proposition 1, without signaling device, the level of investment over time is down to $\Phi\left((1 - \pi)r(\mu_0)\right)$ due to the idiosyncratic shock. With the signaling device, the entrepreneurs failing for idiosyncratic shocks have the incentive to compensate the investors. Consequently, the level of investment over time is at a higher level.

**Proposition 4** $\forall \ t : x_t = \Phi\left(\left[(1 - \pi) + \pi C^{(0,1,0)}\right] r(\mu_0)\right)$

Intuitively, the amount of investment depends on the marginal type which is the “fresh” entrepreneurs. As in proposition 3, those “young” entrepreneurs, who were financed but failed due to the idiosyncratic shock in period $t$, have the incentive to compensate the investors $C^{(0,1,0)}$ to avoid dropping out. Therefore, the investment is lower due to the idiosyncratic shock, but partially alleviated through the signaling process.

4 Benefits of Learning

For entrepreneurs who failed due to unfriendly and unstable policy environments, the experiences of failure provide them not only vivid information on the possibilities and damage making mechanisms of idiosyncratic shocks, but also the incentive to reflect on how to survive against idiosyncratic shocks and on how to mitigate damages when failure is inevitable. In addition, if the failed entrepreneurs send out costly signals and revive under supports of investors, they also acquire knowledge of trust building. Suppose the reviving entrepreneurs learn from their experience and learning by experiencing has certain irreplaceable values, then they should have higher expected returns in business than average. If they apply some lessons learned from policy shocks against other system shocks, they should be at a greater advantage than average in times of such system shocks as business cycle and sector upgrading. Aggregately, there should also be system level unintended consequences: ceteris paribus, compared with an economy where revival is rare, an economy with a pool of seasoned entrepreneurs experienced with revival should have higher expected returns and be more adaptable to system shocks.
4.1 Higher Expected Returns

Suppose revived entrepreneurs learn from their experience, the impact of an idiosyncratic shock on their future projects should be mitigated. To simplify, we measure learning with the number of “transfer” signals $k$. Therefore, the impact of an idiosyncratic shock $\pi(k)$ should be strictly decreasing in $k$.

Obviously, we have the following proposition: compared with the case without learning, the expected return is higher with learning for $k > 0$.

**Proposition 5** \( r(\mu^{(i,j,k)}_t) \) is higher with learning for $k > 0$.

At the beginning of period $t$, with learning, the expected return investing on entrepreneurs with public history $(i, j, k)_t$ is

\[
 r(\mu^{(i,j,k)}_t) = (1 - \pi(k))[\mu^{(i,j,k)}_t \rho_G + (1 - \mu^{(i,j,k)}_t) \rho_B] \\
+ \pi(k)[\mu^{(i,j,k)}_t \rho_G + (1 - \mu^{(i,j,k)}_t) \rho_B]C^{(i,j+1,k)}_t \mathbf{1}_{\text{signaling}} \\
= [\mu^{(i,j,k)}_t \rho_G + (1 - \mu^{(i,j,k)}_t) \rho_B][1 - \pi(k)(1 - C^{(i,j+1,k)}_t \mathbf{1}_{\text{signaling}})]
\]

where $\mathbf{1}_{\text{signaling}}$ is the indicator function of the entrepreneur sending costly signal.\footnote{Without loss of generality, assume $C^{(i,j+1,k)}_t$ is less than 1, the payoff from a successful project.}

Similarly, without learning, for $k > 0$

\[
 r(\mu^{(i,j,k)}_t) = [\mu^{(i,j,k)}_t \rho_G + (1 - \mu^{(i,j,k)}_t) \rho_B][1 - \pi(1 - C^{(i,j+1,k)}_t \mathbf{1}_{\text{signaling}})] \\
< [\mu^{(i,j,k)}_t \rho_G + (1 - \mu^{(i,j,k)}_t) \rho_B][1 - \pi(k)(1 - C^{(i,j+1,k)}_t \mathbf{1}_{\text{signaling}})]
\]

where $\pi = \pi(0) > \pi(k)$.

