Distributed Event Detection in Sensor Networks under Random Spatial Deployment

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***Abstract*—We present a novel event detection algorithm in sensor networks for the case where the sensors are randomly deployed in space. In particular we consider a random sensors deployment according to a Homogeneous Finite Binomial Point Process. We first derive the optimal event detection decision rule. We then develop a novel algorithm to evaluate the intractable marginal likelihood based on the Gram-Charlier series expansion. We evaluate our algorithms through extensive Monte Carlo simulations. Simulation results present the detection and false alarm rates for different system parameters such as number of sensors deployed, deployment region size etc.**
Keywords: Sensor networks, Event detection, Finite Binomial Point Process, Gram-Charlier series expansion.

I. Introduction

Sensor Networks (SN) have attracted considerable attention due to the large number of applications, such as environmental monitoring, weather forecasts [1]–[3], surveillance, health care, and home automation [3], [4]. We consider SN which consist of a set of spatially distributed sensors which monitor a spatial physical phenomenon containing some desired attribute (e.g pressure, temperature, concentrations of substance, sound intensity, radiation levels, pollution concentrations etc.), and regularly communicate their observations to a Fusion Centre (FC) [5]–[7]. The FC collects these observations and fuses them in order to perform event detection, based on which effective actions are made [4]. It is therefore imperative for the SN to be accurate in identifying a valid event (high detection rate) while maintaining as low as possible false detection (low false alarm). For example, in [8] the problem of distributed detection was considered, where the sensors transmit their local decisions over perfectly known wireless channels. Theoretical performance analysis was derived in [9] for detection fusion under conditionally dependent and independent local decisions. Distributed detection in sensor networks over fading channels with multiple receive antennas at the Gateway (GW) was considered in [10].

While the problem of event detection for the case of **deterministic and known sensors deployment** in SN has been widely investigated [4], [9]–[12] and references within, the problem of event detection where the sensors are **randomly deployed in the field** has not been addressed before. This problem is of great practical interest because in many cases the locations of the sensors are unknown to the FC. For example, the sensor nodes may be dropped by airplanes, unmanned aerial vehicles or ships [13]. As a result, new models and algorithms for event detection need to be developed for the case of random deployment models and is the focus of this paper.

In this paper we develop the first reported solution for event detection in SN under random deployment. We consider the practical case where the spatial distribution of the sensor follows a **Homogeneous Finite Binomial Point Process (HFBPP)**. In HFBPP sensor network deployment, fixed and known number of sensors are spatially distributed in a given region according to a spatial uniform distribution. If the target (event) is present/active, it emits energy (acoustic or electromagnetic) which is measured by each of the sensors. We assume an energy decay model in which the amount of energy each sensor measures falls off with distance and obeys an inverse power-law where the exponent is known as the path loss exponent [14]. All the measurements from the sensors are then aggregated to the GW which makes the final decision whether the target is present or absent. In contrast to previous works, since we assume a random spatial deployment, the distance from the target to the sensors is now a random variable. The objective of the SN is to distinguish between two hypotheses, such as the absence (Null Hypothesis), or presence (Alternative Hypothesis) of a certain event [15]–[17]. The ability of a SN to perform such detection and decisions is crucial for various applications, for example the detection of the presence or absence of a target in a surveillance system, detection of missiles, identification of chemical, biological or nuclear plumes and many more [18]–[20]. The resulting optimal detection algorithm involves the to derivation of the likelihood ratio between the two hypotheses.

While deriving the marginal likelihood under the Null hypothesis is trivial, the derivation of the marginal likelihood under the Alternative hypothesis is intractable, as we show, since it involves a multi-variate convolution which cannot be solved exactly. Building on the approach for deriving distance distributions in random networks taken in [21], [22], we develop a novel algorithm to approximate the intractable distribution of the marginal likelihood under the alternative hypothesis. Our solution is based on the Gram-Charlier series expansion. As we show, our algorithm only requires deriving the first four cumulants to obtain good detection performance. The Gram-Charlier series is found to be highly accurate and captures the important tail behaviour of the marginal likelihood under the alternative hypothesis.

II. Sensor Network System Model

In this section we present the model assumptions. We begin with a formal definition of a Homogeneously Finite Binomial Point Process followed by system model assumptions.

Definition 1 (Finite Binomial Point Process (FBPP) [23])**.**

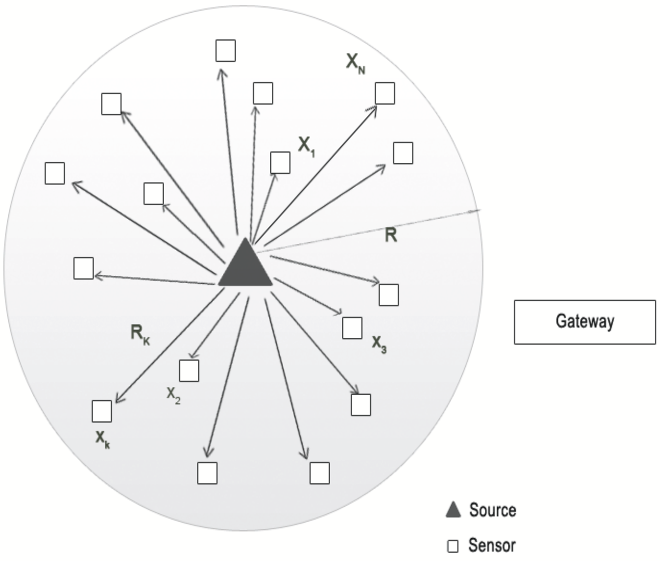

Fig. 1. A realization of sensors deployment according to a Homogeneous Finite Binomial Point Process with 16 sensors in a circular region of radius R. In case of event, the source emits energy which falls off with the distance from each of the sensors.

