Optimal Stackelberg Strategies for Financing A Supply Chain through

Online Peer-to-peer Lending

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Abstract

In recent years, supply chain finance (SCF) through online peer-to-peer (P2P) lending platforms has gained its popularity. We study an SCF system with a manufacturer selling a product to a retailer that faces uncertain demand over a single period. We assume that either the retailer or the manufacturer faces a capital constraint and must borrow capital through an online P2P lending platform. The platform determines a service rate for the loan, the manufacturer sets a wholesale price for the product, and the retailer chooses its order quantity for the product. We identify optimal Stackelberg strategies of the participants in the SCF system. For an SCF system with a capital-constrained retailer, we find that the retailer’s optimal order quantity and the manufacturer’s optimal wholesale price decrease with the platform’s service rate. For an SCF system with a capital-constrained manufacturer, we find that as the platform’s service rate increases, the manufacturer’s optimal wholesale price increases but the retailer’s optimal order quantity decreases. Our analysis suggests that it is important for the retailer and the manufacturer to take the online P2P lending platform’s financial decisions (such as the service rate) into account when making their operational decisions.

Keywords: Supply chain management; Supply chain finance; Online P2P lending; Stackelberg game; Operational and financial decisions

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1. Introduction

Supply chain finance (SCF) is a new financial business for enterprises in a supply chain. The aims of SCF are to diversify the funding sources of capital-constrained enterprises and improve the entire supply chain’s financial efficiency (Buzacott and Zhang, 2004; Yan et al., 2016). As evidenced in the literature, SCF has become a prevailing short-term financing source for thousands of small- and medium-size enterprises (SMEs), which account for more than 90% of all firms (Yan et al., 2016). These SMEs play an important role as they may serve as suppliers or retailers in a supply chain (Singh, 2011). For example, 93.5% of the suppliers in the Korean automobile industry are SMEs (Choi, 2003; Lee, 2008), and most grocery retailers in China are SMEs owned by local companies (IGD, 2013; Eng, 2016).

Regardless of their roles in a supply chain, SMEs often face a shortage of funds to turn around their business (Cai et al., 2014; Yan et al., 2016). Capital-constrained SMEs in a supply chain traditionally borrow loans from two widely used financing sources: external bank credit financing (BCF) and internal trade credit financing (TCF). BCF refers to one enterprise in a supply chain accesses financing from banks, whereas TCF refers to one enterprise in a supply chain extends its credit to an upstream or downstream partner using short-term loans (that is, accounts payable or accounts receivable) (Kouvelis and Zhao, 2012; Cai et al., 2014; Yan et al., 2016).

It is generally challenging for SMEs to finance through BCF. For example, approximately 50% of SMEs facing financial constraints in developing countries (such as China) suffer from limited access to external financial resources (Buzacott and Zhang, 2004; Baas and Schrooten, 2006; Chen, Lai, and Lin, 2014). Furthermore, it is a global phenomenon that banks are generally reluctant to provide credit to SMEs (Baas and Schrooten, 2006; Ni et al., 2017). On the other hand, an upstream or downstream partner in a supply chain would regard an SME with low wealth (low working capital or low collateral) as an enterprise with low creditworthiness and high bankruptcy risk. To mitigate potential risk, the upstream or downstream partner often charges the SME a high interest rate, which results in a high financing cost under TCF. This causes the capital-constrained SMEs to seek other financing schemes such as online peer-to-peer (P2P) lending.
With advances in information technologies and the Internet, online P2P lending has become an important supplement to traditional financing recently (Mild, Waitz, and Wöckl, 2015). Online P2P lending allows firms to directly finance one another on an online platform, which does not involve traditional financial intermediaries (Guo et al., 2016; Chen, Lai, and Lin, 2014; Chen and Han, 2012; Lin, Prabhala, and Viswanathan, 2009; Bachmann et al., 2011). Compared with the traditional financing sources (such as BCF and TCF), online P2P lending functions in a different manner. For example, in an online P2P lending platform, scattered money in the society is consolidated, financing activities are completed through the Internet, borrowers would not be required to submit a high pledge or provide comprehensive information, and borrowers' creditworthiness is only assessed by a simple risk evaluation (Guo et al., 2016).

Specifically, the following characteristics differentiate online P2P lending from BCF and TCF: (1) The source of profit for online P2P lending is commission (determined by its service rate). In contrast to BCF, which benefits from the spread between deposit and loan, online P2P lending benefits from integrating the information from both borrowers and investors (Lin, 2009). Note that online P2P lending platforms may also bear the borrowers’ default risk (Zhang, 2017; Wang et al., 2015; Wang et al., 2014). They promise a principal refund to the investors in case of default or overdue to attract the investors that are conservative about P2P lending (Wang et al., 2015). For example, the P2P lending platforms in China, Hongling Capital (www.my089.com), Hydbest (www.hydbest.com), and Ssrong (www.ssrong.com), attract many investors as they guarantee their investors’ principal and interest. (2) Borrowers and lenders can easily post and search for information, and complete transactions on the online platform with low transaction costs (Lin, 2009; Lin, Prabhala, and Viswanathan, 2009, 2013). (3) SMEs can acquire loans more efficiently. (4) The lack of comprehensive information and risk evaluation of the borrowers leads to information asymmetry between the lenders and the borrowers (Lin, Prabhala, and Viswanathan, 2009).

As a new financing mode, SCF based on online P2P lending has altered the participants’ roles and relationships in a supply chain. This creates new challenges such as identifying the participants’ optimal operational and financial decisions, as well as the interactions between these decisions.
In this paper, we consider a supply chain with a manufacturer selling a product to a retailer that faces uncertain demand over a single period. We assume that either the retailer or the manufacturer faces a capital constraint and must borrow capital through an online P2P lending platform. The platform is responsible for evaluating a borrower’s creditworthiness, and publishing the loan information online to invite individual lenders to bid for the loan. We assume that the borrower will be fully financed. The platform determines a service rate for the loan, the manufacturer sets a wholesale price for the product, and the retailer chooses its order quantity for the product.

At the end of the selling period, the platform will earn a profit if the borrower repays the loan principal and interest, and pays the service commission to the platform. If the borrower’s revenue is not sufficient to cover its loan obligation and the service commission, then it declares bankruptcy and the platform will use the borrower’s liquidated asset and some risk-reserve capital to repay the loan principal and interest to the lenders. This causes a loss to the platform. We determine the online P2P platform’s optimal service rate, the manufacturer’s optimal wholesale price, and the retailer’s optimal order quantity under two different situations described as follows.

In the first situation, we assume a capital-constrained retailer with bankruptcy risk. We formulate a multi-level Stackelberg game model with three participants, where the manufacturer acts as a leader, the online P2P lending platform acts as a subleader, and the capital-constrained retailer acts as a follower. We identify the manufacturer’s optimal wholesale price, the platform’s optimal service rate, and the retailer’s optimal order quantity. We also prove some properties of these optimal decisions.

In the second situation, we assume a capital-constrained manufacturer with bankruptcy risk. Similarly, we construct a multi-level Stackelberg game model with three participants. In this model the platform acts as a leader, the capital-constrained manufacturer acts as a subleader, and the retailer acts as a follower. We determine the optimal service rate, wholesale price, and order quantity.

We make three main contributions in this paper. First, we address the gap in the literature on SCF with online P2P lending by constructing a theoretical model. Second, we obtain the optimal Stackelberg solution for a supply chain with a capital-constrained retailer or with a
capital-constrained manufacturer. Third, we provide insights by analyzing the interactions between the operational and the financial decisions.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 defines notation and specifies assumptions. Sections 4 and 5 analyze the optimal Stackelberg solutions for a supply chain with a capital-constrained retailer and with a capital-constrained manufacturer respectively. Section 6 studies the interactions between the operational and the financial decisions based on realistic parameter values for small-size manufacturing enterprises in China. Section 7 extends the study to include two cases where the online P2P platform is risk averse and where the platform does not bear the borrowers’ default risk. Section 8 gives concluding remarks.

2. Related literature

Our work is related to two streams of literature. The first stream of literature studies the equilibrium strategies for SCF problems. The second stream focuses on online P2P lending. We discuss them separately below.

2.1. Equilibrium strategies for supply chain finance

Many papers in this stream of literature focus on the optimal Stackelberg strategies for a capital-constrained supply chain. For example, Buzacott and Zhang (2004) make the first attempt to incorporate asset-based financing into production decisions by developing game theoretical models that capture the trade-offs between financial and production decisions. They demonstrate that asset-based financing can offer retailers better returns than using only their own capital. Caldentey and Haugh (2009) formulate a Stackelberg game model consisting of a capital-constrained retailer and a manufacturer to analyze the equilibrium strategies for two different supply chain contracts. They find that if the retailer is more cash constrained, the manufacturer will charge a higher wholesale price. Under that scenario, the manufacturer always prefers a flexible contract with hedging. Kouvelis and Zhao (2011) investigate a capital-constrained retailer’s optimal order quantity and a manufacturer’s optimal wholesale price for three types of bankruptcy cost. They formulate a Stackelberg game model in which the manufacturer acts as a leader and the retailer acts as a follower.
They find that in the presence of the retailer’s bankruptcy risk, the increase in the retailer’s wealth leads to the increase in the equilibrium wholesale price. If there is no bankruptcy risk, the increase in the retailer’s wealth leads to the same or a lower equilibrium wholesale price.

To address the problem of a third-party logistics (3PL) firm providing both logistics and financial services to a capital-constrained retailer, Chen and Cai (2011) conduct a Stackelberg game model to obtain the retailer’s optimal operational decisions and the 3PL firm’s optimal financial decisions. Compared with supplier credit financing, the 3PL financing can yield better performance.

