Supplementary Theoretical Appendix to "Instability and the Incentives for Corruption"

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1 A Model with Concavity

In this Appendix, we consider a functional form that allows for both kinds of corruption to exist simultaneously in equilibrium by introducing concavity into the incumbent's utility function. In particular, we take $\pi(L_t, \alpha) = \alpha A_F L_t^{\gamma}$, which has a similar interpretation as in the basic model but with the production function being $f(\cdot) = A_F(\cdot)^{\gamma}$. In order to eschew corner solutions, we specify the returns to embezzlement to be E_t^{γ}, which has the interpretation that the incumbent has to put resources he embezzles through a production process that exhibits diminishing returns. The value function is thus:

$$V(K_0) = \max_{E_0 \geq 0,\ L_0 \geq 0} \{E_0^{\gamma} + \sigma\alpha A_F L_0^{\gamma} + \alpha V(K_1)\} \quad \text{where} \quad K_1 = A(K_0 - E_0 - L_0)^{\gamma} \tag{1}$$

Solving the Bellman equation now yields the following equations (from the first-order conditions and the Envelope Theorem):

$$\gamma E_0^{\gamma-1} = \alpha V'(K_1) A\gamma (K_0 - E_0 - L_0)^{\gamma-1} \tag{2}$$

$$\sigma\alpha A_F \gamma L_0^{\gamma-1} = \alpha V'(K_1) A\gamma (K_0 - E_0 - L_0)^{\gamma-1} \tag{3}$$

$$V'(K_0) = \alpha V'(K_1) A\gamma (K_0 - E_0 - L_0)^{\gamma-1} \tag{4}$$

Let us focus the analysis on a steady state, in which K_t is constant. (4) then implies that the steady-state level of resources will be given once again by:

$$\left.\begin{array}{rcl} K_t - E_t - L_t & = & (A\alpha\gamma)^{\frac{1}{1-\gamma}} \\ K_{t+1} = A(K_t - E_t - L_t)^{\gamma} & = & A^{\frac{1}{1-\gamma}}(\alpha\gamma)^{\frac{\gamma}{1-\gamma}} \end{array}\right\} \tag{5}$$

for all $t \geq 0$, as in the basic model. Using (2) and (3), one can then solve for the values of E_t and L_t:

$$E_t = \frac{1}{1 + (\sigma\alpha A_F)^{\frac{1}{1-\gamma}}} A^{\frac{1}{1-\gamma}} (\alpha\gamma)^{\frac{\gamma}{1-\gamma}} (1 - \alpha\gamma) \tag{6}$$

$$L_t = \frac{(\sigma\alpha A_F)^{\frac{1}{1-\gamma}}}{1 + (\sigma\alpha A_F)^{\frac{1}{1-\gamma}}} A^{\frac{1}{1-\gamma}} (\alpha\gamma)^{\frac{\gamma}{1-\gamma}} (1 - \alpha\gamma) \tag{7}$$

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Note that these expressions have a neat interpretation: From (6), embezzlement takes up a constant share $\frac{1}{1+(\sigma\alpha A_F)^{\frac{\gamma}{1-\gamma}}}$ of total resources for corruption, and moreover, this share falls as α increases. On the other hand, the share that goes towards licensing is an increasing function of α. We can thus see the horizon and demand effects at play very clearly.

Substituting into the definition of Γ_t yields the following expression for corruption:

$$\Gamma_t = \frac{1}{A}\left(\frac{1}{\alpha\gamma} - 1\right)^{\gamma}\left(1 + (\sigma\alpha A_F)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} \tag{8}$$

