Endogenous Transaction Cost, Specialization, and Strategic Alliance

Juyan Zhang
Research Institute of
Economics and Management
Southwestern University of
Finance and Economics

Yi Zhang*
School of Economics
Singapore Management University

October 15, 2008

Abstract

In property rights theory, firm is an organizational response to reduce transaction cost associated with hold-up of using market mechanism. We claim that strategic alliance – without changing firm boundaries or asset ownership – is another type of organizational response. We construct a model to investigate individual firms' strategic choice on specialization or diversification when producing intermediate products and their further choice of organizational form: autarchy or forming strategic alliance. We introduce fixed learning costs as an indicator of scales of economy and show that only if fixed learning costs are large enough, will firms have incentive to be specialization and form strategic alliance. We distinguish between asymmetric strategic alliance and symmetric strategic alliance and show that transaction cost is not monotonic with respect to fixed learning costs. In particular, for asymmetric strategic alliance, there exists overinvestment with un-utilized capacity. Further, asymmetric strategic alliance is always unstable, while symmetric strategic alliance is stable only if fixed learning costs are large enough. The firm who is entitled with higher learning cost gets higher payoff – rewards for the endeavor. If firms are more patient, they are less likely to form strategic alliance.

JEL classification: D23, L14, L22

Keywords: Endogenous Transaction Cost, Hold-up, Specialization, Overinvestment, Strategic Alliance

*E-mail: yizhang@smu.edu.sg

1

1 Introduction

"Hold-up arises when part of the return on an agents relationship-specific investments is ex post expropriable by his trading partner."(Che and Sákovics 2007) This leads to underinvestment ex ante. As an organizational response, the inefficiency and the corresponding transaction cost[1] could be reduced through vertical integration (Klein, Crawford, and Alchian 1978, and Williamson 1979). This was formally incorporated into property rights theory (Grossman and Hart 1986, Hart and Moore 1990), in which the owner of the firm is entitled residual rights of control to minimize the distortion of hold-up.

In contrast, we focus on another type of organizational form – strategic alliance, "where non-integrated parties coordinate activities without changing firm boundaries or asset ownership" (Baker and Gibbons 2002). A notable example is the shoe and garment industry in Wenzhou, Zhejiang, China. Firms specialize on the productions of some intermediate parts, exchange with others, and assemble them into brands of their own.[2] To be specialization, firms enjoy the benefit of scales of economy. To be diversification, firms avoid the possible hold-up afterward. Due to this trade-off, transaction cost is endogenized. We claim that strategic alliance – without changing firm boundaries or asset ownership – is another type of organizational response to reduce the transaction cost associated with hold-up.

In this paper, we construct a model to investigate individual firms' strategic choice on specialization or diversification when producing intermediate products and their further choice of organizational form: autarchy or forming strategic alliance. In our model, we introduce fixed learning costs in production of intermediate products as an indicator of scales of economy. The larger fixed learning costs are, the larger the benefit from specialization and forming strategic alliance are.

The discrepancy between our model and current property rights theory is that in our model firms could form strategic alliance to take advantage of the specialization and the associated scales of economy meanwhile keeping independence of each firm, while in property rights theory firms may integrate to entitle residual rights of control to the owner to reduce the transaction cost. Further, in property rights theory, firms could negotiate for the optimal ownership structure ex ante. In contrast, in our model, firms can not reach a contractible arrangement regarding specialization or diversification ex ante. They have to rely on self-enforceable agreement, i.e. Nash equilibrium.

Our main results are summarized below.

1. Only if fixed learning costs are large enough, will firms have incentive to be

---

[1]Coase (1937) suggested firm is the organizational form to reduce transaction cost of using market mechanism.

[2]See the online report: http://www.efu.com.cn/data/2008/2008-01-29/227157.shtml.

specialization and form strategic alliance.

2. If fixed learning costs are in some middle range, one firm may diversify while the other specializes on the intermediate product with lower learning cost and they form ***asymmetric strategic alliance***. If fixed learning costs are large, both firms will be specialization and form ***symmetric strategic alliance***.

3. We define ***transaction cost*** as the difference between firms' total payoff at the first best and that at the equilibrium choice. We show that transaction cost is not monotonic with respect to fixed learning costs.

4. For the asymmetric strategic alliance, the firm diversified merely increases its bargaining power through diversification during the negotiation afterward. There exists ***overinvestment with un-utilized capacity*** since the firm diversified still specializes on producing the intermediate product with higher learning cost according to the equilibrium agreement of the negotiation. In contrast, if firms are diversification and autarchy, we also have overinvestment but with fully utilized capacity.

5. The firm who is entitled with higher learning cost gets higher payoff – "***rewards for the endeavor***". Intuitively, in the asymmetric strategic alliance, the firm diversified gets higher bargaining power through diversification and therefore gets higher payoff.

6. The asymmetric strategic alliance is always ***unstable***, while the symmetric strategic alliance is unstable if fixed learning costs are small and stable if fixed learning costs are large. Here, unstable means that if the game continues to another stage the strategic alliance will be disbanded to autarchy.[3]

7. If firms are more patient, they are less likely to form strategic alliance since the benefit from diversification is higher by avoiding the possible hold-up both in the current period and in the future periods.

Our model is related to the literature on specialization and division of labor. Yang and Borland (1991) provide a microeconomic mechanism to explain the evolution of specialization. Their model begins with the consumer-producer role for each individual. They point out the monopoly power accruing to producers due to the entry barriers created by learning by doing and increasing returns. However, they nullify the monopoly power by the assumption of the option contract in the future market, which exempts presumptively the possibility for negotiation. Hence, they turn to a Walrasian regime to describe the equilibrium of the division of labor. In our model,

---

[3]This supports the argument of Wang (2005) that most coalitions don't last long. Only if the stakes are high enough, will coalitions be sustainable.

we identify the key role of bargaining power created by fixed learning costs, and provide the possibility for negotiation, which thus enriches the story of the evolution of specialization through a Nash equilibrium scheme. Becker and Murphy (1992) argue that coordination costs and the amount and extent of knowledge are key factors in determining the degree of specialization, which sharply contrast with the traditional view emphasizing the limitations of specialization imposed by the thickness of the market.

The rest of the paper is organized as follows. Section 2 provides the setup of our basic model and presents our main results. Section 3 considers the case if the game continues to an unexpected second stage – myopic case. Section 4 considers the case if the game continues to an expected second stage – foresighted case. Section 5 concludes.

2   The Basic Model

There are two firms $M1$ and $M2$. They need intermediate products x and y to produce their own final goods. Each firm may produce x and y by itself (the case of autarchy) or specialize and produce only one and exchange with the other firm (the case of forming strategic alliance). For simplicity, we assume each firm has one unit of labor endowment in the production of x and y. To produce x and y, each firm has to incur some fixed learning cost a_x and a_y. That is, to be capable to produce x, one has to spend a_x unit of labor in advance; to be capable to produce y, one has to spend a_y unit of labor in advance. Further, we assume $\frac{1}{2} > a_x > a_y > 0$. If one invests both a_x and a_y , this is the diversification case, denoted as D. We assume that if $M1$ does not diversify, he can only choose to specialize on x, denoted as S_x. If $M2$ does not diversify, he can only choose to specialize on y, denoted as S_y.[4]

If one chooses D, the production function for x and y is $x + y = 1 - a_x - a_y$. If $M1$ chooses S_x, the production function for x is $x = 1 - a_x$. If $M2$ chooses S_y, the production function for y is $y = 1 - a_y$. In addition, suppose $M1$ and $M2$ have the same production function for final products, $f(x, y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$.

At date 1.0, $M1$ and $M2$ learn to produce x, y, or both: specialization of diversification. At date 1.1, they negotiate to form strategic alliance. If negotiation breaks down, they will be autarchy. If they agree to form strategic alliance, we assume they will divide the trade surplus based on Nash Bargaining solution, in which the bargaining weight of each firm is equalized to be $1/2$. At date 1.2, x and y are produced and exchanged according to the agreement at date 1.1 if there is any. The timing of the model is illustrated in Figure 1.

Denote the choices of $M1$ and $M2$ at date 1.0 as (C_1, C_2), where $C_1 \in \{S_x, D\}$

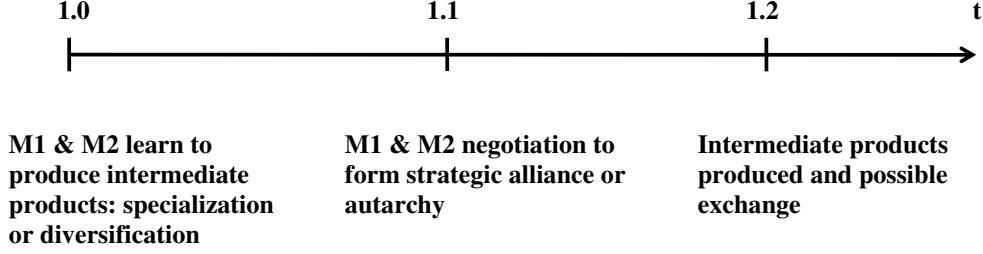[4]This will simplify our analysis and avoid the possible coordination failure from $M1$ and $M2$.

Figure 1: Timing

and $C_2 \in \{S_y, D\}$. To the point view of social planner, clearly the first best solution is (S_x, S_y). That is, $M1$ specializes on x and $M2$ specializes on y. Now we turn to the question: what are the equilibrium choices of $M1$ and $M2$ at date 1.0? The following lemma solves the payoffs at date 1.0 for all possible combinations of (C_1, C_2).