Yan and Sun (2013) formulate a multi-level Stackelberg game model in which a manufacturer acts as a leader, a bank acts as a subleader, and a capital-constrained retailer acts as a follower. Considering the retailer’s bankruptcy risk, they analyze the manufacturer’s optimal wholesale price, the bank’s optimal credit line, and the retailer’s optimal order quantity. Yan, Dai, and Sun (2014) analyze the optimal strategies for a supply chain with a manufacturer and a retailer, who are both capital constrained. They formulate the problem as a bi-level Stackelberg game in which the bank acts as a leader and the manufacturer acts as a subleader. They derive the bank’s optimal interest rate, the manufacturer’s optimal wholesale price, and the retailer’s optimal order quantity. Yan and Sun (2015) conduct a comparative analysis of the Stackelberg equilibrium decisions for SCF participants between two different financing modes. Yan et al. (2016) study the equilibrium strategies for a supply chain with a capital-constrained retailer, a manufacturer, and a commercial bank. They design a partial-credit guarantee contract that combines BCF and TCF, and present the coordination conditions.

In contrast, our paper considers a capital-constrained borrower that obtains loan through an online P2P lending platform. To the best of our knowledge, no papers provide a mathematical analysis of the equilibrium strategies for SCF with online P2P lending, and explore the interaction between the operational and financial decisions.

2.2. Online P2P lending

As a new platform in the Internet-based financial credit market, online P2P lending presents an alternative source of credit for individual borrowers and a potential investment opportunity
for individual lenders (Chaffee and Rapp, 2012; Chen, Zhou, and Wan, 2016). However, there are only limited studies related to online P2P lending. For instance, Berger and Gleisner (2009) empirically examine the role of financial intermediaries in the electronic P2P lending platforms. They find that the intermediaries in the electronic market improve the borrowers’ credit by reducing information asymmetry. The result suggests the attractiveness of online P2P lending. Puro et al. (2010) employ regression models and data-driven query methods to develop a decision support system for borrowers in online P2P lending. The system helps the borrowers to quantify their strategic options. Chen and Han (2012) make a comparative analysis of online P2P lending markets in the USA and China. They classify the credit information into two types: hard information and soft information. The authors find that these two types of credit information may affect lending outcomes in both countries.

Some papers study credit risk assessment in online P2P lending. For example, Chen, Lai, and Lin (2014) develop an integrated trust model to examine trust in online P2P lending. Based on real data collected from a P2P lending platform in China, the study reveals that trust in the borrowers can more efficiently drive the lenders’ intention to lend than trust in the intermediary. Emekter et al. (2015) empirically evaluate the credit risk of online P2P loans. They find that a borrower’s credit score, debt-to-income ratio, FICO score, and utilization of revolving credit lines have significant impact on the likelihood of their loan default. Guo et al. (2016) propose a data-driven investment decision-making framework for online P2P lending to help individual investors to effectively allocate their money across different loans. They propose an instance-based risk assessment model and use a portfolio optimization method to determine investment decisions in online P2P lending.

Other papers on online P2P lending study lenders’ herding behavior and the impact of such behavior on lending outcomes. Herzenstein, Dholakia, and Andrews (2011) conduct an empirical study of online P2P lending and demonstrate that strategic herding behavior exists. They find that the herding behavior has a positive impact on both the lenders and borrowers. Lee and Lee (2012) employ a multinomial logit market-share model to empirically investigate the herding behavior in an online P2P lending market. They find that the lenders in the online P2P lending market tend to exhibit herding behavior. Liu et al. (2015) empirically explore the impact of friendship on economic decision making in online P2P lending. In particular,
friends of the borrowers have a stronger tendency to offer loans than strangers. Endorsements by friends of the borrowers have a negative impact on subsequent lenders’ decisions.

In contrast, this paper mainly focuses on optimal decisions in a supply chain with financing through online P2P lending, which is not studied in the literature. Furthermore, none of the previous papers have explicitly discussed the integration of SCF and online P2P lending.

3. Notation and assumptions

3.1. Notation

The following notation is used to represent the parameters and decision variables.

Parameters:

$P$ : Retailer’s unit retail price, which is normalized to 1. This is consistent with the literature (Burkart and Ellingsen, 2004; Mateut et al., 2006; Cai et al., 2014).

c : Manufacturer’s unit production cost.

$D$ : Market demand, which is a nonnegative random variable following a continuous cumulative distribution function $F(D)$ with density function $f(D) > 0$.

$B_r$ : Retailer’s initial capital.

$B_m$ : Manufacturer’s initial capital.

$i_r$ : Internet financing interest rate for the retailer.

$i_m$ : Internet financing interest rate for the manufacturer.

$\theta_r$ : Creditworthiness of the retailer.

$\theta_m$ : Creditworthiness of the manufacturer.

$\eta$ : Online P2P lending platform’s inability to respond to the borrower’s bankruptcy, $\eta \in [0,1]$ ( $\eta = 1$ means that the platform has no risk reserve capital and $\eta = 0$ corresponds to the case where the platform has the maximum risk reserve capital).

$\xi$ : Online P2P lending platform’s maximum recovery cost when the borrower is bankrupt.
It is worth noting that the Internet financing interest rates \( i_r \) and \( i_m \) depend only on the creditworthiness of the borrowers, and are independent of the decision variables defined as follows.

Decision variables:

\( Q \): Retailer’s order quantity from the manufacturer.

\( s_r \): Online P2P lending platform’s service rate provided to the retailer, \( s_r \in [s_{r-}, s_{r+}] \) and \( 0 \leq s_{r-} < s_r \leq 1 \).

\( s_m \): Online P2P lending platform’s service rate provided to the manufacturer, \( s_m \in [s_{m-}, s_{m+}] \) and \( 0 \leq s_{m-} < s_m \leq 1 \).

\( w \): Manufacturer’s wholesale price for each unit of the product.

We assume that the amount of capital that the retailer needs to borrow is \( wQ-B_r > 0 \).

Similarly, the amount of capital that the manufacturer needs to borrow is \( cQ-B_m > 0 \). To eliminate uninteresting cases, we assume that \( c < w \) and \( w(1+i_r+s_r) < p = 1 \).

3. Assumptions

We make the following assumptions in the paper.

**Assumption 1.** The online P2P lending platform, the manufacturer, and the retailer are all risk-neutral, and they maximize their expected profits (Chen and Cai, 2011; Cai et al., 2014).

**Assumption 2.** All parameters are common knowledge to the three participants (Kouvelis and Zhao, 2012).

**Assumption 3.** Both the retailer and the manufacturer have difficulty in obtaining bank loans and therefore the online P2P lending platform is the only financing option available (Chen and Han, 2012).

**Assumption 4.** Let \( h(D) = f(D)/\overline{F}(D) \) and \( H(D) = Dh(D) \) denote the failure rate and the generalized failure rate, respectively, with \( \overline{F}(D) = 1 - F(D) \). We assume the product’s demand \( D \) satisfies the increasing failure rate (IFR) condition: \( dh(D)/dD > 0 \) (Chen and
Cai, 2011; Cai et al., 2014; Yan et al., 2016).

**Assumption 5.** The capital market is perfect without taxes, operation costs, and bankruptcy costs, and therefore we omit these costs in the paper (Kouvelis and Zhao, 2012).

According to Assumption 3, the retailer and the manufacturer with capital constraints in the supply chain are willing to be financed by the online P2P lending platform, even though the platform has a higher interest rate compared to the commercial banks (Chen and Han, 2012).

4. *Optimal strategies for financing a supply chain with a capital-constrained retailer*

In this section, we consider a situation where the manufacturer has sufficient capital but the retailer is under financial stress and encounters start-up difficulties. Thus, the retailer borrows loans through online P2P lending to maximize its profit. As many loans are not secured by collateral, the platform will assess the retailer’s creditworthiness to determine the Internet financing interest rate \( i_r \), which is a parameter of our model (Mild, Waitz, and Wöckl, 2015).

We derive the corresponding market equilibrium for an SCF system shown in Fig. 1, which consists of three participants (the manufacturer, the online P2P lending platform, and the retailer). We use Stackelberg game theory to analyze the optimal strategies and study the interactions among the three participants. In the multi-level Stackelberg game, the manufacturer acts as a leader, the online P2P lending platform acts as a subleader, and the retailer acts as a follower. We have the following sequence of events. Knowing the optimal responses of the platform and the retailer, the manufacturer first sets a wholesale price \( w \) for the retailer. After observing the wholesale price \( w \), the platform chooses the service rate \( s_r \) by assuming that the retailer will react optimally. Finally, the retailer determines its optimal order quantity \( Q \) given \( w \) and \( s_r \). Fig. 1 illustrates the detailed sequence of the business activities of the SCF system.

We assume that the retailer borrows the capital \( wQ - B_r \) through the platform. For notational convenience, let \( \lambda_r = 1 + i_r + s_r \). The retailer pays the amount \( wQ \) to the manufacturer and makes the total repayment \( L = \lambda_r(wQ - B_r) \) to the platform. Following
previous research (Kouvelis and Zhao, 2011; Yan et al., 2016), we use backward induction to obtain the equilibrium solutions of the SCF system.

