F Supplemental materials

In the numerical studies in Sections F.1–F.4 we consider a warehouse with 50 single-deep racks. Each rack contains 80 sections. Depending on the number of products in the warehouse, the number of levels in each section ranges from 1 to 6. Only one pallet can be stored in each level of a section. All levels of the same section have identical store cost and identical retrieve cost and they belong to the same class. We consider the three layouts shown in Figures 4(b), (c), and (d) in Appendix E.

F.1 Effects of number of classes

As the number of classes (N) in the warehouse increases, the store and retrieve costs of each class become more accurate but the computational complexity of the RLR increases. Table 5 shows the performance of each policy for different numbers of classes. There are 200 products facing random demand with q = 5. The third column shows the computational time of the RLR. The RLR consistently outperforms other heuristics and its costs are very close to EV|PI in all experiments. The DET policy outperforms the TOD and DOS policies, which are more efficient than the TOS policy.

Layout	N	Run Time (sec)	\mathbf{RLR}	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	3	15	$4.89E{+}08$	6.87E + 08	$5.21\mathrm{E}{+08}$	5.96E + 08	5.12E + 08	4.71E + 08
	5	94	4.70E + 08	6.81E + 08	5.08E + 08	5.80E + 08	5.02E + 08	4.50E + 08
	9	353	4.68E + 08	6.83E + 08	5.10E + 08	5.81E + 08	$4.99E{+}08$	$4.49E{+}08$
1	11	709	4.68E + 08	6.82E + 08	5.09E + 08	5.80E + 08	$4.98E{+}08$	4.48E + 08
	16	2474	4.64E + 08	6.82E + 08	5.07E + 08	5.79E + 08	4.96E + 08	4.48E + 08
	21	3176	$4.64\mathrm{E}{+08}$	6.82E + 08	$5.07\mathrm{E}{+08}$	$5.79E{+}08$	4.96E + 08	4.48E + 08
	31	7010	4.66E + 08	6.83E + 08	5.06E + 08	5.79E + 08	$4.97E{+}08$	4.48E + 08
	51	22571	$4.67\mathrm{E}{+08}$	6.84E + 08	$5.06\mathrm{E}{+}08$	$5.81\mathrm{E}{+08}$	4.96E + 08	4.47E + 08
	3	17	$4.72E{+}08$	6.74E + 08	5.06E + 08	5.80E + 08	$4.95E{+}08$	4.56E + 08
	5	98	$4.67\mathrm{E}{+08}$	6.73E + 08	5.06E + 08	5.78E + 08	5.00E + 08	4.53E + 08
	9	443	4.69E + 08	6.73E + 08	5.04E + 08	5.81E + 08	5.02E + 08	4.52E + 08
2	11	770	$4.67\mathrm{E}{+08}$	6.72E + 08	$5.05E{+}08$	$5.80\mathrm{E}{+}08$	$4.97E{+}08$	4.53E + 08
	16	1705	$4.71E{+}08$	6.73E + 08	$5.02E{+}08$	$5.80\mathrm{E}{+}08$	5.00E + 08	4.50E + 08
	21	3093	4.67E + 08	6.73E + 08	5.04E + 08	5.80E + 08	4.96E + 08	$4.49E{+}08$
	31	7142	4.66E + 08	6.72E + 08	$5.04\mathrm{E}{+08}$	5.80E + 08	$4.99E{+}08$	4.50E + 08
	51	23356	4.66E + 08	6.71E + 08	$5.03E{+}08$	5.80E + 08	$4.95E{+}08$	4.50E + 08
	3	15	$5.03E{+}08$	6.99E + 08	$5.34\mathrm{E}{+08}$	$6.08E{+}08$	5.26E + 08	4.84E + 08
	5	93	4.88E + 08	6.99E + 08	5.26E + 08	5.96E + 08	5.20E + 08	4.71E + 08
	9	430	4.89E + 08	6.96E + 08	5.24E + 08	5.97E + 08	5.17E + 08	4.69E + 08
3	11	606	$4.83E{+}08$	6.97E + 08	$5.25E{+}08$	5.97E + 08	5.23E + 08	4.69E + 08
	16	1522	4.85E + 08	6.97E + 08	5.25E + 08	5.97E + 08	5.17E + 08	4.67E + 08
	21	2856	$4.89E{+}08$	6.96E + 08	$5.25E{+}08$	$5.99\mathrm{E}{+}08$	$5.16\mathrm{E}{+08}$	4.69E + 08
	31	7167	$4.85E{+}08$	6.96E + 08	$5.25E{+}08$	$5.98E{+}08$	$5.17\mathrm{E}{+08}$	4.68E + 08
	51	18265	4.83E + 08	6.96E + 08	5.24E + 08	5.98E + 08	$5.19E{+}08$	4.67E + 08