Lemma 1 *The payoff matrix at date 1.0 is as described in table 1, where $U_A = \frac{1}{2}\sqrt{(1 - a_x - a_y)^2}$, $U_{XD} = \frac{1}{2}\sqrt{(1 - a_x)(1 - a_x - a_y)}$, $U_{DY} = \frac{1}{2}\sqrt{(1 - a_x - a_y)(1 - a_y)}$, $U_{XY} = \frac{1}{2}\sqrt{(1 - a_x)(1 - a_y)}$.*

Table 1: Payoff matrix

		$M2$	
		S_y	D
$M1$	S_x	U_{XY}, U_{XY}	$U_{XD} - \frac{1}{2}U_A, U_{XD} + \frac{1}{2}U_A$
	D	$U_{DY} + \frac{1}{2}U_A, U_{DY} - \frac{1}{2}U_A$	U_A, U_A

Proof. See the Appendix. ∎

The following proposition shows that if the fixed learning cost a_x and a_y are small, the equilibrium choice at date 1.0 will be (D, D) and $M1$ and $M2$ end up with autarchy. If a_x and a_y are large, the equilibrium choice at date 1.0 will be (S_x, S_y) and $M1$ and $M2$ end up with forming **symmetric strategic alliance**, in which $M1$ and $M2$ have the symmetric bargaining position and get the same payoff. If a_x and a_y are in some middle range, the equilibrium choice at date 1.0 may be (D, S_y) and $M1$ and $M2$ end up with forming **asymmetric strategic alliance**, in which $M1$ and $M2$ have the asymmetric bargaining position and get different payoff. Finally, (S_x, D) will never be an equilibrium and there may exist multiple equilibria.

Proposition 1 i) *If the fixed learning cost a_x and a_y are small, $M1$ and $M2$ end up with (D, D) at date 1.0.*

ii) *If a_x and a_y are large, M1 and M2 end up with (S_x, S_y) at date 1.0.*

iii) *If a_x and a_y are in some middle range, M1 and M2 may end up with (D, S_y).*

iv) *(S_x, D) will never be an equilibrium.*

v) *There may exist multiple equilibria.*

Proof. See the Appendix. ∎

The intuition is that if learning costs are small, it is worth diversifying to avoid the possible hold-up during the negotiation afterward. If learning costs are large, it is better to specialize and form symmetric strategic alliance, in which the first best is achieved. If learning costs are in some middle range, one may have incentive to diversify while the other specializes on the one intermediate product with low learning cost. In this case, they form asymmetric strategic alliance.[5] (See figure 2 for the MATLAB simulation.)
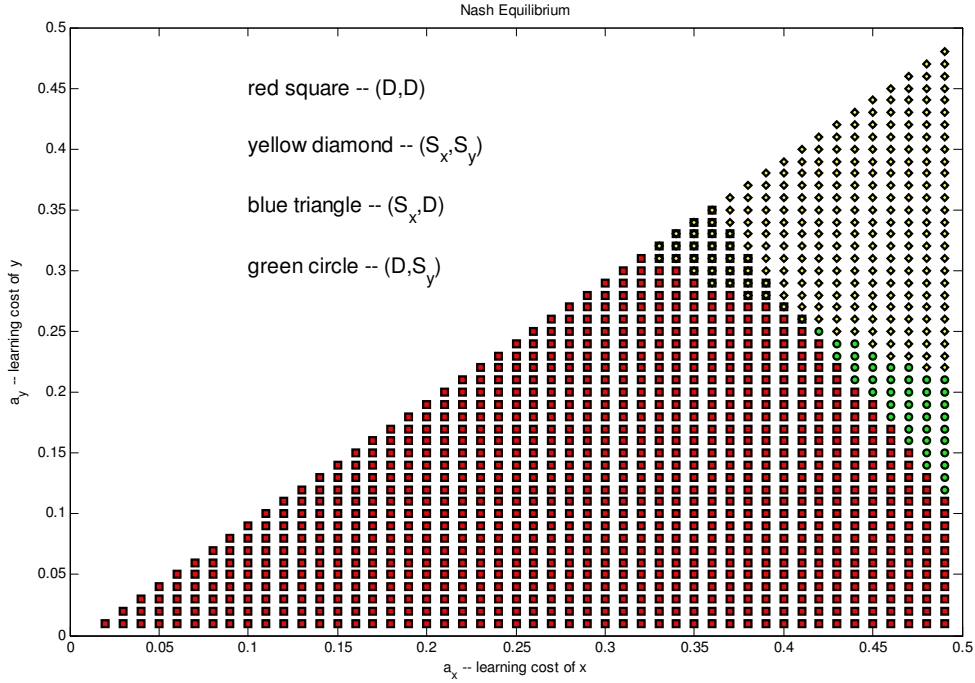

Figure 2: Nash Equilibrium

---

[5]By our assumption, not matter what, M1 has to spend a_x to learn to produce x, while M2 has to spend a_y to learn to produce y. Since $a_x > a_y$, M1 has higher incentive to spend a_y and diversify than M2 to spend a_x and diversify. That is to say, if (S_x, D) is an equilibrium, then for sure (D, S_y) will be an equilibrium.

Define **transaction cost** as the difference between the total payoff for $M1$ and $M2$ at the first best and the total payoff for $M1$ and $M2$ at the equilibrium choice. The following corollary says that transaction cost is not monotonic with respect to fixed learning costs. (See figure 3 for the MATLAB simulation.)

Corollary 1 *(Transaction Cost)*

i) *If a_x and a_y are small, transaction cost increases in a_x and a_y.*

ii) *If a_x and a_y are increasing to some extent, there may exist a jump down of the transaction cost. After that, it increases in a_x and a_y again.*

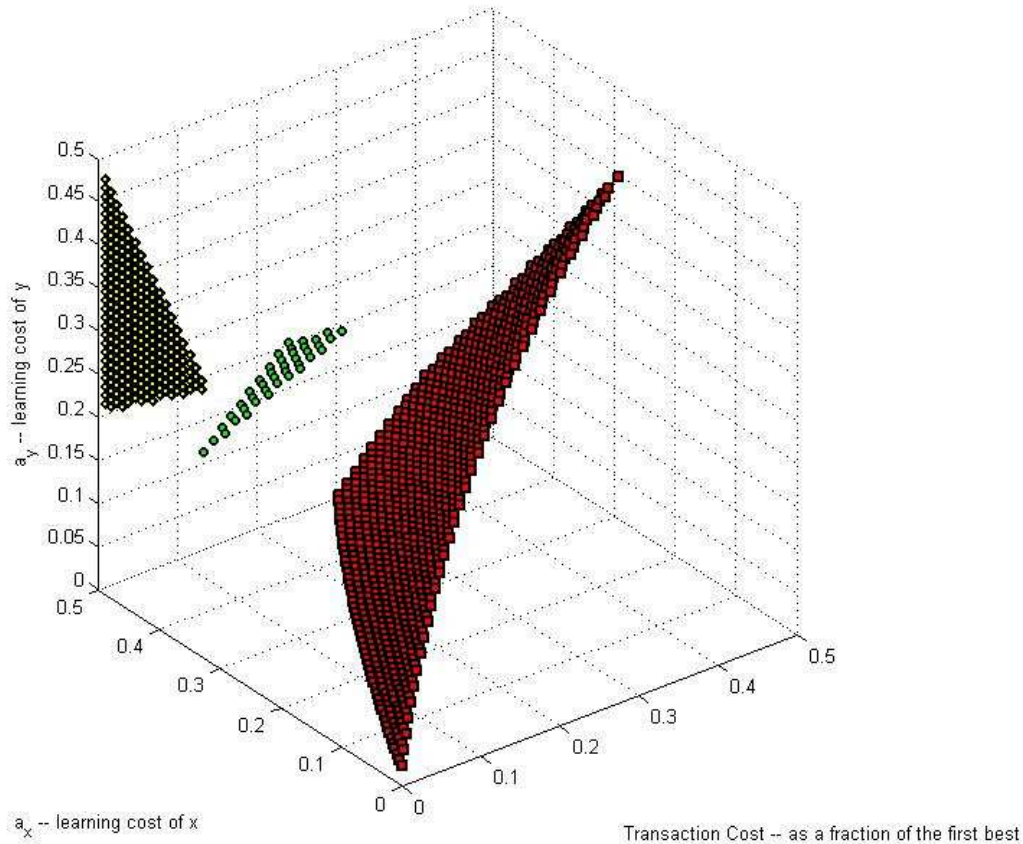**iii)** *If a_x and a_y are large enough, transaction cost jumps down to zero.*

Figure 3: Transaction Cost

Intuitively, if fixed learning costs are small, the equilibrium choice at date 1.0 will be (D, D) and $M1$ and $M2$ end up with autarchy. In this case, transaction cost is

$2U_{XY} - 2U_A$, which is increasing in fixed learning costs. If fixed learning costs are increasing to some extent, the equilibrium choice at date 1.0 may be (D, S_y) and $M1$ and $M2$ end up with forming asymmetric strategic alliance. In this case, transaction cost is $2U_{XY} - 2U_{DY}$, which is increasing in fixed learning costs. Since $U_A < U_{DY}$, there exists a jump down of transaction cost during the transition from (D, D) to (D, S_y). If fixed learning costs are large enough, the equilibrium choice at date 1.0 will be (S_x, S_y) and $M1$ and $M2$ end up with forming symmetric strategic alliance, which is the first best. Consequently, transaction cost jumps down to zero.

The following corollary says that if the equilibrium choice at date 1.0 is (D, S_y) and $M1$ and $M2$ form asymmetric strategic alliance, there exists the case of un-utilized capacity, in which $M1$ spends a_y learns to produce y, but still specializes on producing x during the production afterwards. The diversification of $M1$ merely serves as the purpose of increasing its bargaining power during the negotiation of forming strategic alliance.

