Financial Development and International Capital Flows

Jürgen von Hagen* and Haiping Zhang†

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Abstract

We develop a general equilibrium model with financial frictions in which equity and credit have different rates of return. Financial development raises the loan rate but has a non-monotonic effect on the equity return.

We then show in a two-country model that capital account liberalization leads to outflow of financial capital from the country with less developed financial system. However, the direction of foreign direct investment (FDI, henceforth) depends on the exact degrees of financial development in the two countries as well as the specific capital controls policy.

Our model helps explain the Lucas Paradox (Lucas, 1990). Countries with least developed financial system have the outflows of both financial capital and FDI; countries with most developed financial system witness two-way capital flows, i.e., the inflow of financial capital and the outflow of FDI; countries with intermediate level of financial development have the outflow of financial capital and the inflow of FDI. It is consistent with the fact that FDI flows not to the poorest countries but to the middle-income countries.

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Keywords: Capital controls, Financial frictions, Foreign direct investment

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*University of Bonn, Indiana University and CEPR. Lennestrasse. 37, D-53113 Bonn, Germany. E-mail: vonhagen@uni-bonn.de
†Corresponding author. School of Economics, Singapore Management University. 90 Stamford Road, Singapore 178903. E-mail: hpzhang@smu.edu.sg
1 Introduction

We address two questions on international capital flows in a general equilibrium model with financial frictions. How does domestic financial development affect the respective returns on internal and external capital? How does the difference in the domestic financial development of two countries affect the direction and the magnitude of international flows of financial capital and foreign direct investment (FDI, henceforth) under different capital controls?

According to the classic trade theory, the cross-country difference in factor endowment result in the relative price differential which then gives rise to international trade. The degree of financial development can also be considered as an endowment for an economy as a whole in the short run.\(^1\) Our paper shows that financial development determines the rates of return on external capital (loan) and internal capital (equity). The cross-country differences in the rates of return on two types of capital give rise to capital flows in two forms: financial capital and FDI.

Most of the theoretical papers in the literature on international finance analyze international borrowing and lending in the form of financial capital. Lucas (1990) raises the paradox that too less capital flows into developing economies. He argues that imperfect international capital market (political risk) may explain such a paradox. Caballero and Krishnamurthy (2001, 2003) investigate the dynamic interactions between domestic and international collateral constraints and show that limited financial development reduces the incentives for foreign lenders to enter emerging markets. Aoki, Benigno, and Kiyotaki (2006) investigate how the adjustment to capital liberalization depends upon the domestic and international collateral constraints. von Hagen and Zhang (2006, 2007) consider the efficiency and welfare implications of capital inflows. These papers use a small open economy model and financial capital flows are driven by the interest rate differential between domestic and foreign funds.

FDI is usually analyzed from the perspective of industrial organization, and multinational firms stay in the center of the analysis. According to Blomström and Kokko (2003), there has been a strong consensus in the literature about why multinationals invest in specific locations (Dunning, 1993; Globerman and Shapiro, 1999). Multinational corporations (MNCs) are mainly attracted by strong economic fundamentals in the host economies. The most important of these are market size and the level of real income, with skill levels in the host economy, the availability of infrastructure and other resources that facilitate efficient specialization of production, trade policies, and political and macroe-

\(^1\)Porta, de Silanes, Shleifer, and Vishny (1998) and Simeon Djankov and Shleifer (2006) show that the efficiency of debt enforcement is strongly correlated with per capita income and legal origin and predicts debt market development across countries.
Our paper focuses on the financial motives of FDI and analyzes the joint determination of cross-border flows of both financial capital and FDI in a general equilibrium model. We are not the first to analyze the composition of international capital flows from the perspective of financial development. Ju and Wei (2006) show that the cross-country difference in financial development leads to a unique equilibrium of the outflow of financial capital from and the inflow of FDI to the country with less developed financial system. They argue that their model can explain the fact that net capital flows to less developed economies are too small. Antras and Caballero (2007) focus on the relationship between trade and capital flows. They show that in a world with heterogeneous financial development, trade and capital mobility are complements in less financially developed economies. In a dynamic framework, the complementarity carries over to financial capital flows. Such an interaction implies that deepening trade integration in developing economies raises net capital inflows. Antras and Caballero (2007) only analyze financial capital flows without explicit analysis on FDI.

Normally, a unit of capital has different rates of return in the hand of different persons. Those who have profitable projects become entrepreneurs. If their project has a higher rate of return than the loan rate, entrepreneurs prefer to borrow as much as possible. However, due to moral hazard problem, they cannot fully pledge the project outcomes to external financiers. As a result, they are subject to borrowing constraints and have to put own funds in the projects. The binding borrowing constraints for individual firms has a general equilibrium effect on the loan rate, i.e., the insufficient effective credit demand keeps the loan rate lower than the rate of return to equity (internal capital). In comparison with external financier, entrepreneurs have the privilege owning and controlling the production project which essentially gives rise to the spread between the rates of return on equity and loan.

The strictness of the borrowing constraints depends not only on the characteristics of the individual projects, but also on financial development in the economy. The same project has a larger external value in the economy with better protection of creditors, more efficient legal system, and more liquid asset market.

Intuitively, in the countries with more developed financial system, entrepreneurs can pledge a larger fraction of their project outcomes for loans and thus, they can get more external funds. In the case of international financial autarky, the loan rate is higher because domestic saving is scarcer in comparison with the effective credit demand on the aggregate level. So is the deposit rate. In this sense, financial development has an unambiguous positive effect on the loan rate and the deposit rate.

However, financial development may have non-monotonic effect on the rate of return on equity. First, it enables entrepreneurs to borrow more loans and expand their project
investment. This way, financial development has a positive “scale” effect on equity return. Second, it has a general equilibrium effect: financial development leads to an increasing demand for credit and then the rise in the loan rate. This way, financial development has a negative “cost” effect on equity return. As our first contribution, we show that the rate of return on equity has a hump-shaped pattern with respect to financial development: it initially rises and then declines in financial development.

We then analyze two types of capital flows in a general equilibrium two-country model. The two countries differ in financial development. In the case of international financial autarky, the rate of return on external capital (the loan rate) is surely higher in the country with more developed financial system, while it may not be true for the rate of return on internal capital, given its hump-shaped pattern to financial development.

Capital account liberalization leads to international factor movement: the outflow of financial capital (bank loans) from the country with less developed financial system and FDI into the country with the higher equity return. As our second contribution, we show that the direction of FDI is not straightforward and depends on the exact levels of financial development in the two countries and specific capital controls policy. Note that FDI is involved not only the flow of capital but, more importantly, the movement of profitable projects. The redistribution of entrepreneurs in the two countries has asymmetric competition effect on two credit markets and the output gains in the country with FDI inflow may not fully compensate the output loss in the country with FDI outflow. Surprisingly, the world output may be lower in the case of free mobility of FDI than in the case of international financial autarky. In this sense, given financial frictions, production efficiency may be worsened with free movement of productive factors.

As our third contribution, we show that if the public regulator in the country with less developed financial system lifts controls on FDI only, there may be FDI inflow. However, additionally lifting controls on financial capital results in FDI outflows. In this sense, the sequence of different deregulation policy may matter for the direction of capital flows as well as aggregate domestic production.

The inflow of financial capital affects the credit supply while the inflow of equity capital affects the credit demand. In this sense, different forms of capital inflows can have opposite effects on the loan rate. Therefore, as our fourth contribution, we show that lifting capital controls on equity capital and financial capital have opposite production and welfare effect on both micro- and macro-level. In this sense, more attention should be paid to the motives and effects of different types of capital flows on the micro level instead of to aggregate capital flows on the macro level only.

