An OLG Model of Two-Way Capital Flows: 
The Role of Financial Development

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Abstract
We develop a two-country overlapping-generations model with financial frictions and show that cross-country difference in financial development can explain two recent empirical puzzles in the literature.

First, our model shows the two-way capital flows between developing and developed countries, i.e., financial capital flows from the country with less developed financial sector to the country with more developed financial sector, while foreign direct investment flows in the opposite direction. Second, our model shows that capital flows “uphill” from the poor country with less developed financial sector to the rich country with more developed financial sector in the net term.

Financial capital and FDI flows affect aggregate investment and output in the different way. The unequal or even opposite welfare effects exist within as well as across generations in the same country. It may explain why capital account liberalization encounters support and opposition in the developing economy. The short-run and long-run gains and losses in the intra- and inter-generational dimensions play an important role in determining the policy sequence of capital account liberalization in the developing economies.

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1 Introduction

This paper develops a two-country overlapping generations model with financial frictions and addresses two empirical puzzles. First, capital has been recently flowing “uphill” from poor to rich countries. Second, two-way capital flows, i.e., emerging-market economies have constantly accumulated large stocks of US treasury bills and received large inflows of foreign direct investment (FDI) and portfolio investment at the same time.

Nearly two decades ago, Lucas (1990) raised the question why so little capital flows from rich to poor countries. Prasad, Rajan, and Subramanian (2006, 2007) show empirically that the average per-capita income of countries running current account surpluses has been trending downwards in the past three decades, while that of deficit countries has been trending upward. Since 1998, the relative income per capita of the surplus countries has actually been below that of the deficit countries. In other words, capital flows from poor countries to rich countries in the net term. It is contrary to the prediction of neoclassical growth model. Second, rather than simply being recipients of net capital inflows or outflows, many emerging markets have witnessed growth in gross external financial assets and liabilities at rates that are much larger than the growth rates of their net positions (Lane and Milesi-Ferretti, 2001, 2006, 2007). Emerging-market economies have constantly accumulated large stocks of US treasury bills and received large inflows of foreign direct investment (FDI) and portfolio investment at the same time. Ju and Wei (2007) observe that many developing economies, e.g., China, Malaysia, and South Africa, are net importers of FDI and net exporters of financial capital at the same time; while many developed countries, e.g., France, the United Kingdom, and the United States find themselves in the reversed position, exporting FDI but importing financial capital. Thus, it seems that two-way capital flows are important for understanding the Lucas paradox and global imbalances.

Some theoretical models are developed to explain these empirical puzzles. For example, Mendoza, Quadrini, and Ríos-Rull (2007) present a multi-country dynamic general equilibrium model with incomplete asset markets and show that differences in financial development can explain why countries with more advanced financial markets accumulate foreign liabilities versus countries with less developed financial systems in a gradual, long-lasting process. In their model, differences in financial development also affect the composition of foreign portfolios, so that a country with a negative net foreign asset position like the US can still receive positive flows of factor incomes. Caballero, Farhi, and Gourinchas (2008) use a dynamic model of an endowment economy to explain the fact that the United States are running large current account deficits despite of the low interest rate, which is another version of the ”uphill” paradox. In these models, international capital flows are driven by the cross-country risk-sharing motives and foreign equity in-
vestment takes the form of portfolio investment instead of FDI. In contrast, our model focuses on the production side of the economy and FDI is driven by the financial factor in a deterministic model.

Several recent papers investigate the role of financial market imperfections for determining the composition of international capital flows. Goldstein and Razin (2006) and Razin and Sadka (2007) analyze the choice between FDI and portfolio investment from the perspective of information asymmetry. For the outside investor, FDI provides an opportunity to obtain information and control rights over firms in the recipient economy, which portfolio investment does not. Thus, low corporate transparency in the recipient country induces potential foreign investors to opt for FDI rather than portfolio investment. Our model is closely related with Ju and Wei (2006, 2007). They develop a static two-country model and show that cross-country difference in financial development and property right protection can give rise to two-way capital flows in the sense that households in the less developed economy may benefit from investing their funds in the financial sector of the more developed ones which are then transformed into FDI in the less developed economy. As a result, net capital inflows into developing economy are much smaller than the gross flows. In contrast, our model shows that cross-country difference in financial development alone is enough to generate two-way capital flows and the less developed economy has net capital outflow instead of net inflow. The intuition is as follows.

Due to financial frictions, the more productive agents are subject to credit constraints and production is inefficient. Financial development relaxes credit constraints and improves production efficiency. Under international financial autarky, the loan rate and the equity rate have non-monotonic patterns with respect to financial development. Given cross-country difference in financial development, the loan rate is different across countries and so is the equity rate. Under perfect capital mobility, financial capital flow is driven by the cross-country difference in the loan rate while FDI by that in the equity rate.

von Hagen and Zhang (2007) develop a two-country static model and show the non-monotonic pattern of the equity rate with respect to financial development. However, under perfect capital mobility, both financial capital and FDI flow to the country with more developed financial sector. Thus, von Hagen and Zhang (2007) cannot generate the empirical facts of two-way capital flows. Our model incorporates their basic mechanism into an overlapping-generations framework à la Diamond (1965) and Bernanke and Gertler (1989). Endogenous capital accumulation together with the non-monotonic patterns of the loan rate and the equity rate can generate the two-way capital flows as well as net capital outflow from the poor country under perfect capital mobility.

Mendoza, Quadrini, and Ríos-Rull (2007) and Caballero, Farhi, and Gourinchas (2008) analyze the role of financial development in improving the risk-sharing of idiosyncratic endowment risk. In contrast, we develop a deterministic dynamic model of the production
economy with individuals differing in productivity and financial development improves resource allocation and production efficiency. The country with the high degree of financial development endogenously accumulates a high capital stock in the long run and thus becomes the rich country.

We analyze two types of capital controls policy and their respective impacts on the domestic credit market, aggregate investment and output in the two-country model. Be specific, we take the allocation under international financial autarky as the starting point and compare it with the allocation under free mobility of FDI (controls on financial capital flow) and perfect capital mobility (no capital controls). Interestingly, financial capital inflow and FDI inflow affect domestic credit supply and credit demand, respectively. Thus, they have different effects on the loan rate and the equity rate. In our model, individuals as the owners of different types of capital may be affected in the opposite way by financial capital and FDI inflows. In addition, we also show that the welfare implications of capital account liberalization may be unequal or even opposite to different individuals in the same country both in the intra-generational and the inter-generational dimensions. It may explain why capital account liberalization encounters support and opposition in the developing economy.

The short-run and long-run gains and losses in the intra- and intergenerational dimensions also play an important role in determining the policy sequence of capital account liberalization in the developing economies.

The rest of this paper is structured as follows. Section 2 describes the basic model under international financial autarky and discusses the long-run patterns of the loan rate and the equity rate with respect to financial development. Section 3 analyzes the long-run and short-run efficiency and welfare implications of capital account liberalization given the two countries with different degrees of financial development. Section 4 concludes with the main findings. Appendix collects some technical issues.

2 The Basic Model: International Financial Autarky

The basic framework used is the overlapping generation model with two-period lives à la Diamond (1965) and Bernanke and Gertler (1989). The world economy consists of two countries, $H$ (Home) and $F$ (Foreign). There is no population growth and the population size of each generation in each country is normalized to one. Each generation consists of two types of agents in each country, i.e., entrepreneurs and households, each of mass $\eta$ and $1-\eta$, respectively. There are two types of goods: the final good which is internationally tradable and chosen as the numeraire, and the capital good which is internationally non-traded and $v^i_t$ denotes its price in period $t$ in country $i \in \{H,F\}$. Final goods can be consumed or transformed into capital goods. In the following, we use the superscript $i$
to denote the country-specific variables. There is no aggregate uncertainty in the model economy.