Corollary 2 *(overinvestment with un-utilized capacity)*

If at date 1.0 the equilibrium choice is (D, S_y), then at date 1.2 the equilibrium result is $M1$ specializes on producing x and $M2$ specializes on producing y.

From the proof of proposition 1 part (iii), we have the above corollary. In this case, there exists overinvestment with un-utilized capacity. In contrast, if $M1$ and $M2$ end up with (D, D) at date 1.0 and being autarchy, we also have overinvestment but with fully utilized capacity.

The following corollary says that $M1$'s payoff will always be higher than or at least equal to $M2$'s payoff. That is to say, the firm who is entitled with higher learning cost gets higher payoff – "rewards for the endeavor".

Corollary 3 *(Rewards for the Endeavor)*

At any equilibrium, $M1$'s payoff will always be higher than or at least equal to $M2$'s payoff.

Intuitively, if at date 1.0 the equilibrium choice is (D, S_y) and $M1$ and $M2$ end up with forming asymmetric strategic alliance, $M1$ diversifies and gets higher bargaining power and therefore gets higher payoff. If at date 1.0 the equilibrium choice is (D, D) or (S_x, S_y), $M1$ and $M2$ get the same payoff from autarchy or symmetric strategic alliance. Moreover, from proposition 1, (S_x, D) will never be an equilibrium choice at date 1.0. Thus, we have the corollary above.

Intuitively, to let (S_x, D) be an equilibrium at date 1.0, U_{XD} has to be large enough. From the proof of proposition 1 part (iv), we must have

$$\begin{cases} U_{XD} \geq \frac{3}{2}U_A \\ U_{XD} \geq U_{XY} - \frac{1}{2}U_A \end{cases}$$

But according to our setting, if a_x and a_y are large enough such that first inequality holds, then U_{XY} will be too large such that the second inequality will not hold. If a_x and a_y are small enough such that second inequality holds, then U_A will be too large such that the first inequality will not hold.

3   Two-stage Model: Myopic Case

Now consider the case if the game continues to a second stage. In this section, suppose the second stage is unexpected at the beginning of the game. That is equivalent to say $M1$ and $M2$ are myopic at the first stage. In this case, the fist stage game is same as the basic model in section 2. The question is: if $M1$ and $M2$ are forming strategic alliance at stage 1, do they have incentive to change to diversification and autarchy at the second stage? In other words, is the strategic alliance at stage 1 stable?

Obviously, if the equilibrium choice at date 1.0 is (D, D), the payoffs at date 2.0 for $M1$ and $M2$ are $(\frac{1}{2}, \frac{1}{2})$. The following lemma solves the payoffs at date 2.0 given the equilibrium choice at date 1.0 is (D, S_y), (S_x, S_y), and (S_x, D) respectively.

Lemma 2 *If the equilibrium choice at date 1.0 is (D, S_y), (S_x, S_y), and (S_x, D) respectively, the payoff matrix at date 2.0 is as described in table 2.*

Proof. See the Appendix. ∎

The following proposition shows that if learning costs are high, $M1$ and $M2$ may stick with specialization and the strategic alliance survives. Otherwise, they will change to diversification and the strategic alliance will be disbanded to autarchy at stage 2.

Proposition 2 i) *If the equilibrium choice at date 1.0 is (D, S_y) and M1 and M2 form asymmetric strategic alliance, then at the second stage M2 will change to diversification and M1 and M2 end up with autarchy. That is to say, the asymmetric strategic alliance at stage 1 is always unstable.*

ii) *If the equilibrium choice at date 1.0 is (S_x, S_y) and M1 and M2 form symmetric strategic alliance, then at the second stage M1 and M2 will change to diversification and end up with autarchy if $a_y < 2\sqrt{3} - 3$; they may stick with (S_x, S_y) and the symmetric strategic alliance at stage 1 survives otherwise.*

Proof. See the Appendix. ∎

The intuition is that only if learning costs are large enough, will $M1$ and $M2$ have incentive to stick with specialization and the strategic alliance survives. Since at the beginning of the first stage $M1$ and $M2$ have incurred at least some learning cost,

Table 2: Payoff matrix at date 2.0

i) given (D, S_y) at date 1.0

		M2	
		S_y	D
$M1$	D	$\frac{3}{4}, \frac{1}{4}$	$\frac{1}{2}, \frac{1-a_x}{2}$

ii) given (S_x, S_y) at date 1.0

		M2	
		S_y	D
$M1$	S_x	$\frac{1}{2}, \frac{1}{2}$	$\frac{\sqrt{1-a_x}}{2} - \frac{1-a_x}{4}, \frac{\sqrt{1-a_x}}{2} + \frac{1-a_x}{4}$
	D	$\frac{\sqrt{1-a_y}}{2} + \frac{1-a_y}{4}, \frac{\sqrt{1-a_y}}{2} - \frac{1-a_y}{4}$	$\frac{1-a_y}{2}, \frac{1-a_x}{2}$

iii) given (S_x, D) at date 1.0

		M2
		D
$M1$	S_x	$\frac{1}{4}, \frac{3}{4}$
	D	$\frac{1-a_y}{2}, \frac{1}{2}$

at date 2.0 the cost of becoming diversification is low. Thus, at the second stage, $M1$ and $M2$ have higher incentive to diversify to avoid the possible hold-up during the negotiation afterward. Moreover, if the equilibrium choice at date 1.0 is (S_x, S_y) and learning costs are large enough such that (S_x, S_y) is still a Nash equilibrium at date 2.0, (D, D) is also a Nash equilibrium at date 2.0, even though (D, D) is Pareto dominated by (S_x, S_y). In this case, we have a coordination game. In contrast, if the equilibrium choice at at date 1.0 is (D, S_y), according to proposition 1 this implies learning costs are not large enough. Then for sure $M2$ will change to diversification at date 2.0 and $M1$ and $M2$ end up with autarchy and the asymmetric strategic alliance formed at stage 1 is always unstable. (See figure 4 for the MATLAB simulation.)

Similar to the one stage static case in section 2, we have the corollary for "non-monotonic transaction cost" and "rewards for the endeavor" at the second stage as follows. But we don't have the corollary for "un-utilized capacity" since at date 2.0 $M1$ and $M2$ will either stick with the symmetric strategic alliance or change to diversification and end up with autarchy.

Corollary 4 *(Transaction Cost – Myopic Case)*

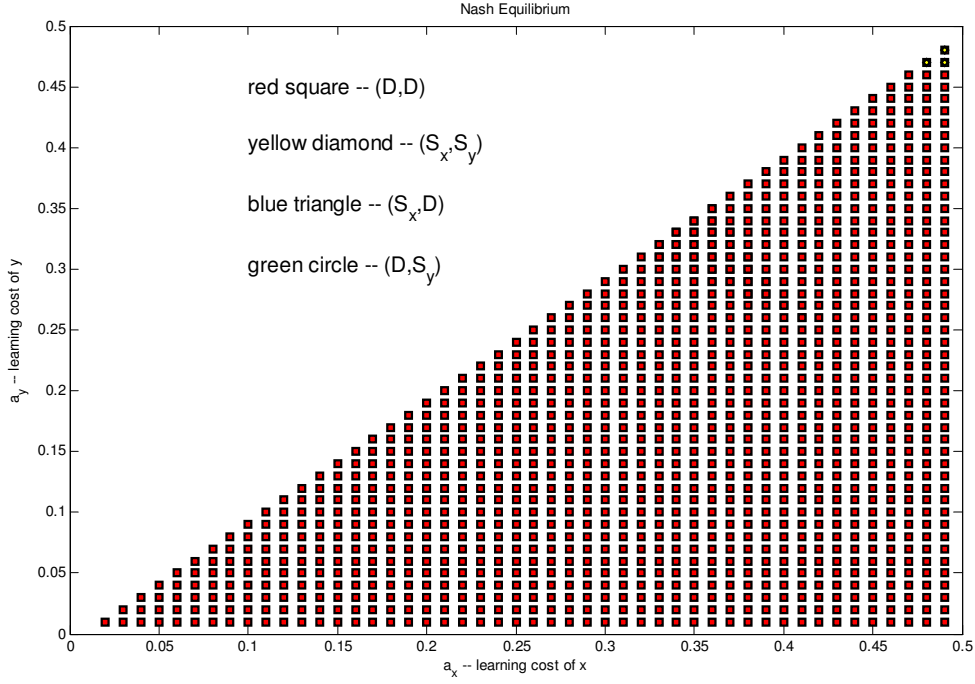**i)** *If a_x and a_y are small, transaction cost is zero.*

Figure 4: Nash Equilibrium at the Second Stage – Myopic Case

ii) *If a_x and a_y are increasing to some extent, there may exist a jump up of transaction cost. After that, it increases in a_x and a_y again.*

iii) *If a_x and a_y are large enough but still $a_y < 2\sqrt{3} - 3$, there exists a jump up of transaction cost and it increases in a_x and a_y again after the jump.*

iv) *If $a_y \geq 2\sqrt{3} - 3$, transaction cost jumps down to zero.*

Proof. See the Appendix. ∎

Corollary 5 *(Rewards for the Endeavor – Myopic Case)*

At any equilibrium, $M1$'s payoff will always be higher than or at least equal to $M2$'s payoff.

From lemma 2 and proposition 2, clearly at any equilibrium, $M1$'s payoff will always be higher than or at least equal to $M2$'s payoff.

4   Two-stage Model: Foresighted Case

In this section, consider the case if the game continues to a second stage and the second stage is expected at the beginning of the game. That is equivalent to say

$M1$ and $M2$ are foresighted at the first stage. In this case, we need to use backward induction, starting from the second stage.