Essentially, what matters in our model for the direction and the size of different types of capital flows is not aggregate capital stock in the economy but who owns the capital stock and how much. The ownership of capital stock matters not only for aggregate out-
put but also the respective rates of return on different forms of capital. It is financial underdevelopment that leads to the lack of the effective demand for credit and depresses the domestic loan rate in the case of international financial autarky. Improving the domestic financial system can affect the amount and direction of international capital flows but it is a long-run issue.

The rest of this paper is organized as follows. Section 2 sets up the basic model of a closed economy and analyzes how financial development affects the rates of return on internal and external capital. Section 3 extends the basic model into a two-country setting and discusses the welfare implications of international flows of two types of capital. Section 4 concludes with some final remarks.

2 The Closed-Economy Model

Consider a two-period closed economy with two types of agents, entrepreneurs and households, with mass \( \eta \) and \((1 - \eta)\), respectively. Each agent is endowed with one unit of consumption goods in period 1. It takes one period for them to produce using their respective projects. The consumption good is chosen as the numeraire. There is no aggregate uncertainty in the model economy. Households and entrepreneurs have linear preference over consumption in two periods,

\[
U = c_1 + c_2.
\]

In equilibrium, the marginal product of the entrepreneurs’ project is larger than that of the households’ project. Thus, entrepreneurs prefer to borrow from households. However, households do not have the relevant monitoring technology and financial intermediaries (banks) emerge as delegated monitors (Diamond, 1984). In equilibrium, banks collect deposits from households and lend to entrepreneurs. The efficiency of domestic legal and financial system determines the fraction of the future project outcome that entrepreneurs can pledge to banks for external financing. In other words, entrepreneurs are subject to borrowing constraints.

Households invest \( i \) units of goods in their project and deposit \( d \) units of goods at the banks in period 1. In period 2, the project of households yields \( G(i) = Ri - 0.5i^2 \) units of goods, where \( R \geq 2 \), and the gross return on their deposits is \( rd \). The production function of households is chosen in such a way that the loan rate is strictly larger than unity in equilibrium. As a result, both households and entrepreneurs prefer to postpone consumption to period 2. Households maximize their period-2 consumption subject to the budget constraints,

\[
\max_{i,d} \quad c = G(i) + rd, \quad (1)
\]
\[
i + d = 1. \quad (2)
\]
In equilibrium, the marginal product of their project is equal to the gross deposit rate,

\[ G'(i) = R - i = r. \]  

(3)

Entrepreneurs have linear production project, \( y^e = R^e i^e \), which implies that their project is more productive than that of households in the case of autarky. They finance the project investment \( i^e \) using their endowment and bank loans \( i^e = 1 + z \) in period 1. The loan rate is also defined as the rate of return on external capital. \( R^e \) units of goods are produced in period 2. After repaying \( rz \) to banks, entrepreneurs consume \( c^e \) in period 2. Let the rate of return on internal capital or the equity rate denote the rate of return on the entrepreneurs’ endowment,

\[ \Gamma \equiv R^e - r(i^e - 1). \]  

(4)

In equilibrium, the rate of return on internal capital is no less than that on external capital, \( \Gamma \geq r \); otherwise, entrepreneurs would deposit at the banks instead of borrow from the banks. Entrepreneurs maximize their period-2 consumption subject to the period-2 budget constraint, borrowing constraints (6) and participation constraints (7):

\[
\begin{align*}
& \max_{i^e} c^e = R^e - r(i^e - 1) \\
& r(i^e - 1) \leq \theta R^e, \\
& \Gamma \geq r \Rightarrow r = R - i \leq R.
\end{align*}
\]  

(5, 6, 7)

Following Matsuyama (2004, 2007, 2008), we use \( \theta \in [0,1] \) to measure the degree of financial development in the economy. \( \theta \) is higher in the country with more sophisticated financial and legal system, better creditor protection, and etc.\(^2\) Note that only one of the two constraints (6) and (7) is strictly binding in equilibrium.

Markets for consumption goods and credit clear in two periods,

\[
\begin{align*}
\eta z &= (1 - \eta)d, \\
\eta i^e + (1 - \eta)i &= 1, \\
\eta c^e + (1 - \eta)c &= \eta R^e + (1 - \eta)(R^e - 0.5 i^2).
\end{align*}
\]  

(8, 9, 10)

**Definition 1.** Market equilibrium is a set of allocations of households, \( \{i,c,d\} \), entrepreneurs, \( \{i^e,c^e,z\} \), together with the rates of return on external capital and internal capital, \( \{r,\Gamma\} \), satisfying equations (1)-(9).

\(^2\)The pledgeability, \( \theta \), can be argued in various forms of agency costs story, e.g., the inalienability of human capital of entrepreneurs by Hart and Moore (1994) or costly state verification by Townsend (1979), or unobservable project (effort) choices by Holmstrom and Tirole (1997). See Tirole (2006) for a comprehensive overview of different models of financial contracting. Here, we focus more on the implications of legal and financial development on the borrowing constraints of individuals. What matters here is the borrowing constraints that restrict the project investment of the more productive agents. As a result, we choose the simplest form of borrowing constraints.
Lemma 1. There exists $\theta^U \equiv (1-\eta)$ such that for any $\theta \in [\theta^U, 1]$, the loan rate is constant at $r = R$ and the project investments of households and entrepreneurs are constant at $i = 0$ and $i^e = \frac{1}{\eta}$, respectively. For $\theta \in (0, \theta^U)$, the loan rate rises monotonically in $\theta$; there exists $\theta^*$ such that the equity rate $\Gamma$ reaches its global maximum at $\theta = \theta^*$.

In the case of $\theta = 0$, entrepreneurs cannot borrow and they invest all endowment into their project. The equity rate is simply the marginal product of their project, $\Gamma = R$. For $\theta \in (\theta^U, 1]$, the borrowing constraints of entrepreneurs are slack because the equity rate is equal to the loan rate and the marginal product of their project, $\Gamma = r = R$. Thus, economic allocation is unaffected by further increase in $\theta$.

For $\theta \in (0, \theta^U]$, the equity rate of entrepreneurs exceeds the loan rate, $\Gamma(\theta) > r$. Entrepreneurs prefer to invest all their endowment into the project and borrow to the limit. Due to the leverage effect, the equity rate is even higher than the marginal product of their project, $\Gamma > R > r$. Therefore, we can shown that $\Gamma'(\theta = 0) > 0$ and $\Gamma'(\theta = \theta^U) < 0$. There exist at least a maximum for $\Gamma$. For $\theta \in (0, \theta^U]$, the following three equations hold,

$$\eta i^e + (1-\eta)i = 1, \quad R - i = r, \quad r(i^e - 1) = \theta R i^e.$$ 

By solving the following quadratic equation,

$$(xi^e + R - 1 - x)(i^e - 1) = \theta Ri^e,$$ (11)

we get

$$i^e = 1 + \sqrt{[(1-\theta)R - 1]^2 + 4\theta Rx - [(1-\theta)R - 1]}$$ (12)

where $x \equiv \frac{R}{1-\eta}$. In the case of binding borrowing constraints, i.e., $\theta \in [0, \theta^U]$, for each unit of project output in period 2, an entrepreneur has to invest $\frac{1}{R}$ in period 1. By pledging a fraction $\theta$ of the period-2 project output, the entrepreneur can borrow $\frac{\theta}{r}$ to finance his project investment and he only has to invest $\frac{1}{R} - \frac{\theta}{r}$ from his own pocket. In period 2, after paying $\theta$ to banks, the entrepreneur gets $1 - \theta$ as net return. The rate of return on the entrepreneurs’ internal capital can also be defined as

$$\Gamma \equiv \frac{1 - \theta}{R - \frac{\theta}{r}}.$$ (13)

Our qualitative results is independent of the parameter values of $R$ and $\eta$. We set $R = 2$ and $\eta = 0.2$ in the numerical analysis. Entrepreneurs account for 20% of the population in the economy. Figure 1 shows how financial development affects the allocation in the model economy. The horizontal axis denotes $\theta \in [0, 1]$.