**Fig. 1.** An SCF system with a capital-constrained retailer based on online P2P lending

**4.1. Retailer’s optimal decision**

Assuming symmetric information, the capital-constrained retailer borrows the amount \( wQ - B_r \) through the online P2P lending platform. The retailer uploads its authentication information required by the platform. After evaluating the retailer’s creditworthiness \( \theta_r \), the platform chooses an Internet financing interest rate \( i_r(\theta_r) \). Furthermore, assuming the retailer will react optimally, the platform also sets a service rate \( s_r \) for the loans. Given the wholesale price \( w \) and the service rate \( s_r \), the retailer determines the order quantity \( Q \) to maximize its expected profit:

\[
\Pi_r(Q) = E\{p \min[D,Q] + (wQ - B_r)(1 - s_r) - wQ - (wQ - B_r)(1 + i_r(\theta_r))\}. \tag{1}
\]

Eq. (1) can be rewritten as

\[
\Pi_r(Q) = E\{[\min[D,Q] - L]^+ - B_r\}. \tag{2}
\]

From Eq. (2), we know that if \( \min[D,Q] \geq L \) the retailer will not go bankrupt (that is, at the end of the selling season, the retailer’s revenue from the market can cover its loan
obligation and the service charge). Otherwise, the retailer announces bankruptcy and pays the amount \( \min[D,Q] \) to the platform. In the latter case, the platform will repay lenders using its risk reserve capital and the retailer’s liquidated assets.

The following proposition characterizes the retailer’s optimal order quantity and its basic properties. All proofs can be found in the Appendix.

**Proposition 1.** Given the wholesale price \( w \), the Internet financing interest rate \( i_r(\theta_r) \), and the service rate \( s_r \), we have the following results:

1. The retailer’s optimal order quantity is \( Q^*(w,s_r) = F^{-1}[w\lambda_r F(L)] \), where \( \lambda_r = 1 + i_r(\theta_r) + s_r \).

2. Given the initial capital \( B_r \), the retailer’s optimal order quantity \( Q^*(w,s_r) \) decreases with the wholesale price \( w \).

3. The retailer’s optimal order quantity \( Q^*(w,s_r) \) decreases with the Internet financing interest rate \( i_r \) and the service rate \( s_r \), but increases with its creditworthiness \( \theta_r \).

4. The retailer’s optimal order quantity \( Q^*(w,s_r) \) decreases with its initial capital \( B_r \).

Proposition 1 shows that the retailer’s optimal order quantity depends not only on the operational decision (the wholesale price), but also on the Internet financing interest rate and the service charge. This is one of the key differences between online P2P lending and offline traditional financing (e.g., BCF).

Part 2 of Proposition 1 shows that if the wholesale price increases, then the retailer will order less. Since the manufacturer’s profit is proportional to both the wholesale price \( w \) and the order quantity \( Q \), this implies that the manufacturer faces a trade-off when choosing the wholesale price \( w \). This trade-off is addressed in Section 4.3.

Under online P2P lending, both the Internet financing interest rate and the service rate determine the retailer’s financing cost and bankruptcy risk. Part 3 of Proposition 1 shows that the retailer will order less (hence borrow less) when the Internet financing interest and service rates are higher. Fortunately, Part 3 of Proposition 1 ensures that the retailer can increase its
order quantity by improving its creditworthiness.

Part 4 of Proposition 1 shows that the retailer’s order quantity decreases with its initial capital, which implies the retailer will borrow less when its initial capital is higher. For example, if the retailer’s initial capital is sufficiently large such that \( B_r \geq wF^{-1}(w) \), then it does not need to borrow through online P2P lending, and the retailer’s problem reduces to the classic newsvendor problem without capital constraints (Chen and Cai, 2011).

4.2. Online P2P lending platform’s optimal decision

As one of the participants in the SCF system, the online P2P lending platform acts as the subleader of the multi-level Stackelberg game. At the end of the sales season, the retailer should pay the loan principal and interest \( ((wQ - B_r)(1+i_r)) \) and the service charge \( ((wQ - B_r)s_r) \) to the platform from its revenue. The platform then earns the service charge, and repays the principal and interest to the investors. If the retailer’s revenue is not sufficient to cover its loan obligation and the service charge, the platform will use the retailer’s liquidated asset \( p\min[D,Q] \) as a repayment. If the liquidated asset is still insufficient to repay the principal, the interest, and the service charge, then the platform will use the risk reserve capital to fill the gap.

We assume the platform has limited risk reserve capital and has a significant recovery cost for the retailer’s bankruptcy. Let \( \eta \in [0,1] \) denote the platform’s inability to respond to the retailer’s bankruptcy, and let \( \xi \) denote the cost per recovery for the platform. We define the expected recovery cost as \( \xi F(\eta L / p) \). Note that if \( \eta = 1 \), it means that the platform has no risk reserve capital. Thus, the platform has to seek other capital to repay the investors, which incurs an expected recovery cost \( \xi F(L / p) \). If \( \eta = 0 \), it means that the platform has the maximum ability to respond to the retailer’s bankruptcy. This corresponds to the case where the platform has sufficient risk reserve capital such that it does not need any extra capital to fill the gap, which incurs an expected recovery cost \( \xi F(0) \).

It is worth noting that in practice the risk reserve capital is managed by an independent trust
institution. This is to ensure the safety and benefits of the Internet investors. Thus, the risk reserve capital cannot be viewed as the platform’s profit. Given the retailer’s optimal response \( Q^*(w,s_r) \) and its initial capital, the online P2P lending platform’s expected profit can be expressed as follows:

\[
\Pi_o(s_r) = E \{ \min[\min[D,Q^*],L] - (wQ^* - B_r)(1+i_r(\theta_r)) \} - \xi F(\eta L)
\]

\[
= \int_0^L F(D)dD - (wQ^* - B_r)(1+i_r(\theta_r)) - \xi F(\eta L).
\]  

(3)

The first term of Eq. (3) represents the expected revenue of the platform. Specifically, if \( \min[D,Q^*] \geq L \), then the platform earns a revenue of \( (wQ^* - B_r)s_r \). Otherwise, the retailer is bankrupt, and the platform liquidates the retailer and obtains an amount of \( \min[D,Q^*] - (wQ^* - B_r)(1+i_r(\theta_r)) \). The second term of Eq. (3) represents the expected recovery cost.

The following proposition characterizes the P2P lending platform’s optimal service rate and its basic properties.

**Proposition 2.** Given the retailer’s best response, the initial capital \( B_r \), the manufacturer’s wholesale price \( w \), and the Internet financing interest rate \( i_r(\theta_r) \), the online P2P lending platform’s optimal service rate is

\[
s_r^* = \begin{cases} 
  s_r & \text{if } \mu(s_r) \geq 1+i_r(\theta_r) \\
  s_r & \text{if } \mu(s_r) \leq 1+i_r(\theta_r) \\
  \hat{s}_r & \text{if } \mu(s_r) < 1+i_r(\theta_r) \text{ and } \mu(\hat{s}_r) > 1+i_r(\theta_r)
\end{cases}
\]

In the middle case, where \( \mu(s_r) < 1+i_r(\theta_r) \) and \( \mu(\hat{s}_r) > 1+i_r(\theta_r) \), a unique \( \hat{s}_r \) is implied by \( \mu(s_r) = 1+i_r(\theta_r) \), where

\[
\mu(s_r) = \frac{\bar{F}(Q^*)(1-\xi h(\eta L))[1-wQ^* - B_r h(Q^*)]}{w[1-Lh(L)]}
\]

increases with \( s_r \in [s_r, \hat{s}_r] \).

**Proposition 2** suggests that the P2P lending platform’s optimal service rate heavily relies on \( \mu(s_r) \) which can be regarded as the coefficient that indicates the sensitivity of the order
quantity to the service rate change. When \( \mu(s_r) \geq 1 + i_r(\theta) \), the order quantity is considered very sensitive to the service rate change, which implies that a minor increase in the service rate will result in a major decrease in order quantity and the amount of the retailer’s loan. In this case, the online P2P lending platform’s profit decreases with the service rate, such that \( \frac{d \Pi_s(s_r)}{ds_r} \leq 0 \). Thus, the online P2P lending platform will set the service rate at the lowest level \( s_r \) to encourage the retailer to place more orders. When \( \mu(s_r) < 1 + i_r(\theta) \), the order quantity is insensitive to the service rate change, and the retailer’s order decision is thus only slightly affected by the service rate. In this case \( \frac{d \Pi_s(s_r)}{ds_r} \geq 0 \), and the online P2P lending platform will set the service rate at the highest level \( s_r \) to maximize its profit. Except for the above two extreme cases, when \( \mu(s_r) < 1 + i_r(\theta) \) and \( \mu(s_r) > 1 + i_r(\theta) \), a unique service rate \( s_r^* \), defined in Proposition 2, can balance the trade-off between the online P2P lending platform’s profit and the retailer’s order quantity.

4.3. Manufacturer’s optimal decision

As the upstream firm in the supply chain, the manufacturer without capital constraints aims to choose an optimal wholesale price to maximize its expected profit. Hence, the manufacturer’s decision problem can be formulated as follows.

\[
\Pi_m(w) = (w - c)Q^*.
\]  

**Proposition 3.** Given the retailer’s optimal order quantity \( Q^* \), the optimal service rate \( s_r^* \) and the Internet financing interest rate \( i_r(\theta) \), the manufacturer would set the optimal wholesale price that uniquely satisfies

\[
w^* = c - \frac{w^* Q^* [w^* \lambda_r h(L) - h(Q^*)]}{1 - w^* \hat{\lambda} Q^* h(L)}.
\]

**Proposition 3** suggests that the expression of the optimal wholesale price under online P2P lending is more complex than that under traditional offline financing. As the leader of the multi-level Stackelberg game, the manufacturer needs to decide how to set a suitable
wholesale price in response to both the retailer and the online P2P lending platform. Therefore, the wholesale price depends not only on the retailer’s order quantity but also on the online P2P lending platform’s service rate and the Internet financing interest rate (i.e., the retailer’s creditworthiness). In addition, the optimal wholesale price can effectively guarantee that the manufacturer’s profit is over zero.