Suppose $M1$ and $M2$ have the common discount factor is δ. Combining lemma 1 and lemma 2, table 3 solves the discounted two-stage payoffs at date 1.0.[6]

Table 3: Discounted two-stage payoff matrix

i) if $a_y < 2\sqrt{3} - 3$

		$M2$	
		S_y	D
$M1$	S_x	$U_{XY} + \delta\frac{1-a_y}{2}, U_{XY} + \delta\frac{1-a_x}{2}$	$U_{XD} - \frac{1}{2}U_A + \delta\frac{1-a_y}{2}, U_{XD} + \frac{1}{2}U_A + \delta\frac{1}{2}$
	D	$U_{DY} + \frac{1}{2}U_A + \delta\frac{1}{2}, U_{DY} - \frac{1}{2}U_A + \delta\frac{1-a_x}{2}$	$U_A + \delta\frac{1}{2}, U_A + \delta\frac{1}{2}$

ii) if $a_y \geq 2\sqrt{3} - 3$

		$M2$	
		S_y	D
$M1$	S_x	$U_{XY} + \delta\frac{1}{2}, U_{XY} + \delta\frac{1}{2}$	$U_{XD} - \frac{1}{2}U_A + \delta\frac{1-a_y}{2}, U_{XD} + \frac{1}{2}U_A + \delta\frac{1}{2}$
	D	$U_{DY} + \frac{1}{2}U_A + \delta\frac{1}{2}, U_{DY} - \frac{1}{2}U_A + \delta\frac{1-a_x}{2}$	$U_A + \delta\frac{1}{2}, U_A + \delta\frac{1}{2}$

The following proposition shows that, if the fixed learning cost a_x and a_y are small, $M1$ and $M2$ end up with (D, D) at date 1.0 and stick with (D, D) at date 2.0, denoted as $(D, D; D, D)$, where the first pair is the choice at date 1.0 and the second pair is the choice at date 2.0. If a_x and a_y are large, $M1$ and $M2$ may end up with $(S_x, S_y; S_x, S_y)$. If a_x and a_y are in some middle range, $M1$ and $M2$ may end up with $(D, S_y; D, D)$. $M1$ and $M2$ are more likely to end up with $(D, D; D, D)$ if $M1$ and $M2$ are more patient.

Proposition 3 i) *If the fixed learning cost a_x and a_y are small, $M1$ and $M2$ end up with $(D, D; D, D)$.*

ii) *If $a_x > a_y \geq 2\sqrt{3} - 3$, $M1$ and $M2$ may end up with $(S_x, S_y; S_x, S_y)$. And the symmetric strategic alliance at stage 1 survives.*

iii) *If a_x and a_y are in some middle range, $M1$ and $M2$ may end up with $(D, S_y; D, D)$ or $(S_x, S_y; S_x, S_y)$. And the strategic alliance at stage 1, either symmetric or asymmetric, is unstable.*

---

[6]For simplicity, we assume if there are multiple equilibria, $M1$ and $M2$ will end up with the Poreto dominant one.

iv) *As δ increases, $M1$ and $M2$ are more likely to end up with $(D, D; D, D)$.*

v) *There may exist multiple equilibria.*

vi) *(S_x, D) will never be an equilibrium choice at date 1.0.*

Proof. See the Appendix. ∎

The intuition is that if learning costs are large enough such that $(S_x, S_y; S_x, S_y)$ is a Subgame Perfect Nash equilibrium, $(D, D; D, D)$ is also a Subgame Perfect Nash equilibrium, even though $(D, D; D, D)$ is Pareto dominated by $(S_x, S_y; S_x, S_y)$. In this case, we have a coordination game. If $M1$ and $M2$ are more patient, they are less likely to form strategic alliance since the benefit from diversification is higher by avoiding the possible hold-up both in the current period and in the future period. The exception is that if learning costs are large enough ($a_x > a_y \geq 2\sqrt{3} - 3$), the symmetric strategic alliance is sustainable. (See figure 5 and 6 for the MATLAB simulation for $\delta = 0.1$ and $\delta = 0.5$ respectively.)
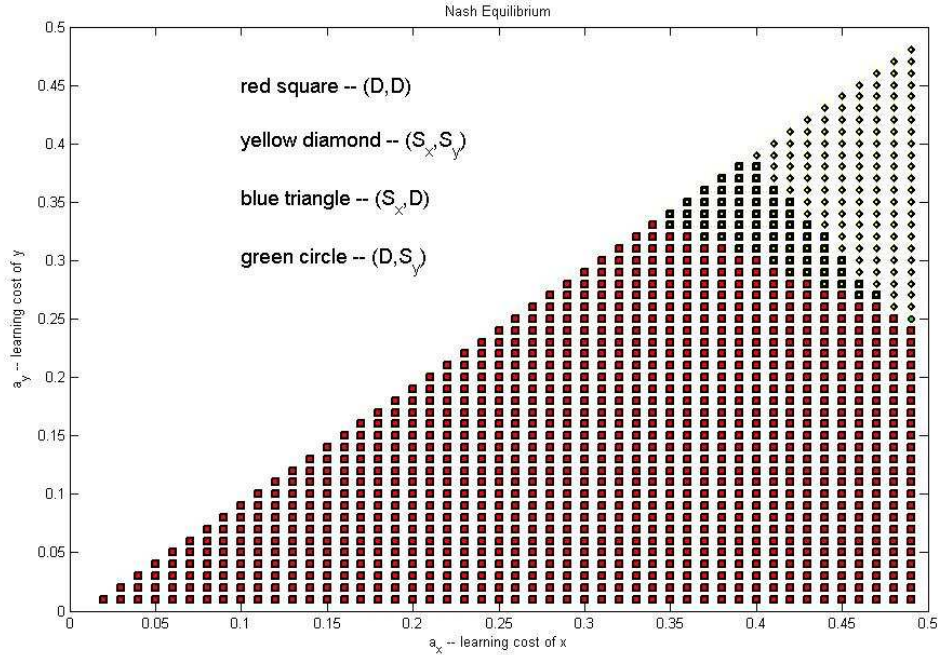

Figure 5: Nash Equilibrium at date 1.0 – foresighted Case: $\delta = 0.1$

Similar to the one stage static case in section 2, we have the corollary for "non-monotonic transaction cost", "overinvestment – un-utilized capacity", and "rewards for the endeavor" as follows.

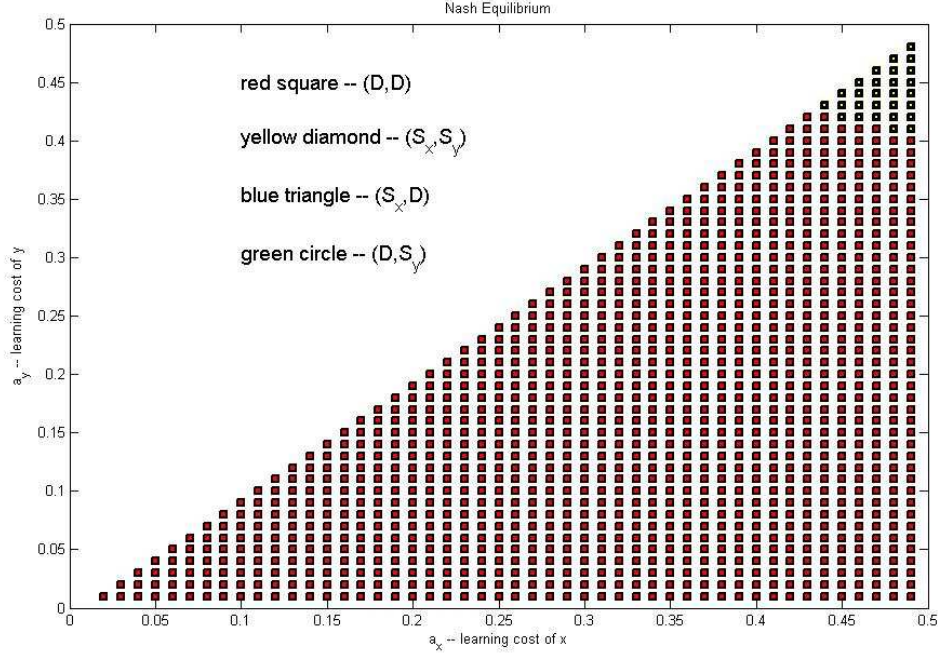**Corollary 6** *(Transaction Cost – foresighted Case)*

13

Figure 6: Nash Equilibrium at date 1.0 – foresighted Case: $\delta = 0.5$

i) If $a_y \geq 2\sqrt{3} - 3$, transaction cost is zero.

ii) If $a_y < 2\sqrt{3} - 3$, Given δ transaction cost is not monotonic with respect to a_x and a_y; given a_x and a_y, transaction cost is non-decreasing with respect to δ.

Proof. See the Appendix. ∎

Corollary 7 *(overinvestment with un-utilized capacity – foresighted Case)*

If at date 1.0 $M1$ and $M2$ end up with (D, S_y), then at date 1.2 the equilibrium result is $M1$ specializes on producing x and $M2$ specializes on producing y.

From the proof of proposition 3, if at date 1.0 $M1$ and $M2$ end up with (D, S_y), then they form asymmetric strategic alliance, in which $M1$ specializes on producing x and $M2$ specializes on producing y at date 1.2. But this asymmetric strategic alliance is unstable. It will be disbanded to autarchy at stage 2 and consequently, overinvestment with un-utilized capacity disappears.

Corollary 8 *(Rewards for the Endeavor – foresighted Case)*

At any equilibrium, $M1$'s payoff will always be higher than or at least equal to $M2$'s payoff.