Table 5: Performance of each policy for different numbers of classes.

As N increases the cost of each policy becomes more accurate. However, as we can see below, the improvement in accuracy becomes marginal when N exceeds a small value. Define the *percentage*



Figure 5: **Percentage refinement**. The percentage refinement of each policy for layouts 1, 2, and 3 is shown in (a), (b), and (c) respectively. The percentage refinement of EV|PI for the three layouts is shown again in (d).

refinement of a policy for a warehouse with N classes as $\psi(N) = |Z(N + \Delta N) - Z(N)|/Z(N) \times 100\%$, where Z(N) represents the cost of the policy for N classes. The percentage refinement $\psi(N)$ of a policy determines the improvement in its cost accuracy as the number of classes increases from N to $N + \Delta N$.

Figures 5(a), (b), and (c) show the percentage refinement of each policy for layouts 1, 2, and 3 respectively. We also show the percentage refinement of EV|PI for all the three layouts in Figure 5(d). The percentage refinement maintains significantly below 2% for $N \ge 5$. This suggests that, for practical purposes, it suffices to set N = 5 for the three layouts as increasing the number of classes beyond this value does not render any significant cost refinement. However, to show that our approach can handle a reasonably large N without causing the computational time unnecessarily long, we set N = 11 in most of the numerical experiments.

F.2 Effects of number of products

We investigate the effects of the number of products (M) in the warehouse in this section. Table 6 compares the performance of all policies for different numbers of products with N = 11 and q = 5. The RLR significantly outperforms other heuristics for all numbers of products and its results are close to EV|PI in many cases. Among the other heuristics the DET policy is the best, followed by the TOD and DOS policies, which outperform the TOS policy.

Layout	M	Run Time (sec)	\mathbf{RLR}	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	500	4700	1.37E + 09	$1.98E{+}09$	$1.51E{+}09$	$1.62E{+}09$	1.46E + 09	1.29E + 09
	1000	23777	7.11E + 09	9.61E + 09	7.75E + 09	7.98E + 09	7.33E + 09	6.30E + 09
1	1500	44669	3.02E + 09	$4.53E{+}09$	3.48E + 09	3.58E + 09	3.15E + 09	2.75E + 09
	2000	82305	5.97E + 09	$9.13E{+}09$	6.85E + 09	7.26E + 09	6.15E + 09	5.09E + 09
	2500	187256	$4.63E{+}09$	7.02E + 09	5.25E+09	5.56E+09	$4.92E{+}09$	$3.85E{+}09$
	500	4398	1.37E + 09	$1.96E{+}09$	$1.50E{+}09$	$1.62E{+}09$	1.45E + 09	1.29E + 09
	1000	23970	$6.95E{+}09$	9.36E + 09	7.55E + 09	7.89E + 09	7.22E + 09	6.24E + 09
2	1500	45718	3.01E + 09	$4.49E{+}09$	3.44E + 09	3.56E + 09	3.12E + 09	2.74E + 09
	2000	92495	5.93E + 09	$9.09E{+}09$	6.81E + 09	7.23E + 09	6.17E + 09	5.01E + 09
	2500	186385	$4.63E{+}09$	6.96E + 09	5.20E + 09	$5.53E{+}09$	4.89E + 09	3.81E + 09
	500	4236	1.40E + 09	$1.99E{+}09$	$1.53E{+}09$	$1.65E{+}09$	1.50E + 09	1.32E + 09
	1000	20157	7.55E + 09	9.88E + 09	8.15E + 09	8.37E + 09	7.79E + 09	6.84E + 09
3	1500	43319	3.07E + 09	4.57E + 09	3.54E + 09	3.65E + 09	3.25E + 09	2.84E + 09
	2000	87133	6.06E + 09	9.18E + 09	$6.93E{+}09$	7.33E + 09	6.31E + 09	5.20E + 09
	2500	128486	4.77E + 09	$7.07\mathrm{E}{+}09$	5.34E + 09	5.65E+09	$4.99E{+}09$	$3.97E{+}09$

Table 6: Performance of each policy for different numbers of products.