Consequently, with learning, those entrepreneurs sending out costly signals have higher expected returns and therefore are less likely to drop out and account for a larger share among the survivors. Hence, we have a virtuous circle. For the economy as a whole, the expected return will be even higher on average. These are summarized in the following corollary.

**Corollary 1** With learning, those entrepreneurs sending out costly signals account for a larger share among the survivors. For the economy as a whole, the expected return will be higher on average.

4.2 System Shock Alleviation

Since the “process of Creative Destruction is the essential fact about capitalism,” system shocks such as economic cycle and sector upgrading are inevitable for modern
market economies (Schumpeter 1994 [1942], 83). Suppose in each period with probability \( \psi \in (0, 1) \) an economy will be hit by a system shock, in which case the entire economy may collapse. Assume failed entrepreneurs sending out costly signals may learn from their failures about policy shocks and reflect on possible ways to handle such shocks. Therefore, the impact of the system shock on the economy can be mitigated with learning: \( \psi(\bar{k}) \) strictly decreasing in \( \bar{k} \), where \( \bar{k} \) is the average number of “transfer” signals among the population of entrepreneurs who are financed.

From corollary 1, with learning, those entrepreneurs sending out costly signals account for a larger share among the survivals. Consequently, we have the following proposition: compared with the case without learning, the chance for the economy to collapse is decreasing in case of crisis.

**Proposition 6** With learning, \( \bar{k} \) is larger and consequently \( \psi(\bar{k}) \) is smaller.

This indicates that struggling in the unfriendly and unstable policy environments may not be that bad. Even though the entrepreneurs may suffer from idiosyncratic shocks, they could develop some signaling devices to alleviate the disadvantages. Further, through learning, there will be a larger share of “experienced” entrepreneurs in the economy and therefore a lower chance for the economy to collapse in case of crisis.

5 Conclusion

Our model offers a new theoretical explanation, reputation building through failure, to a largely overlooked phenomenon of entrepreneur revival. In societies, such as China, with unfriendly and unstable policy environments where business failures per se cannot discern the competence of entrepreneurs, entrepreneurs fail due to policy shocks have the incentive to make extra efforts to signal their competence and trustworthiness while investors have the incentive to support such entrepreneurs for future businesses. The spontaneously generated signaling mechanism of reputation building through failure helps partially alleviate the disadvantages of the unfriendly and unstable policy environments. As an unintended consequence, the “experienced” entrepreneurs produced through this mechanism also help to decrease the probability of system failure in case of such inevitable system shocks as economic cycle and sector upgrading. More empirical evidence can be applied to test our results. In addition, since pure risk is another reason, besides system shock, for competent entrepreneurs to fail, institutional arrangements friendly for revival to encourage risk taking entrepreneurship can bring similar reputation building mechanism without paying the costs of policy shocks. The comparison between these two mechanisms and their impacts should be significant theoretically.
Appendix

Proof of Lemma 1

At the beginning of period 0, there is no historical record, \( \mu^{(0,0)_0} = \mu_0 \). The marginal type of entrepreneurs \((i,j) = (0,0)\). The corresponding expected return is \( r(\mu^{(0,0)_0}) = r(\mu_0) = \mu_0 \rho_G + (1 - \mu_0) \rho_B \). Investors with the opportunity cost of participation \( m \leq \mu_0 \rho_G + (1 - \mu_0) \rho_B \) will have incentive to invest. The total amount of investment in period 0, \( x_0 = \Phi(r(\mu_0)) = \sum_{i,j} N^{(i,j)_0} \). As assumed, entrepreneurs alive in period 0, \( m \leq \mu_0 \rho_G + (1 - \mu_0) \rho_B \) will have incentive to invest. The total amount of investment in period 0, \( x_0 = \Phi(r(\mu_0)) = \sum_{i,j} N^{(i,j)_0} \).