As $\theta$ rises from 0 to $\theta^U$, entrepreneurs can pledge an increasing fraction of their project output to the banks. The rise in the demand for loans pushes up the loan rate. Due to
perfect competition, banks do not make profit. In equilibrium, the deposit rate is equal
to the loan rate. The rise in the deposit rate induces households to deposit more at the
banks and reduce investment in their own project. The flow of funds from the households’
sector to the entrepreneurs’ sector affects the rate of return on internal capital in two ways.
First, entrepreneurs can increase their investment scale which has the positive effect on
$\Gamma = (1 - \theta) R_i^c$; second, the rise in the loan rate implies that entrepreneurs have to pay the
higher cost for external funds as they borrow more from the banks. The first is called the
investment scale effect and the second the cost effect. The hump-shaped pattern of the
rate of return on internal capital results from the interaction of these two effects. Given a
small initial value of $\theta \in [0, \theta^*)$, the investment scale effect dominates the loan rate effect
for an increase in $\theta$ and $\Gamma$ rises in $\theta$. For a large initial value of $\theta \in (\theta^*, \theta^U)$, the loan rate
effect dominates the investment scale effect and $\Gamma$ declines in $\theta$. 

Figure 1: Allocation in the Case of Autarky: $\theta \in [0, 1]$
3 A Two-Country Model

Suppose that the world consists of two countries, country H (Home) and country F (Foreign). We follow the notation in section 2 and the variables in country $j$ are denoted with subscript $j \in \{H, F\}$. If the two countries have the same degree of financial development, $\theta_H = \theta_F$, there is no capital flows between them even in the case of perfect capital mobility and the world economy is simply the sum of two autarky economy. If the two countries have the different degrees of domestic financial development, $\theta_H \neq \theta_F$, figure 1 shows that the loan rate and the equity rate in the two countries may be different in the case of international financial autarky. Without capital controls, financial capital may flow to the country with the higher loan rate and entrepreneurs will move their projects to the country with the higher equity return.

Let $D^* \in [\eta - 1, 1 - \eta]$ and $\rho \in [-\eta, \eta]$ denote the outflows of financial capital and entrepreneurs from country H. The negative values of $D^*$ and $\rho$ represent the inflows of financial capital and entrepreneurs into country H.

Three cases are analyzed in the following subsections: capital controls on FDI only ($\rho = 0$), capital controls on financial capital only ($D^* = 0$), and no capital controls at all.

3.1 Free Mobility of Financial Capital Only

As shown in figure 1, if two countries differ in the degree of financial development, e.g., $\theta_j \in (0, \theta^U)$ and $\theta_j < \theta_i$, where $i,j \in \{H, F\}$ and $i \neq j$, the loan rate in country $i$ is strictly higher than that in country $j$, $r_i > r_j$, in the case of international financial autarky. Suppose that the international flow of financial capital is allowed but FDI is not. In equilibrium, capital flows equalize the loan rate in the two countries, as long as capital flows do not fully exhaust deposits in one country, i.e., $D^* \in (\eta - 1, 1 - \eta)$.

$$G'(i_H) = r_H = r_F = G'(i_F).$$

The consumption of entrepreneurs and the resource constraints in the two countries are,

$$c_H = Ri_H - r_H(i_H - 1) = \Gamma_H, \quad c_F = Ri_F - r_F(i_F - 1) = \Gamma_F,$$

$$\eta i_H + (1 - \eta)i_H = 1 - D^*, \quad \eta i_F + (1 - \eta)i_F = 1 + D^*,$$

$$\eta c_H + (1 - \eta)c_H = \eta R i_H + (1 - \eta)G(i_H) + r_F D^*,$$

$$\eta c_F + (1 - \eta)c_F = \eta R i_F + (1 - \eta)G(i_F) - r_F D^*.\quad (18)$$

Given the parameter values, it can be shown $D^* \in (\eta - 1, 1 - \eta)$ and no need to consider the corner solution here.
Entrepreneurs are subject to borrowing constraints and participation constraints in the two countries,

\[ r_H(i_H^e - 1) \leq \theta_H R i_H^e, \quad (19) \]
\[ r_F(i_F^e - 1) \leq \theta_F R i_F^e, \quad (20) \]
\[ r_H \leq R, \quad (21) \]
\[ r_F \leq R. \quad (22) \]

Note that only one of the two equations (19) and (21) is strictly binding in equilibrium. Similar claim applies to equations (20) and (22).

**Definition 2.** Market equilibrium is a set of allocations of households, \( \{i_j, c_j\} \), entrepreneurs, \( \{i_e^j, c_e^j\} \), capital flows, \( \{D^*\} \), together with the rates of return on external and internal capital, \( \{r_j, \Gamma_j\} \) in country \( j \in \{H, F\} \), satisfying equations (14)-(22).

We first analyze the allocation in the two-country model with \( \theta_H = 0.3 \) and \( \theta_F \in [0, 1] \). Afterwards, we will analyze the allocation in the two-country model for the complete set of parameter values in figure 4.

**Lemma 2.** Given \( \theta_H \in [0, \theta^U] \), there exists \( \theta^U_F \equiv 1 - \frac{1}{\eta - 1 - \theta_H} \in (\theta^U, 1) \) such that for \( \theta_F \in (\theta^U_F, 1] \), the entrepreneurs’ borrowing constraints specified by equations (19) and (20) are slack, the loan rate is constant at \( r = R \), the project investments of households and entrepreneurs are constant at \( i_H = i_F = 0 \), \( i_e^F = \frac{1}{1-\theta^U_F} \), and \( i_e^H = \frac{1}{1-\theta_H} \), respectively.

We make comparative static analysis of the two-country model in the case of free flow of financial capital. Given \( \theta_H = 0.3 \), figure 2 shows the values of relevant variables and figure 3 shows the percentage difference of these variables in comparison with the corresponding case of international financial autarky. The horizontal axis denotes \( \theta_F \in [0, 1] \). For \( \theta_F \in (\theta^U_F, 1] \), the borrowing constraints of entrepreneurs in country F are slack and economic allocation is unaffected by any change in \( \theta_F \).

The following discussion focuses on the case of \( \theta_F \in (0, \theta^U_F] \). Let us start from the case of \( \theta_F = \theta_H = 0.3 \). The loan rate is same in the two countries in the case of financial autarky. Therefore, there is no capital flows even if there is no capital controls. As the financial system in country F becomes more developed, the loan rate is higher in country F and households in country H prefer to make deposits abroad. According to figure 1, a larger difference between \( \theta_F \) and \( \theta_H \) implies the larger interest rate differential between the two countries in the case of international financial autarky. Thus, in the case of free flow of financial capital, capital flow from country H to country F increases in \( \theta_F \), as shown in the fourth panel of figure 2.

Due to capital outflow, the loan rate in country H converges upwards to the world interest rate and it forces entrepreneurs to reduce their borrowing and project investment.
The decline in their leverage ratio leads to a decrease in the rate of return on their internal capital, $\Gamma_H = (1 - \theta_H) R_i^e_H$. The increase in the deposit rate also induces households in country $H$ to invest less in their own project but deposit more. In the aggregate, capital outflow results in the decline in aggregate output in country $H$.