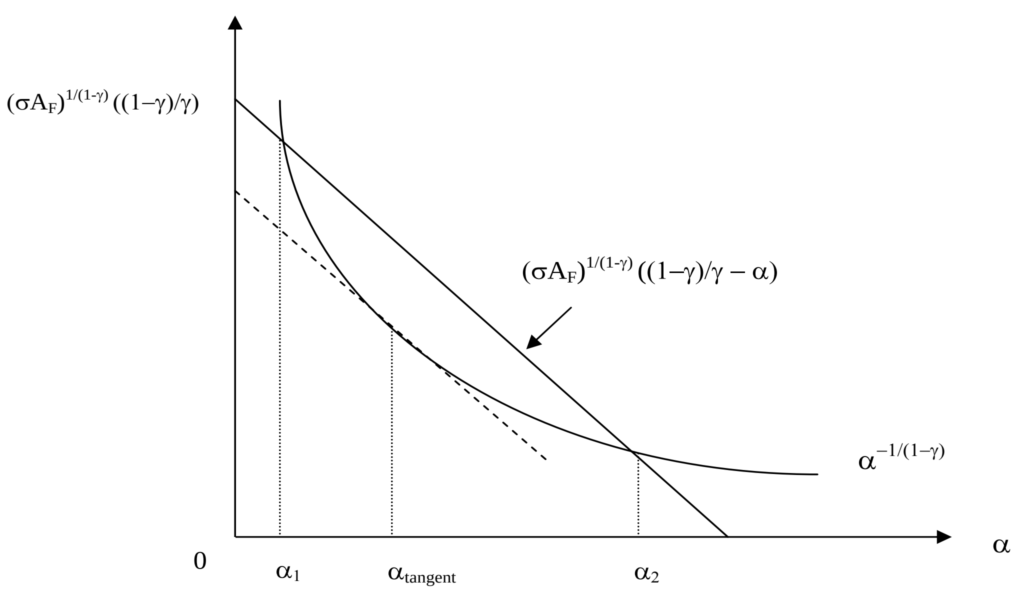
To see how corruption varies with stability, differentiating (8) with respect to α gives:

$$\frac{d\Gamma_t}{d\alpha} = \frac{1}{A}\left(\frac{1}{1+(\sigma\alpha A_F)^{\frac{1}{1-\gamma}}}\right)^{\gamma}\left(\frac{\alpha\gamma}{1-\alpha\gamma}\right)^{1-\gamma}\left[\frac{(\sigma A_F)^{\frac{1}{1-\gamma}}(\frac{1-\gamma}{\gamma}\alpha^{\frac{1}{1-\gamma}} - \alpha^{\frac{1}{1-\gamma}+1}) - 1}{\alpha^2}\right] \tag{9}$$

The critical term for the sign of this derivative is the expression in square brackets. Observe that in the limit as $\alpha \to 0^+$, this expression is clearly negative, so that corruption decreases in α at low levels of stability, ie the horizon effect dominates. To analyze the behavior of this term over the entire relevant range of α, observe that: $(\sigma A_F)^{\frac{1}{1-\gamma}}(\frac{1-\gamma}{\gamma}\alpha^{\frac{1}{1-\gamma}} - \alpha^{\frac{1}{1-\gamma}+1}) - 1 > 0$ if and only if:

$$(\sigma A_F)^{\frac{1}{1-\gamma}}(\frac{1-\gamma}{\gamma} - \alpha) > \alpha^{-\frac{1}{1-\gamma}} \tag{10}$$

Note that the right-hand side of (10) is a hyperbola in α, whereas the left-hand side is linearly decreasing. We illustrate these curves in the figure below for the interesting case where the two curves intersect:



Denoting the two points of intersection by $0 < \alpha_1 < \alpha_2$, observe that (10) holds when $\alpha \in (\alpha_1, \alpha_2)$. From (9), this implies that $\frac{d\Gamma_t}{d\alpha} < 0$ for $\alpha \in (0, \alpha_1)$, ie the horizon effect prevails in this lowest range of α. Conversely, as stability increases, $\frac{d\Gamma_t}{d\alpha} > 0$ for $\alpha \in (\alpha_1, \alpha_2)$, so that there is some range of α for which the demand effect dominates and corruption now increases with stability. A final case is where $\frac{d\Gamma_t}{d\alpha} < 0$ for values α greater than α_2; thus, it is possible for the horizon effect to re-emerge as incumbents in very stable regimes may opt to set more resources aside for the future, instead of diverting them towards corrupt activities today.