FBPP Θ is formed as a result of independently uniformly distributing N points in a compact set W. The density of the BPP at any location $\mathbf{x}$ is defined to be $\lambda(\mathbf{x}) = (N/|W|)\,\mathbf{1}(\mathbf{x})$. For any set $V \in W$, the number of points in V, i.e. $\Theta(V)$, is binomial (n,p) with parameters $n = N$ and $p = |V \cap W| / |W|$

$$\Theta(V)\,|n = N \sim \textit{binomial}\,(V,n,p)\,. \tag{1}$$

By this property, the number of nodes in disjoint sets is joined via a multinomial distribution

We now present the system model for the sensor network (see Fig. 1)

1) The SN consists of N sensors which are deployed according to a homogenous FBPP in a 2 dimensional circle with radius R, according to Definition 1.
2) The unknown random location of the k^{th} sensor $(k = \{1,\cdots,N\})$ is $\mathbf{X}_k = [X_k, Y_k]$.
3) The known location of the source (if present) is $\mathbf{X}_s = [X_0, Y_0]$. We assume without loss of generality, that it is located in the center of circle.
4) The source is present $(\mathcal{H}_1)$ or absent $(\mathcal{H}_0)$. Under $\mathcal{H}_1$, the source transmits constant power P_0. Under $\mathcal{H}_0$, the source does not transmit power.
5) The amount of energy the k^{th} sensors measures is given by $\sqrt{P_0}R_k^{-\alpha/2}$. The random variable R_k represents the random distance between the k^{th} sensor and the source. The parameter α is the path-loss exponent.
6) The observed signal at the GW from N sensors (k=1,..., N) at the s^{th} time slot $(s = \{1,\cdots,S\})$ is given by:

$$\begin{cases} \mathcal{H}_0 : Y(s) = \sum_{k=1}^{N} W_k(s) \\ \mathcal{H}_1 : Y(s) = \sum_{k=1}^{N} \sqrt{P_0} R_k(s)^{-\alpha/2} + \sum_{k=1}^{N} W_k(s)\,, \end{cases}$$

where $R_k(s)$ is the minimum radius of the ball with center from $\mathbf{X}_s$ that contains at least k points in the ball, i.e., $R_k(s) = \inf\{r : \{R_{(1)}(s), R_{(2)}(s), \cdots, R_{(k)}(s)\} \in B_{\mathbf{X}_s}(r)\}$. $B_{\mathbf{X}_s}(r)$ is the ball with radius r and center at $\mathbf{X}_s$. The random variable $W_k(s)$ is the i.i.d additive Gaussian noise $\mathcal{N}(0, \sigma^2_{W_k(s)})$.

III. Event Detection Algorithm

In this section we develop the algorithm for event detection in randomly deployed sensor networks. We first present the optimal decision rule. We then derive the various components required in order to evaluate the optimal decision rule.

A. Optimal Event Detection Decision Rule

The optimal decision rule is a threshold test based on the likelihood ratio [24]. We consider a frame-by-frame detection, where the length of each frame is S. The decision rule is then given by:

$$\Lambda\left(Y(1:S)\right) \triangleq \frac{p(Y(1:S)\,|\mathbf{X}_s, \mathcal{H}_0)}{p\left(Y(1:S)\,|\mathbf{X}_s, \mathcal{H}_1\right)} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \gamma, \tag{2}$$

where the threshold γ can be set to assure a fixed system false-alarm rate under the Neyman-Pearson approach or can be chosen to minimize the overall probability of error under the Bayesian approach [25]. We can decompose the full marginals under each hypothesis, $p\left(Y(1:S)\,|\mathbf{X}_s, \mathcal{H}_k\right)$, $k = 0,1$, as

$$p(Y(1:S)\,|\mathbf{X}_s, \mathcal{H}_k) = \prod_{s=1}^{S} p(Y(s)\,|\mathbf{X}_s, \mathcal{H}_k). \tag{3}$$

This decomposition is useful as it allows us to work on a lower dimensional space, resulting in efficiency gains for the algorithm we develop and requiring no memory storage for data.