From the welfare perspective, as the project return is the only income of entrepreneurs in country $H$, their consumption declines in $\theta_F$; while households in country $H$ have two sources of income in period 2: project return and deposit return. The increase in their deposit return overcompensates the decline in their project return. Overall, households in country $H$ consume more than in the case of international financial autarky. Social welfare is defined as the weighted average of the consumption of entrepreneurs and households, $\Omega_j \equiv \eta c_j^e + (1 - \eta)c_j$, in country $j \in \{H, F\}$. As $\theta_F$ rises from 0.3 to $\theta_{U_F}$, the social welfare in country $H$ has a non-monotonic pattern. Intuitively, a slight increase in $\theta_F$ from 0.3 has only a small positive effect on the loan rate in country $H$. The increase in the deposit return of households in period 2 is also small. However, the increase in the loan rate has a larger negative effect on the project investment and the rate of return on internal capital of entrepreneurs. In sum, the social welfare in country $H$ falls in $\theta_F$. For a higher initial value of $\theta_F$, the interest rate differential between the two country in the case of international financial autarky is larger. A further increase in $\theta_F$ has a much stronger effect on the loan rate and thus the deposit return for households. Therefore, the increase in the households’ income overcompensates the decline in the project return.
of entrepreneurs. As a result, social welfare in country H rises in $\theta_F$.

How do international capital flows affect the allocation and welfare in country F? The inflow of cheap foreign funds reduces the loan rate in country F. Entrepreneurs expand their project investment by borrowing at a lower rate, while households prefer to deposit less but invest more in their own project. The first panel of figure 2 shows that the project investment of households in country F decreases in $\theta_F$. It may be misleading. For the comparative static analysis, we should compare the cases with and without capital flows for the same $\theta_F$. In other words, the reference point for the analysis is the corresponding case of international financial autarky, as shown in figure 1.

As the fourth panel of figure 3 shows, for $\theta_F \in (\theta_H, \theta_U)$, the project investments of households and entrepreneurs in country F are larger than their corresponding values under international financial autarky and so is aggregate output in country F; the larger $\theta_F$ is, the loan rate declines more and the difference in project investment between the cases with and without capital flows is larger.\(^4\)

From the welfare perspective, entrepreneurs in country F consume more than in the

\(^4\)Note that there is a kink for the project investment of entrepreneurs at $\theta_U$. In the case without (with) capital flows, economic allocation is constant for $\theta \in (\theta_U, 1]$ ($\theta \in (\theta_U^F, 1]$). $\theta_U^F > \theta_U$ for any $\theta_H \in (0, \theta_U)$. As shown in the first panel in figure 2, the project investment of entrepreneurs in country F rises in $\theta_F$ for $\theta_F \in (\theta_U^F, \theta_U^F]$. The project investment of households in country F declines in $\theta_F$ for $\theta_F \in (\theta_U^F, \theta_U^F]$ in the case with capital flows, while it is constant in the case without capital flows.
case of international financial autarky, due to the increase in their project return. House-
holds have two income sources: project return and deposit returns. The inflow of cheap foreign funds reduces the loan rate and crowds out the bank deposits of households in country F. The increase in their project return cannot compensate the decline in their deposit return. Thus, households consume less than under international financial autarky. Social welfare in country F has a hump-shaped pattern mainly driven by the hump-shaped pattern of the entrepreneurs’ consumption.

Consider the world as a whole. Capital flows improve production efficiency and the increase in aggregate output in country F overcompensates the decline in aggregate output in country H. Thus, the world output rises in $\theta_F$. In the case of international financial autarky, domestic consumption is equal to domestic output; while in the case of capital flows, output and social welfare do not have to move together. As shown in figure 2, although output in country H declines due to capital outflow, social welfare is strictly higher than output, due to the interest payment from abroad.

In the case of $\theta_F \in (0, \theta_H)$, households in country F make deposits abroad and the analysis follows almost exactly as above. Note that capital flows improve production efficiency in the world economy and the improvement increases strictly in the difference in financial development in the two countries, $|\theta_F - \theta_H|$.

Figure 4: Free Flow of Financial Capital: Threshold Values

Now, we consider the allocation in the two-country model for the complete set of parameter values of $\theta_H, \theta_F \in [0, 1]$. Figure 4 shows some threshold values. The horizontal
axis denotes \( \theta_H \) and the vertical axis denotes \( \theta_F \).

In region A, both \( \theta_H \) and \( \theta_F \) are larger than \( \theta^U \equiv 1 - \eta \) defined in Lemma 1. In this case, according to figure 1, production is efficient in both countries in the sense that marginal rates of return on the project of households and entrepreneurs are equal, \( G'(i_j) = R \), where \( j \in \{H, F\} \). As a result, the loan rate and the rate of return on the internal capital (equity return) are equal, \( r_H = r_F = R \); there is no capital flows between the two countries, \( D^* = 0 \), even if international flows of financial capital is allowed. The curve splitting region B and E (\( B' \) and \( E' \)) specifies the relationship of the degrees of financial development in the two countries,

\[
\frac{1}{1 - \theta_H} + \frac{1}{1 - \theta_F} = \frac{2}{\eta}.
\]

(23)

As mentioned in Lemma 2, given \( \theta_H \in [0, \theta^U) \), changes in \( \theta_F \in (\theta^U, 1] \) or in region B do not affect capital flows and economic allocation. Similarly, given \( \theta_F \in [0, \theta^U) \), changes in \( \theta_H \) in region \( B' \) do not affect capital flows and economic allocation, either.

As shown in figure 2, given \( \theta_H \in [0, \theta^U) \), an increase in \( \theta_F \in (\theta_H, \theta^U_F) \) or in region \( E \) leads to an increase in capital outflow from country H. The economic allocation for parameter values in region \( E' \) is symmetric to that in region \( E \).

Table 1 summarizes the sign and size of capital flows in the five regions.

<table>
<thead>
<tr>
<th>Region</th>
<th>A</th>
<th>B</th>
<th>B'</th>
<th>E</th>
<th>E'</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^* )</td>
<td>0</td>
<td>( D_{\max}^*(\theta_H) )</td>
<td>(-D_{\max}^*(\theta_F) )</td>
<td>((0, D_{\max}^*(\theta_H)) )</td>
<td>((-D_{\max}^*(\theta_F), 0) )</td>
</tr>
</tbody>
</table>

where \( D_{\max}^*(\theta) \equiv \left( \frac{1 - \eta}{1 - \theta} \right) > 0 \) for \( \theta \in [0, \theta^U) \).

**Proposition 1.** If international flows of financial capital are allowed but not FDI, financial capital (bank loans) monotonically flows to the country with higher degree of financial development unless production is efficient in the two countries. The size of capital flows increases and production efficiency in the world economy improves monotonically in the difference between the degrees of financial development in the two countries.

### 3.2 Free Mobility of Foreign Direct Investment Only

According to figure 1, the rate of return on internal capital has a non-monotonic pattern with respect to the degree of financial development. If two countries differ in the degree of financial development, e.g., \( \theta_j \in (0, \theta^U) \) and \( \theta_j < \theta_i \), where \( i, j \in \{H, F\} \) and \( i \neq j \), the rate of return on internal capital may be different. Suppose that the international flow of financial capital is allowed but FDI is not. In equilibrium, capital flows equalize the rate
of return on internal capital in the two countries, as long as entrepreneurs in one country do not all move to the other country, i.e., \( \rho \in (-\eta, \eta) \).
\[
\Gamma_H = \Gamma_F. \tag{24}
\]
The equilibrium loan rates in the two countries are
\[
G'(i_H) = r_H, \quad G'(i_F) = r_F. \tag{25}
\]
The consumption of entrepreneurs and the resource constraints in the two countries are,
\[
c_e^H = Ri_e^H - r_H(i_e^H - 1) = \Gamma_H, \quad c_F^F = Ri_F^F - r_F(i_F^F - 1) = \Gamma_F, \tag{26}
\]
\[
(\eta - \rho)i_e^H + (1 - \eta)i_H = 1 - \rho, \quad (\eta + \rho)i_e^F + (1 - \eta)i_F = 1 + \rho, \tag{27}
\]
\[
(\eta - \rho)c_e^H + (1 - \eta)c_H = (\eta - \rho)Ri_e^H + (1 - \eta)G(i_H), \tag{28}
\]
\[
(\eta + \rho)c_e^F + (1 - \eta)c_F = (\eta + \rho)Ri_e^F + (1 - \eta)G(i_F). \tag{29}
\]

We assume that entrepreneurs who move their project abroad in period 1 will move back their home country in period 2. Social welfare is calculated according to the nationality principle, \( \Omega = \eta c_j^H + (1 - \eta) c_j \), where \( j \in \{H, F\} \). Given free mobility of FDI, the rate of return on internal funds is equal in the two countries and so is entrepreneurs’ consumption, \( c_H^e = c_F^e \). Therefore, the repatriation of entrepreneurs affects the welfare analysis.