5. Optimal strategies for financing a supply chain with a capital-constrained manufacturer

In this section, we analyze another situation where the retailer has sufficient capital to procure products, but the manufacturer has capital constraints in the production process. To overcome financing difficulties, the manufacturer obtains capital from online P2P lending. Fig. 2 describes the interactions among the online P2P lending platform, the capital-constrained manufacturer, and the retailer in the SCF system. We formulate the problem as a multi-level Stackelberg game in which the online P2P lending platform acts as a leader in setting the service rate $s_m$ and the Internet financing interest rate $i_m$. In response to the platform’s decision, the manufacturer, acting as a subleader, decides the wholesale price $w$. Finally, the retailer, acting as a follower, decides the order quantity $Q$ based on the decisions of the platform and the manufacturer. For notational convenience, the repayment of the full loan principal, the interest, and the service charge is expressed as $\lambda_m(cQ - B_m)$, where $\lambda_m = 1 + i_m + s_m$.

Similar to Section 4, we use backward induction to obtain the equilibrium solutions of the SCF system. We first determine the retailer’s and manufacturer’s optimal decisions. Then, anticipating the optimal responses of the retailer and the manufacturer, we find the optimal decisions of the platform.
5.1. Retailer’s and manufacturer’s optimal decisions

We first solve the retailer’s problem, which is a classic newsvendor problem where the retailer procures the product from the manufacturer and then sells it to the market without knowing the actual demand. At the end of the sales season, the retailer yields a revenue of \( \min[D, Q] \) and pays the procurement cost \( wQ \) to the manufacturer. The retailer chooses its order quantity \( Q \) to maximize its expected profit expressed as follows:

\[
\Pi_r(Q) = E\{\min[D, Q] - wQ\} = \int_0^Q F(D)dD - wQ.
\]

Since the retailer has no capital constraints, its optimal decision is identical to the classic newsvendor’s, which is given in the following proposition.

**Proposition 4.** Given the wholesale price \( w \), the retailer’s optimal order quantity is \( Q^* = F^{-1}(w) \), and \( Q^* \) decreases with the wholesale price \( w \).

Given the order quantity \( Q \) by the retailer, the manufacturer incurs a production cost \( cQ \) and borrows \( cQ - B_m \) from the platform. After receiving the payment \( wQ \) from the retailer, the manufacturer pays the platform the loan principal, the interest, and the service charge: \( \lambda_m(cQ - B_m) \). The manufacturer optimizes its wholesale price \( w \) to maximize its expected profit expressed as follows:
The following proposition provides an equilibrium solution to the manufacturer’s and the retailer’s problems.

**Proposition 5.** Given the wholesale price \( w \), the retailer’s optimal order quantity satisfies

\[
Q^* = \frac{\bar{F}(Q') - \lambda_m c}{f(Q')},
\]

which decreases with \( i_m \) and \( s_m \). The capital-constrained manufacturer’s unique optimal wholesale price is

\[
w^* = Q' f(Q') + \lambda_m c.
\]

Proposition 5 suggests that the retailer’s optimal order quantity depends on the interest rate \( i_m \) and the service rate \( s_m \). This is quite different from the traditional supply chain without capital constraints. As the interest rate \( i_m \) and the service rate \( s_m \) increase, the capital-constrained manufacturer transfers the financing cost to the retailer by setting a higher wholesale price, which in turn causes the retailer to place a smaller order. This exhibits an inextricable relationship between financing and operational decisions.

### 5.2. Online P2P lending platform’s optimal decision

For the online P2P lending platform’s decision problem, we consider a situation where the platform collects a full repayment \( \lambda_m (cQ - B_m) \) from the manufacturer at the end of the sales season. This occurs only if \( wQ > \lambda_m (cQ - B_m) \), which leads to conditions on \( B_m \) stated in the following lemma.

**Lemma 1.** Given the wholesale price \( w \), the online P2P lending platform collects a full repayment if the manufacturer’s initial capital satisfies

\[
\frac{c(\sqrt{\bar{F}(Q')} - \sqrt{w})}{f(Q')} < B_m < \frac{c(\sqrt{\bar{F}(Q')} + \sqrt{w})}{f(Q')}.
\]

It is worth noting that if the manufacturer’s initial capital is sufficient to cover the production cost \( B_m \geq \frac{c(\sqrt{\bar{F}(Q')} + \sqrt{w})}{f(Q')} \), there is no need to borrow from the platform. If the manufacturer’s initial capital is relatively small \( B_m \leq \frac{c(\sqrt{\bar{F}(Q')} - \sqrt{w})}{f(Q')} \), the manufacturer
cannot make the full repayment to the platform.

Anticipating the manufacturer’s and the retailer’s best responses, the platform, acting as a leader, optimizes the service rate $s_m$ to maximize its expected profit:

$$\Pi_m(s_m) = (cQ^* - B_m)s_m. \quad (7)$$

Under the condition in Lemma 1, the following proposition determines the platform’s optimal decision.

**Proposition 6.** Given the manufacturer’s and the retailer’s best responses, the online P2P lending platform’s optimal service rate is

$$s_m^* = \frac{2f(Q^*)(cQ^* - B_m)}{c^2}.$$

Proposition 6 suggests that the optimal service rate increases with both the order quantity and the loan size. The former is determined by the retailer, while the latter is determined by the capital-constrained manufacturer. This explains the different service rates charged by online P2P lending platforms, such as Renrendai (www.renrendai.com) and CreditEase (www.yirendai.com), for different borrowers in practice.

6. Numerical study

We perform a numerical study to explore the relationships between operational and financial decisions for the two different financing scenarios corresponding to Sections 4 and 5. In the Stackelberg game models, we set the values of the parameters based on real data from China's small-size industrial enterprises in the manufacturing sector.

6.1. Parameters

We focus on China's small-size industrial enterprises in the manufacturing sector with data in the period of 2011-2016. We list the parameters used in the numerical study as follows.

- **Service rate:** Following "The guidance on the P2P platforms' fee charge standards" published by the Internet Financial Association of China, we set the online P2P lending platform’s service rate provided to the borrower ($s_r$ or $s_m$) in the range of [6%, 12%].

- **Internet financing interest rate:** According to the data published by the Ministry of Industry and Information Technology of China (http://www.sme.gov.cn) in 2015-2016, we
set the Internet financing interest rate \( (i_r \text{ or } i_m) \) in the range of \([8\%,12\%]\), where 8\% indicates a low level for interest rate, 10\% stands for a moderate level, and 12\% means a high level.

- Initial capital: Following the literature (see, for example, Cai et al., 2014), we use the average liquid assets to approximate a firm's initial capital. Table 1 shows the detailed data on the total liquid assets, the number of enterprises, and the average liquid assets of China's small-size industrial enterprises in the manufacturing sector in 2011-2015. We use the data in 2015 and set the borrower's initial capital equal to the average liquid assets \( (B_r = B_m = 0.45) \). We observe similar results in our numerical study when we use the data in other years.

- Supplier’s unit production cost: Following Buzacott and Zhang (2004), we set \( c = 0.4 \). Based on this setting, we have the wholesale price \( w \in [0.4, 1] \).

- Market demand: We assume the demand \( D \) follows an exponential distribution with a mean of 10 units, which is consistent with the literature (see Buzacott and Zhang, 2004; Yan et al., 2016).

**Table 1. Liquid assets of China's small-size industrial enterprises in 2011-2015**

<table>
<thead>
<tr>
<th>Year</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total liquid assets</td>
<td>87,273.31</td>
<td>103,549.79</td>
<td>124,501.82</td>
<td>134,721.44</td>
<td>143,742.87</td>
</tr>
<tr>
<td>Total number of enterprises</td>
<td>264,262</td>
<td>280,455</td>
<td>304,299</td>
<td>312,587</td>
<td>319,445</td>
</tr>
<tr>
<td>Average liquid assets</td>
<td>0.33</td>
<td>0.37</td>
<td>0.41</td>
<td>0.43</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Data source: National Bureau of Statistics of China (http://data.stats.gov.cn). The unit of total and average liquid assets is 100 million Yuan.

6.2. Numerical results

Based on the above parameter setting, we study the impacts of financial decisions (the platform's Internet interest rate and service rate) on the operational decisions (the retailer’s order quantity and the manufacturer's wholesale price).

6.2.1. Interactions between operational and financial decisions for the SCF system with a capital-constrained retailer

Our first set of experiments illustrates the interactions between the operational and financial
decisions for the SCF system with a capital-constrained retailer. Fig. 3 shows how the retailer’s optimal order quantity varies with the platform’s service rate for different Internet financing interest rates \( (i_r = 0.08, 0.1, 0.12) \). We set the wholesale price equal to the mid-point value \( \bar{w} = 0.7 \) of its range. Fig 3 suggests that for a fixed \( i_r \), the retailer’s optimal order quantity decreases with the platform’s service rate \( s_r \). Furthermore, for a given \( s_r \), the optimal order quantity decreases with \( i_r \). We have these observations because a higher service rate or a higher Internet financing interest rate implies a higher financing cost, which results in a smaller loan amount and order quantity. This is consistent with Proposition 1.