An agent born in period \( t \) in country \( i \) has the additive logarithm preference over consumption in period \( t \) and \( t + 1 \) as follows,

\[
U^{i,j}_t = \ln c^{i,j}_{1,t} + \ln c^{i,j}_{2,t+1},
\]

where the superscript \( j \in \{h, e\} \) refers to household or entrepreneur, respectively; \( c^{i,j}_{1,t} \) and \( c^{i,j}_{2,t+1} \) denote the consumption of agent \( j \) born in period \( t \) in country \( i \) when young and when old, respectively\(^1\). Agents born in period \( t \) are endowed with a unit of labor and earn the wage income only when young. At the end of period \( t \), young agents invest final goods in their respective projects\(^2\). At the beginning of period \( t + 1 \), their projects produce capital goods. Final goods are then produced contemporaneously by capital and the labor of the young generation.

By assumption, the project of entrepreneurs has a higher marginal product than that of households in equilibrium. Thus, young entrepreneurs prefer to borrow from young households to finance their investment. However, due to limited commitment problem, young entrepreneurs can only borrow against a fraction of their future project output. The strictness of their borrowing constraint depends on domestic financial development. In other words, entrepreneurs can pledge a larger fraction of their future project output for loans in the country with better protection of creditors, more efficient legal system, and more liquid asset market. The two countries differ only in financial development. The basic model describes the economic allocation under international financial autarky where agents are not allowed to borrow or lend abroad or move their projects abroad. Thus, the world economy can be considered as the sum of two closed economies.

2.1 Households

In period \( t \), a representative young household born in country \( i \) earns the wage income \( w^i_t \), consume \( c^{i,h}_{1,t} \), invest \( i^{i,h}_t \) in their project, and lend \( d^i_t = w^i_t - i^{i,h}_t - c^{i,h}_{1,t} \) at the gross loan rate of \( r^i_t \). His project has the decreasing return to scale and produces \( G(i^{i,h}_t) = R i^{i,h}_t - 0.5(i^{i,h}_t)^2 \) units of capital in period \( t + 1 \), where \( G'(i^{i,h}_t) = R - i^{i,h}_t > 0 \) and \( G''(i^{i,h}_t) = -1 < 0 \). When the household gets old in period \( t + 1 \), he also gets the loan repayment \( r^i_t d^i_t \). In period \( t \), he chooses \( i^{i,h}_t, c^{i,h}_{1,t}, c^{i,h}_{2,t+1} \) to maximize their lifetime utility (1) subject to the lifetime

\(^1\)von Hagen and Zhang (2008) show that adding the bequest motive does not change the results qualitatively.

\(^2\)Both types of agents invest in our model economy while some agents invest and others do not in the model of Boyd and Smith (1997). Such a difference in technology setting enables us to distinguish between the loan rate and the equity rate.
budget constraints,
\[ c_{1,t}^{i,h} + \frac{c_{2,t+1}^{i,h}}{r_t^i} = w_t^i - i_t^{i,h} + \frac{v_{t+1}^i G(i_t^{i,h})}{r_t^i}. \] (2)
The first order conditions are,
\[ r_t^i = v_{t+1}^i G'(i_t^{i,h}) \quad \text{and} \quad \frac{v_{t+1}^i G(i_t^{i,h})}{c_{1,t}^{i,h}} = r_t^i. \] (3)
Combine equations (2) and (3), the household life-time budget constraint is rewritten as
\[ c_{1,t}^{i,h} = \frac{w_t^i + \Psi_t^i}{2}, \quad \text{where} \quad \Psi_t^i \equiv \frac{v_{t+1}^i G(i_t^{i,h})}{r_t^i} - i_t^{i,h}. \] (4)

2.2 Entrepreneurs
In period \( t \), a representative young entrepreneur born in country \( i \) finances the project investment \( i_t^{e,i} \) using own funds \( n_t^i = w_t^i - c_{1,t}^{i,e} \) and loans \( z_t^i = i_t^{i,e} - n_t^i \). In period \( t+1 \), the project produces \( R_t^{i,e} \) units of capital; after repaying the debt of \( r_t^i z_t^i \), the old entrepreneur consumes \( c_{2,t+1}^{i,e} = R_t^{i,e} v_{t+1}^i - r_t^i z_t^i \).

His project has a higher marginal product than that of households, \( v_{t+1}^i R > v_{t+1}^i (R - i_t^{i,h}) = r_t^i \), as long as \( i_t^{i,h} > 0 \). Thus, the young entrepreneur prefers strictly to borrow and finance the project investment in period \( t \). However, due to limited commitment problem, he can only borrow against a fraction of the future project output,
\[ r_t^i z_t^i \leq \theta_t^i R_t^{i,e} v_{t+1}^i. \] (5)
Following Matsuyama (2004, 2007, 2008), we use \( \theta_t^i \in [0, 1] \) to measures the degree of financial development in country \( i \) in period \( t \). \( \theta_t^i \) is higher in the country with more sophisticated financial and legal system, better creditor protection, and etc.\(^3\)

The equity rate in period \( t \) is defined as the rate of return to the net worth of the young entrepreneur invested in the project in period \( t \),
\[ \Gamma_t^i \equiv \frac{R_t^{i,e} v_{t+1}^i - r_t^i z_t^i}{w_t^i - c_{1,t}^{i,e}} \] (6)
In equilibrium, the equity rate is no less than the loan rate, \( \Gamma_t^i \geq r_t^i \); otherwise, the young entrepreneur would rather lend than borrow. It can be considered as the entrepreneur’s

\(^3\)The pledgeability, \( \theta \), can be argued in various forms of agency costs story, e.g., the inalienability of human capital of entrepreneurs by Hart and Moore (1994) or costly state verifcation by Townsend (1979), or unobservable project (effort) choices by Holmstrom and Tirole (1997). See Tirole (2006) for a comprehensive overview of different models of financial contracting. This paper analyzes the implications of financial development on the borrowing constraints of different individuals. Thus, we choose the simplest form of borrowing constraints.
participation constraint. Rewrite equation (6) into \( \Gamma^i_t = r^i_t + \frac{(Rv^i_{t+1} - r^i_t)z^i_t}{c^i_{1,t} - z^i_t} \) and the participation constraint is equivalent to \( r^i_t \leq Rv^i_{t+1} \). Intuitively, only if the loan rate is lower than the marginal revenue of the entrepreneur’s project, the entrepreneur would like to finance his project using external debt and the equity rate is higher than the loan rate; if the loan rate is equal to the marginal revenue of the entrepreneur’s project, the equity rate is equal to the loan rate and the entrepreneur may not borrow to the limit.

In period \( t \), the young entrepreneur chooses \( i^i_{1,t}, z^i_t, c^i_{1,t}, c^i_{2,t+1} \) to maximize his life-time utility (1) subject to the period budget constraints (7) and (8), the borrowing constraints (5) and the participation constraints (9):

\[
c^i_{1,t} + i^i_{1,t} = w^i_t + z^i_t, \tag{7}
\]

\[
c^i_{2,t+1} + r^i_t z^i_t = R^i_{t} v^i_{t+1}, \tag{8}
\]

\[
r^i_t \leq R^i_{t+1}. \tag{9}
\]

Note that only one of the two constraints (5) and (9) is strictly binding in equilibrium.