F.3 Effects of demand variability

Since the robust optimization formulation takes demand uncertainty into consideration, we expect the RLR to outperform other heuristics when demand becomes more variable. Table 7 shows the performance of each policy under different levels of demand variability for a warehouse with 200 products and 11 classes. Figure 6 shows the impact of demand variability under each layout.

The cost of each policy generally increases as q increases. The RLR significantly outperforms all other heuristics and produces results that are very close to EV|PI over a wide range of demand variability q. For each layout even the worst result given by the RLR (when q = 20) is still better than the best results given by all other existing heuristics (when q = 1). Note that the performance of the DET policy deteriorates significantly as q increases. These results strongly indicate that the RLR is superior to all other heuristics over a wide range of demand variability and clearly manifest the value of considering demand uncertainty for solving the storage assignment problem.

Layout	q	Run Time (sec)	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	1	352	$1.81E{+}09$	2.17E + 09	2.02E + 09	$2.19E{+}09$	1.82E + 09	$1.80E{+}09$
	5	457	1.82E + 09	$2.17\mathrm{E}{+09}$	2.02E + 09	$2.19E{+}09$	1.85E + 09	1.80E + 09
1	10	573	1.83E + 09	2.17E + 09	2.02E + 09	$2.19E{+}09$	1.88E + 09	1.80E + 09
	15	607	1.83E + 09	2.18E + 09	2.04E + 09	$2.19E{+}09$	1.92E + 09	1.80E + 09
	20	528	1.84E + 09	$2.17\mathrm{E}{+09}$	2.04E + 09	$2.19E{+}09$	$1.99E{+}09$	$1.80E{+}09$
	1	385	1.82E + 09	2.13E + 09	$1.99E{+}09$	$2.15E{+}09$	1.82E + 09	$1.81E{+}09$
	5	500	1.83E + 09	2.13E + 09	1.99E + 09	2.15E + 09	1.84E + 09	1.81E + 09
2	10	659	1.83E + 09	2.13E + 09	1.99E + 09	2.15E + 09	1.87E + 09	1.81E + 09
	15	645	$1.83E{+}09$	2.14E + 09	2.01E + 09	$2.15E{+}09$	$1.91E{+}09$	$1.81E{+}09$
	20	502	1.84E + 09	2.13E + 09	2.01E + 09	2.16E + 09	1.94E + 09	$1.81E{+}09$
	1	360	1.86E + 09	2.20E + 09	2.06E + 09	2.22E + 09	1.87E + 09	1.85E + 09
	5	458	1.87E + 09	2.21E + 09	2.06E + 09	2.22E + 09	1.89E + 09	1.85E + 09
3	10	645	1.88E + 09	2.20E + 09	2.06E + 09	2.22E + 09	1.91E + 09	1.85E + 09
	15	682	1.88E + 09	2.21E + 09	2.08E + 09	2.23E + 09	$1.95E{+}09$	$1.85E{+}09$
	20	507	1.89E + 09	2.21E + 09	2.08E + 09	2.23E + 09	2.00E + 09	1.85E + 09

Table 7: Performance of each policy under different levels of demand variability.



Figure 6: The impact of demand variability. The cost of each policy under various values of q for layouts 1, 2, and 3 is shown in (a), (b), and (c) respectively. The cost of the RLR maintains very close to EV|PI and is significantly lower than the costs of all other heuristics for a wide range of q.