Entrepreneurs are subject to participation constraints and borrowing constraints in the two countries,
\[
r_H(i_e^H - 1) \leq \theta_H Ri_e^H, \tag{30}
\]
\[
r_F(i_e^F - 1) \leq \theta_F Ri_e^F, \tag{31}
\]
\[
r_H \leq R, \tag{32}
\]
\[
r_F \leq R. \tag{33}
\]

**Definition 3.** Market equilibrium is a set of allocations of households, \( \{i_j, c_j\} \), entrepreneurs, \( \{i_e^j, c_e^j\} \), capital flows, \( \{\rho\} \), together with the rates of return on external and internal capital, \( \{r_j, \Gamma_j\} \) in country \( j \in \{H, F\} \), satisfying equations (24)-(33).

For a complete understanding of the sign and size of FDI flows in the two-country model, we identify three types of threshold values. Figure 5 shows some threshold values. The horizontal axis denotes \( \theta_H \) and the vertical axis denotes \( \theta_F \).

As mentioned in subsection 3.1, for the two parameters in region A, the equal rate of return on internal capital in the two countries, \( \Gamma_H = \Gamma_F = R \), implies zero FDI, \( \rho = 0 \).

\(^5\)As shown later, if all entrepreneurs move to the other country, \( \rho \in (-\eta, \eta) \), the rate of return on internal capital may not be same in the two countries and the corner solution has to be considered.
In addition, for the parameter values on the 45° line, the two countries have same degree of financial development and thus, there is no FDI flows, either.

The curve splitting region $B$ and $E$ ($B'$ and $E'$) shows the relationship of $\theta_H$ and $\theta_F$,

$$\frac{1}{\theta_H} + \frac{1}{\theta_F} = \frac{2}{1 - \eta}. \quad (34)$$

Similar as stated in Lemma 2, given $\theta_H$, changes in $\theta_F$ in region $B$ do not affect allocation because production in both countries is efficient in the sense that the marginal products of the projects of households and entrepreneurs are equal $G'_{i_H} = G'_{i_F} = R$. The project investment of households in the two countries is constant at $i_H = i_F = 0$, that of entrepreneurs in country H is $\frac{1}{1 - \theta_H}$. FDI flow to country F is constant at $\rho = 1 - \frac{1 - \eta}{\theta_H} < 0$, and the project investment of entrepreneurs in country F is $i^e_F = \frac{1}{1 - \theta_F}$, where $\theta^U_F$ satisfies equation (34) for given $\theta_H$.

The curve splitting region $E$ and $J$ ($E'$ and $J'$) specifies the relationship of $\theta_H$ and $\theta_F$,

$$(1 - \theta_H)i^e_{H, Aut} = (1 - \theta_F)i^e_{F, Aut} \quad (35)$$

where $i^e_{j, Aut}$ denotes the project investment of entrepreneurs in country $j \in \{H, F\}$ in the case of financial autarky as specified in equation (12). For the parameter values on this curve, the rate of return on the internal capital is same in both countries and thus, there is no FDI flows, $\rho = 0$, although the two countries may differ significantly in the degree
of financial development. For the parameter values in region $E$, the rate of return on internal capital in country $H$ is larger than that in country $F$, $\rho \in (-\eta, 0)$; while for the parameter values in region $J$, the opposite is true and FDI flows from country $H$ to country $F$, $\rho \in (0, \eta)$.

The curve splitting region $J$ and $L$ specifies the relationship of $\theta_H$ and $\theta_F$ as follows,

$$\frac{1 - \theta_H}{R - \theta_H} = \Gamma_H = \frac{1 - \theta_F}{R - \theta_F}$$

(36)

where $i_H = 1$, $i_F = 0$, $r_H = 1$, and $r_F = 2x \left\{ \sqrt{\left\{ \left( 1 - \theta_F \right) R - 1 \right\}^2 + 8 \theta_F R x - \left\{ (1 - \theta_F) R - 1 \right\} } \right\} + 1$. For the parameter values in region $L$, given $\theta_F$, changes in $\theta_H$ do not affect the allocation in the two countries; the rate of return on the internal capital in country $F$ is higher than that in country $H$ even though all entrepreneurs move their project to country $F$, $\rho = \eta$.

The curve splitting region $J'$ and $L'$ is indeed a symmetric case of that mentioned above and the only difference is the change in the subscript.

Table 2 summarizes the sign and size of FDI flows in the nine regions, where $\rho_{\text{max}}(\theta) \equiv \min\{\eta, \frac{1-n}{\sigma} - 1\}$.

<table>
<thead>
<tr>
<th>Region</th>
<th>$A$</th>
<th>$B$</th>
<th>$B'$</th>
<th>$E$</th>
<th>$E'$</th>
<th>$J$</th>
<th>$J'$</th>
<th>$L$</th>
<th>$L'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0</td>
<td>$-\rho_{\text{max}}(\theta_H)$</td>
<td>$\rho_{\text{max}}(\theta_F)$</td>
<td>$(\eta, 0)$</td>
<td>$(0, \eta)$</td>
<td>$(0, \eta)$</td>
<td>$(0, \eta)$</td>
<td>$\eta$</td>
<td>$-\eta$</td>
</tr>
</tbody>
</table>

In the following, we analyze the allocation in the two-country model with $\theta_H = 0.3$ and $\theta_F \in [0, 1]$. Given $\theta_H = 0.3$, figure 6 show the values of relevant variables and figure 7 shows the percentage difference of these variables in comparison with the corresponding case of the closed economy. The horizontal axis denotes $\theta_F \in [0, 1]$.

Let us start from the case of $\theta_F = \theta_H = 0.3$. The rate of return on internal capital is same in the two countries in the case of financial autarky. Therefore, there is no FDI flows even if there is no capital controls on FDI. Given the hump-shaped pattern of the rate of return on internal capital shown in figure 1, for any $\theta_H \in (0, \theta_U)$, there exists $\theta_F \in (0, \theta_U)$ and $\theta_F \neq \theta_H$ such that the rate of return on internal capital in the two countries is same. Therefore, there could be no FDI even if the level of financial development differs significantly in the two countries.

As the degree of financial development in country $F$ rises slightly, the rate of return on internal capital is higher there and entrepreneurs in country $H$ prefer to move their

---

6Since no entrepreneur borrows and produces in country $F$, the loan rate in country $F$ equals the marginal rate of return on the households’ project, $r_H = G'(1) = 1$. According to equation (13), the underlying rate of return on internal capital in country $H$ is $\Gamma_H = \frac{1 - \theta_H}{\frac{\theta_H}{\rho} - 1}$, which can be shown lower than $\Gamma_F$ given the parameter values in region $L$.
endowment and project abroad. According to figure 1, as long as $\theta_F \in (\theta_H, \theta^*)$, a larger difference between $\theta_F$ and $\theta_H$ widens the difference in the rate of return on internal capital between the two countries in the case of international financial autarky. Thus, capital outflow from country H is larger as $\theta_F$ rises. See the fourth panel of figure 6. For $\theta_F \in (\theta^*, \theta')$, entrepreneurs in country H still move to country F but at a smaller magnitude, where $\theta'$ is defined as the level of financial development in country F where the rate of return on internal capital is equal to that of $\theta_H$ in the case of international financial autarky.