The equilibrium condition is,

\[
\frac{c^i_{2,t+1}}{c^i_{1,t}} = \Gamma^i_t = \begin{cases} 
\frac{1 - \theta^i_t}{\eta R^i_{t} v^i_{t+1} - \theta^i_t}, & \text{if } r^i_t < R^i_{t+1}, \\
r^i_t, & \text{if } r^i_t = R^i_{t+1}.
\end{cases} \tag{10}
\]

According to equations (6), (7) and (10), the entrepreneur’s consumption in the two periods and the period-\( t \) debt are

\[
c^i_{1,t} = \frac{w^i_t}{2} \quad \text{and} \quad c^i_{2,t+1} = \frac{\Gamma^i_t w^i_t}{2}, \tag{11}
\]

\[
z^i_t = i^i_{1,t} - \frac{w^i_t}{2}. \tag{12}
\]

### 2.3 Aggregate Production and Market Equilibrium

In period \( t \), final goods are produced from capital \( K^i_t \) and the labor input of young generation \( L^i \) in country \( i \). Capital fully depreciates after production. The wage rate and the price of capital are equal to the marginal products of labor and capital, respectively,

\[
Y^i_t = (K^i_t)\alpha (L^i)^{1-\alpha}, \quad \text{where} \quad K^i_t = \eta R^i t_{t-1} + (1 - \eta) G(i^i_{t-1}), \quad \text{and} \quad L^i = 1, \tag{13}
\]

\[
v^i_t K^i_t = \alpha Y^i_t, \quad \text{and} \quad w^i_t L^i = (1 - \alpha) Y^i_t. \tag{14}
\]

The credit market clears in period \( t \),

\[
\eta z^i_t = (1 - \eta) d^i_t \quad \text{or} \quad \eta [i^i e_t - (w^i_t - c^i_{1,t})] = (1 - \eta) [w^i_t - (i^i h_t - c^i_{1,t})], \tag{15}
\]
Definition 1. Given the degree of financial development $\theta^i_t$, market equilibrium is a set of allocations of households, $\{i^{h1}_t, i^{h2}_t, c^{h1}_t, c^{h2}_t\}$, entrepreneurs, $\{i^{e1}_t, i^{e2}_t, z^i_t, c^{e1}_t, c^{e2}_t\}$, aggregate variables, $\{Y^i_t, K^i_t, w^1_t, v^i_t\}$, together with the loan rate and the equity rate $\{r^i_t, \Gamma^i_t\}$, satisfying equations (3)-(6), (9)-(15),

2.4 Parameterization

Our paper intends to provide a conceptual framework to think about the efficiency and welfare implications of capital account liberalization. Thus, we focus here more on its qualitative results instead of its quantitative relevance. As an analytical solution is not obtainable, we use a numerical example to show the intuition explicitly. We set $\alpha = 0.36$ so that the wage income accounts for 64% of aggregate output, in line with the empirical fact. The values of $R$ and $\eta$ do not matter for our qualitative results. We set $R = 1$ implying that entrepreneurs produce capital goods one-to-one from final goods. We set $\eta = 0.2$ implying that entrepreneurs account for 20% of the population.

2.5 Long-Run Effects of Financial Development

This section analyzes how financial development affects the patterns of the loan rate and the equity rate in the long run under international financial autarky. Thus, we drop the country superscript and the time subscript of the relevant variables. Figure 1 shows the steady-state values of some endogenous variables and the horizontal axis denotes $\theta \in [0, 1]$. Let $\Delta X \equiv \left[\frac{X(\theta \in [0, 1])}{X(\theta = 0)} - 1\right] 100$ denote the percentage difference of variable $X$ in the case of $\theta \in [0, 1]$ versus the case of domestic financial autarky $\theta = 0$.

In the case of domestic financial autarky, $\theta = 0$, entrepreneurs cannot borrow against their future project outcome and have to finance their project investment using own funds only, $\hat{i}^e = w - c^{e,y} = \frac{w}{2}$. As a result, the equity rate is simply the marginal revenue of their project, $\Gamma = Rv$. Despite of inactive credit market, the (underlying) loan rate is equal to the marginal revenue of the households project, $r = vG'(i^h) = v(R - i^h)$. Due to the logarithm utility function, households prefer to have positive consumption when old. Under domestic financial autarky, the project revenue is the only source of their consumption when old. Thus, households make positive project investment, $i^h > 0$ and the (underlying) loan rate is smaller than the equity rate, $r = v(R - i^h) < vR = \Gamma$.

Financial development is measured by an increase in $\theta$ which enables entrepreneurs to borrow against a larger fraction of their future project revenue and expand their current project investment. As long as the loan rate is lower than the marginal revenue of their project, $r < vR$, entrepreneurs always borrow up to the limit. The improvement in

\footnote{Such patterns essentially explain international capital flows in section 3.}
resource allocation increases aggregate output of capital goods and final goods. Given the constant labor input in the final good production, the rise in the input of capital goods increases the wage rate and reduces the price of capital good.

As shown later, if financial development is above a threshold value so that entrepreneurs are not credit constrained, project investment will be undertaken only by entrepreneurs and households keep all their savings in the form of loan to entrepreneurs instead of investment in own project. Given that financial development is below this threshold value, the project investment of households is positive, which is inefficient and can be approximately regarded as potential credit supply.

Financial development has a non-monotonic impact on the loan rate in the long run due to the interactions of the credit demand and the credit supply, as shown in the last panel of figure 1. On the one hand, the rise in $\theta$ enables entrepreneurs to borrow against a larger fraction of their future project revenue and the rise in the credit demand tends to push up the loan rate; on the other hand, the decrease in the price of capital makes the project investment less attractive for households and they prefer to save more in the form of lending to entrepreneurs rather than investing in own projects, and the rise in the credit supply tends to reduce the loan rate. For a small initial value of $\theta$, households still have a large project investment and the potential credit supply is relatively abundant.
For a marginal increase in $\theta$, the rise in the credit supply due to the decline in the price of capital dominates the rise in the credit demand due to the increase in $\theta$, and the loan rate falls. In contrast, for a large initial value of $\theta$, households have a small project investment and the potential credit supply is relatively scarce. For a marginal increase in $\theta$, the rise in the credit demand dominates the rise in the credit supply, and the loan rate falls.

Financial development also has a non-monotonic impact on the equity rate in the long run, as shown in the third panel of figure 1. We can decompose the equity rate by substituting $i^e_t = n_t + z_t$ into the definition of the equity rate and rewriting it as follows,

$$\Gamma_t \equiv \frac{Ri_t v_{t+1} - r_t z_t}{n_t} = Rv_{t+1} + \left( Rv_{t+1} - r_t \right) \frac{z_t}{n_t}. \quad (16)$$

Intuitively, for each unit of net worth invested in the project, the entrepreneur can obtain $Rv_t$ as the marginal revenue of his own funds. Additionally, he can get $\frac{n_t}{z_t}$ units of loan. After repaying the debt at the loan rate $r_t$, the entrepreneur can obtain the extra return of $(Rv_{t+1} - r_t) \frac{z_t}{n_t}$. Thus, the equity rate is affected by three factors: it rises in the debt-equity ratio and the price of capital but decreases in the loan rate.

**Lemma 1.** Let $\theta^U \equiv 1 - \eta$ denote the threshold value of financial development. For any $\theta \in [\theta^U, 1]$, economic allocation is independent of $\theta$ and efficient in the sense that entrepreneurs are not credit constrained, capital goods are produced only by entrepreneurs in the steady state, $i^h_t = 0$, and the loan rate is equal to the equity rate at $r = \Gamma = Rv$.