14

From lemma **??** and proposition 3, clearly at any equilibrium, $M1$'s payoff will always be higher than or at least equal to $M2$'s payoff.

5   Conclusion

In property rights theory, firm is an organizational response to reduce transaction cost associated with hold-up of using market mechanism. We claim that strategic alliance – without changing firm boundaries or asset ownership – is another type of organizational response. We construct a model to investigate individual firms' strategic choice on specialization or diversification when producing intermediate products and their further choice of organizational form: autarchy or forming strategic alliance. To be specialization, firms enjoy the benefit of scales of economy. To be diversification, firms avoid the possible hold-up afterward. Due to this trade-off, transaction cost is endogenized. We introduce fixed learning costs as an indicator of scales of economy. We show that only if fixed learning costs are large enough, will firms have incentive to be specialization and form strategic alliance. We distinguish between asymmetric strategic alliance and symmetric strategic alliance and show that transaction cost is not monotonic with respect to fixed learning costs. In particular, for asymmetric strategic alliance, there exists overinvestment with un-utilized capacity and the firm diversified merely increases its bargaining power through diversification during the negotiation afterward. Further, asymmetric strategic alliance is always unstable, while symmetric strategic alliance is stable only if fixed learning costs are large enough. The firm who is entitled with higher learning cost gets higher payoff – rewards for the endeavor. If firms are more patient, they are less likely to form strategic alliance.

Appendix

Proof of Lemma 1

By backward induction, if $(C_1, C_2) = (S_x, S_y)$, at date 1.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ and $M2$ is zero. Since we assume if $M1$ and $M2$ form strategic alliance, they will divide the trade surplus through Nash Bargaining solution and the bargaining weight of each firm is equalized to be $1/2$, we need to solve the following maximization problem.

$$\max_{x_1, x_2, y_1, y_2} [U_1(S_x, S_y) - 0][U_2(S_x, S_y) - 0] = \sqrt{x_1 y_1}\sqrt{x_2 y_2}$$

$$\text{s.t.} \quad x_1 + x_2 = 1 - a_x$$

$$y_1 + y_2 = 1 - a_y$$

The solution is $x_1 = x_2 = \frac{1-a_x}{2}$ and $y_1 = y_2 = \frac{1-a_y}{2}$. Thus, $U_1(S_x, S_y) = U_2(S_x, S_y) = U_{XY}$, where $U_{XY} = \frac{1}{2}\sqrt{(1-a_x)(1-a_y)}$.

If $(C_1, C_2) = (D, D)$, at date 1.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ and $M2$ is autarchy. The payoff from autarchy for each firm, denoted as U_A, is equal to $\frac{1}{2}\sqrt{(1-a_x-a_y)^2}$. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_1^p,x_2^p}[U_1(D,D)-U_A][U_2(D,D)-U_A] = (\sqrt{x_1 y_1} - U_A)(\sqrt{x_2 y_2} - U_A)$$

$$\text{s.t.} \quad x_1 + x_2 = x_1^p + x_2^p$$

$$y_1 + y_2 = 2(1 - a_x - a_y) - x_1^p - x_2^p$$

where x_1^p and x_2^p are the amount of x produced by $M1$ and $M2$ respectively. The solution is $x_1 = x_2 = y_1 = y_2 = \frac{1-a_x-a_y}{2}$. Thus, $U_1(D,D) = U_2(D,D) = \frac{1}{2}\sqrt{(1-a_x-a_y)^2} = U_A$.

If $(C_1, C_2) = (S_x, D)$, at date 1.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ is zero and the outside option for $M2$ is U_A. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_2^p}[U_1(S_x,D)-0][U_2(S_x,D)-U_A] = (\sqrt{x_1 y_1})(\sqrt{x_2 y_2} - U_A)$$

$$\text{s.t.} \quad x_1 + x_2 = 1 - a_x + x_2^p$$

$$y_1 + y_2 = 1 - a_x - a_y - x_2^p$$

where x_2^p is the amount of x produced by $M2$. The solution is $x_1 = \frac{\sqrt{1-a_x}}{2}\frac{\sqrt{(1-a_x)(1-a_x-a_y)}-U_A}{\sqrt{1-a_x-a_y}}$,

$y_1 = \frac{\sqrt{1-a_x-a_y}}{2}\frac{\sqrt{(1-a_x)(1-a_x-a_y)}-U_A}{\sqrt{1-a_x}}$, $x_2 = 1 - a_x - \frac{\sqrt{1-a_x}}{2}\frac{\sqrt{(1-a_x)(1-a_x-a_y)}-U_A}{\sqrt{1-a_x-a_y}}$, and

$y_2 = 1 - a_x - a_y - \frac{\sqrt{1-a_x-a_y}}{2}\frac{\sqrt{(1-a_x)(1-a_x-a_y)}-U_A}{\sqrt{1-a_x}}$. Thus, $U_1(S_x, D) = U_{XD} - \frac{1}{2}U_A$ and $U_2(S_x, D) = U_{XD} + \frac{1}{2}U_A$, where $U_{XD} = \frac{1}{2}\sqrt{(1-a_x)(1-a_x-a_y)}$.

If $(C_1, C_2) = (D, S_y)$, at date 1.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ is U_A and the outside option for $M2$ is zero. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,y_1^p}[U_1(D,S_y)-U_A][U_2(D,S_y)-0] = (\sqrt{x_1 y_1} - U_A)(\sqrt{x_2 y_2})$$

$$\text{s.t.} \quad x_1 + x_2 = 1 - a_x - a_y - y_1^p$$

$$y_1 + y_2 = 1 - a_y + y_1^p$$

where y_1^p is the amount of y produced by $M1$. The solution is $x_1 = \frac{\sqrt{1-a_x-a_y}}{2}\frac{\sqrt{(1-a_x-a_y)(1-a_y)}+U_A}{\sqrt{1-a_y}}$,

$y_1 = \frac{\sqrt{1-a_y}}{2}\frac{\sqrt{(1-a_x-a_y)(1-a_y)}+U_A}{\sqrt{1-a_x-a_y}}$, $x_2 = 1 - a_x - a_y - \frac{\sqrt{1-a_x-a_y}}{2}\frac{\sqrt{(1-a_x-a_y)(1-a_y)}+U_A}{\sqrt{1-a_y}}$,

16

and $y_2 = 1 - a_y - \frac{\sqrt{1-a_y}\sqrt{(1-a_x-a_y)(1-a_y)}+U_A}{2\sqrt{1-a_x-a_y}}$. Thus, $U_1(D, S_y) = U_{DY} + \frac{1}{2}U_A$ and

$U_2(D, S_y) = U_{DY} - \frac{1}{2}U_A$, where $U_{DY} = \frac{1}{2}\sqrt{(1-a_x-a_y)(1-a_y)}$. ∎

Proof of Proposition 1

    i) If $a_x \to 0$ and $a_y \to 0$, then the payoff matrix in lemma 1 will converge to table 4.

Table 4: Payoff matrix: $a_x \to 0$ and $a_y \to 0$

		M2	
		S_y	D
M1	S_x	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{4}, \frac{3}{4}$
	D	$\frac{3}{4}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{2}$

    D will be the dominant strategy for both $M1$ and $M2$. And the unique Nash Equilibrium is (D, D) at date 1.0.

    ii) If $a_x \to \frac{1}{2}$ and $a_y \to \frac{1}{2}$, then the payoff matrix in lemma 1 will converge to table 5.

Table 5: Payoff matrix: $a_x \to \frac{1}{2}$ and $a_y \to \frac{1}{2}$

		M2	
		S_y	D
M1	S_x	$\frac{1}{4}, \frac{1}{4}$	$0, 0$
	D	$0, 0$	$0, 0$

    Also, if $a_x \to \frac{1}{2}$ and $a_y \to \frac{1}{2}$, we have $U_{XD} - \frac{1}{2}U_A > U_A$ and $U_{DY} - \frac{1}{2}U_A > U_A$. Thus, S_x will be the dominant strategy for $M1$ and S_y will be the dominant strategy for $M2$. The unique Nash Equilibrium is (S_x, S_y) at date 1.0.

    iii) For instance, if $a_x = 0.47$ and $a_y = 0.2$, we have $\frac{3}{2}U_A < U_{DY}$ and $U_{XY} - \frac{1}{2}U_A < U_{DY}$. Then (D, S_y) is a Nash Equilibrium.

    iv) To let (S_x, D) be a Nash Equilibrium, we must have

$$\begin{cases} U_{XD} - \frac{1}{2}U_A \geq U_A \\ U_{XD} + \frac{1}{2}U_A \geq U_{XY} \end{cases}$$

The first inequality implies $\sqrt{(1-a_x)} \geq \frac{3}{2}\sqrt{(1-a_x-a_y)}$. Rearrange the second inequality and we have $\frac{1}{2}(1 - a_x - a_y) \geq \sqrt{(1-a_x)}[\sqrt{(1-a_y)} - \sqrt{(1-a_x-a_y)}] \geq$

$\frac{3}{2}\sqrt{(1-a_x-a_y)}[\sqrt{(1-a_y)}-\sqrt{(1-a_x-a_y)}]$. This implies $\sqrt{(1-a_x-a_y)} \geq \frac{3}{4}\sqrt{(1-a_y)}$. That is to say, $\sqrt{(1-a_x)} \geq \frac{3}{2}\sqrt{(1-a_x-a_y)} \geq \frac{9}{8}\sqrt{(1-a_y)} > \sqrt{(1-a_y)}$. Since $a_x > a_y$, this is a contradiction.

v) For instance, if $a_x = 0.36$ and $a_y = 0.3$, we can check that both (D,D) and (S_y, S_y) are Nash Equilibrium. ∎

Proof of Lemma 2

i) If the equilibrium choice at date 1.0 is (D, S_y), $M1$ has learned to produce both x and y and $M2$ has learned to produce y at the first stage. The question is whether $M2$ will stick with S_y or learn to produce x and become diversification at date 2.0.