For $\theta_F \in (\theta_H, \theta')$, due to outflow of entrepreneurs, the effective credit demand of entrepreneurs in country H is smaller than in the case of international financial autarky and so is the loan rate. The entrepreneurs who stay in country H get cheaper credit and their project investment rises, as shown in the first panel of figure 7. The decline in the loan rate in country H induces households to deposit less but invest more in their own project, given that they are not allowed to save abroad. Due to outflow of entrepreneurs, aggregate output in country H is less than in the case of international financial autarky despite of the increase in the per capita project investment of households and entrepreneurs.

For $\theta_F \in (\theta', 1)$, the rate of return on internal capital is lower in country F than in country H. As a result, entrepreneurs in country F move their own funds and projects to country H for a higher return. The inflow of FDI raises the credit demand in country H and the loan rate rises. On the one hand, households reduce their project investment
but make more deposits at the banks. Entrepreneurs born in country H have to borrow at a higher rate and their project investment is smaller than in the case of international financial autarky. As more entrepreneurs produce in country H, aggregate output is more than in the case of international financial autarky, despite of a smaller per capita project investment of households and entrepreneurs.

For $\theta_F \in (0, \theta_H)$, the allocation is simply opposite to that in the case of $\theta_F \in (\theta_H, \theta^*)$, i.e., entrepreneurs in country F move their project to country H for a higher rate of return on internal capital. Note that there is a threshold value of $\theta_L$ below which all entrepreneurs move from country F to country H. It corresponds to region L in figure 5. As shown in the second panel in figure 6, the (underlying) rate of return on internal capital in country F is smaller than that in country H which also justifies the claim mentioned above.\(^7\)

Let us consider the welfare implication of free flow of FDI for country H. As the only income of entrepreneurs, their project return is linear to their project investment, $\Gamma_H = (1-\theta_H)R_i^H$. For $\theta_F \in (\theta_H, \theta^*)$, entrepreneurs in country H benefit strictly from free flow of FDI due to the investment scale effect. Households have two sources of income in period 2: project return and deposit return. The decline in the loan rate and their deposits lead to the fall in their deposit return and the increase in their project return cannot fully compensate it. Overall, households in country H consume less than in the

\(^7\)In this case, the consumption of entrepreneurs in country F is different from the rate of return on internal capital in country F, $c_F^e \neq \Gamma_F$, but equal to that in country H, $c_F^e = c_H^e$. 

Figure 7: Free Flow of FDI: $\theta_H = 0.3$
case of international financial autarky. For $\theta_F \in (\theta', 1)$, inflow of foreign entrepreneurs has negative scale effect on the project of entrepreneurs and they consume less than under international financial autarky. In contrast, households benefit from the higher deposit return and their overall consumption is also higher. As the consumption variation of entrepreneurs is much larger than that of households, social welfare in country H has a hump-shaped pattern similar as that of entrepreneurs’ consumption.

How does free flow of FDI affect economic allocation and welfare in country F? For $\theta_F \in (\theta_H, \theta')$, entrepreneurs move their endowment and project to country F because of larger pledgeable project outcome ($\theta_F > \theta_H$) despite of a higher loan rate ($r_F > r_H$). The inflow of FDI raises the effective credit demand in country F and the loan rate is higher than under international financial autarky, as shown in the fifth panel of figure 7. The rise in the loan rate has negative scale effect on their project investment and the rate of return on internal capital. Households prefer to make more deposit and invest less in their project. As more entrepreneurs produce in country F, aggregate output in country F is higher than under international financial autarky despite of the smaller per capita project investment of households and entrepreneurs.

For $\theta_F \in (\theta', 1)$, the loan rate is so high that dominates the scale effect of $\theta$. As a result, entrepreneurs move out of country F. The decline in the effective loan demand reduces the loan rate. On the one hand, households reduce their deposits and invest more in their own project; on the other hand, entrepreneurs who stay in country F can borrow at a lower rate and the expansion of their project investment has a positive scale effect on their equity return, as shown in the fifth panel in figure 7. The kink at $\theta_F = \theta^U$ can be explained in a similar way as in subsection 3.1. As less entrepreneurs produce in country F, aggregate output in country F is lower than under international financial autarky despite of the larger per capita project investment of households and entrepreneurs.

Let us consider the welfare implication of free flow of FDI for country F. For $\theta_F \in (\theta_H, \theta')$, households benefit from the higher loan rate due to the inflow of FDI. While the competition of foreign entrepreneurs on the credit market in country F has a negative welfare effect on local entrepreneurs. For $\theta_F \in (\theta', 1)$, households lose from the lower loan rate while entrepreneurs benefit. As the size of the consumption variation of entrepreneurs dominates that of households, social welfare in country F mainly follows the consumption pattern of entrepreneurs.

Consider the world as a whole. In contrast to the case of free flow of financial capital in subsection 3.1, world output is surprisingly lower in the case of free flow of FDI than in the case of international financial autarky for $\theta_F \in (\theta_H, \theta')$. It implies that allowing intertemporal trade (in the form of FDI) may not necessarily improve production efficiency. Intuitively, the outflow of entrepreneurs from country H to country F has two opposite effects on aggregate output in country H: fewer entrepreneurs producing in country H
and higher per capita project investment of local individuals. The first effect dominates the second effect and aggregate output in country H is smaller. The inflow of FDI also has two opposite effect on aggregate output in country F: more entrepreneurs producing and lower per capita project investment of entrepreneurs. The first effect dominates the second effect and aggregate output in country F is larger. Intuitively, FDI reduces the mass of entrepreneurs in country H and raises the mass of entrepreneurs in country F. Although the total mass of entrepreneurs is unchanged from the world perspective, the distribution of entrepreneurs has asymmetric competition effect on the credit market in the two countries. For $\theta_F = \theta^*$, the loan rate declines in country H by 3.4%, while the loan rate rises in country F by 3.6%. These also reflect the competition pressure on the two credit markets. In sum, the output gains in country F cannot fully compensate the output loss in country H. As a result, the world output is lower. This phenomenon exists for $\theta_F \in (0.20, 0.66)$, given $\theta_H = 0.3$.

In the case of international financial autarky, domestic consumption is equal to domestic output; while in the case of free flow of FDI, output and social welfare do not move together. As shown in figure 6, although output in country H declines due to capital outflow for $\theta_F \in (\theta_H, \theta')$, social welfare is strictly higher than in the case of international financial autarky, due to the repatriation of entrepreneurs from abroad.

Comparing figures 3 and 7, we find that lifting controls on different forms of capital flows may have different welfare effects. For example, if $\theta_F$ is slightly higher than $\theta_H = 0.3$, allowing free flow of financial capital reduces social welfare in country H and improve social welfare in country F, while we get the opposite result by allowing free flow of FDI. Intuitively, lifting controls on different forms of capital flows directly affect the financing and investment decisions of different agents. In this sense, more attention should be paid to the motive of different types of capital flows on the micro-level, before we discuss the aggregate implications of capital account liberalization.

**Proposition 2.** If international flows of FDI are allowed but not financial capital, FDI does not necessarily flow to the country with higher degree of financial development and may lead to the deterioration of production efficiency in the world economy.

### 3.3 Perfect Capital Mobility

After analyzing economic allocation and welfare implication of different capital controls policy in subsections 3.1 and 3.2, we consider the case of perfect capital mobility in this subsection.