**Proof.** Let $\theta^U$ denote the threshold value where capital goods are only produced by entrepreneurs, $i^h_t = 0$, and the entrepreneur’s borrowing constraint (5) is binding. In this case, the loan rate is equal to the equity rate at the threshold, $r_t = v_{t+1}(R - i^h_t) = v_{t+1}R = \Gamma_t$. According to equation (4), the first-period consumption pattern of households is equal to that of entrepreneurs, $c^h_1 = \frac{w}{2} = c^e_1$. The credit market clearing implies $D = (1 - \eta)\frac{w}{2} = Z = \eta z$. Aggregate investment is only undertaken by the young entrepreneurs, $I = \frac{w}{2} = \eta^e$. Given per capita investment and borrowing of young entrepreneurs, $i^e = \frac{w}{2\eta}$ and $z = \frac{(1 - \eta)w}{2\eta}$, the binding borrowing constraint $r z = \theta^U R v^e$ implies $\frac{(1 - \eta)w}{2\eta} = \frac{\theta^U v^e}{2\eta}$, or $\theta^U = (1 - \eta)$. \hfill \Box

As $\theta$ rises from 0 to $\theta^U$, the debt-equity ratio increases and the price of capital declines monotonically. As shown above, the loan rate first declines but then rises. The net effect of financial development on the equity rate depends on the interactions of the three factors. For a small initial value of $\theta$, the increase in debt-equity ratio and the decline in the loan rate dominate the decrease in the price of capital so that the equity rate rises in $\theta$. For a relatively large initial value of $\theta$, the rise in the loan rate and the decrease in the price of capital dominate the rise in the debt-equity ratio so that the equity rate decreases in $\theta$. As $\theta$ rises further, the equity rate and the loan rate tend to converge. See the third
panel of figure 1 for the hump-shaped pattern of the equity rate. As shown in Lemma 1, when financial development is at its threshold value, \( \theta^U \), capital is produced only by entrepreneurs, \( i^h = 0 \), and the loan rate is equal to the equity rate, \( r = vR = \Gamma \). Any further increase in \( \theta \) does not affect allocation. Despite that the equity rate has the same form, \( \Gamma = vR \), in the case of domestic financial autarky and in the case of unconstrained equilibrium, the equity rate is lower in the latter case, due to the lower price of capital.

The second panel of figure 1 shows that social welfare defined as the weighted sum of households’ and entrepreneurs’ lifetime utility, \( U^s_t = \eta U^e_t + (1 - \eta) U^h_t \), increases in the degree of financial development. However, financial development may have unequal or opposite welfare implications to individual household and entrepreneur both in the long run and in the short run. See von Hagen and Zhang (2008) for detailed discussion on the welfare implications of financial development.

3 The Full Model: International Capital Flows

The non-monotonic patterns of the loan rate and the equity rate with respect to financial development under international financial autarky are essentially the driving forces of international capital flows in the two-country framework. This section considers two cases of international capital flows: free mobility of FDI (capital controls on financial capital) in the sense that entrepreneurs are allowed to move their project abroad but households are not allowed to lend abroad, and perfect capital mobility (no capital control) in the sense that individuals are allowed to lend or produce abroad. Technically speaking, there is the third case: free mobility of financial capital (capital controls on FDI) in the sense that individuals are only allowed to borrow or lend abroad but not shift their project and produce abroad. However, capital controls if any are normally imposed on financial capital but not on FDI. Therefore, we take international financial autarky (capital controls on financial capital and FDI) as the starting point and analyze the efficiency and welfare implications of free mobility of FDI and perfect capital mobility. Please see the detailed discussion about free mobility of financial capital in appendix A.

Since we focus on the implications of capital account liberalization to emerging market economies, country H is taken as an emerging economy with \( \theta^H = 0.3 \) and country F as a developed economy with \( \theta^F = 0.75 \). Endogenously, country F has a higher income due to a higher degree of financial development. Let \( \Upsilon_i^f \) and \( \Omega_i^f \) denote capital outflows from country \( i \) in the form of financial capital and FDI, respectively.
3.1 Free Mobility of FDI

The equilibrium conditions specifying the world economy under free mobility of FDI are almost same as under international financial autarky except for a few equations.

Let $\lambda^i_t \equiv \frac{1}{1 - \eta^i_t w^i_t - \Omega^i_t} \theta^i_t$ denote the investment-equity ratio in country $i$ in the case of binding borrowing constraint. The debt-to-equity ratio is $\lambda^i_t - 1$. Under free mobility of FDI, aggregate output of capital goods produced by entrepreneurs in country $i$, $R\lambda^i_t (\eta^t w^i_t - \Omega^i_t)$, and their aggregate credit demand, $(\lambda^i_t - 1)(\eta^w_t - \Omega^i_t)$, are both linear in aggregate net worth of entrepreneurs invested domestically, $(\eta^w_t - \Omega^i_t)$.

In equilibrium, FDI flows sum up to zero, the equity rate is equal across the border, the credit-market-clearing condition, i.e., equation (15), is reformulated as equation (18), and aggregate output of capital goods is calculated by equation (19),

$$\Omega^H_t + \Omega^F_t = 0, \quad \Gamma^H_t = \Gamma^F_t,$$

$$\Omega^H_t = \Omega^F_t,$$

$$K^i_t = R\lambda^i_t (\eta^i_t w^i_t - \Omega^i_t) + (1 - \eta) G(i^i_t),$$

$$K^i_t = R\lambda^i_t (\eta^i_t w^i_t - \Omega^i_t) + (1 - \eta) G(i^i_t).$$

(17)

(18)

(19)

Note that the outflows of FDI reduces domestic credit demand as shown by equation (18). Under free mobility of FDI, due to the cross-country difference in the wage income, entrepreneurs investing in country $i$ may come from country $i$ or from country $m$ and thus may have different net worth, where $i, m \in \{H, F\}$ and $i \neq m$. Entrepreneurs investing in country $i$ are subject to the same borrowing constraint, $\theta^i$, no matter where they come from. Due to the cross-country equalization of the equity rate, entrepreneurs born in the same country have the same wage income when young and thus the same consumption pattern as well as the lifetime utility, no matter where they produce.

3.1.1 Long-Run Effect of Free Mobility of FDI

We first analyze the steady-state allocation in the two-country model under free mobility of FDI. Figure 2 shows the patterns of the interest rates and capital outflows from country $H$, given $\theta^H = 0.3$ and $\theta^F \in [0, 1]$. The horizontal axis denotes $\theta^F \in [0, 1]$. Given $\theta^H = 0.3$ and $\theta^F = 0.75$, the equity rate is higher in country $H$ than in country $F$ under international financial autarky, as shown in figure 1. If allowed, FDI flows from country $F$ to country $H$ and the equity rate is equal across the border. See the second panel of figure 2. Note that the loan rate is different in the two countries due to controls on financial capital flows. See appendix B for a detailed description of the direction and size of FDI flows for the complete combinations of $\theta^H$ and $\theta^F$ under free mobility of FDI.

In order to evaluate the long-run efficiency and welfare implications, figure 3 shows the percentage differences of major economic variables under free mobility of FDI versus
under international financial autarky, given $\theta^H = 0.3$ and $\theta^F \in [0, 1]$. The horizontal axis denotes $\theta^F \in [0, 1]$. Given $\theta^H = 0.3$ and $\theta^F = 0.75$, the inflow of FDI raises aggregate output of capital goods and final goods in country H. As a result, the wage income of young generation rises.

Due to free mobility of FDI, the equity rate declines in country H mainly because more entrepreneurs are active in country H and the increase in aggregate output of capital reduces the price of capital and thus the equity rate. The opposite is true in country F. See the fifth and eighth panels of figure 3.

The inflow of FDI affects both credit demand and credit supply in country H. On the one hand, it raises credit demand because more entrepreneurs borrow from the credit market; on the other hand, the decline in the price of capital induces households to save more in the form of loan instead of investment in their own projects. Since $\theta^H$ is relatively low, the potential credit supply is abundant. As shown in the fifth panel of figure 3, the increase in the credit supply dominates the increase in the credit demand and the loan rate in country H declines in the long run due to free mobility of FDI. By the same logic, the loan rate rises in country F. Since the loan rate is higher in country F than in country H under international financial autarky, free mobility of FDI further enhances the cross-country difference in the loan rate, as shown in the first panel of figure 2.