Fig 4 illustrates the impact of the platform’s service rate and Internet financing interest rate on the manufacturer’s optimal wholesale price. We set the retailer’s order quantity equal to its mean value \( \bar{Q} = 10 \). For a fixed \( i_r \), the optimal wholesale price decreases with \( s_r \). Furthermore, by comparing three scenarios of \( i_r = 0.08, \ i_r = 0.1, \) and \( i_r = 0.12 \), we find that the optimal wholesale price decreases with \( i_r \) for a given \( s_r \). These observations can be explained as follows. As the platform’s service rate or Internet financing interest rate increases, the capital-constrained retailer's financing cost increases. This forces the manufacturer to reduce its wholesale price in order to maintain its sales. This is consistent with Proposition 3.
6.2.2. Interactions between operational and financial decisions for the SCF system with a capital-constrained manufacturer

We also conduct numerical experiments to analyze the interactions between the operational and financial decisions for the SCF system with a capital-constrained manufacturer. Fig. 5 illustrates the impact of the platform’s service rate and Internet financing interest rate on the manufacturer's optimal wholesale price. For a given $i_m$, the optimal wholesale price increases with $s_m$. Similarly, for a fixed $s_m$, the optimal wholesale price increases with $i_m$. As the service rate or the Internet financing interest rate increases, the financing cost of the manufacturer becomes higher. As a result, the manufacturer will transfer the financing cost to the retailer by setting a higher wholesale price. These observations are consistent with Proposition 5.

\[
\text{Fig. 5. The capital-constrained manufacturer’s optimal wholesale price under different service rates and Internet financing interest rates}
\]
As the wholesale price increases with both the service rate and the Internet financing interest rate, the retailer responds to the higher wholesale price by lowering its order quantity. This is evidenced in Fig. 6, which shows that the retailer’s optimal order quantity decreases with both $s_m$ and $i_m$. This observation is also confirmed in Proposition 5.

![Fig. 6: The retailer’s optimal order quantity under different service rates and Internet financing interest rates](image)

The above results demonstrate that a capital-constrained borrower’s operational decision depend not only on their upstream or downstream partner’s decision in a supply chain, but also on the online P2P lending platform's financial decisions.

7. Extensions

We further study two extensions of the basic model of the SCF system. The first extension considers a risk-averse platform. The second extension considers an online P2P lending platform that does not bear the borrowers' default risk.

7.1. A risk-averse online P2P lending platform

Our basic model assumes that the online P2P lending platform is risk neutral. However, to protect individual investors’ benefits, the platform can be risk averse in practice. In this section, we consider a risk-averse platform that has a decreasing marginal utility (Fibich, Gabious, and Sela, 2006; Giri, 2011; Kouvelis and Zhao, 2012). We model the risk aversion with a general utility function $u(\cdot)$ that is increasing and concave. That is, $u'(\cdot) \geq 0$ and $u''(\cdot) \leq 0$. The following corollaries illustrate the results of the SCF system with a risk-averse platform.
Corollary 1. For the SCF system with a risk-averse P2P lending platform and a capital-constrained retailer, the online P2P lending platform’s optimal service rate is identical to that found in Proposition 2.

Corollary 2. For the SCF system with a risk-averse P2P lending platform and a capital-constrained manufacturer, the online P2P lending platform’s optimal service rate is identical to that found in Proposition 6.

7.2. An online P2P lending platform that does not bear the borrowers’ default risk

The online P2P lending platform in our model is responsible for investors’ loss in case of borrowers’ default. However, some platforms serve only as information intermediary without bearing the borrowers’ default risk (Zhang, 2017). It is worth noting that the case where the platform does not bear the borrowers’ default risk corresponds to the case with \( \eta = 0 \) in our model. The following corollary determines the optimal service rate for this case.

Corollary 3. For the SCF system with a capital-constrained retailer, if \( \min[D, Q^*] \geq L \) and \( \eta = 0 \), then the platform’s optimal service rate is

\[
s^*_r = \frac{1}{(wQ^* - B_r)h(L)} - (1 + i_r(\theta_r)).
\]

Corollary 3 shows that the optimal service rate depends on the loan size and the Internet financing interest rate. In practice, a platform first chooses an Internet financing interest rate by evaluating a borrower’s creditworthiness, and then charges the borrower a service fee according to the loan size.

Corollary 4. For the SCF system with a capital-constrained manufacturer, if \( \eta = 0 \), then the platform’s optimal service rate is

\[
s^*_m = \frac{2f(Q^*)(cQ^* - B_m)}{c^2}.
\]

Similar to Proposition 6, Corollary 4 shows that the platform’s optimal service rate depends not only on the retailer’s order quantity, but also on the manufacturer’s loan size.

8. Conclusion

We study an SCF system that consists of a manufacturer, an online P2P lending platform, and a retailer. We analyze the optimal Stackelberg decisions of the participants in the SCF
system. Specifically, we consider two different scenarios: (i) an SCF system with a capital-constrained retailer and (ii) an SCF system with a capital-constrained manufacturer. Our analysis shows that the optimal strategies are quite different under these two scenarios.

For the SCF system with a capital-constrained retailer, we find that the retailer’s optimal order quantity decreases with the platform’s service rate and Internet financing interest rate. This is because a higher service rate or a higher Internet financing interest rate implies a higher financing cost, which results in a smaller loan amount and order quantity. We also find that the manufacturer’s optimal wholesale price decreases with the platform’s service rate and Internet financing interest rate. This can be explained as follows. As the platform’s service rate or Internet financing interest rate increases, the capital-constrained retailer’s financing cost increases. This forces the manufacturer to reduce its wholesale price in order to maintain its sales. The above observations are proved in Propositions 1 and 3, and confirmed in our numerical study.

For the SCF system with a capital-constrained manufacturer, the manufacturer’s optimal wholesale price increases with the platform’s service rate and Internet financing interest rate. As the service rate or the Internet financing interest rate increases, the financing cost of the manufacturer becomes higher. As a result, the manufacturer will transfer the financing cost to the retailer by setting a higher wholesale price. In response to the higher wholesale price, the retailer lowers its order quantity. These observations are proved in Proposition 5 and confirmed in our numerical study.

We extend our model to consider a risk-averse online P2P lending platform. We find that even if the platform is risk averse, it still sets the same service rate as that of the risk-neutral platform. Furthermore, we study the case where the online P2P lending platform does not bear the borrowers' default risk. We identify the platform’s optimal service rate for this case.

Our model captures the interactions between the operational and the financial decisions. Through our analytical and numerical studies, we conclude that it is important for the retailer and the manufacturer to take the platform’s financial decisions (the service rate and the Internet financing interest rate) into consideration when making their operational decisions.

This paper has some limitations, which may serve as avenues for future research. First, similar to the majority of the literature, we assume that information is symmetrical among the
participants of the SCF system. In reality, only the borrower knows exactly its capital need and the other participants in the supply chain may not have the access to this information. This implies that the information is asymmetrical in the SCF system. To study an SCF system with asymmetric information, we need an entirely different model that may provide additional insights. For example, the online P2P lending platform may charge a higher service rate to the borrower to mitigate the effect of information asymmetry in the lending process.

Second, we assume a perfect capital market in our model. However, the capital market in practice contains taxes, operation costs, and bankruptcy costs. The analysis of decision making for an SCF system with an imperfect capital market including these complications would be meaningful and intriguing.

Third, this paper studies the optimal Stackelberg strategies for an SCF system either with a capital-constrained retailer or with a capital-constrained manufacturer. In practice, both the retailer and the manufacturer may be capital-constrained at the same time. An analysis of this more complex situation would be interesting.

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References


**Appendix**

**Proof of Proposition 1**

*Part (1):* From Eq. (2), the retailer’s expected profit function can be rewritten as

\[ \Pi_r(Q) = \int_L^Q (D - L) dF(D) + \int_Q^\infty (Q - L) dF(D) - B_r = \int_L^Q F(D) dD - B_r. \]  

(A.1)

Taking the first-order and the second-order derivative of \( \Pi_r(Q) \) with respect to \( Q \) in Eq. (A.1), we have

\[ \frac{d \Pi_r(Q)}{dQ} = F(Q) - w \lambda_r F(L) \quad \text{and} \quad \frac{d^2 \Pi_r(Q)}{dQ^2} = -f(Q) + (w \lambda_r)^2 f(L). \]

Applying the implicit function theorem of \( F(Q) = w \lambda_r F(L) \), we obtain

\[ \frac{d^2 \Pi_r(Q)}{dQ^2} = F(Q)\left[ -\frac{f(Q)}{F(Q)} + \frac{(w \lambda_r)^2 f(L)}{F(Q)} \right] = F(Q)\left[ -h(Q) + w \lambda_r h(L) \right]. \]  

(A.2)

As IFR distributions of demand and \( w \lambda_r < 1 \), we have \( h(Q) > w \lambda_r h(L) \). Hence, \( \frac{d^2 \Pi_r(Q)}{dQ^2} < 0 \) holds, which means that the retailer’s expected profit function is concave and that there exists a unique optimal solution \( Q^* \). From the first-order condition, i.e.,

\[ \frac{d \Pi_r(Q)}{dQ} = 0, \quad \text{we can obtain the retailer’s optimal order quantity, i.e.,} \quad Q^* = F^{-1}[w \lambda_r F(L)]. \]
Part (2): Differentiating $Q^*$ with respect to $w$ and applying the implicit function theorem of $\bar{F}(Q) = w\lambda, \bar{F}(L)$, we have

$$\frac{dQ^*}{dw} = \frac{\lambda_c[\bar{F}(L) - w\lambda_c Q^* f(L)]}{(w\lambda_c)^2 f(L) - f(Q^*)} = \frac{\lambda_c[1 - w\lambda_c Q^* h(L)]}{w\lambda_c[w\lambda_c h(L) - h(Q^*)]}.$$  \hspace{1cm} (A.3)

Similar to Chen and Cai (2011), we also use proof by contradiction to imply that $\frac{dQ^*}{dw} < 0$.