Perfect capital mobility tends to equalize the rate of return on external capital as well as internal capital in the two countries. As long as $-(1 - \eta) < D < (1 - \eta)$ and
$-\eta < \rho < \eta$, the loan rate and the equity rate are same in the two countries,

$$r_H = r_F = r, \quad \Gamma_H = \Gamma_F.$$  \hfill (37)

The period-1 resource constraints and the entrepreneurs’ borrowing and participation constraints in the two countries are

$$\begin{align*}
(\eta - \rho)(i^e_H - 1) &= (1 - \eta)(1 - i_H) - D^*, \quad (R - i_H)(i^e_H - 1) \leq \theta_H R i^e_H, \quad r_H \leq R \quad (38) \\
(\eta + \rho)(i^e_F - 1) &= (1 - \eta)(1 - i_F) + D^*, \quad (R - i_F)(i^e_F - 1) \leq \theta_F R i^e_F, \quad r_F \leq R. \quad (39)
\end{align*}$$

The consumption of entrepreneurs and the period-2 resource constraints in the two countries are,

$$\begin{align*}
c^e_H &= R i^e_H - r_H (i^e_H - 1) = \Gamma_H, \quad (40) \\
c^e_F &= R i^e_F - r_F (i^e_F - 1) = \Gamma_F, \quad (41) \\
(\eta - \rho)c^e_H + (1 - \eta)c_H &= (\eta - \rho) R i^e_H + (1 - \eta) G(i_H) + r D^*, \quad (42) \\
(\eta + \rho)c^e_F + (1 - \eta)c_F &= (\eta + \rho) R i^e_F + (1 - \eta) G(i_F) - r D^*. \quad (43)
\end{align*}$$

**Definition 4.** Market equilibrium is a set of allocations of households, $\{i_j, c_j\}$, entrepreneurs, $\{i^*_j, c^*_j\}$, capital flows, $\{D^*, \rho\}$, together with the rates of return on external and internal capital, $\{r_j, \Gamma_j\}$ in country $j \in \{H, F\}$, satisfying equations (37)-(43).

If $\theta_H = \theta_F$, there is no capital flows between the two economies and the allocation in the two-country model is simply the sum of two countries in autarky. If $\theta_H \neq \theta_F$, the equilibrium solution implies that production is efficient in both countries,

$$\begin{align*}
i_H &= i_F = 0, \quad r_H = r_F = \Gamma_H = \Gamma_F = R \quad (44) \\
i^e_H &= \frac{1}{1 - \theta_H}, \quad \rho = \frac{2(1 - \eta) - \eta \left(\frac{\theta_F}{1 - \theta_F} + \frac{\theta_H}{1 - \theta_H}\right)}{\frac{\theta_F}{1 - \theta_F} - \frac{\theta_H}{1 - \theta_H} \quad (45) \\
i^e_F &= \frac{1}{1 - \theta_F}, \quad D = \frac{(1 - \eta) \left(\frac{\theta_F}{1 - \theta_F} + \frac{\theta_H}{1 - \theta_H}\right) - 2\eta \left(\frac{\theta_F}{1 - \theta_F} - \frac{\theta_H}{1 - \theta_H}\right)}{\frac{\theta_F}{1 - \theta_F} - \frac{\theta_H}{1 - \theta_H}}. \quad (46)
\end{align*}$$

For a complete understanding of the sign and size of capital flows in the two country-model, we identify three types of threshold values. Figure 8 shows the threshold values. The horizontal and vertical axes denote $\theta_H$ and $\theta_F$, respectively.

As mentioned in subsection 3.1, for the two parameters in region $A$, the equal rate of return on internal and external capital in the two countries, $r_H = \Gamma_H = r_F = \Gamma_F = R$, implies no capital flows, $\rho = 0$. In addition, for the parameter values on the 45° line, the two countries have same degree of financial development and thus, there is no capital flows, either.

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The curve splitting region $B$ and $E$ ($B'$ and $E'$) shows the relationship of $\theta_H$ and $\theta_F$, \[
\frac{1}{\theta_H} + \frac{1}{\theta_F} = \frac{2}{1 - \eta}.
\] (47)
For the parameter values on this curve, there is no financial capital flows, $D^* = 0$; for the parameter values in region B (E), financial capital flows to country H (F).

The curve splitting region $E$ and $J$ ($E'$ and $J'$) specifies the relationship of $\theta_H$ and $\theta_F$, \[
\frac{1}{1 - \theta_H} + \frac{1}{1 - \theta_F} = \frac{2}{\eta}.
\] (48)
For the parameter values on this curve, there is no FDI flows, $\rho = 0$; for the parameter values in region E (J), financial capital flows to country H (F), $\rho < 0$ ($\rho > 0$).

The line splitting region $E$ and $J$ ($E'$ and $J'$) is $\theta_F = \theta_U$ ($\theta_H = \theta_U$). For the parameter values in region L, all entrepreneurs move their project to country F, $\rho = \eta$. The solution mentioned in equations (44)-(46) does not apply for this case. However, the solution in this case is rather simple. Since all entrepreneurs move to country F and the loan rate in the two countries is same, the two-country model economy can be considered as a closed economy with households and entrepreneurs, each of mass 2, and the degree of financial development $\theta_F$.

Table 3 summarizes the sign and size of FDI and financial capital flows in the nine regions, where $D^*_{aut}(\theta) \equiv \eta[i^{*}_{aut}(\theta) - 1]$. $i^{*}_{aut}(\theta)$ denotes the entrepreneurs’ project investment in a closed economy with the degree of financial development $\theta$, which is calculated
Table 3: Sign and Size of FDI and Financial Capital Flows

<table>
<thead>
<tr>
<th>Region</th>
<th>$A$</th>
<th>$B$</th>
<th>$B'$</th>
<th>$E$</th>
<th>$E'$</th>
<th>$J$</th>
<th>$J'$</th>
<th>$L$</th>
<th>$L'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>$\eta$</td>
<td>$-\eta$</td>
</tr>
<tr>
<td>$D^*$</td>
<td>0</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>$D^*_{aut}(\theta_F)$</td>
<td>$-D^*_{aut}(\theta_H)$</td>
<td></td>
</tr>
</tbody>
</table>

according to equation (12). Note that capital flows in regions $B'$, $E'$, $J'$, $L'$ are symmetric but opposite cases of those in regions $B$, $E$, $J$, $L$, respectively.

In the following, we analyze the allocation in the two-country model with $\theta_H = 0.3$ and $\theta_F \in [0, 1]$ under perfect capital mobility. Given $\theta_H = 0.3$, figure 9 shows the values of relevant variables and figure 10 shows the percentage difference of these variables in comparison with the corresponding case of the closed economy. The horizontal axis denotes $\theta_F \in [0, 1]$.

Let us start from the case of $\theta_F = \theta_H = 0.3$. The rate of return on internal capital is same in the two countries in the case of financial autarky. Therefore, there is no FDI flows even in the case of perfect capital mobility.

For $\theta_F \in [0, \theta_H)$, the rates of return on internal and external capital are both lower in country $F$ than in country $H$ in the case of financial autarky, according to figure 1. Thus, perfect capital mobility results in the inflows of both FDI and financial capital to country
H. As mentioned in table 3, for the parameter values in region $L'$, all entrepreneurs move from country F to country H. Intuitively, for $\theta_F < \theta_H = 0.3$, the loan rate is higher in country H than in country F in the case of financial autarky. Perfect capital mobility allows households in country F to make deposit abroad. On the one hand, capital outflows reduce the credit supply in country F. The rise in the loan rate reduces the borrowing capacity and the project investment of entrepreneurs. It tends to reduce the rate of return on internal capital in country F. On the other hand, the inflow of cheap foreign funds from country F tends to reduce the loan rate in country H, which has positive effect on the rate of return on internal capital in country H. As a result, entrepreneurs in country F prefer to move their project to country H. As shown in the second panel of figure 8, the underlying rate of return on internal capital in country F is still lower than that in country H even though all entrepreneurs move their projects to country H.

Perfect capital mobility does not affect the allocation in country H. In fact, what matters for the allocation is the degree of financial development in country H in this case. Given all entrepreneurs moving from country F to country H, the outflows of financial capital from country F match exactly the credit demand of these entrepreneurs in country H. In equilibrium, entrepreneurs who move their project to country H pledge a larger fraction of their project outcome for external financing of their project investment, while the loan rate in country F matches upwards to that in country H. In this sense, both households and entrepreneurs free ride on the higher degree of financial development in
country H.