F.4 Increasing number of periods

We also consider a planning horizon of T = 30 periods, which corresponds to monthly planning if each period represents a day. We set q = 5. Table 8 shows the performance of all policies for different numbers of products and classes. The costs of the RLR remain significantly lower than that of all other heuristics and they are very close to EV|PI. The DET outperforms the TOD and DOS policies, which are generally better than the TOS policy.

Layout	M	N	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	150	11	4.77E + 09	9.49E + 09	$8.98E{+}09$	9.46E + 09	5.13E + 09	$4.52\mathrm{E}{+09}$
1	80	31	1.43E + 09	3.27E + 09	2.36E + 09	3.1E + 09	$1.5E{+}09$	1.39E + 09
	50	101	$1.5E{+}09$	2.68E + 09	$2.34\mathrm{E}{+09}$	$2.9E{+}09$	1.56E + 09	1.46E + 09
	150	11	$4.74E{+}09$	9.48E + 09	8.97E + 09	9.45E + 09	5E + 09	$4.51\mathrm{E}{+09}$
2	80	31	1.42E + 09	3.26E + 09	2.35E + 09	3.09E + 09	1.45E + 09	1.40E + 09
	50	101	$1.49E{+}09$	2.68E + 09	$2.33E{+}09$	$2.9E{+}09$	$1.51E{+}09$	1.46E + 09
	150	11	$4.83E{+}09$	9.5E + 09	$8.99E{+}09$	9.47E + 09	5.18E + 09	$4.59E{+}09$
3	80	31	$1.47E{+}09$	3.28E + 09	$2.38E{+}09$	3.11E + 09	1.54E + 09	$1.43E{+}09$
	50	101	1.52E + 09	2.69E + 09	2.35E + 09	2.91E + 09	1.55E + 09	1.48E + 09

Table 8: Performance of each policy for a planning horizon of 30 periods.

F.5 Effects of different class generation methods

We investigate the effects of different class generation methods on the performance of all policies. We consider the layouts shown in Figures 4(b), (c), and (d) in Appendix E and assume the warehouse has 8 racks. Each rack contains 10 sections and each section has 3 levels. Only one pallet can be stored in each level of a section. All levels of the same section have identical store cost and identical retrieve cost and they belong to the same class. We assume 40 products and set q = 1. Note that the results for N = 81 in Tables 9–12 correspond to the case where actual location-to-location distances are used.

F.5.1 Grid-based class generation method

Under a grid-based class generation method, the warehouse is partitioned into classes with a grid. Storage locations that are close to each other with similar store cost and similar retrieve cost are grouped together in a class. Figure 7 shows the distributions of storage classes with N = 5 under the grid-based class generation method for layouts 1, 2, and 3. Note that the distributions of classes are the same for different layouts under this method.

Table 9 shows the performance of all policies. The RLR consistently outperforms other heuristics and its results are very close to EV|PI across different numbers of classes and different layouts. The DET policy generally outperforms the DOS and TOD policies, which in turn outperform the TOS policy.



Figure 7: Storage classes under the grid-based class generation method. (a) Layout 1. (b) Layout 2. (c) Layout 3.

Layout	N	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	5	6.44E + 07	9.57E + 07	8.16E + 07	$6.91E{+}07$	6.94E + 07	6.26E + 07
	21	6.40E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	$6.98E{+}07$	$6.92E{+}07$	6.26E + 07
1	41	$6.39E{+}07$	9.57E + 07	8.16E + 07	7.20E + 07	$6.92E{+}07$	6.26E + 07
	61	6.38E + 07	9.57E + 07	8.15E + 07	7.61E + 07	$6.93E{+}07$	6.26E + 07
	81	6.36E + 07	9.57E + 07	$8.15E{+}07$	8.00E + 07	7.01E + 07	6.27E + 07
	5	6.43E + 07	$9.57E{+}07$	8.16E + 07	$6.91E{+}07$	$6.95E{+}07$	6.27E + 07
	21	6.43E + 07	9.57E + 07	8.16E + 07	7.00E + 07	6.96E + 07	6.27E + 07
2	41	$6.41E{+}07$	$9.58E{+}07$	8.16E + 07	7.21E + 07	7.04E + 07	6.27E + 07
	61	$6.39E{+}07$	9.57E + 07	8.16E + 07	7.61E + 07	$6.95E{+}07$	6.27E + 07
	81	6.40E + 07	9.57E + 07	8.16E + 07	$8.01E{+}07$	$6.95E{+}07$	6.27E + 07
	5	6.43E + 07	9.57E + 07	8.16E + 07	$6.92E{+}07$	$6.92E{+}07$	6.27E + 07
	21	6.43E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	7.00E + 07	$6.93E{+}07$	6.27E + 07
3	41	6.40E + 07	9.57E + 07	$8.15E{+}07$	7.21E + 07	6.88E + 07	6.27E + 07
	61	6.40E + 07	9.57E + 07	8.16E + 07	7.62E + 07	6.96E + 07	$6.27E{+}07$
	81	6.37E + 07	9.57E + 07	8.16E + 07	8.00E + 07	6.96E + 07	$6.27E{+}07$