Assume that the inequality $\frac{dQ^*}{dw} \geq 0$ holds. Then, we have that the inequalities

$$1 - w\lambda_c Q^* h(L) \leq 0 \text{ and } 1 - Q^* h(Q^*) \leq 1 - w\lambda_c Q^* h(L) \leq 0$$

hold. Furthermore, we can obtain

that $\frac{dL}{dw} = \frac{\lambda_c[1 - Q^* h(Q^*)]}{w\lambda_c h(L) - h(Q^*)} \geq 0$, i.e., $\frac{dL}{dw} \geq 0$ is equivalent to $\frac{dQ^*}{dw} \geq 0$. Let $w_0$ satisfy

$$1 - Q^* h(Q^*) = 0,$$

i.e., $\frac{dL}{dw_0} = 0$. Then, we have $\frac{dL}{dw} < 0$ when $w < w_0$, and $\frac{dL}{dw} > 0$ when $w > w_0$, which implies that $L$ achieves its minimum value at $w = w_0$. Recalling that the wholesale price feasible region is $[c, \frac{1}{\lambda_c}]$ by assumption, we will consider the following three cases:

a) $w_0 \geq \frac{1}{\lambda_c}$. In this case, for any $w \in [c, \frac{1}{\lambda_c}]$, we have $\frac{dL}{dw} \leq \frac{dL}{dw} \mid_{w=\frac{1}{\lambda_c}} \leq \frac{dL}{dw_0}$, which implies that $\frac{dL}{dw} \leq 0$. It contradicts the assumption, i.e., $\frac{dL}{dw} \geq 0$.

b) $c < w_0 < \frac{1}{\lambda_c}$. In this case, we have $\frac{dL}{dw} < 0$ when $w \in [c, w_0]$. It also contradicts the assumption, i.e., $\frac{dL}{dw} \geq 0$.

c) $w_0 = c$. In this case, for any $w \geq c$ and $L \leq Q^*$, we can obtain

$$w\lambda_c Q^* h(L) < Q^* h(Q^*) = 1.$$ As we suppose that $\frac{dQ^*}{dw} \geq 0$, then $w\lambda_c Q^* h(L) \geq 1$. Again, this is a contradiction.
For the contradictions in the above three cases, we must have that the assumption is invalid, and we have the result that the optimal order quantity decreases with $w$, i.e., $\frac{dQ^*}{dw} < 0$.

**Part (3):** Applying the implicit function theorem of $F(Q) = w\lambda_r F(L)$ and taking the first-order derivative of $Q^*$ with respect to both $i_r(\theta_r)$ and $s_r$, we have

$$\frac{dQ^*}{di_r(\theta_r)} = \frac{dQ^*}{ds_r} = \frac{w[F(L) - Lf(L)]}{w[1 - L_h(L)]} = \frac{w[1 - L_h(L)]}{w[\lambda_r h(L) - h(Q^*)]}.$$  (A.4)

Similarly, differentiating $L$ with respect to both $i_r(\theta_r)$ and $s_r$, we have

$$\frac{dL}{di_r(\theta_r)} = \frac{dL}{ds_r} = \frac{w - (wQ^* - B_r)h(Q^*)}{w[1 - (wQ^* - B_r)/w h(Q^*)]}.$$  (A.5)

As $L = w\lambda_r Q^* - \lambda_r B < w\lambda_r Q^*$ and $\frac{d^2 L}{dQ^2} = (w\lambda_r)^2 f(L) - f(Q^*) < 0$, we can obtain

$$\frac{w[F(L) - Lf(L)]}{(w\lambda_r)^2 f(L) - f(Q^*)} < \frac{w[F(L) - w\lambda_r Q^* f(L)]}{(w\lambda_r)^2 f(L) - f(Q^*)},$$

which implies that $\frac{dQ^*}{di_r(\theta_r)} = \frac{dQ^*}{ds_r} < \frac{w}{\lambda_r} \frac{dQ^*}{dw}$. As $\frac{dQ^*}{dw} < 0$ proved above, then the inequality $\frac{dQ^*}{di_r(\theta_r)} = \frac{dQ^*}{ds_r} < 0$ holds. Moreover, because the Internet financing interest rate $i_r(\theta_r)$ decreases with the creditworthiness of the retailer $\theta_r$, $Q^*$ increases with $\theta_r$.

Therefore, we have the property that the retailer’s optimal order quantity decreases with both the Internet financing interest rate and the service rate but increases with its creditworthiness.

**Part (4):** Applying the implicit function theorem of $F(Q) = w\lambda_r F(L)$ and taking the first-order derivative of $Q^*$ with respect to $B_r$, we have

$$\frac{dQ^*}{dB_r} = \frac{w\lambda_r^2 f(L)}{(w\lambda_r)^2 f(L) - f(Q^*)}.$$  (A.6)

Because of $w\lambda_r h(L) < h(Q^*)$ proved above, the denominator is negative. Besides, as the numerator is positive, thus, we can obtain the property that the retailer’s optimal order
quantity decreases with its initial capital, i.e., \( \frac{dQ^*}{dB_r} < 0 \).

This completes the proof of **Proposition 1.** 

**Proof of Proposition 2**

In Eq. (3), taking the first-order derivative of \( \prod_{s_r}(s_r) \) with respect to \( s_r \), we have

\[
\frac{d \prod_{s_r}(s_r)}{ds_r} = \frac{\partial \prod_{s_r}(s_r)}{\partial Q^*} \frac{dQ^*}{ds_r} + \frac{\partial \prod_{s_r}(s_r)}{\partial s_r}
\]

\[
= \left[ w\lambda_r(\bar{F}(L) - \eta \xi f(\eta L)) - w(1 + i_r(\theta_r)) \right] \frac{dQ^*}{ds_r} + \left( wQ^* - B_r(\bar{F}(L) - \eta \xi f(\eta L)) \right)
\]

\[
= \left[ \frac{w\lambda_r(\bar{F}(L) - \eta \xi f(\eta L)) - w(1 + i_r(\theta_r))}{\lambda_r[w\lambda_r h(L) - h(Q^*)]} \right] [1 - Lh(L)] \frac{wQ^* - B_r h(Q^*)}{w} - (1 + i_r(\theta_r))
\]

\[
= w \frac{\frac{dQ^*}{ds_r} [\mu(s_r) - (1 + i_r(\theta_r))]}{w[1 - Lh(L)]}, \quad (A.7)
\]

where \( \mu(s_r) = \frac{\bar{F}(Q^*)[1 - \xi h(\eta L)][1 - \frac{wQ^* - B_r h(Q^*)}{w}]}{w[1 - Lh(L)]} \).

As \( Q'' = \frac{dQ^*}{ds_r} < 0 \) proved in **Proposition 1**, we have \( \frac{d\bar{F}(Q^*)}{ds_r} > 0 \). Let

\[ L' = \frac{dL}{ds_r} = (wQ^* - B_r) + w\lambda_r Q'' \]

as \( L = \lambda_r(wQ^* - B_r) < \frac{(wQ^* - B_r)}{w} < Q^* \), thus

\[ 1 - \frac{(wQ^* - B_r)}{w} h(Q^*) < 1 - Lh(L) \]

i.e., \( 1 - H(Q^*) + \frac{B_r}{w} h(Q^*) < 1 - H(L) \). Recall that

\[ \frac{dL}{dw} = \frac{\lambda_r[1 - Q'h(Q^*)]}{w\lambda_r h(L) - h(Q^*)} < 0 \]

proved in **Proposition 1**, and we have \( 1 - Q'h(Q^*) > 0 \). As

\[ 1 - \frac{(wQ^* - B_r)}{w} h(Q^*) > 1 - Q'h(Q^*) > 0 \]

then we have \( 1 - H(Q^*) + \frac{B_r}{w} h(Q^*) > 0 \) and
\[ L' = \frac{dL}{ds_r} = \frac{w[1 - ((wQ^* - B_r)/w)h(Q^*)]}{w\lambda_r h(L) - h(Q^*)} < 0. \]

As IFR distributions of demand, we can obtain that \( \frac{dh(L)}{ds_r} < 0 \) and \( \frac{dh(\eta L)}{ds_r} < 0. \) Hence, \( F(Q^*)[1 - \xi h(\eta L)] \) increases with \( s_r. \)

Furthermore, let \( \phi(s_r) = \frac{1 - \frac{wQ^* - B_r}{w} h(Q^*)}{1 - Lh(L)}. \) If the relation between \( \phi(s_r) \) and \( s_r \) can be determined, then we will obtain the relation between \( \mu(s_r) \) and \( s_r. \)

Similar to the proof in Chen and Cai (2011), taking the first-order derivative of \( \phi(s_r) \) with respect to \( s_r, \) we have

\[
\frac{d\phi(s_r)}{ds_r} = \frac{[\frac{B}{w} h'(Q^*)Q'' + B h'(Q^*)][1 - H(L)] + [1 - H(Q^*) + \frac{B}{w} h(Q^*)]H'(L)L'}{[1 - H(L)]^2},
\]

where \( H'(Q^*) = \frac{dH(Q^*)}{dQ^*}, \) \( h'(Q^*) = \frac{dh(Q^*)}{dQ^*}, \) \( Q'' = \frac{dQ^*}{dr}, \) \( H'(L) = \frac{dH(L)}{dL}, \) \( h'(L) = \frac{dh(L)}{dL} \)

and \( L' = \frac{dL}{ds_r} = (wQ^* - B_r) + w\lambda_r Q'' < 0. \)