Similar to the proof of lemma 1, if $M2$ sticks with S_y, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ is autarchy and the outside option for $M2$ is zero. The payoff from autarchy for $M1$ conditional on diversification at the first stage, denoted as $U_{A|D}$, is equal to $\frac{1}{2}$. We need to solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,y_1^p}[U_1(D,S_y|D,S_y)-U_{A|D}][U_2(D,S_y|D,S_y)-0] = (\sqrt{x_1y_1}-\frac{1}{2})(\sqrt{x_2y_2})$$

$$\text{s.t.} \quad x_1 + x_2 = 1 - y_1^p$$
$$y_1 + y_2 = 1 + y_1^p$$

where y_1^p is the amount of y produced by $M1$ at date 2.2. The solution is $x_1 = y_1 = \frac{3}{4}$, $x_2 = y_2 = \frac{1}{4}$. Thus, $U_1(D,S_y|D,S_y) = \frac{3}{4}$ and $U_2(D,S_y|D,S_y) = \frac{1}{4}$.

If $M2$ learns to produce x and becomes diversification at date 2.0, when $M1$ and $M2$ negotiate to form strategic alliance at date 2.1, the outside option for $M1$ and $M2$ are both autarchy. The payoff from autarchy for $M1$ conditional on diversification at the first stage is $U_{A|D}$, which is equal to $\frac{1}{2}$. The payoff from autarchy for $M2$ conditional on specialization on y at the first stage, denoted as $U_{A|S_y}$, is equal to $\frac{1-a_x}{2}$. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_1^p,x_2^p}[U_1(D,D|D,S_y)-U_{A|D}][U_2(D,D|D,S_y)-U_{A|S_y}] = (\sqrt{x_1y_1}-\frac{1}{2})(\sqrt{x_2y_2}-\frac{1-a_x}{2})$$

$$\text{s.t.} \quad x_1 + x_2 = x_1^p + x_2^p$$
$$y_1 + y_2 = (2-a_x) - x_1^p - x_2^p$$

where x_1^p and x_2^p are the amount of x produced by $M1$ and $M2$ at date 2.2 respectively. The solution is $x_1 = y_1 = \frac{1}{2}$, $x_2 = y_2 = \frac{1-a_x}{2}$. Thus, $U_1(D,D|D,S_y) = \frac{1}{2} = U_{A|D}$ and $U_2(D,D|D,S_y) = \frac{1-a_x}{2} = U_{A|S_y}$.

ii) If the equilibrium choice at date 1.0 is (S_x, S_y), $M1$ has learned to produce x and $M2$ has learned to produce y at the first stage. The question is whether $M1$ and/or $M2$ will stick with specialization or become diversification at date 2.0.

If $M1$ and $M2$ both stick with specialization, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ and $M2$ are zero. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2} [U_1(S_x, S_y|S_x, S_y) - 0][U_2(S_x, S_y|S_x, S_y) - 0] = (\sqrt{x_1y_1})(\sqrt{x_2y_2})$$

$$\text{s.t.} \quad x_1 + x_2 = 1$$
$$y_1 + y_2 = 1$$

The solution is $x_1 = y_1 = x_2 = y_2 = \frac{1}{2}$. Thus, $U_1(S_x, S_y|S_x, S_y) = U_2(S_x, S_y|S_x, S_y) = \frac{1}{2}$.

If $M1$ sticks with specialization and $M2$ becomes diversification, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ is zero and the outside option for $M2$ is autarchy. The payoff from autarchy for $M2$ conditional on specialization on y at the first stage is $U_{A|S_y}$, which is equal to $\frac{1-a_x}{2}$. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_2^p} [U_1(S_x, D|S_x, S_y) - 0][U_2(S_x, D|S_x, S_y) - U_{A|S_y}] = (\sqrt{x_1y_1} - 0)(\sqrt{x_2y_2} - \frac{1-a_x}{2})$$

$$\text{s.t.} \quad x_1 + x_2 = 1 + x_2^p$$
$$y_1 + y_2 = (1 - a_x) - x_2^p$$

where x_2^p is the amount of x produced by $M2$ at date 2.2. The solution is $x_1 = \frac{1}{2} - \frac{\sqrt{1-a_x}}{4}$, $y_1 = (1 - a_x)(\frac{1}{2} - \frac{\sqrt{1-a_x}}{4})$, $x_2 = \frac{1}{2} + \frac{\sqrt{1-a_x}}{4}$, $y_2 = (1 - a_x)(\frac{1}{2} + \frac{\sqrt{1-a_x}}{4})$. Thus, $U_1(S_x, D|S_x, S_y) = \frac{\sqrt{1-a_x}}{2} - \frac{1-a_x}{4}$ and $U_2(S_x, D|S_x, S_y) = \frac{\sqrt{1-a_x}}{2} + \frac{1-a_x}{4}$.

If $M1$ becomes diversification and $M2$ sticks with specialization, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ is autarchy and the outside option for $M2$ is zero. The payoff from autarchy for $M1$ conditional on specialization on x at the first stage is $U_{A|S_x}$, which is equal to $\frac{1-a_y}{2}$. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,y_1^p} [U_1(D, S_y|S_x, S_y) - U_{A|S_x}][U_2(D, S_y|S_x, S_y) - 0] = (\sqrt{x_1y_1} - \frac{1-a_y}{2})(\sqrt{x_2y_2} - 0)$$

$$\text{s.t.} \quad x_1 + x_2 = (1 - a_y) - y_1^p$$
$$y_1 + y_2 = 1 + y_1^p$$

where y_1^p is the amount of y produced by $M1$ at date 2.2. The solution is $x_1 = (1 - a_y)(\frac{1}{2} + \frac{\sqrt{1-a_y}}{4})$, $y_1 = \frac{1}{2} + \frac{\sqrt{1-a_y}}{4}$, $x_2 = (1 - a_y)(\frac{1}{2} - \frac{\sqrt{1-a_y}}{4})$, $y_2 = \frac{1}{2} - \frac{\sqrt{1-a_y}}{4}$. Thus, $U_1(D, S_y|S_x, S_y) = \frac{\sqrt{1-a_y}}{2} + \frac{1-a_y}{4}$ and $U_2(D, S_y|S_x, S_y) = \frac{\sqrt{1-a_y}}{2} - \frac{1-a_y}{4}$.

If both $M1$ and $M2$ become diversification, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ and $M2$ are autarchy. The payoff from autarchy for $M1$ conditional on specialization on x at the first stage is $U_{A|S_x}$, which is equal to $\frac{1-a_y}{2}$. The payoff from autarchy for $M2$ conditional on specialization on y at the first stage is $U_{A|S_y}$, which is equal to $\frac{1-a_x}{2}$. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_1^p,x_2^p}[U_1(D,D|S_x,S_y)-U_{A|S_x}][U_2(D,D|S_x,S_y)-U_{A|S_y}]=(\sqrt{x_1y_1}-\frac{1-a_y}{2})(\sqrt{x_2y_2}-\frac{1-a_x}{2})$$

$$\text{s.t.} \quad x_1+x_2=x_1^p+x_2^p$$
$$y_1+y_2=(2-a_x-a_y)-x_1^p-x_2^p$$

where x_1^p and x_2^p are the amount of x produced by $M1$ and $M2$ at date 2.2 respectively. The solution is $x_1=y_1=\frac{1-a_y}{2}$, $x_2=y_2=\frac{1-a_x}{2}$. Thus, $U_1(D,D|S_x,S_y)=\frac{1-a_y}{2}=U_{A|S_x}$ and $U_2(D,D|S_x,S_y)=\frac{1-a_x}{2}=U_{A|S_y}$.

iii) If the equilibrium choice at date 1.0 is (S_x,D), $M1$ has learned to produce x and $M2$ has learned to produce both x and y at the first stage. The question is whether $M1$ will stick with S_x or learn to produce y and become diversification at date 2.0.

Similar to the proof of lemma 1, if $M1$ sticks with S_x, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ is zero and the outside option for $M2$ is autarchy. The payoff from autarchy for $M2$ conditional on diversification at the first stage is $U_{A|D}$, which is equal to $\frac{1}{2}$. We need to solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_2^p}[U_1(S_x,D|S_x,D)-0][U_2(S_x,D|S_x,D)-U_{A|D}]=(\sqrt{x_1y_1})(\sqrt{x_2y_2}-\frac{1}{2})$$

$$\text{s.t.} \quad x_1+x_2=1+x_2^p$$
$$y_1+y_2=1-x_2^p$$

where x_2^p is the amount of x produced by $M2$ at date 2.2. The solution is $x_1=y_1=\frac{1}{4}$, $x_2=y_2=\frac{3}{4}$. Thus, $U_1(S_x,D|S_x,D)=\frac{1}{4}$ and $U_2(S_x,D|S_x,D)=\frac{3}{4}$.