Table 9: Performance of all policies under the grid-based class generation method.

F.5.2 Distance-based class generation method I

Under this class generation method, define the travel cost of each storage location as the sum of its store and retrieve costs. Storage locations are sorted according to their travel costs. Locations with the least travel cost are assigned to class 1 and locations with the largest travel cost are assigned to class N - 1. In this method we assume the storage locations are evenly distributed among the classes. Figure 8 shows the distributions of storage classes with N = 5 under the distance-based class generation method I for layouts 1, 2, and 3.

Table 10 shows that the RLR outperforms all other heuristics and produces results that are very close to EV|PI for different values of N and different layouts. The DET policy generally outperforms the DOS and TOD policies, which in turn outperform the TOS policy.



Figure 8: Storage classes under the distance-based class generation method I. (a) Layout 1. (b) Layout 2. (c) Layout 3.

Layout	N	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	5	6.44E + 07	9.57E + 07	8.16E + 07	$6.91E{+}07$	$6.91E{+}07$	6.26E + 07
	21	6.42E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	6.99E + 07	6.92E + 07	6.26E + 07
1	41	6.42E + 07	9.57E + 07	8.15E + 07	7.21E + 07	$6.99E{+}07$	6.26E + 07
	61	6.38E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	7.61E + 07	$6.91\mathrm{E}{+07}$	6.26E + 07
	81	6.36E + 07	$9.57\mathrm{E}{+07}$	$8.15E{+}07$	8.00E + 07	7.01E + 07	6.27E + 07
	5	6.42E + 07	$9.58\mathrm{E}{+07}$	8.15E + 07	6.92E + 07	$6.91E{+}07$	6.27E + 07
	21	6.42E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	7.00E + 07	$6.90E{+}07$	6.27E + 07
2	41	6.42E + 07	9.57E + 07	8.15E + 07	7.21E + 07	7.04E + 07	6.27E + 07
	61	6.36E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	7.61E + 07	7.00E + 07	6.27E + 07
	81	6.40E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	$8.01E{+}07$	$6.95E{+}07$	6.27E + 07
	5	6.42E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	$6.92E{+}07$	$6.93E{+}07$	6.27E + 07
	21	6.44E + 07	9.58E + 07	8.16E + 07	7.00E + 07	6.89E + 07	6.27E + 07
3	41	6.40E + 07	$9.58\mathrm{E}{+07}$	8.16E + 07	7.21E + 07	$6.91\mathrm{E}{+07}$	$6.28E{+}07$
	61	$6.37E{+}07$	$9.57\mathrm{E}{+07}$	8.16E + 07	7.61E + 07	6.94E + 07	6.27E + 07
	81	6.37E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	8.00E + 07	6.96E + 07	6.27E + 07

Table 10: Performance of all policies under the distance-based class generation method I.

F.5.3 Distance-based class generation method II

This method is similar to the distance-based class generation method I except the size of class 1 is approximately two times the size of all other classes. Figure 9 shows the distributions of storage classes with N = 5 under the distance-based class generation method II for layouts 1, 2, and 3.

Table 11 shows that the RLR continues to outperform all other heuristics and consistently produces results that are close to EV|PI. Among the other heuristics the DET policy is the best, followed by the DOS policy, followed by the TOD policy, and the TOS policy is the least efficient.