Continue to period 1 game. The historical record \((i,j) \in \{(1,0)_1, (0,0)_1, (0,1)_1\}\) at the beginning of period 1. By remark 1, we have \( r(\mu^{(1,0)_1}) > r(\mu^{(0,0)_1}) > r(\mu^{(0,1)_1}) \). As assumed, entrepreneurs alive in period \( t - 1 \) remain in the economy in period \( t \) with probability \( \lambda \in (0,1) \). By Bayesian updating, the mass of entrepreneurs with \((1,0)_1\) is \( \lambda x_0 [\mu_0 \rho_G + (1 - \mu_0) \rho_B] = \lambda r(\mu_0)x_0 < x_0 \). Further, as the population size of entrepreneurs are sufficiently large, entrepreneurs with \((0,1)_1\) drop out. Investors will turn to the pool of “fresh” entrepreneurs rather than giving a second chance to the failed “young” entrepreneurs. Consequently, the marginal type of entrepreneurs \((i,j)_1 = (0,0)_1\). Entrepreneurs with \((1,0)_1\) will be continually financed in period 1.

Due to competition among entrepreneurs, there is no premium paid to the entrepreneurs with \((0,0)_1\): \( d^{(0,0)_1} = 0 \). Since \( N^{(1,0)_1} = \lambda r(\mu_0)x_0 < x_0 \), entrepreneurs with \((1,0)_1\) will get some premium. Market clearing makes \( d^{(0,0)_1} = r(\mu^{(1,0)_1}) - r(\mu^{(0,0)_1}) \), such that the expected returns from both \((0,0)_1\) and \((1,0)_1\) are \( r(\mu_0) = \mu_0 \rho_G + (1 - \mu_0) \rho_B \). Investors with the opportunity cost of participation \( m \leq r(\mu_0) = \mu_0 \rho_G + (1 - \mu_0) \rho_B \) will have incentive to invest on the entrepreneurs with \((0,0)_1\) and \((1,0)_1\). The total amount of investment in period 1, \( x_1 = \Phi(r(\mu^{(1,0)_1})) = \Phi(r(\mu_0)) = x_0 \).

By induction, assume at the beginning of period \( t - 1 \) the marginal type of entrepreneurs \((i,j)_{t-1} = (0,0)_{t-1}\)\). Let’s check if it is true that at the beginning period \( t \) the marginal type of entrepreneurs \((i,j)_t = (0,0)_t\). As \((i,j)_{t-1} = (0,0)_{t-1}, x_{t-1} = x_0 \). Again, entrepreneurs alive in period \( t - 1 \) remain in the economy in period \( t \) with probability \( \lambda \in (0,1) \). By Bayesian updating, the mass of entrepreneurs with \((i,j)_t\) such that \( r(\mu^{(i,j)_t}) > \mu_0 \) is less than \( \lambda x_0 \). Similarly, as the population size of entrepreneurs are sufficiently large, entrepreneurs with \((0,1)_t\) drop out. Investors will turn to the pool of “fresh” entrepreneurs rather than giving a second chance to the failed “young” entrepreneurs. Consequently, the marginal type of entrepreneurs \((i,j)_t = (0,0)_t\). Entrepreneurs with \((i,j)_t\) such that \( r(\mu^{(i,j)_t}) > \mu_0 \) will be continually financed in period 1.

Due to competition among entrepreneurs, there is no premium paid to the entrepreneurs with \((0,0)_t\): \( d^{(0,0)_t} = 0 \). Since \( \sum_{i,j} N^{(i,j)_t} |_{\mu^{(i,j)_t} > \mu_0} < \lambda x_0 < x_0 \), en-
trepreneurs with history \((i,j)_t\), where \(\mu^{(i,j)_t} \geq \mu^{(i,j)_t} \geq \mu_0\) will get some premium. Market clearing makes \(d^{(i,j)_t} = \mu^{(i,j)_t} - \mu_0\). Investors with the opportunity cost of participation \(m \leq r(\mu_0) = \mu_0\rho_B + (1 - \mu_0)\rho_B\) will have incentive to invest on the entrepreneurs with history \((i,j)_t\) such that \(\mu^{(i,j)_t} \geq \mu^{(0,0)} = \mu_0\). There are some “fresh” entrepreneurs being financed in period \(t\). Still, \(x_t = \Phi(r(\mu^{(0,0)_t})) = \Phi(r(\mu_0)) = x_0\). □