Therefore, for this specification, we also find that corruption is a non-monotonic function of α that can be approximated by a U-shape (although the function may bend downwards for very high values of α). From (9), the derivative of corruption with respect to stability is positive at $\alpha = 1$ if and only if:

$$(\sigma A_F)^{\frac{1}{1-\gamma}} \left(\frac{1-2\gamma}{\gamma} \right) > 1 \tag{11}$$

This requires that $\gamma < \frac{1}{2}$, in order for the left-hand side of (11) is positive. Also, corruption is an increasing function of stability for all $\alpha \in (\alpha_1, 1]$ (with $\alpha_2 > 1$) only if σA_F is sufficiently large, since we need to ensure that licensing and the accompanying demand effect dominates for the entire range of high α.

The above discussion is predicated on the validity of the above figure. In order to ensure this, we require that the left-hand side function of (10) have a large enough vertical intercept to generate two points of intersection with the hyperbola. Moreover, the larger σA_F is, the closer the root α_1 is to 0, thus ensuring that the turning point of any potential U-shape lies in the interior of $[0, 1]$.

To pin down these conditions formally, note that $\alpha_{tangent} = (\sigma A_F)^{\frac{1}{\gamma-2}} (1-\gamma)^{\frac{1-\gamma}{\gamma-2}}$ is the value of α where the tangent to the hyperbola is parallel to the linear function on the left-hand side of (10). The necessary and sufficient condition for the existence of the two points of intersection is thus that the vertical intercept of this linear function be higher than the vertical intercept of the tangent line through $\alpha_{tangent}$. Some algebraic manipulation delivers this condition to be:

$$\sigma A_F > \frac{(\gamma(2-\gamma))^{2-\gamma}}{(1-\gamma)^{3-2\gamma}} \tag{12}$$

Note that (12) alone does not guarantee that the turning point α_1 will lie in the interior of $[0, 1]$. A sufficient condition for this is: $\alpha_{tangent} < 1 \iff \sigma A_F > (1-\gamma)^{-(1-\gamma)}$, which ensures that corruption will be increasing over some range in $[0, 1]$. Collecting this with (12) yields the condition:

$$\sigma A_F > \max \left\{ \frac{(\gamma(2-\gamma))^{2-\gamma}}{(1-\gamma)^{3-2\gamma}},\ (1-\gamma)^{-(1-\gamma)} \right\} \tag{13}$$

To summarize, we now have a result that parallels Proposition 1 in the paper: Suppose that (13) holds. Then steady-state corruption is decreasing in stability for $\alpha \in (0, \alpha_1)$, increasing in α for $\alpha \in (\alpha_1, \alpha_2)$, and decreasing in α for $(\alpha_2, 1)$. If in addition, $\gamma < \frac{1}{2}$ and (11) holds, then steady-state corruption is a U-shaped function of α (since $\alpha_2 > 1$.) This generalizes our baseline model to allow for embezzlement and licensing to coexist in equilibrium.

2 Extension with Endogenous Stability

Proof of Proposition 3. Let us restate the optimization program with the introduction of public goods provision:

$$\begin{aligned} V(K_0) &= \max_{E_0,L_0}\{E_0+\sigma\alpha(\zeta,P_0)A_F L_t+\alpha(\zeta,P_0)V(K_1)\} \\ \text{where } K_1 &= A(K_0-E_0-L_0-P_0)^\gamma \end{aligned} \tag{14}$$

We will first prove that the cleaning-up property holds subject to some general conditions. The first-order conditions and the Envelope Theorem yield:

$$1 = \alpha V'(K_1)A\gamma(K_0-E_0-L_0-P_0)^{\gamma-1} \tag{15}$$

$$\sigma\alpha A_F = \alpha V'(K_1)A\gamma(K_0-E_0-L_0-P_0)^{\gamma-1} \tag{16}$$

$$\alpha'(P_0)(\sigma A_F L_0+V(K_1)) = \alpha V'(K_1)A\gamma(K_0-E_0-L_0-P_0)^{\gamma-1} \tag{17}$$