Figure 9: Storage classes under the distance-based class generation method II. (a) Layout 1. (b) Layout 2. (c) Layout 3.

Layout	N	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	5	6.42E + 07	9.57E + 07	8.15E + 07	$6.91E{+}07$	6.90E + 07	6.27E + 07
	21	6.40E + 07	9.57E + 07	8.15E + 07	$6.99E{+}07$	6.94E + 07	6.26E + 07
1	41	$6.38E{+}07$	9.57E + 07	8.16E + 07	7.21E + 07	$6.89E{+}07$	6.27E + 07
	61	6.38E + 07	9.57E + 07	8.15E + 07	7.60E + 07	$6.91E{+}07$	6.26E + 07
	81	6.36E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	8.00E + 07	7.01E + 07	6.27E + 07
	5	6.44E + 07	9.57E + 07	8.15E + 07	$6.91E{+}07$	6.90E + 07	6.27E + 07
	21	$6.41E{+}07$	$9.57\mathrm{E}{+07}$	8.15E + 07	7.00E + 07	6.88E + 07	6.27E + 07
2	41	6.42E + 07	9.57E + 07	8.16E + 07	7.20E + 07	6.94E + 07	6.27E + 07
	61	6.38E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	7.61E + 07	6.96E + 07	6.27E + 07
	81	6.40E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	$8.01E{+}07$	$6.95E{+}07$	6.27E + 07
	5	6.45E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	$6.91E{+}07$	$6.89E{+}07$	6.27E + 07
	21	6.46E + 07	9.58E + 07	8.15E + 07	$6.99E{+}07$	$6.95E{+}07$	6.27E + 07
3	41	$6.39E{+}07$	$9.57\mathrm{E}{+07}$	8.16E + 07	7.21E + 07	7.02E + 07	6.27E + 07
	61	6.40E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	7.61E + 07	$6.95E{+}07$	6.27E + 07
	81	6.37E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	8.00E + 07	6.96E + 07	6.27E + 07

Table 11: Performance of all policies under the distance-based class generation method II.

F.5.4 Distance-based class generation method III

This method is similar to the distance-based class generation method I except the size of class N - 1 is approximately two times the size of all other classes. Figure 10 shows the distributions of storage classes with N = 5 under the distance-based class generation method III for layouts 1, 2, and 3.



Figure 10: Storage classes under the distance-based class generation method III. (a) Layout 1. (b) Layout 2. (c) Layout 3.

Table 12 shows that the RLR continues to outperform all other heuristics and consistently produces results that are very close to EV|PI. Among the other heuristics the DET policy is the best, followed by the DOS policy, followed by the TOD policy, and the TOS policy is the least efficient.

Layout	N	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	5	$6.45E{+}07$	$9.57E{+}07$	$8.15E{+}07$	$6.91E{+}07$	6.94E + 07	$6.27\mathrm{E}{+07}$
	21	$6.41E{+}07$	$9.56\mathrm{E}{+07}$	$8.15E{+}07$	$6.99E{+}07$	6.88E + 07	6.26E + 07
1	41	6.41E + 07	9.57E + 07	8.15E + 07	7.20E + 07	$6.98E{+}07$	6.27E + 07
	61	6.36E + 07	$9.57\mathrm{E}{+07}$	$8.15E{+}07$	7.61E + 07	7.03E + 07	6.26E + 07
	81	6.36E + 07	$9.57\mathrm{E}{+07}$	8.15E + 07	8.00E + 07	7.01E + 07	6.27E + 07
	5	$6.41E{+}07$	$9.57\mathrm{E}{+07}$	$8.15E{+}07$	$6.91E{+}07$	$6.91E{+}07$	6.27E + 07
	21	6.40E + 07	9.57E + 07	8.15E + 07	$6.99E{+}07$	$6.93E{+}07$	6.27E + 07
2	41	6.40E + 07	9.57E + 07	$8.15E{+}07$	7.21E + 07	6.94E + 07	6.27E + 07
	61	6.37E + 07	$9.57\mathrm{E}{+07}$	$8.15E{+}07$	7.61E + 07	$6.98E{+}07$	6.27E + 07
	81	6.40E + 07	$9.57\mathrm{E}{+07}$	8.16E + 07	$8.01E{+}07$	$6.95E{+}07$	6.27E + 07
	5	6.41E + 07	9.57E + 07	8.16E + 07	6.92E + 07	$6.91E{+}07$	6.27E + 07
	21	6.42E + 07	$9.58\mathrm{E}{+07}$	$8.15E{+}07$	7.00E + 07	$6.92E{+}07$	6.27E + 07
3	41	6.42E + 07	9.57E + 07	8.16E + 07	7.21E + 07	6.94E + 07	6.27E + 07
	61	6.40E + 07	$9.58E{+}07$	8.16E + 07	7.61E + 07	6.90E + 07	6.27E + 07
	81	$6.37E{+}07$	$9.57\mathrm{E}{+07}$	8.16E + 07	8.00E + 07	6.96E + 07	$6.27\mathrm{E}{+07}$