Proof of Proposition 2

Consider the surviving rate of the competent and incompetent entrepreneurs beyond “natural death” in the long run. As a tie breaking rule, assume if investors are indifferent between entrepreneurs with historical record \((i,j)_t\) and the "fresh" entrepreneurs with record \((0,0)_t\), they will choose to invest on the former. From lemma 1, the marginal type is always the “fresh” entrepreneurs: \((i,j)_t = (0,0)_t\). Therefore, entrepreneurs with historical record \((i,j)_t\) such that \(\mu^{(i,j)_t} \geq \mu_0\) will be continually financed, while entrepreneurs with historical record \((i,j)_t\) such that \(\mu^{(i,j)_t} < \mu_0\) will not be financed.

Thus, entrepreneurs will drop out in period \(t\) only if the historical record is \((i,j)_t\) such that

\[
\begin{align*}
\mu^{(i,j-1)_{t-1}} & \geq \mu_0 \\
\mu^{(i,j)_t} & < \mu_0
\end{align*}
\]

Once an entrepreneur drops in period \(t\), he will never be financed again in the future. Investors will turn to the pool of “fresh” entrepreneurs.

Consider a competent entrepreneur who is financed in period \(0\). Suppose he is continually financed till the beginning of period \(t\). Let \(i_t\) represent the number of successes for this entrepreneur till the beginning of period \(t\). Then the number of failures for this entrepreneur at the same time will be \(t - i_t\). From inequality 3, we can identify the sequence of \(\{i_1^*, i_2^*, \ldots, i_t^*, \ldots\}\) such that if \(i_t < i_t^*\), he will drop out in period \(t\).

By Bayes’ rule, from inequality 3, \(i_t^*\) must satisfy the following inequalities.

\[
\begin{align*}
\mu^{i_t,t-i_t-1} &= \frac{\mu_0 \rho_G^{i_t}(1 - \rho_G)^{t-i_t-1}}{\mu_0 \rho_G^{i_t}(1 - \rho_G)^{t-i_t-1} + (1 - \mu_0) \rho_B^{i_t}(1 - \rho_B)^{t-i_t-1}} \geq \mu_0 \\
\mu^{i_t,t-i_t} &= \frac{\mu_0 \rho_G^{i_t}(1 - \rho_G)^{t-i_t}}{\mu_0 \rho_G^{i_t}(1 - \rho_G)^{t-i_t} + (1 - \mu_0) \rho_B^{i_t}(1 - \rho_B)^{t-i_t}} < \mu_0
\end{align*}
\]

These imply

\[
\rho_G^{i_t}(1 - \rho_G)^{t-i_t} < \rho_B^{i_t}(1 - \rho_B)^{t-i_t} \leq \rho_G^{i_t}(1 - \rho_G)^{t-i_t} \left(\frac{1 - \rho_B}{1 - \rho_G}\right)
\]
Denote \( y^*_t = \frac{i^*_t}{t} \) as the share of successes in the amount of trials. The inequality above can be rewritten as
\[
\left( \frac{1 - \rho_G}{1 - \rho_B} \right) < \left( \frac{\rho_B (1 - \rho_G)}{\rho_G (1 - \rho_B)} \right)^{y^*_t} \leq \left( \frac{1 - \rho_G}{1 - \rho_B} \right)^{1 - 1/t} \tag{4}
\]

Define \( q^*_G \) as the ex ante probability of competent entrepreneurs who were financed in period 0 dropping out in period \( t \). At the beginning of period 1, if he failed in period 0, with record \((0, 1)\) he will drop out; if he succeeded in period 0, with record \((1, 0)\) he will be continually financed. The probability of dropping out in period 1 is \( q^*_1 = 1 - \rho_G \). If he was financed in period 1, at the beginning of period 2, his record will be either \((2, 0)\) or \((1, 1)\). As we assumed before, he will be continually financed in period 2. The probability of dropping out in period 2 is \( q^*_2 = 0 \). Continuing to period \( t \), we have \( q^*_t \geq 0 \).