If $M1$ learns to produce y and becomes diversification, at date 2.1 when $M1$ and $M2$ negotiate to form strategic alliance, the outside option for $M1$ and $M2$ are both autarchy. The payoff from autarchy for $M1$ conditional on specialization on x at the first stage is $U_{A|S_x}$, which is equal to $\frac{1-a_y}{2}$. The payoff from autarchy for $M2$ conditional on diversification at the first stage is $U_{A|D}$, which is equal to $\frac{1}{2}$. Similarly, solve the following maximization problem.

$$\max_{x_1,x_2,y_1,y_2,x_1^p,x_2^p}[U_1(D,D|S_x,D)-U_{A|S_x}][U_2(D,D|S_x,D)-U_{A|D}]=(\sqrt{x_1y_1}-\frac{1-a_y}{2})(\sqrt{x_2y_2}-\frac{1}{2})$$

$$\text{s.t.} \quad x_1+x_2=x_1^p+x_2^p$$
$$y_1+y_2=(2-a_y)-x_1^p-x_2^p$$

20

where x_1^p and x_2^p are the amount of x produced by $M1$ and $M2$ at date 2.2 respectively. The solution is $x_1 = y_1 = \frac{1-a_y}{2}$, $x_2 = y_2 = \frac{1}{2}$. Thus, $U_1(D, D|S_x, D) = \frac{1-a_y}{2} = U_{A|S_x}$ and $U_2(D, D|S_x, D) = \frac{1}{2} = U_{A|D}$. ■

Proof of Proposition 2

i) Clearly, since $a_x < \frac{1}{2}$, $\frac{1-a_x}{2} > \frac{1}{4}$. The dominant strategy for $M2$ at date 2.0 is D.

ii) Since $0 < a_y < a_x < \frac{1}{2}$, $\frac{\sqrt{1-a_x}}{2} - \frac{1-a_x}{4} < \frac{1-a_x}{2} < \frac{1-a_y}{2}$. Further, $\frac{\sqrt{1-a_y}}{2} - \frac{1-a_y}{4}$ is decreasing in a_y and its supremum is $\frac{1}{4}$, while $\frac{1-a_x}{2}$ is decreasing in a_x and its infimum is $\frac{1}{4}$. That is to say $\frac{\sqrt{1-a_y}}{2} - \frac{1-a_y}{4} < \frac{1-a_x}{2}$. Thus, (D, D) is always a Nash equilibrium at date 2.0.

Clearly, if $a_x > a_y \geq 2\sqrt{3} - 3$, $\frac{\sqrt{1-a_x}}{2} - \frac{1-a_x}{4} < \frac{\sqrt{1-a_y}}{2} + \frac{1-a_y}{4} \leq \frac{1}{2}$. This implies if $a_x > a_y \geq 2\sqrt{3} - 3$, (S_x, S_y) is a Nash equilibrium at date 2.0. Otherwise, if $a_y < 2\sqrt{3} - 3$, $\frac{\sqrt{1-a_y}}{2} + \frac{1-a_y}{4} > \frac{1}{2}$ and definitely $M1$ will become diversification. In this case, (D, D) is the unique Nash equilibrium at date 2.0. ■

Proof of Corollary 4

Intuitively, if fixed learning costs are small, $M1$ and $M2$ will stick with (D, D). In this case, transaction cost is zero. If fixed learning costs are increasing to some extent, $M1$ and $M2$ may switch from (D, S_y) to (D, D) at date 2.0. By the proof of lemma 2 part (i), transaction cost is $(\frac{1}{2} + \frac{1}{2}) - (\frac{1}{2} + \frac{1-a_x}{2}) = \frac{a_x}{2}$. If fixed learning costs are large enough but still $a_y < 2\sqrt{3} - 3$, $M1$ and $M2$ will switch from (S_x, S_y) to (D, D) at date 2.0. By the proof of lemma 2 part (ii), transaction cost is $(\frac{1}{2} + \frac{1}{2}) - (\frac{1-a_x}{2} + \frac{1-a_y}{2}) = \frac{a_x+a_y}{2}$. If $a_y \geq 2\sqrt{3} - 3$, $M1$ and $M2$ will stick with (S_x, S_y) at date 2.0. In this case, transaction cost is zero. (See figure 7 for the MATLAB simulation.) ■

Proof of Proposition 3

i) Similar to the proof of proposition 1, if $a_x \to 0$ and $a_y \to 0$, then the payoff matrix in lemma 1 will converge to table 6. Clearly, D is the dominant strategy for both $M1$ and $M2$ at date 1.0.

Since $\delta\frac{1-a_x}{2} < \delta\frac{1-a_y}{2} < \delta\frac{1}{2}$, comparing table 1 and the top half of table 3, we find that D will always be the dominant strategy for both $M1$ and $M2$ at date 1.0 in the discounted two stage game if D is the dominant strategy for both $M1$ and $M2$ at date 1.0 in the static one stage game. Adding the second stage discounted payoff only reinforces the dominance of D. In this case, the unique Nash Equilibrium is
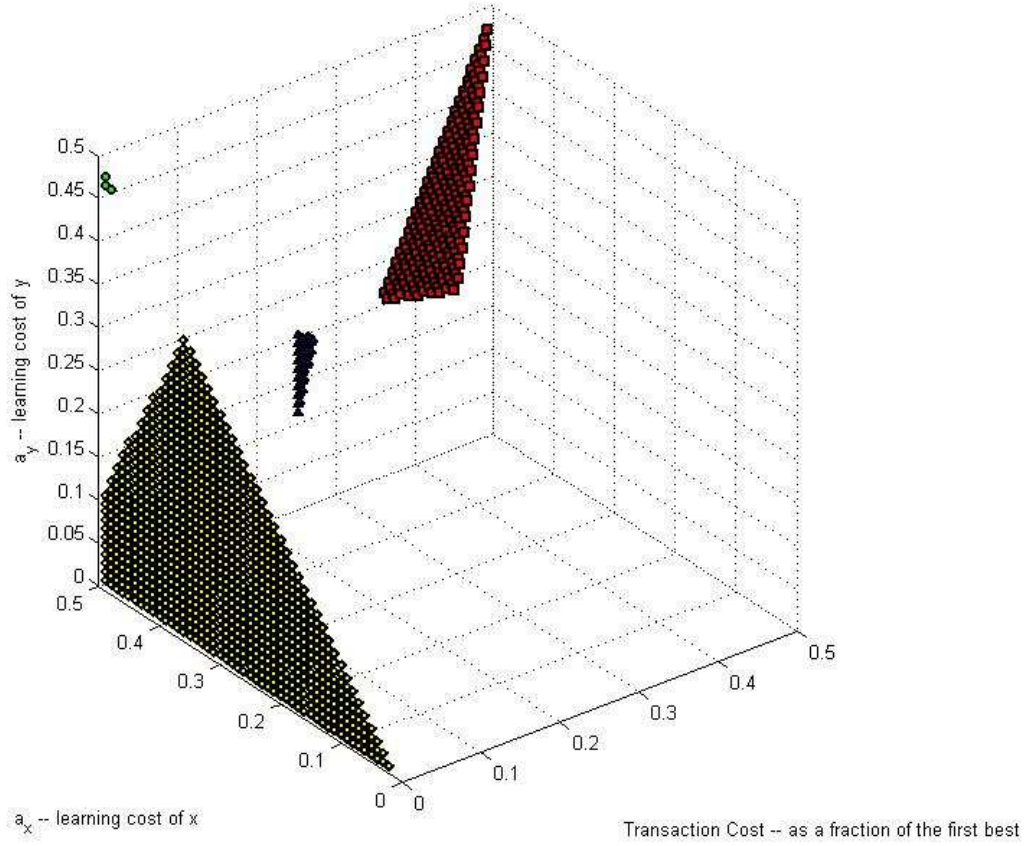
Figure 7: Transaction Cost – Myopic Case

Table 6: Payoff matrix: $a_x \to 0$ and $a_y \to 0$

		M2	
		S_y	D
M1	S_x	$\frac{1}{2}+\delta\frac{1}{2}, \frac{1}{2}+\delta\frac{1}{2}$	$\frac{1}{4}+\delta\frac{1}{2}, \frac{3}{4}+\delta\frac{1}{2}$
	D	$\frac{3}{4}+\delta\frac{1}{2}, \frac{1}{4}+\delta\frac{1}{2}$	$\frac{1}{2}+\delta\frac{1}{2}, \frac{1}{2}+\delta\frac{1}{2}$

$(D, D; D, D)$.

ii) Similar to the proof of proposition 1, if $a_x \to \frac{1}{2}$ and $a_y \to \frac{1}{2}$, then the pay-off matrix in lemma 1 will converge to table 7. Clearly, $(S_x, S_y; S_x, S_y)$ is a Nash equilibrium.

In this case, we may have multiple equilibria. $(D, D; D, D)$ may still be a Nash equilibrium, even though it is Pareto dominated by $(S_x, S_y; S_x, S_y)$.

Table 7: Payoff matrix: $a_x \to \frac{1}{2}$ and $a_y \to \frac{1}{2}$

		M2	
		S_y	D
M1	S_x	$\frac{1}{4}+\delta\frac{1}{2}, \frac{1}{4}+\delta\frac{1}{2}$	$0+\delta\frac{1}{4}, 0+\delta\frac{1}{2}$
	D	$0+\delta\frac{1}{2}, 0+\delta\frac{1}{4}$	$0+\delta\frac{1}{2}, 0+\delta\frac{1}{2}$

iii) For instance, if $\delta = 0.1$, $a_x = 0.49$ and $a_y = 0.25$, we can check that $(D, S_y; D, D)$ is a Nash equilibrium.