Table 12: Performance of all policies under the distance-based class generation method III.

From Figures 7 to 10, we can see that the distributions of classes are very different under different class generation methods for each layout. Our numerical experiments suggest that the performance of the RLR relative to other heuristics is rather insensitive to the way the classes are defined. The RLR significantly outperforms all other heuristics and consistently produces results that are very close to

EV|PL for different numbers of classes and different layouts, regardless of the class generation method used.

Furthermore, Tables 9–12 suggest that the cost given by each policy (except the DOS policy) does not change significantly as N increases and is very close to the cost when actual location-to-location distances are used (that is, when N = 81). These results suggest that the savings obtained by the RLR over other heuristics based on the average store cost and average retrieve cost of each class serves as a reasonably good approximation to the savings obtained using the actual location-to-location distances.

F.6 Using actual location-to-location distances

We investigate the performance of all policies when actual location-to-location distances are used to compute the travel cost. We consider the layouts shown in Figures 4(b), (c), and (d) in Appendix E and assume the warehouse has 8 racks. Each rack contains 10 sections and each section has 3 levels. Only one pallet can be stored in each level of a section. All levels of the same section have identical store cost and identical retrieve cost.

Table 13 shows that the RLR significantly outperforms other heuristics and its results are extremely close to EV|PI for some cases. This suggests that the RLR remains significantly more efficient than other heuristics even if actual location-to-location distances are used to compute the travel cost. Among the other heuristics the DET policy is the best, followed by the DOS and TOD policies, which outperform the TOS policy.

Layout	M	q	RLR	TOS	TOD	DOS	DET	$\mathbf{EV} \mathbf{PI}$
	40	1	6.36E + 07	9.57E + 07	$8.15E{+}07$	8.00E + 07	7.01E + 07	6.27E + 07
1	30	2	$1.70E{+}07$	$2.55E{+}07$	$2.06E{+}07$	$2.19E{+}07$	1.87E + 07	$1.67\mathrm{E}{+}07$
	20	3	7.22E + 06	$1.16E{+}07$	9.76E + 06	$1.10E{+}07$	$8.05E{+}06$	6.54E + 06
	40	1	6.40E + 07	9.57E + 07	$8.16E{+}07$	$8.01E{+}07$	$6.95E{+}07$	6.27E + 07
2	30	2	$1.71E{+}07$	$2.55E{+}07$	2.07E + 07	$2.19E{+}07$	$1.90E{+}07$	$1.67\mathrm{E}{+}07$
	20	3	7.27E + 06	$1.16E{+}07$	9.82E + 06	$1.10E{+}07$	8.06E + 06	6.57E + 06
	40	1	6.37E + 07	9.57E + 07	$8.16E{+}07$	8.00E + 07	6.96E + 07	6.27E + 07
3	30	2	1.70E + 07	2.56E + 07	2.07E + 07	$2.19E{+}07$	1.88E + 07	1.67E + 07
	20	3	7.29E + 06	$1.16E{+}07$	9.84E + 06	$1.10E{+}07$	$8.09E{+}06$	6.61E + 06

Table 13: Performance of all policies using actual location-to-location distances.