Here, we use proof by contradiction to imply that \( L' > Q''. \) Assume that the inequality \( L' \leq Q'' \) holds. Substituting \( L' = (wQ^* - B_r) + w\lambda_r Q'' \) into \( L' \leq Q'', \) we know that \( (wQ^* - B_r) + (w\lambda_r - 1)Q'' \leq 0 \) holds. Because of \( wQ^* - B_r > 0 \) in Section 3.1 and \( (w\lambda_r - 1)Q'' > 0 \) obtained in the proof of Proposition 1, we have \( (wQ^* - B_r) + (w\lambda_r - 1)Q'' > 0. \) This contradicts the assumption, i.e.,

\( (wQ^* - B_r) + (w\lambda_r - 1)Q'' \leq 0. \) Therefore, we have \( L' > Q''. \)

Based on the above analysis, we can obtain that

\[
\frac{d\phi(s_r)}{ds_r} > \frac{[1 - H(Q^*) + \frac{B}{w} h(Q^*)][-H'(Q^*)Q'' + \frac{B}{w} h'(Q^*)Q'' + H'(L)L']}{[1 - H(L)]^2}.
\]
\[
\begin{align*}
&\frac{[1 - H(Q') + \frac{B}{w} h(Q')] - H'(Q') - \frac{B}{w} h'(Q') + H'(L)]Q''}{[1 - H(L)]^2} \\
&= \frac{[1 - H(Q') + \frac{B}{w} h(Q')] - \frac{wQ' - B}{w} h'(Q') + h(L) + Lh'(L)]Q''}{[1 - H(L)]^2} \\
&> 0.
\end{align*}
\] (A.8)

Therefore, by \( \frac{d \varphi(s_r)}{ds_r} > 0 \) and \( \frac{d F(Q')}{ds_r} > 0 \), we have the property that \( \mu(s_r) \) increases with \( s_r \). Furthermore, based on the above analysis, we know that there exists a unique optimal service rate \( s_r^* \) by maximizing \( \Pi_\alpha(s_r) \), \( s_r \in [0,1] \), where the value of \( s_r^* \) is considered as the following three cases.

a) Given \( \mu(s_r) \geq 1 + i_\gamma(\theta_r) \), we know that \( \mu(s_r) \geq \mu(s_r) \geq 1 + i_\gamma(\theta_r) \) holds. For this case, we have \( \frac{d \Pi_\alpha(s_r)}{ds_r} \leq 0 \), and the optimal service rate is \( s_r^* = s_r \).

b) Given \( \mu(s_r) \leq 1 + i_\gamma(\theta_r) \), we know that \( \mu(s_r) \leq \mu(s_r) \leq 1 + i_\gamma(\theta_r) \) holds. For this case, we have \( \frac{d \Pi_\alpha(s_r)}{ds_r} \geq 0 \), and the optimal service rate is \( s_r^* = s_r \).

c) Given \( \mu(s_r) < 1 + i_\gamma(\theta_r) \) and \( \mu(s_r) > 1 + i_\gamma(\theta_r) \), we know that \( \mu(s_r) \leq \mu(s_r) \leq \mu(s_r) \) holds. For this case, we can obtain that \( \frac{d \Pi_\alpha(s_r)}{ds_r} \bigg|_{s_r = s_r} > 0 \) and \( \frac{d \Pi_\alpha(s_r)}{ds_r} \bigg|_{s_r = \bar{s}_r} < 0 \), which implies that there exists a unique \( \tilde{s}_r \) that satisfies \( \frac{d \Pi_\alpha(s_r)}{ds_r} \bigg|_{s_r = \tilde{s}_r} = 0 \), \( \tilde{s}_r \in [s_r, \bar{s}_r] \), i.e., \( \mu(\tilde{s}_r) = 1 + i_\gamma(\theta_r) \). Thus, the optimal service rate is

\[
\tilde{s}_r = \frac{1}{(wQ' - B)(1 + i_\gamma(\theta_r))h(L)} - \frac{\xi h(\eta L)}{w}[1 - \frac{wQ' - B}{w} h(Q')] - (1 + i_\gamma(\theta_r)).
\]

This completes the proof of Proposition 2. #
Proof of Proposition 3

Following the proof of Proposition 1, we know that \( \frac{dQ*}{ds_r} = \frac{w[1-Lh(L)]}{w \lambda_r[h(L) - h(Q^*)]} < 0 \) and \( w \lambda_r h(L) - h(Q^*) < 0 \). By the above two inequalities, we can obtain that \( h(L) < \frac{1}{L} \) and \( F(L) - Lf(L) > 0 \), and we can also obtain that \( h(Q) < \frac{1}{Q} \) and \( F(Q) - Qf(Q^*) > 0 \) if \( L \) is substituted by \( Q \). Besides, as \( L = \lambda_r(wQ^* - B_r) < (wQ^* - B_r)/w < Q^* \), thus \( F(Q^*) - Q^* f(L) > 0 \). Meanwhile, because of \( (w \lambda_r)^2 < 1 \), we have \( F(Q^*) - (w \lambda_r)^2 Q^* f(L) > 0 \).

In Eq. (4), taking the first-order and the second-order derivative of \( \Pi_m(w) \) with respect to \( w \), we have \( \frac{d \Pi_m(w)}{dw} = \frac{\partial \Pi_m(w)}{\partial Q^*} \frac{dQ^*}{dw} + \frac{\partial \Pi_m(w)}{\partial w} = (w-c) \frac{dQ^*}{dw} + Q^* \) and \( \frac{d^2 \Pi_m(w)}{dw^2} = 2 \frac{dQ^*}{dw} + (w-c) \frac{d^2 Q^*}{dw^2} \), respectively. Based on the above analysis, it is obvious that \( \frac{d^2 Q^*}{dw^2} = -\frac{2w \lambda_r^2 f(L)[2F(Q^*) - Q^* f(Q^*) - (w \lambda_r)^2 Q^* f(L)]}{w[(w \lambda_r)^2 f(L) - f(Q^*)]^2} < 0 \). Hence, we can conclude that \( \frac{d^2 \Pi_m(w)}{dw^2} < 0 \), namely, the manufacturer’s expected profit function is concave, and there exists a unique optimal solution \( w^* \). The optimal wholesale price can be obtained by solving \( \frac{d \Pi_m(w)}{dw} = 0 \), i.e., \( w^* = c - \frac{Q^*}{dQ^*/dw} = c - \frac{w^* Q^*[w^* \lambda_r h(L) - h(Q^*)]}{1 - w^* \lambda_r Q^* h(L)} \), and \( w^* > c \).

Intuitively, the optimal wholesale price \( w^* \) can guarantee that the manufacturer’s profit is over zero.

This completes the proof of Proposition 3. #

Proof of Proposition 4
From the retailer’s expected profit function, we have \( \frac{d \Pi_r(Q)}{dQ} = F(Q) - w \) and
\[
\frac{d^2 \Pi_r(Q)}{dQ^2} = -f(Q) < 0.
\]
Thus, the retailer’s optimal order quantity is \( Q^* = F^{-1}(w) \) and
\[
\frac{dQ^*}{dw} = -\frac{1}{f(Q)} < 0.
\]
This completes the proof of Proposition 4. #

**Proof of Proposition 5**

Substituting \( Q^* = F^{-1}(w) \) into Eq. (6), the manufacturer’s expected profit function can be rewritten as \( \Pi_m(Q^*) = [\bar{F}(Q^*) - \lambda_m c]Q^* + (i_m(\theta_m) + s_m)B_m \). Taking the first-order derivative of \( \Pi_m(Q^*) \) with respect to \( Q^* \), we have \( \frac{d \Pi_m(Q^*)}{dQ^*} = \bar{F}(Q^*) - Q^* f(Q^*) - \lambda_m c \). As \( \bar{F}(Q^*) \) decreases with \( Q^* \) and \( Q^* f(Q^*) \) increases with \( Q^* \), we know that \( \frac{d \Pi_m(Q^*)}{dQ^*} \) monotonously decreases with \( Q^* \) and that \( \Pi_m(Q^*) \) is a unimodal concave function with respect to \( Q^* \). Thus, a unique \( Q^* \) can be obtained by solving \( \frac{d \Pi_m(Q^*)}{dQ^*} = 0 \), i.e.,
\[
Q^* = \frac{\bar{F}(Q^*) - \lambda_m c}{f(Q^*)}.
\]
Differentiating \( Q^* \) with respect to both \( s_m \) and \( i_m \), we have
\[
\frac{dQ^*}{ds_m} = \frac{dQ^*}{di_m} = -\frac{c}{2f(Q^*)} < 0.
\]
Moreover, substituting \( Q^* = \frac{\bar{F}(Q^*) - \lambda_m c}{f(Q^*)} \) into \( Q^* = F^{-1}(w) \), we can obtain a unique wholesale price, i.e., \( w^* = Q^* f(Q^*) + \lambda_m c \).