Define \( p^*_t \) as the probability of competent entrepreneurs surviving after \( t \) periods “trial.” From the analysis above, we have \( p^*_1 = 1 - \sum_{s=1}^{t} q^*_s \).

Similarly, define \( q^*_B \) as the ex ante probability of incompetent entrepreneurs who was financed in period 0 dropping out in period \( t \). We have \( q^*_1 = 1 - \rho_B \), \( q^*_2 = 0 \), and continuing to period \( t \), \( q^*_t \geq 0 \). Define \( p^*_t \) as the probability of incompetent entrepreneurs surviving after \( t \) periods “trial” and we have \( p^*_1 = 1 - \sum_{s=1}^{t} q^*_s \).

Let us consider the asymptotic property of \( p^*_t \) and \( p^*_t \) as \( t \to \infty \). Clearly, both \( p^*_t \) and \( p^*_t \) are decreasing functions of \( t \) and bounded below by 0. Thus, they must converge. Suppose as \( t \to \infty \), \( p^*_t \to p^*_G \) and \( p^*_t \to p^*_B \).

By the Log Sum Inequality, we have
\[
\rho_B \ln \left( \frac{\rho_B}{\rho_G} \right) + (1 - \rho_B) \ln \left( \frac{1 - \rho_B}{1 - \rho_G} \right) > 0
\]
which implies
\[
\left( \frac{\rho_B}{\rho_G} \right)^{\rho_B} > \left( \frac{1 - \rho_G}{1 - \rho_B} \right)^{1 - \rho_B}
\]
Rearrange and for \( y^*_t \leq \rho_B \) we have
\[
\left( \frac{\rho_B (1 - \rho_G)}{\rho_G (1 - \rho_B)} \right)^{y^*_t} \geq \left( \frac{\rho_B (1 - \rho_G)}{\rho_G (1 - \rho_B)} \right)^{\rho_B} > \left( \frac{1 - \rho_G}{1 - \rho_B} \right)
\]
As \( t \to \infty \), the inequality above implies for \( y^*_t \leq \rho_B \) we have
\[
\left( \frac{\rho_B (1 - \rho_G)}{\rho_G (1 - \rho_B)} \right)^{y^*_t} \geq \left( \frac{1 - \rho_G}{1 - \rho_B} \right)^{1 - 1/t}
\]
To satisfy inequality 4, we must have: as $t \to \infty$, $y^*_t > \rho_B$. But by the Law of Large Numbers, for the incompetent entrepreneurs $y_t = i_t/t$ (the share of successes in the amount of trials) must converge to $\rho_B$, which is less than $y^*_t$. I.e., beyond “natural death,” eventually the incompetent entrepreneurs will drop out in the long run: as $t \to \infty$, $p^B_t \to p^B = 0$.

For the competent entrepreneurs, as $t \to \infty$, $p^C_t \to p^C$. We know

$$p^C_t - p^B_t = (1 - \sum_{s=1}^{t} q^C_s) - (1 - \sum_{s=1}^{t} q^B_s) = \sum_{s=1}^{t} (q^B_s - q^C_s)$$

Note, $q^B_s \geq q^C_s$ for all $s$. Since $q^C_1 = 1 - \rho_G$ and $q^B_1 = 1 - \rho_B$, $p^C - p^B \geq q^C_1 - q^B_1 = \rho_G - \rho_B$

As $p^B = 0$, $p^C \geq \rho_G - \rho_B$. ■

References