For $\theta_F \in (\theta_H, \theta^U]$, the allocation is opposite to that in the case of $\theta_F \in [0, \theta_H)$. As mentioned in section 2, the rate of return on internal capital, $\Gamma = \frac{1 - \theta}{1 - R}$, has the hump-shaped pattern with respect to the degree of financial development due to two effects, the scale effect and the cost effect. In the case of free mobility of financial capital, the loan rate is same in the two countries. Without the cost effect, the rate of return on internal capital is strictly higher in the country with a higher degree of financial development, $\frac{\partial \Gamma}{\partial \theta} = \frac{1}{r} - \frac{1}{R} > 0$, where $r \leq R$. As shown in the second panel of figure 2, the rate of return on internal capital is strictly higher in country F than that in country H in the case of free mobility of financial capital. Allowing additionally free mobility of FDI induces entrepreneurs move to the country with a higher degree of financial development. Thus, both FDI and financial capital flow to country F where financial development is higher. As shown in the second panel of figure 8, the underlying rate of return on internal capital in country H is still lower than that in country F even though all entrepreneurs move their projects to country F.

Note that for $\theta_F \in (0, \theta^U)$, the allocation in the two-country model resembles that in the closed-economy model with $\theta = \max\{\theta_H, \theta_F\}$ where production is not efficient in the sense that the marginal products of the projects of households and entrepreneurs are different.

For $\theta_F \in (\theta^U, 1]$, production is efficient and the entrepreneurs’ borrowing constraints is slack in country F in the case of international financial autarky. In other words, entrepreneurs do not borrow to the limit at the prevailing loan rate. Consider first the case of $\theta_F$ slightly larger than $\theta^U$. Allowing free mobility of financial capital leads to capital inflows into country F. The extra supply of credit tends to reduce the loan rate and induces entrepreneurs to borrow more in country F. Allowing additionally free mobility of FDI induces entrepreneurs in country H to move their projects to country F for higher rate of return on internal capital.

As $\theta_F$ rises to unity, FDI flows to country F at a smaller size and even changes direction for a very large $\theta_F$. According to the solution in equations (44)-(46), for $\theta_H < \theta^U < \theta_F$, production is efficient at both countries in the case of perfect capital mobility, $i_H = i_F = \frac{1}{1 - R}$, and aggregate credit supply is constant at $2(1 - \eta)(1 - i) = 2(1 - \eta)(R - 1)$. While, the project investments of entrepreneurs in the two countries are $i_H^e = \frac{1}{1 - \theta_H}$ and $i_F^e = \frac{1}{1 - \theta_F}$; the aggregate credit demand in the two countries is

$$
(\eta - \rho)(i_H^e - 1) + (\eta + \rho)(i_F^e - 1) = (\eta - \rho) \frac{1}{\theta_H} + (\eta + \rho) \frac{1}{\theta_F}.
$$

Intuitively, the rise in $\theta_F$ towards unity raises the borrowing capacity of entrepreneurs in country F and tends to pushes up the loan rate there. As a result, less entrepreneurs
in country H move their projects to country F. For \( \theta_F \) close to unity, entrepreneurs in country F may move their project to country H in equilibrium. In that case, we can observe the two-way capital flows: FDI flows to the country with lower degree of financial development and financial capital flows in the opposite direction. In other words, we can observe a small net capital flows with large gross capital flows in the different forms.

As mentioned in figure 8 and table 3, for the parameter values in region \( B \), the loan rate differential in the two countries is small and so is the incentive for households making deposits abroad. Similar as the argument mentioned above, for \( \theta_F \) close to unity, we can observe that both FDI and financial capital flow to country H.

Let us first consider the welfare implications of perfect capital mobility for country H. For \( \theta_F \in [0, \theta_H] \), allowing perfect capital mobility does not affect the allocation and the welfare of individual household and entrepreneur in country H. For \( \theta_F \in (\theta_H, \theta_U) \), perfect capital mobility induces all entrepreneurs in country H to move their projects into country F and households in country H to make deposits abroad. On the one hand, the more developed financial system in country F enables entrepreneurs to borrow more but at a higher loan rate than in country H. As \( \theta_F \) approaches \( \theta_U \), the cost effect dominates the scale effect, and the consumption of entrepreneurs from country H has a hump-shaped pattern with regards to \( \theta_F \). On the other hand, as \( \theta_F \) rises, households in country H can make more deposits both at home and abroad at a higher rate than in the case of financial autarky. Despite of the decline in their project investment, the consumption of households rises strictly in \( \theta_F \). The positive effect of perfect capital mobility on the households' welfare strictly dominates its non-monotonic effect on the entrepreneurs' welfare. Thus, the overall welfare of country H rises in \( \theta_F \). For \( \theta_F \in [\theta_U, 1] \), the rates of return on internal and external capital are both constant at \( r_H = \Gamma_H = R \). Thus, changes in \( \theta_F \) do not further affect the welfare of households and entrepreneurs.

The welfare implication of perfect capital mobility for country F is almost the symmetric but opposite case of that for country H. For \( \theta_F \in [0, \theta_H] \), perfect capital mobility improves the welfare of households and entrepreneurs in country F. While for \( \theta_F \in (\theta_H, \theta_U) \), perfect capital mobility does not affect the allocation and the welfare of individual agents in country F.

Consider the world as a whole. As shown in the ninth panel in figure 10, perfect capital mobility improve the world production efficiency. Comparing the fourth panels of figures 6 and 9, we can see that entrepreneurs from country F move their projects to country H for \( \theta_F \in (\theta', \theta_U) \) in the case of free mobility of FDI. Allowing additionally free mobility of financial capital reverses the direction of FDI, i.e., FDI flows from country H to country F. From the welfare perspective, perfect capital mobility improves the welfare of country H as a whole in comparison with the case of free mobility of FDI only. While, from the production perspective, the reversal of FDI reduces
domestic production in country H. If a government in country H can levy tax on GDP and make transfer to domestic agents, the government may prefer to encourage FDI but impose capital controls on financial capital in order to maximize domestic production. In this sense, we can rationalize certain capital controls policy in some developing economies.

**Proposition 3.** In most cases, both financial capital and FDI tend to flow to the country with higher degree of financial development. In some cases, there may exist two-way capital flows, i.e., financial capital flows to the country with higher degree of financial development, while FDI flows to the country with lower degree of financial development. It is also possible that both financial capital and FDI flow to the country with lower degree of financial development. Perfect capital mobility improves production efficiency in comparison with the case of financial autarky. From a dynamic perspective, lifting capital controls may change the direction of capital flows.

## 4 Final Remarks

We develop a general equilibrium model with financial frictions in which equity and loans have different rates of return. Financial development raises the loan rate but has a non-monotone effect on equity return.

We then show in a two-country model that capital account liberalization leads to outflow of financial capital from the country with less developed financial system. However, the direction of foreign direct investment (FDI, henceforth) is not straightforward and depends on capital controls policy and the exact degrees of financial development in the two countries. Lifting controls on different types of capital controls may have opposite welfare effect on the macro-level as well as the micro-level in the two countries.

Interestingly, free flow of FDI changes the distribution of entrepreneurs in the two countries and has asymmetric competition effect on the credit markets in the two countries. As a result, the world output may decrease. In this sense, allowing intertemporal trade between two countries may not necessarily improve the world production efficiency.

According to our model, countries with least developed financial system witness outflow of both financial capital and FDI; countries with most developed financial system may have two-way capital flows, i.e., the inflow of financial capital and the outflow of FDI; countries with intermediate level of financial development have the inflows of both financial capital and FDI. It is consistent with the fact that FDI flows not to the poorest countries but to the middle-income countries. Developing countries with less developed financial system may impose controls on financial capital flows and attract FDI inflows.
References