This completes the proof of Proposition 5. #

**Proof of Lemma 1**

For notational convenience, let \( \hat{\Pi}_n(\lambda_m) = wQ^* - \lambda_m (cQ^* - B_m) \), and substituting
\[ Q^* = \frac{F(Q^*) - \lambda_m c}{f(Q^*)} \] into \( \hat{\Pi}_o(\lambda_m) \), we have

\[ \hat{\Pi}_o(\lambda_m) = c^2 \lambda_m^2 + [B_m f(Q^*) - c\bar{F}(Q^*) - wc] \lambda_m + w\bar{F}(Q^*) > 0. \] (A.9)

It can be seen from \( c^2 > 0 \) that the first-order derivative of \( \hat{\Pi}_o(\lambda_m) \) with respect to \( \lambda_m \) is strictly convex. Let \( \Delta \) represent the criterion of the quadratic function \( \hat{\Pi}_o(\lambda_m) \), then we have

\[ \Delta = [f(Q^*)B_m]^2 - 2B_m f(Q^*)[c\bar{F}(Q^*) + wc] + [c\bar{F}(Q^*) + wc]^2 - 4wc^2 \bar{F}(Q^*) < 0. \] (A.10)

Furthermore, we can obtain that \( \frac{c(\sqrt{F(Q^*)} - \sqrt{w})}{f(Q^*)} < B_m < \frac{c(\sqrt{F(Q^*)} + \sqrt{w})}{f(Q^*)} \) by solving Eq. (A.10).

This completes the proof of Lemma 1. #

Proof of Proposition 6

In Eq. (7), taking the first-order and the second-order derivative of \( \Pi_o(s_m) \) with respect to \( s_m \), we have

\[ \frac{d \Pi_o(s_m)}{ds_m} = \frac{\partial \Pi_o(s_m)}{\partial Q} \frac{dQ^*}{ds_m} + \frac{\partial \Pi_o(s_m)}{\partial s_m} = cQ - B_m + cs_m \frac{dQ^*}{ds_m} \]

and

\[ \frac{d^2 \Pi_o(s_m)}{ds_m^2} = 2c \frac{dQ^*}{ds_m} + cs_m \frac{d^2 Q^*}{ds_m^2} = -\frac{c^2}{f(Q^*)} < 0, \] respectively.

Thus, the P2P lending platform’s expected profit function is concave. By solving \( \frac{d \Pi_o(s_m)}{ds_m} = 0 \), we can obtain the optimal service rate, i.e., \( s_m^* = \frac{2f(Q^*)(cQ^* - B_m)}{c^2} \).

This completes the proof of Proposition 6. #

Proof of Corollary 1

Taking the risk-averse P2P platform's expected utility function as the form of \( u(\Pi_o(s_r)) \), and taking the first-order derivative of \( u(\Pi_o(s)) \) with respect to \( s_r \), we have
\[
\frac{du(\Pi_o(s_r))}{ds_r} = \frac{du(\Pi_o(s_r))}{d\Pi_o(s_r)} \frac{d\Pi_o(s_r)}{ds_r}
\]

\[
= \frac{du(\Pi_o(s_r))}{d\Pi_o(s_r)} \frac{w[1-Lh(L)]}{\lambda_o[w\lambda_o(h(L)-h(Q^*))]} \left\{ \frac{\bar{F}(Q^*)[1-\xi h(\eta L)][1-\frac{wQ^*}{w} h(Q^*)]}{w[1-Lh(L)]} - (1+\iota_r(\theta_r)) \right\}
\]

\[
= \frac{w}{d\Pi_o(s_r)} \frac{dQ^*}{ds_r} [\mu(s_r) - (1+\iota_r(\theta_r))],
\]

where \( \mu(s_r) = \frac{\bar{F}(Q^*)[1-\xi h(\eta L)][1-\frac{wQ^*}{w} h(Q^*)]}{w[1-Lh(L)]} \).

Recalling \( u'(\Pi_o(s_r)) = \frac{du(\Pi_o(s_r))}{d\Pi_o(s_r)} > 0, \frac{dQ^*}{ds_r} < 0 \) and \( \frac{d\mu(s_r)}{ds_r} > 0 \) proved in Proposition 1 and Proposition 2, respectively, we know that there exists a unique optimal service rate \( s_r^* \) by maximizing \( \Pi_o(s_r), \ s_r^* \in [0,1] \), where the value of \( s_r^* \) is considered as the following three cases.

a) Given \( \mu(s_r) \geq 1 + \iota_r(\theta_r) \), we know that \( \mu(s_r) \geq \mu(s_r) \geq \mu(s_r) \geq 1 + \iota_r(\theta_r) \) holds. For this case, we have \( \frac{du(\Pi_o(s_r))}{ds_r} \leq 0 \), and the optimal service rate is \( s_r^* = s_r \).

b) Given \( \mu(s_r) \leq 1 + \iota_r(\theta_r) \), we know that \( \mu(s_r) \leq \mu(s_r) \leq \mu(s_r) \leq 1 + \iota_r(\theta_r) \) holds. For this case, we have \( \frac{du(\Pi_o(s_r))}{ds_r} \geq 0 \), and the optimal service rate is \( s_r^* = s_r \).

c) Given \( \mu(s_r) < 1 + \iota_r(\theta_r) \) and \( \mu(s_r) > 1 + \iota_r(\theta_r) \), we know that \( \mu(s_r) \leq \mu(s_r) \leq \mu(s_r) \) holds. For this case, we can obtain that \( \frac{du(\Pi_o(s_r))}{ds_r} |_{s_r = \hat{s}_r} > 0 \) and \( \frac{du(\Pi_o(s_r))}{ds_r} |_{s_r = \hat{s}_r} < 0 \), which implies that there exists a unique \( \hat{s}_r \) that satisfies \( \frac{du(\Pi_o(s_r))}{ds_r} |_{s_r = \hat{s}_r} = 0, \ \hat{s}_r \in [s_r, s_r] \), i.e., \( \mu(\hat{s}_r) = 1 + \iota_r(\theta_r) \).

This completes the proof of Corollary 1. #
Proof of Corollary 2

Taking the first-order and the second-order derivative of \( u(\Pi_o(s_m)) \) with respect to \( s_m \), we have \( \frac{du(\Pi_o(s_m))}{ds_m} = \frac{du(\Pi_o(s_m))}{d \Pi_o(s_m)} \frac{d \Pi_o(s_m)}{ds_m} \) and
\[
\frac{d^2 u(\Pi_o(s_m))}{ds_m^2} = \frac{d^2 u(\Pi_o(s_m))}{d \Pi_o(s_m)^2} \left( \frac{d \Pi_o(s_m)}{ds_m} \right)^2 + \frac{du(\Pi_o(s_m))}{d \Pi_o(s_m)} \frac{d^2 \Pi_o(s_m)}{ds_m^2},
\]
respectively. As
\[
\frac{d u(\Pi_o(s_m))}{d \Pi_o(s_m)} \geq 0, \quad \frac{d^2 u(\Pi_o(s_m))}{d \Pi_o(s_m)^2} \leq 0 \quad \text{and} \quad \frac{d^2 \Pi_o(s_m)}{ds_m^2} < 0
\]
proved in Proposition 6, we have \( \frac{d^2 u(\Pi_o(s_m))}{ds_m^2} \leq 0 \).

Thus, the P2P lending platform’s expected utility function is concave. By solving \( \frac{du(\Pi_o(s_m))}{ds_m} = 0 \), we can obtain the optimal service rate, i.e., \( s_m^* = \frac{2 f(Q^*)(cQ^* - B_\tau)}{c^2} \).

This completes the proof of Corollary 2. 

Proof of Corollary 3

In the case of the platform not bearing the retailer's default risk, given the retailer’s optimal response \( Q^*(w, s_\tau) \) and its initial capital, the online P2P lending platform’s expected profit is \( \Pi_o(s_\tau) = (wQ^* - B_\tau)s_\tau \).

Taking the first-order and the second-order derivative of \( \Pi_o(s_\tau) \) with respect to \( s_\tau \), we have
\[
\frac{d \Pi_o(s_\tau)}{ds_\tau} = \frac{\partial \Pi_o(s_\tau)}{\partial Q^*} \frac{dQ^*}{ds_\tau} + \frac{\partial \Pi_o(s_\tau)}{\partial s_\tau} = wQ - B_\tau + ws_\tau \frac{dQ^*}{ds_\tau}
\]
and
\[
\frac{d^2 \Pi_o(s_\tau)}{ds_\tau^2} = 2w \frac{dQ^*}{ds_\tau} + ws_\tau \frac{d^2 Q^*}{ds_\tau^2},
\]
respectively. \( \frac{dQ^*}{ds_\tau} = \frac{w [\bar{F}(L) - Lf(L)]}{(w\lambda)^2 f(L) - f(Q^*)} < 0 \) obtained from Proposition 1 and \( \bar{F}(Q^*) - Q^* f(Q^*) > 0 \) as well as \( \bar{F}(L) - Lf(L) > 0 \) proved in Proposition 3, we have
\[
\frac{d^2 Q^*}{ds_\tau^2} = -\frac{2w^2 f(L)[2\bar{F}(Q^*) - Q^* f(Q^*) - w\lambda Lf(L)]}{[(w\lambda)^2 f(L) - f(Q^*)]^2} < 0.
\]
Thus, the P2P lending platform’s expected profit function is concave. By solving \( \frac{d \Pi_o(s_r)}{ds_r} = 0 \), we can obtain the optimal service rate, i.e.,

\[
s_r^* = \frac{1}{(wQ^* - B_r)h(L)} - (1 + i_r(\Theta_r)).
\]

This completes the proof of Corollary 3. #

**Proof of Corollary 4**

In the case of the platform not bearing the manufacturer's default risk, given the retailer’s optimal response \( Q^*(w, s_m) \), the manufacturer's initial capital and its production cost, the online P2P lending platform’s expected profit is

\[
\Pi_o(s_m) = (cQ^* - B_m)s_m.
\]

Following the proof of Proposition 6, we can have the optimal service rate

\[
s_m^* = \frac{2f(Q^*)(cQ^* - B_m)}{c^2}.
\]

This completes the proof of Corollary 4. #