From the welfare perspective, the increase in the wage income dominates the decline in the loan rate and the equity rate in country H. Thus, both households and entrepreneurs in country H are better off under free mobility of FDI than under international financial autarky. The rise in the equity rate dominates the decline in the wage income in country $F$ and entrepreneurs strictly benefit from free mobility of FDI. In contrast, the decline in the wage income dominates the rise in the loan rate in country $F$ and households in
country $F$ suffer from free mobility of FDI. The welfare results are more explicit in the next subsection on the dynamic analysis.

On the country level, social welfare is positively correlated with aggregate output, according to the second panel of figure 3. In other words, country $H$ benefits while country $F$ loses from free mobility of FDI, given $\theta^H = 0.3$ and $\theta^F = 0.75$. Thus, it is optimal for country $H$ (emerging economy) to allow free flow of FDI but impose controls on financial capital flows. However, in the case of a moderate $\theta^F$, e.g., $\theta^F = 0.5$, FDI flows from country $H$ to country $F$ and country $H$ loses. Thus, country $H$ may impose controls on FDI flow.

On the world level, given $\theta^H = 0.3$ and $\theta^F = 0.75$, world output $Y^W = Y^H + Y^F$ is higher but world welfare $U^w = U^H + U^F$ is lower under free mobility of FDI than under international financial autarky. In this sense, free mobility of FDI may be welfare-deteriorating for the world economy despite of output-enhancing. Note that this result may change under different parameter combinations of $\theta^H$ and $\theta^F$.

In sum, free mobility of FDI may have opposite long-run welfare implications on the individual level and on the country level.
3.1.2 Dynamic Effects of Free Mobility of FDI

Suppose that the world economy is at its long-run steady state under international financial autarky before period 0. From period 0 on, entrepreneurs are allowed to bring their projects and own funds to produce abroad. Figure 4 shows the impulse responses of relevant economic variables in the percentage point, given $\theta^H = 0.3$ and $\theta^F = 0.75$. Note that the vertical axis of the tenth panel entitled “Capital Flows” is in terms of levels instead of percentage change as there is no FDI flows under international financial autarky.

Given the model structure of overlapping generations, free mobility of FDI in period 0 do not affect the production and welfare of individuals born before period 0, even if the policy change is announced before period 0. In period 0, free mobility of FDI equalizes the equity rate in the two countries. Thus, the equity rate declines in country H and rises in country F. As mentioned in the subsection 3.1.1, FDI inflows affect both credit demand and credit supply in country H. Overall, the rise in the credit supply dominates the rise in the credit demand and the loan rate decreases in country H in period 0, as shown in the fifth panel of figure 4. By the same logic, the loan rate rises in country F.

In terms of the project investment, since households born in country H lend more domestically instead of investing in own projects, per capita project investment of households born in country H declines in period 0; since some entrepreneurs from country F...
bring their own funds and projects into country H, competition on the product market reduces the price of capital and thus, per capita project investment of entrepreneurs born in country H declines in period 0. Despite of a smaller project sizes of both households and entrepreneurs in country H, aggregate output of capital goods and final goods actually rises in period 1 in country H because more entrepreneurs produce in country H. Thus, the wage income of individuals born in period 1 in country H is higher than previously.

From the welfare perspective, given the predetermined wage income in period 0, both entrepreneurs and households born in period 0 in country H suffer from the decline in the equity rate and the loan rate, respectively. It takes two periods before the capital-accumulation effect is large enough so that the positive wage-income effect dominates the negative loan-rate and equity-rate effects. Thus, both households and entrepreneurs born from period 2 on are better off than their respective ancestors.

Entrepreneurs born in period 0 in country F benefit strictly from the rise in the equity rate, given the predetermined wage income in period 0. For their descendants, the positive equity-rate effect still dominates the negative wage-income effect and the entrepreneurs born later are still better off than those born before free mobility of FDI but at a decreasing magnitude than their ancestors. Given the predetermined wage income in period 0, households born in period 0 in country F benefit from the rise in the loan rate in period 0; while for their descendants, the negative wage-income effect dominates the positive loan-rate effect and households born later are worse off than their ancestors.

According to the second panel of figure 4, such opposite intergenerational welfare implications also exist on the country level.

3.2 Perfect Capital Mobility

The equilibrium conditions specifying the world economy under perfect capital mobility are almost same as under international financial autarky except for a few equations.

In equilibrium, cross-border flows of financial capital (FDI) sum up to zero, the loan rate (the equity rate) is equal across the border, the credit-market-clearing condition, i.e., equation (15), is reformulated as equation (21), and aggregate output of capital goods is calculated by equation (22),

$$\begin{align*}
\Upsilon^H_t + \Upsilon^F_t &= 0, \\
\Omega^H_t + \Omega^F_t &= 0, \\
\lambda^H_t = \lambda^F_t, \\
\Gamma^H_t = \Gamma^F_t, \\
\end{align*}$$

$$\begin{align*}
(\lambda^H_t - 1)(\eta \frac{u^i_t}{2} - \Omega^H_t) &= (1 - \eta)[w^i_t - (i^{i,h}_t + c^{i,h}_t)] - \Upsilon^H_t, \\
\end{align*}$$

$$\begin{align*}
K^i_t &= R\lambda^i_t(\eta \frac{u^i_t}{2} - \Omega^i_t) + (1 - \eta)G(i^{i,h}_{t-1}).
\end{align*}$$
3.2.1 Long-Run Effect of Perfect Capital Mobility

We first analyze the steady-state allocation in the two-country model under perfect capital mobility. Figure 5 shows the patterns of the interest rates and capital outflows from country H, given $\theta^H = 0.3$ and $\theta^F \in [0, 1]$. The horizontal axis denotes $\theta^F \in [0, 1]$. Given $\theta^H = 0.3$ and $\theta^F = 0.75$, the equity rate is equal but the loan rate is higher in country F than in country H under free mobility of FDI, as discussed in subsection 3.1.1 and shown in figure 2. If allowed, financial capital flows from country H to country F and the loan rate is then equal across the border. See the second panel of figure 5. This way, our model can explain the two-way capital flow. Note that in most cases, financial capital and FDI flow in the opposite direction. See appendix C for a detailed description of the direction and size of FDI and financial capital flows for the complete combinations of $\theta^H$ and $\theta^F$ under perfect capital mobility.

In order to evaluate the long-run efficiency and welfare implications, figure 7 shows the percentage differences of major economic variables under perfect capital mobility versus under free mobility of FDI, given $\theta^H = 0.3$ and $\theta^F \in [0, 1]$. The horizontal axis denotes $\theta^F \in [0, 1]$. Given $\theta^H = 0.3$ and $\theta^F = 0.75$, the outflow of financial capital reduces aggregate output of capital goods and final goods in country H. As a result, the wage income of young generation declines.

Given free mobility of FDI, allowing additionally free mobility of financial capital enables households born in country H to lend abroad which directly increases the loan rate in country H and reduces it in country F. See the fifth and eighth panels of figure 7.

Financial capital flows from country H to country F affects indirectly the equity rate in country F. On the one hand, the inflow of financial capital into country F reduces the
loan rate which tends to increase the equity rate in country F; on the other hand, the inflow of financial capital raises aggregate output of capital goods which tends to reduce the price of capital and the equity rate in country F. Overall, the second effect would dominate the first effect and the equity rate would decline in country F if the flow of FDI had not changed. The opposite would be true in country H due to the outflow of financial capital. However, due to free mobility of FDI, entrepreneurs born in country F further move their projects to country H for a higher equity rate. In equilibrium, the equity rate is higher in both countries under perfect capital mobility than under free mobility of FDI. See the fifth and eighth panels of figure 7. Thus, financial capital flow and FDI are complements instead of substitutes in the sense that allowing additionally free mobility of financial capital promotes cross-country FDI flows.