$$V'(K_0) = \alpha V'(K_1)A\gamma(K_0-E_0-L_0-P_0)^{\gamma-1} \tag{18}$$

However, only one of equations (15) and (16) can be satisfied at the same time (except in a knife-edge case). Suppose that (15) binds, which means that there is no licensing. We deduce from (15) and (18) that $V'(K)=1$, for all $K\geq K_t$. From (15), it follows that $K_0-E_0-P_0=(A\alpha\gamma)^{\frac{1}{1-\gamma}}$, and therefore $K_t=A^{\frac{1}{1-\gamma}}(\alpha\gamma)^{\frac{\gamma}{1-\gamma}}$, which is exactly the same formula as in the model without public goods. The cleaning-up property follows since this formula does not depend on K_0. Analogously, when (16) binds and there is no embezzlement, (16) and (18) imply that $V'(K)=\alpha\sigma A_F$, for all $K\geq K_t$. Then (16) implies that $K_t=A^{\frac{1}{1-\gamma}}(\alpha\gamma)^{\frac{\gamma}{1-\gamma}}$, which leads us once again to the cleaning-up property.

Now consider the specific functional form of $\alpha=\min(\zeta,g(P))$. Consider first the case where $\alpha=g(P)$. Here, the optimization program does not depend on intrinsic stability, ζ:

$$\begin{aligned} V(K_0) &= \max\{E_0+\sigma A_F g(P_0)L_0+g(P_0)V(K_1)\} \\ \text{where } K_1 &= A(K_0-E_0-L_0-P_0)^\gamma \end{aligned} \tag{19}$$

Under some mild conditions on $g(P)$ (e.g. $g(P)<1-\epsilon<1$ for some small $\epsilon>0$), this problem yields an optimal sequence of public goods levels, P_t, which we denote more precisely by $\bar{P}(K_t)$; similarly, define $\bar{\zeta}(K_t)=g(\bar{P}(K_t))$. Because of the cleaning-up property, K_t ($t\geq 1$) does not depend on K_0 for sufficiently large K_0. So $\bar{P}$ and $\bar{\zeta}$ are constant and do not depend on K_0 from period 1 onwards.

Let P_ζ be the inverse function of g, i.e. $g(P_\zeta)=\zeta$. With the cleaning-up property, the program reaches its steady state after just one period. We focus on what happens to the steady state value of corruption Γ_t by considering two cases:

- If $\zeta\geq\bar{\zeta}$: The polity displays a high enough level of intrinsic stability, so that $\alpha=\min(\zeta,g(P))=g(P)$. Here, the incumbent essentially solves (19) and thus chooses $P=\bar{P}$. In particular, $\alpha=\bar{\zeta}$, and thus corruption does not depend on ζ in this case.

- If $\zeta < \bar{\zeta}$: Because of the cleaning-up property for $t \geq 1$, $\bar{\zeta}$ only depends on the exogenous parameters σ, A_F, A and γ. As stability cannot reach the level $\bar{\zeta}$, the government chooses $P = P_\zeta$, so $\alpha = \zeta$.[1] The optimization program now reduces to:

$$\begin{aligned} V(K_0) &= \max\{E_0 + \sigma A_F \zeta L_0 + \zeta V(K_1)\} \\ \text{where } K_1 &= A(K_0 - E_0 - L_0 - P_\zeta)^\gamma \end{aligned} \tag{20}$$

As P_ζ is fixed, we are essentially back to the baseline model with exogenous stability presented in Section 2.1. It follows immediately that $V'(K_t)$ is a constant for all $t \geq 0$. Assuming that $\frac{1}{\sigma A_F} < \bar{\zeta}$, we now have two sub-cases:

  - If $\sigma A_F \zeta < 1$: The government chooses to embezzle and there is no licensing. The FOCs imply that:

$$\begin{aligned} (K_0 - E_0 - P_\zeta)^{1-\gamma} &= A\zeta\gamma \\ \Rightarrow \qquad K_1 = A(K_0 - E_0 - P_\zeta)^{1-\gamma} &= A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}} \\ \Rightarrow \quad \Gamma_t = \frac{A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}} - (A\zeta\gamma)^{\frac{1}{1-\gamma}} - P_\zeta}{A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}}} &= 1 - \zeta\gamma - \frac{P_\zeta}{A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}}} \end{aligned}$$

    for all $t \geq 1$. Let us call this last expression for corruption Γ_E.