If $\delta = 0.3$, $a_x = 0.45$ and $a_y = 0.35$, we can check that $(S_x, S_y; D, D)$ is a Nash equilibrium.

iv) Since $\delta\frac{1-a_x}{2} < \delta\frac{1-a_y}{2} < \delta\frac{1}{2}$ and the absolute value of differences between $\delta\frac{1-a_x}{2}$, $\delta\frac{1-a_y}{2}$, and $\delta\frac{1}{2}$ are increasing in δ, comparing table 1 and table 3, clearly D is more likely to be the choice for both $M1$ and $M2$ at date 1.0 if δ increases. Therefore, $M1$ and $M2$ are more likely to end up with $(D, D; D, D)$ if they are more patient.

v) According to the proof in part ii) above, if $a_y \geq 2\sqrt{3} - 3$, $(S_x, S_y; S_x, S_y)$ is a Nash equilibrium. Similarly, according to 7, $(D, D; D, D)$ is also a Nash equilibrium. In this case, we have a coordination game.

vi) If $a_y \geq 2\sqrt{3} - 3$, according to the bottom half of table 2, to let (S_x, D) be an equilibrium choice at date 1.0, we must have

$$\begin{cases} U_{XD} - \frac{1}{2}U_A + \delta\frac{1-a_y}{2} \geq U_A + \delta\frac{1}{2} \\ U_{XD} + \frac{1}{2}U_A + \delta\frac{1}{2} \geq U_{XY} + \delta\frac{1}{2} \end{cases}$$

This implies

$$\begin{cases} U_{XD} - \frac{1}{2}U_A \geq U_A + \delta\frac{a_y}{2} > U_A \\ U_{XD} + \frac{1}{2}U_A \geq U_{XY} \end{cases}$$

Back to the proof in proposition 1, clearly this is impossible for $0 < a_y < a_x < \frac{1}{2}$.

If $a_y < 2\sqrt{3} - 3$, according to the top half of table 2, to let (S_x, D) be an equilibrium choice at date 1.0, we must have

$$\begin{cases} U_{XD} - \frac{1}{2}U_A + \delta\frac{1-a_y}{2} \geq U_A + \delta\frac{1}{2} \\ U_{XD} + \frac{1}{2}U_A + \delta\frac{1}{2} \geq U_{XY} + \delta\frac{1-a_x}{2} \end{cases}$$

Using the exhaustion method, by Matlab simulation (see figure 8), the above two inequalities will not hold at the same time for $0 < a_y < a_x < \frac{1}{2}$ and $0 \leq \delta \leq 1$. ∎
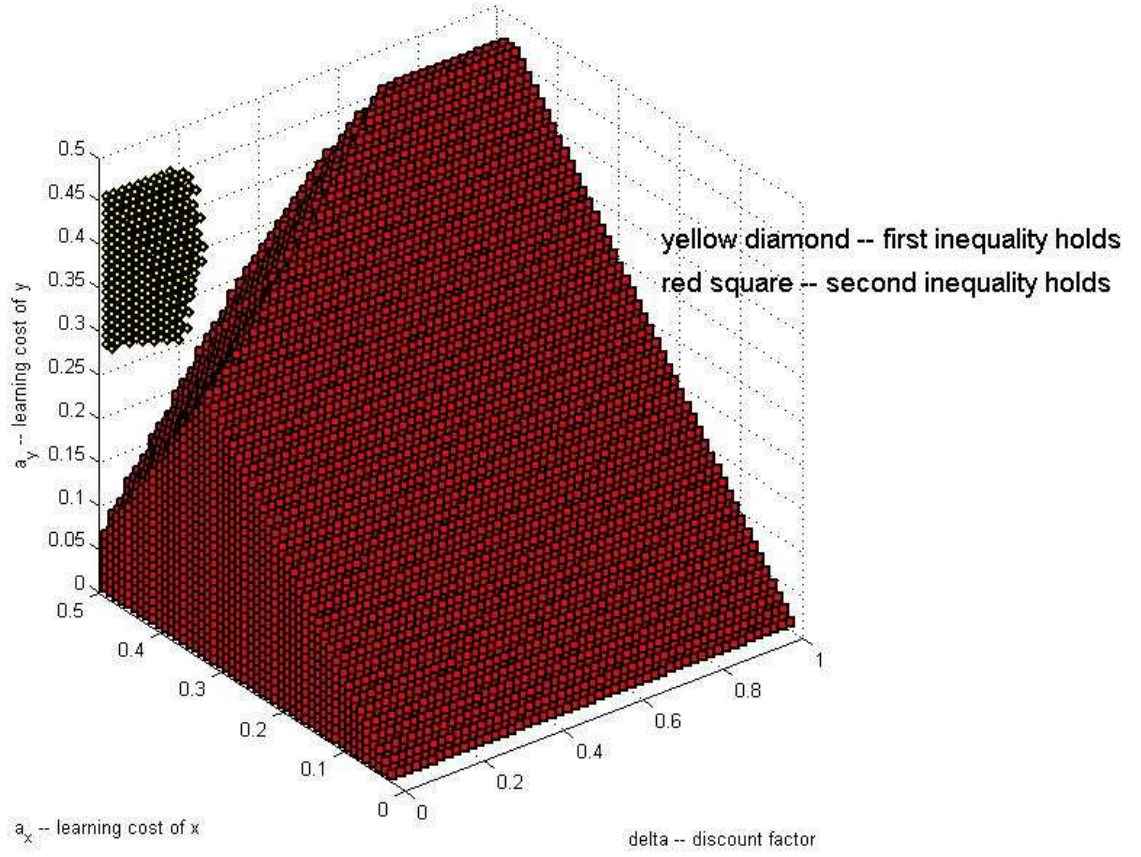
Figure 8: Areas of Inequalities Hold

Proof of Corollary 6

i) If $a_y \geq 2\sqrt{3} - 3$, according to the payoff matrix in table 3, the first best is $2U_{XY} + \delta$. Thus, the transaction cost under different combination of choices at date 1.0 are as described in table 8.

Table 8: Transcation Cost – Foresighted Case: $a_y \geq 2\sqrt{3} - 3$

		$M2$	
		S_y	D
$M1$	S_x	0	$2(U_{XY} - U_{XD}) + \delta\frac{a_y}{2}$
	D	$2(U_{XY} - U_{DY}) + \delta\frac{a_x}{2}$	$2(U_{XY} - U_A)$

According to proposition 3, if $a_y \geq 2\sqrt{3} - 3$ the symmetric strategic alliance is

24

sustainable. Thus, transaction cost is zero.

ii) If $a_y < 2\sqrt{3} - 3$, according to the payoff matrix in table 3, the first best is $2U_{XY} + \delta$. Thus, the transaction cost under different combination of choices at date 1.0 are as described in table 9.

Table 9: Transcation Cost – Foresighted Case: $a_y < 2\sqrt{3} - 3$

		M2	
		S_y	D
M1	S_x	$\delta(\frac{a_x}{2} + \frac{a_y}{2})$	$2(U_{XY} - U_{XD}) + \delta\frac{a_y}{2}$
	D	$2(U_{XY} - U_{DY}) + \delta\frac{a_x}{2}$	$2(U_{XY} - U_A)$

Similar to the proof of corollary 1, given δ, transaction cost is not monotonic with respect to a_x and a_y. In particular, if fixed learning costs are small, the equilibrium choice at date 1.0 will be (D, D) and $M1$ and $M2$ end up with autarchy. In this case, transaction cost is $2U_{XY} - 2U_A$, which is increasing in fixed learning costs. If fixed learning costs are increasing to some extent, the equilibrium choice at date 1.0 may be (D, S_y) and $M1$ and $M2$ may end up with forming asymmetric strategic alliance. In this case, transaction cost is $2U_{XY} - 2U_{DY} + \delta\frac{a_x}{2}$, which is increasing in fixed learning costs. Using the exhaustion method, by Matlab simulation, there exists a jump down of transaction cost during the transition from (D, D) to (D, S_y). If fixed learning costs are large enough but still $a_y < 2\sqrt{3} - 3$, the equilibrium choice at date 1.0 will be (S_x, S_y) and $M1$ and $M2$ end up with forming symmetric strategic alliance (unstable – disbanded to autarchy at stage 2). In this case, transaction cost is $\delta(\frac{a_x}{2} + \frac{a_y}{2})$, which is increasing in fixed learning costs. Using the exhaustion method, by Matlab simulation, there exists a jump down of transaction cost during the transition from (D, D) or (D, S_y) to (S_x, S_y).

Further, given a_x and a_y, transaction cost is non-decreasing with respect to δ. From the proof of proposition 3, as δ increases, the possible transition of equilibrium choice at date 1.0 is from (S_x, S_y) to (D, S_y) then to (D, D). Using the exhaustion method, by Matlab simulation, transaction cost is non-decreasing with respect to δ during these transitions. ∎

References

[1] Baker, George and Kevin J. Murphy. 2002. "Relational Contracts in Strategic Alliances." mimeo.

[2] Becker, Gary S. and Kevin M. Murphy. 1992. "The Division of Labor, Coordination Costs, and Knowledge." *Quarterly Journal of Economics*, 107(4): 1137-60.

[3] Che,Y. K., and Sákovics. 2007. "Hold-up Problem." *The New Palgrave Dictionary of Economics*, Second Edition.

[4] Coase, Ronald H. 1937. "The Nature of the Firm." *Economica*, 4: 386-405.

[5] Grossman, Sanford and Oliver Hart. 1986. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy*, 94(4): 691-719.

[6] Hart, Oliver and John Moore. 1990. "Property Rights and the Nature of the Firm." *Journal of Political Economy*, 98(6): 1119-58.

[7] Klein, B., Crawford, R. and Alchian, A. 1978. "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process." *Journal of Law and Economics*, 21: 297-326.

[8] Wang, Huan. 2005. *Essays on Group Politics*. Dissertation, New York University, Department of Politics.

[9] Williamson, Oliver. 1979. "Transactions-Cost Economics: The Governance of Contractual Relations." *Journal of Law and Economics*, October 22(2): 233-62.

[10] Yang, Xiaokai and Jeff Borland. 1991. "A Microeconomic Mechanism for Economic Growth." *Journal of Political Economy*, 99(3): 460-82.