From the welfare perspective, the decline in the wage income dominates the rise in the equity rate in country H. Thus, entrepreneurs born in country H are worse off under perfect capital mobility than under free mobility of FDI. In contrast, the rise in the loan rate slightly dominates the decline in the wage income and households born in country H are slightly better off. The opposite is true for individuals born in country F. The welfare results will become more explicit in the next subsection on the dynamic analysis.

On the country level, country H loses while country F benefits from perfect capital mobility in comparison with free mobility of FDI, given $\theta^H = 0.3$ and $\theta^F = 0.75$. Thus, it
is optimal for country H (emerging economy) to allow free flow of FDI but keep controls on financial capital flows. However, in the case of a very high $\theta^F$, e.g., $\theta^F = 0.95$, country H also benefits and perfect capital mobility would be implemented.

On the world level, given $\theta^H = 0.3$ and $\theta^F = 0.75$, world output $Y^W = Y^H + Y^F$ and world welfare $U^W = U^H + U^F$ are higher under perfect capital mobility than under free mobility of FDI. In this sense, allowing additionally free mobility of financial capital is both output-enhancing and welfare-improving for the world economy.

In order to see the joint effects of free mobility of FDI and free mobility of financial capital, figure 7 shows the percentage differences of major economic variables under perfect capital mobility versus under international financial autarky, given $\theta^H = 0.3$ and $\theta^F \in [0, 1]$. The horizontal axis denotes $\theta^F \in [0, 1]$. Given $\theta^H = 0.3$ and $\theta^F = 0.75$, the net output effect of FDI and financial capital flows is positive for country F and negative for country H. However, if $\theta^F$ is very high, e.g., $\theta^F = 0.9$, the net output effect is positive for both countries.

In comparison with international financial autarky, perfect capital mobility results in the cross-country equalization of the loan rate and the equity rate, respectively, and its opposite long-run welfare implications of perfect capital mobility to households and entrepreneurs exist in the two countries, respectively.

In sum, financial capital and FDI are complements instead of substitutes and allowing
3.2.2 Dynamic Effects of Perfect Capital Mobility

Suppose that the world economy is at its long-run steady state under free mobility of FDI before period 0. From period 0 on, financial capital is additionally allowed to flow freely across the border. Figure 8 shows the impulse responses of relevant economic variables in percentage points, given $θ^H = 0.3$ and $θ^F = 0.75$. Note that the vertical axis of the tenth panel entitled “Capital Flows” is in terms of levels instead of percentage change.

Given the model structure of overlapping generations, allowing additionally free mobility of financial capital in period 0 do not affect the production and welfare of individuals born before period 0, even if the policy change is announced before period 0. In period 0, free mobility of financial capital equalizes the loan rate in the two countries. Thus, the loan rate rises in country H and declines in country F. As mentioned in the subsection 3.2.1, additional free mobility of financial capital affects aggregate production and the price of capital in both countries. In equilibrium, the equity rate rises in the two countries in period 0. See the fifth and eighth panel of figure 8.

As mentioned in the previous subsection, the outflow financial capital promotes the
inflow of FDI in country H. Despite of the increase in FDI inflow, additional free mobility of financial capital makes country H from a net capital importer under free mobility of FDI into a net capital exporter under perfect capital mobility in the sense that financial capital outflow exceeds FDI inflow. See the tenth panel of figure 8.

From the welfare perspective, given the predetermined wage income in period 0, both entrepreneurs and households born in period 0 in country H benefit from the rise in the equity rate and the loan rate, respectively. However, the decline in aggregate output reduces the wage income in period 1 which dominates the rise in the equity rate and the loan rate. Thus, entrepreneurs and households born in period 1 in country H are both worse off than their ancestors before period 0. Given the predetermined wage income in period 0, entrepreneurs born in period 0 in country F benefit from the rise in the equity rate while households suffer from the decline in the loan rate. Due to the rise in aggregate output, individuals born in period 1 in country H benefit from a higher wage income. Entrepreneurs are even better off than their ancestors, while households are better off than those born in period 0 but still worse off than those born before the policy change.

According to the second panel of figure 8, such opposite intergenerational welfare implications also exist on the country level.

We also consider the case of moving from international financial autarky to perfect capital mobility in period 0. Suppose that the world economy is at its long-run steady
state under international financial autarky before period 0. From period 0 on, both financial capital and FDI are allowed to flow freely across the border. Figure 9 shows the impulse responses of relevant economic variables in the percentage point, given $\theta^H = 0.3$ and $\theta^F = 0.75$. Note that the vertical axis of the tenth panel entitled “Capital Flows” is in terms of levels instead of percentage change.

Similar as discussed above, perfect capital mobility has unequal intergenerational welfare implications. For example, entrepreneurs born in period 0 in country H are slightly worse off than their ancestors due to the decline in the equity rate, while entrepreneurs born later in country H are quite worse off than their ancestors due to the endogenous decline in the wage income.

In sum, given $\theta^H = 0.3$ and $\theta^F = 0.75$, various capital controls policies normally have opposite or uneven welfare implications in the intergenerational dimension. Contrary to the prediction of the standard economic theory, capital flows from the poor country (country H) to the rich country (country F) in the net term. This way, our model explains the “uphill” puzzle.

### 3.3 Three-Country Model

This subsection shows the long-run patterns of capital flows in a three-country OLG model. The three countries are identical except their respective degree of financial development. Let $i \in \{L, M, H\}$ denote countries with low, middle, and high degree of financial development, respectively, i.e., $0 < \theta^L < \theta^M < \theta^H < 1$. Let $\Upsilon^i_t$ and $\Omega^i_t$ denote capital outflows from country $i$ in the form of financial capital and FDI, respectively. We consider the case of perfect capital mobility where the loan rate is same across countries and so is the equity rate. Given $\theta^L = 0.1$, $\theta^M \in [0.1, 0.8]$, and $\theta^H = 0.8$, figure 10 shows the patterns of international capital flows as well as their welfare implications in three countries. Note that the vertical axes of the three panels on the first row are in terms of levels while those of the rest panels are in terms of the percentage difference of relevant variables under perfect capital mobility versus under international financial autarky.

Take the case of $\theta^M = 0.4$ as an example. Financial capital flows from country M to country H and country L, while FDI flows from country H and country L to country M. This way, cross-country difference in financial development may explain the fact that FDI does not flow to the poorest country but more to the middle-income country. The dotted lines in the three panels of the first row show the net capital flows in the three countries, respectively. Similar as mentioned in the previous subsection, the country with more developed financial sector (country H) witnesses net capital inflows while countries with less developed financial sector (M and L) experience net capital outflow.

Perfect capital mobility raises aggregate output in country H and M but reduces ag-
Figure 10: Long-Run Effects of Perfect Capital Mobility: A Three-Country Model

ggregate output in country L. Social welfare in the three countries is positively correlated with aggregate output because the wage income dominates other factors in the long run on the country level. It may explain the fact why developed economies prefer to promote perfect capital mobility while developing economies prefer to impose controls on capital flows. Note that entrepreneurs born in the country with highest degree of financial development are those who benefit in most cases from perfect capital mobility.

4 Conclusion

We develop a two-country overlapping-generations model with financial frictions and show that cross-country difference in financial development can explain two empirical puzzles in the literature, i.e., the “two-way capital flow” puzzle and the “uphill” puzzle.

Financial development has non-monotonic effects on the loan rate and the equity rate in a closed economy. Cross-country difference in financial development endogenously generates cross-country difference in the loan rate as well as cross-country difference in the equity rate. It then leads to financial capital and FDI flows depending on the exact capital controls policy. We show that financial capital and FDI normally flow in the opposite direction and they are complements instead of substitutes. In the net term, the poor country exports capital to the rich country, which is contrary to the neoclassical
economic theory. In a simple extension of the three-country OLG model, we find that FDI flows to the middle income country while financial capital mainly flows to the rich country. Both the middle income country and the rich country benefit while the poor country suffers from perfect capital mobility in the long run.