  - If $\sigma A_F \zeta > 1$: The government chooses to license and there is no embezzlement. The FOCs now imply that:

$$\begin{aligned} \Rightarrow \qquad \zeta\gamma A(K_0 - L_0 - P_\zeta)^{\gamma-1} &= 1 \\ \Rightarrow \qquad K_1 = A(K_0 - E_0 - P_\zeta)^{1-\gamma} &= A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}} \\ \Rightarrow \quad \Gamma_t = \frac{\sigma A_F \zeta [A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}} - (A\zeta\gamma)^{\frac{1}{1-\gamma}} - P_\zeta]}{A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}}} &= \sigma A_F \zeta \left[1 - \zeta\gamma - \frac{P_\zeta}{A^{\frac{1}{1-\gamma}}(\zeta\gamma)^{\frac{\gamma}{1-\gamma}}}\right] \end{aligned}$$

    for all $t \geq 1$. Let us call this last expression Γ_L.

We can now show that under certain reasonable conditions on the stability function $g(P)$, the steady state level of corruption has a generalized U-shape, or a "root-shape" $\sqrt{}$, with respect to ζ. That is, when the level of intrinsic stability ζ goes up, steady state corruption first declines, then increases, before stabilizing at a constant level. To establish this, we show that Γ_E is decreasing, while Γ_L is increasing in ζ.

We limit ourselves to the class of increasing, concave functions $g(P) = (cP)^\rho$, with parameters $c > 0$ and $0 < \rho < 1$. Then $P_\zeta = \left(\frac{\zeta}{c}\right)^{\frac{1}{\rho}}$ and:

$$\Gamma_E = 1 - \zeta\gamma - \frac{\zeta^{\frac{1}{\rho} - \frac{\gamma}{1-\gamma}}}{c^{\frac{1}{\rho}} A^{\frac{1}{1-\gamma}} \gamma^{\frac{\gamma}{1-\gamma}}} = 1 - \zeta\gamma - B\zeta^{\frac{1}{\rho} - \frac{\gamma}{1-\gamma}}.$$

[1] The value function is concave in P if $g(P)$ is concave, and therefore has a unique maximum at $\bar{P}$. The value function is thus increasing in the interval $[0, \bar{P}]$.

where $B \equiv c^{-\frac{1}{\rho}} A^{-\frac{1}{1-\gamma}} \gamma^{-\frac{\gamma}{1-\gamma}}$. A sufficient condition for Γ_E to be decreasing in ζ is thus $\frac{1}{\rho} \geq \frac{\gamma}{1-\gamma}$, or equivalently:

$$\rho \leq \frac{1-\gamma}{\gamma} \tag{21}$$

Intuitively, for the horizon effect to exist on this "decreasing arm" of the "root-shape", public goods provision must display sufficient diminishing returns so that there is not too much of an incentive to increase its provision and invest in future stability. (Note that the conditions $\gamma < \frac{1}{2}$ and $\rho < 1$ are sufficient to ensure that (21) is satisfied.)

On the other hand, we have $\frac{\partial}{\partial \zeta} \Gamma_L \geq 0$ if and only if:

$$\frac{1 - 2\zeta\gamma}{\zeta^{\frac{1}{\rho} - \frac{\gamma}{1-\gamma} - 1} \left(\zeta + \frac{1}{\rho} - \frac{\gamma}{1-\gamma} \right)} \geq B \tag{22}$$

This last inequality, combined with (21), generates the generalized U-shape of corruption. It is worthwhile stressing that this last condition is in fact quite reasonable. To illustrate, suppose that $\rho = \frac{1-\gamma}{\gamma}$. (22) then simplifies to $1 - 2\zeta\gamma \geq B$, which is satisfied when B is sufficiently small (or equivalently, when c and A are large enough), bearing in mind that $\gamma < \frac{1}{2}$. In words, we get the "increasing arm" of the non-monotonic relationship when the economy's accumulation technology and that for public goods provision are sufficiently efficient; there is thus an incentive not to embezzle everything immediately, and some resources are available for licensing instead. ■