We also find that international capital flows may have opposite welfare implications to different individuals within and across generations. Thus, the next question will be whether and how the inter- or intra-generational transfer can be applied to achieve Pareto improvement during the process of capital account liberalization.

References


A Free Mobility of Financial Capital

The equilibrium conditions specifying the world economy under free mobility of financial capital are almost same as under international financial autarky except for a few equations.

Under free mobility of financial capital, aggregate output of capital goods produced by entrepreneurs in country $i$, $R\lambda^i_t\eta^{w_i}_t$, and their aggregate credit demand, $(\lambda^i_t - 1)\eta^{w_i}_t$, are both linear in aggregate net worth of entrepreneurs invested domestically, $\eta^{w_i}_t$.

In equilibrium, financial capital flows sum up to zero, the loan rate is equal across the border, and the credit-market-clearing condition, i.e., equation (15), is reformulated as equation (24),

$$\Upsilon^H_t + \Upsilon^F_t = 0,$$  

$$r^H_t = r^F_t,$$  

$$2(\lambda^i_t - 1)\eta^{w_i}_t = (1 - \eta)[w^i_t - (i^{i,h}_t + c^{i,h}_1t)] - \Upsilon^i_t.$$  

According to equation (24), the outflows of financial capital reduces domestic credit supply.

A.1 Long-Run Effect of Free Mobility of Financial Capital

We first analyze the steady-state allocation in the two-country model under free mobility of financial capital. Figure 11 shows the patterns of the interest rates and capital outflows from country H, given $\theta^H = 0.3$ and $\theta^F \in [0, 1]$. The horizontal axis denotes $\theta^F \in [0, 1]$. Given $\theta^H = 0.3$ and $\theta^F = 0.75$, the loan rate is higher in country F than in country H under international financial autarky, as shown in figure 1. If allowed, financial capital flows from country H to country F and the loan rate is equal across the border. See the second panel of figure 11. Note that the equity rate is different in the two countries due to controls on equity capital flows (FDI).

Figure 12 shows some threshold values, where the horizontal and vertical axes denote the degrees of financial development in country $H$ and in country $F$, $\theta^i \in [0, 1]$, respectively.

If the two countries have the same degree of financial development, i.e., the parameter combinations are on the 45 degree line, the allocation in the two countries are exactly same and there is no capital flows even if allowed, $\Upsilon^H_t = \Upsilon^F_t = 0$.

For the parameter combination in region A, both $\theta^H$ and $\theta^F$ are larger than $\theta^U \equiv 1 - \eta$ as defined in Lemma 1. According to figure 1, production is efficient in both countries under international financial autarky in the sense that the marginal revenue of the projects of households and entrepreneurs are equal, $vG^i(j^i) = vR$, and capital goods are produced only by entrepreneurs, $i^{i,h} = 0$. Economic allocation is identical in the two countries, especially $r^H = r^F$.

As a result, there is no international financial capital flows even if allowed.

The curve splitting region $B$ and $E$ represents a set of threshold values $\theta^F_{FC,U} \in (1 - \eta, 1)$ as a function of $\theta^H$. Be specific, given $\theta^H \in [0, 1 - \eta)$, production is efficient and the equity rate is equal to the loan rate in country F for any $\theta^F \in [\theta^F_{FC,U}, 1]$. Any further increase in $\theta^F$ does not affect the economic allocation and capital flows any more. Similarly, the curve splitting region $B'$ and $E'$ represents a set of threshold values $\theta^H_{FC,U} \in (1 - \eta, 1)$ as the function of $\theta^F \in [0, 1 - \eta)$.

The curve splitting region $E$ and $J$ represents a set of threshold values $\theta^FC,0$ as a function of $\theta^H$. Be specific, given $\theta^H$ and $\theta^F = \theta^FC,0$, the loan rate is equal in the two countries under
international financial autarky and thus, there is no international financial capital flows even if allowed. The intuition has been explained in subsection 2.5 and is straightforward from the last panel of figure 1. Similarly, the curve splitting region $E'$ and $J'$ represents a set of threshold values $\theta_{FC}^H$ as a function of $\theta_F$.

Table 1 summarizes the direction and size of financial capital flows from country H to country F in the seven regions. Financial capital flows from country F is simply the opposite, $\Upsilon_F = -\Upsilon_H$.

Table 1: Direction and Size of Financial Capital Flows from Country H to Country F

<table>
<thead>
<tr>
<th>Region</th>
<th>$A$</th>
<th>$B$</th>
<th>$B'$</th>
<th>$E$</th>
<th>$E'$</th>
<th>$J$</th>
<th>$J'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon_H$</td>
<td>0</td>
<td>$\Upsilon_H(\theta_H) &gt; 0$</td>
<td>$\Upsilon_H(\theta_F) &lt; 0$</td>
<td>$(0, \Upsilon_H(\theta_H))$</td>
<td>$(\Upsilon_H(\theta_F), 0)$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

where $\Upsilon_i(\theta_i)$ implies that given the parameter combination in region $B$ and $B'$, financial capital flows depends only on $\theta_i$ not on $\theta_m$, where $i, m \in \{H, F\}$ and $i \neq m$.

In order to evaluate the long-run efficiency and welfare implications of free mobility of financial capital, figure 13 shows the percentage differences of major economic variables under free mobility of financial capital versus under international financial autarky, given $\theta_H = 0.3$ and $\theta_F \in [0, 1]$. The horizontal axis denotes $\theta_F \in [0, 1]$. Given $\theta_H = 0.3$ and $\theta_F = 0.75$, the outflow of financial capital reduces aggregate output of capital goods and final goods in country H. As a result, the wage income of young generation declines.
Due to free mobility of financial capital, the loan rate rises in country H mainly because households born in country H lend abroad and the decrease in aggregate output of capital increases the price of capital and thus the equity rate. However, the opposite is true in country F. See the fifth and eighth panels of figure 13.

The outflow of financial capital affects the equity rate through the price-of-capital effect and the loan-rate effect, according to equation (16). As shown in the fifth panel of figure 13, the price-of-capital effect dominates the loan-rate effect and the equity rate rises in country H. By the same logic, the equity rate declines in country F. Since the equity rate is higher in country F than in country H under international financial autarky, free mobility of financial capital further enhances the cross-country difference in the equity rate, as shown in the first panel of figure 11.

From the welfare perspective, the decline in the wage income dominates the rise in the loan rate and the equity rate in country H. Thus, both households and entrepreneurs in country H are worse off under free mobility of financial capital than under international financial autarky. The rise in the wage income dominates the decline in the equity rate in country F and entrepreneurs strictly benefit from free mobility of financial capital, while the decline in the loan rate slightly dominates the rise in the wage income in country F and households suffer from free mobility of FC. The welfare results are more explicit in the next subsection on the dynamic analysis.

On the country level, social welfare is positively correlated with aggregate output. See the second panel of figure 3. In other words, country H suffers while country F benefits from free mobility of financial capital, given $\theta^H = 0.3$ and $\theta^F = 0.75$. Thus, it is optimal for country H
On the world level, given $\theta_H = 0.3$ and $\theta_F = 0.75$, world output $Y^W = Y^H + Y^F$ and world welfare $U^w = U^H + U^F$ are both higher under free mobility of financial capital than under international financial autarky. In this sense, free mobility of financial capital promotes both world output and world welfare.

In sum, free mobility of financial capital may have opposite long-run welfare implications on the individual level and on the country level, similar as free mobility of FDI.

### A.2 Dynamic Effects of Free Mobility of Financial Capital

Suppose that the world economy is at its long-run steady state under international financial autarky before period 0. From period 0 on, individuals are allowed to borrow or lend abroad. Figure 14 shows the impulse responses of relevant economic variables in the percentage point, given $\theta_H = 0.3$ and $\theta_F = 0.75$. Note that the vertical axis of the tenth panel entitled “Capital Flows” is in terms of levels instead of percentage change because there is no financial capital flows under international financial autarky.

Given the model structure of overlapping generations, free mobility of financial capital in period 0 do not affect the production and welfare of individuals born before period 0, even if the policy change is announced before period 0. In period 0, free mobility of financial capital equalizes the loan rate in the two countries in the sense that the loan rate rises in country H.
Figure 14: The Dynamics of Free Mobility of Financial Capital from Period 0 on

and declines in country F. As mentioned in the subsection A.1, financial capital outflows affect both the price of capital and the loan rate. Overall, the rise in the price of capital dominates the rise in the loan rate and the equity rate rises in country H in period 0. See the fifth panel of figure 14. By the same logic, the equity rate declines in country F.

In terms of the project investment, since households born in country H lend abroad, they reduce their own project investment in period 0. The outflow of financial capital reduces domestic credit supply and pushes up the loan rate in country H. Entrepreneurs have to reduce their project investment in country H. Due to lower project investment, aggregate output declines in period 1 in country H. The decline in the wage income in period 1 reduces the net worth of the young entrepreneurs born in period 1 and they reduce their project investment to a larger scale than the young entrepreneurs born in period 0.

From the welfare perspective, given the predetermined wage income in period 0, both entrepreneurs and households born in period 0 in country H benefit from the rise in the equity rate and the loan rate, respectively. From period 1 on, the decline in the wage income is large enough so that the negative wage-income effect dominates the positive loan-rate and equity-rate effects. Thus, both households and entrepreneurs born from period 1 on are worse off than their respective ancestors even before period 0.

Entrepreneurs and households born in period 0 in country F suffer strictly from the declines in the equity rate and the loan rate, respectively, given the predetermined wage income in period 0. For their descendants, the positive wage-income effect dominates the negative equity-rate and
loan-rate effects, respectively, and the individuals born later are better off than those born before period 0.

According to the second panel of figure 14, such opposite intergenerational welfare implications also exist on the country level and on the world level.

**B Free Mobility of FDI**

Figure 15 shows some threshold values, where the horizontal and vertical axes denote the degrees of financial development in country $H$ and in country $F$, $\theta^i \in [0, 1]$, respectively.

![Figure 15: Long-Run Effects of Free mobility of FDI: Threshold Values](image)

By the similar logic as in subsection A.1, for the parameter combination in region A as well as on the 45 degree line, the equity rate is same in the two countries under international financial autarky and there is no FDI flows across the border even if allowed, $\Omega^H = \Omega^F = 0$.

The curve splitting region $B$ and $E$ represents a set of threshold values $\theta^{FDI,U}_F \in (1 - \eta, 1)$ as a function of $\theta^H$. Be specific, given $\theta^H \in [0, 1 - \eta)$, production is efficient and the equity rate is equal to the loan rate in country $F$ for any $\theta^F \in [\theta^{FDI,U}_F, 1]$. Any further increase in $\theta^F$ does not affect the economic allocation and capital flows any more. Similarly, the curve splitting region $B'$ and $E'$ represents a set of threshold values $\theta^{FDI,U}_H \in (1 - \eta, 1)$ as the function of $\theta^F \in [0, 1 - \eta)$.

The curve splitting region $E$ and $J$ represents a set of threshold values $\theta^{FDI,0}_F$ as a function of $\theta^H$. Be specific, given $\theta^H$ and $\theta^F = \theta^{FDI,0}_F \neq \theta^H$, the equity rate is equal in the two
countries under international financial autarky and thus, there is no FDI flows even if allowed. The intuition has been explained in subsection 2.5 and is straightforward from the fifth panel of figure 1. Similarly, the curve splitting region $E'$ and $J'$ represents a set of threshold values $\theta_F^{FDI,0}$ as a function of $\theta^F$. Note that $\theta^*$ refers to the degree of financial development corresponding to the highest equity rate under international financial autarky.

Table 2 summarizes the direction and size of FDI flows from country H to country F in the seven regions. FDI flows out of country F is simply the opposite, $\Omega = -\Omega_H$.

Table 2: Direction and Size of FDI Flows from Country H to Country F

<table>
<thead>
<tr>
<th>Region</th>
<th>A</th>
<th>B</th>
<th>B'</th>
<th>E</th>
<th>E'</th>
<th>J</th>
<th>J'</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_H$</td>
<td>0</td>
<td>$\Omega^H(\theta^H) &lt; 0$</td>
<td>$\Omega^H(\theta^F) &gt; 0$</td>
<td>$(\Omega^H(\theta^H),0)$</td>
<td>$(0,\Omega^H(\theta^F))$</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

where $\Omega^H(\theta^H)$ implies that given the parameter combination in region $B$ and $B'$, FDI flows depends only on $\theta^i$ not on $\theta^m$, where $i, m \in \{H,F\}$ and $i \neq m$.

C  Perfect Capital Mobility

Figure 16 shows some threshold values, where the horizontal and vertical axes denote the degrees of financial development in country $H$ and in country $F$, $\theta^i \in [0,1]$, respectively.

By the similar logic in subsection A.1, for the parameter combination in region A as well as on the 45 degree line, there is no capital flows across the border, $\Omega^H = \Omega^F = 0$ and $\Upsilon^H = \Upsilon^F$.

The curve splitting region $B$ and $E$ represents the relationship between $\theta^H$ and $\theta^F$,

$$\theta^H + \theta^F = 2(1 - \eta).$$

(25)

Be specific, given $\theta^H \in [1 - 2\eta, 1 - \eta]$, economic allocation is identical and first-best in the two countries in the sense that capital goods are produced only by entrepreneurs for any $\theta^F \in [2(1 - \eta) - \theta^H, 1]$. Any further increase in $\theta^F$ does not affect the economic allocation and capital flows any more. Similarly, the curve splitting region $B'$ and $E'$ also represents the relationship between $\theta^H$ and $\theta^F$ as specified in equations (25).

Region $J$ ($J'$) is the belt region between region $E$ and $M$ ($E'$ and $M'$). The curve splitting region $E$ and $J$ represents a set of threshold values $\theta_F^{FC,0}$ as a function of $\theta^H$. Be specific, given $\theta^H$ and $\theta^F = \theta_F^{FC,0} \neq \theta^H$, the equity rate is equal in the two countries under free mobility of financial capital and thus, there is no FDI flows even if additionally allowed.

The curve splitting region $J$ and $M$ represents a set of threshold values $\theta_F^{FDI,0}$ as a function of $\theta^H$. Be specific, given $\theta^H$ and $\theta^F = \theta_F^{FDI,0} \neq \theta^H$, the loan rate is equal in the two countries under free mobility of FDI and thus, there is no financial capital flows even if additionally allowed.

Similarly, the curve splitting region $E'$ and $J'$ ($J'$ and $M'$) represents a set of threshold values $\theta_H^{FDI,0}$ ($\theta_H^{FC,0}$) as a function of $\theta^F$. 

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Table 3 summarizes the direction of financial capital and FDI flows from country H to country F in the nine regions. The outflow of financial capital and FDI from country F is simply the opposite, i.e., $\Upsilon^F = \Upsilon^H$, $\Omega^F = \Omega^H$.

where $\Upsilon^H_i(\theta^i)$ and $\Omega^H_i(\theta^i)$ imply that given the parameter combination in region $B$ and $B'$, financial capital and FDI flows depend only on $\theta^i$ not on $\theta^m$, where $i, m \in \{H, F\}$ and $i \neq m$. 