B. Marginal Likelihood Calculation

The optimal decision rule in (2) involves calculating the marginal likelihood under each model. The marginal likelihood under $\mathcal{H}_0$ can be easily calculated as it follows a Normal distribution. Obtaining the marginal likelihood under the alternative hypothesis, $p\left(Y(s)\,|\mathbf{X}_s, \mathcal{H}_1\right)$, is not attainable in closed form because in involves solving the $2N$-fold convolution. The first N terms of the convolution follow a non-standard distribution as presented in Theorem 1 below, which is not closed under convolution. To overcome this problem, we derive a novel approximation for the marginal likelihood using the Gram-Charlier series expansion which approximates a probability distribution in terms of its cumulants [26]. The Gram-Charlier series expansion for any density $f(x)$ is given by:

$$f(x) \approx \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \times \left(1 + \frac{\kappa_3}{6\sigma^3} H_3\left(\frac{x-\mu}{\sigma}\right) + \frac{\kappa_4}{24\sigma^4} H_4\left(\frac{x-\mu}{\sigma}\right)\right), \tag{4}$$

where $\mu = \kappa_1$, $\sigma^2 = \kappa_2$, κ_3 and κ_4 are the third and fourth cumulants of $f(x)$. $H_3(x) = x^3 - 3x$ and $H_4(x) = x^4 - 6x^2 + 3$ are the Hermite polynomials.

In our model we set $f(x) = p\left(Y(s) | \mathbf{X}_s, \mathcal{H}_1\right)$. To this end we need to find the first four cumulants of $p\left(Y(s) | \mathbf{X}_s, \mathcal{H}_1\right)$. To obtain that we perform the following procedure:

1) Find the distance probability density function of each sensor reading $R_k(s)$ under the alternative hypothesis $f\left(R_k(s) | \mathbf{X}_s, \mathcal{H}_1\right)$.
2) Derive the probability density function of the transformed random variable $f(Z_k)(s) | \mathbf{X}_s, \mathcal{H}_1$ where $Z_k(s) := R_k(s)^{-\alpha/2}$.
3) Calculate the Moment Generating function (MGF) $(M_{Z_k(s)}(t))$ for $Z_k(s)$.
4) Calculate the Moment Generating function (MGF) $(M_{Y(s)}(t))$ for $Y(s) = \sum_{k=1}^{N} \sqrt{P_0} Z_k(s) + \sum_{k=1}^{N} W_k(s)$.
5) Use Gram-Charlier series expansions to approximate the probability density function of $Y(s)$.

We begin with obtaining the distance distribution $f(R_k(s) | \mathbf{X}_s, \mathcal{H}_1)$. In [21], the distance $R_k(s)$ is shown to follow a generalized beta distribution, as follows:

$$\begin{aligned} & f_{R_k(s)}\left(r(s) | \mathbf{X}_s, \mathcal{H}_1\right) \\ &= \frac{2}{R} \frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})} \beta\left(\frac{r(s)^2}{R^2}; k+\frac{1}{2}, N-k+1\right) \end{aligned}$$

where N points uniformly randomly distributed in 2-dimensional circle with radius R and $0 < r(s) < R$.

Next, we derive the density $f(Z_k(s) | \mathbf{X}_s, \mathcal{H}_1)$.

Theorem 1. *The density* $f\left(Z_k(s) | \mathbf{X}_s, \mathcal{H}_1\right) = f_{Z_k(s)}\left(r(s)^{-\alpha/2} | \mathbf{X}_s, \mathcal{H}_1\right)$ *is given by*

$$\begin{aligned} f\left(Z_k(s) | \mathbf{X}_s, \mathcal{H}_1\right) = & \frac{4}{R\alpha} \frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})} z_k(s)^{-2/\alpha - 1} \\ & \times \beta\left(\frac{z_k(s)^{-4/\alpha}}{R^2}; k+\frac{1}{2}, N-k+1\right) \end{aligned}$$

where $Z_k(s) \in (R^{-\alpha/2}, +\infty)$

Proof: See Appendix A ■

The MGF of $Z_k(s)$ is given by (5). Solving this integral directly is difficult. Instead, we calculate the m^{th} moment for $Z_k(s)$ and then derive the MGF based on the moments.

Theorem 2. *The* m^{th} *moment of* $Z_k(s)$ *is given by:*

$$\mathbb{E}[Z_k(s)^m] = \begin{cases} R^{-\frac{\alpha}{2}m} \frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}, & k - \frac{\alpha}{4},\ m \notin \mathbb{Z}_{\leq 0} \\ \infty, & \textit{otherwise} \end{cases} \tag{6}$$

Proof: See Appendix B ■

Now that we have derived a general expression for the moments of $Z_k(s)$, we derive its MGF:

Theorem 3. *The moment generating function* $M_{Z_k(s)}(t) | \mathbf{X}_s, \mathcal{H}_1$ *is given by:*

$$M_{Z_k}(t) = 1 + \sum_{m=1}^{\infty} \frac{t^m}{m!} R^{-\frac{\alpha}{2}m} \frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}$$

Proof: See Appendix C ■

Next, we find the Moment Generating Function $M_{Y(s)}(t)$ for $\underbrace{\sum_{k=1}^{N} \sqrt{P_0} Z_k(s) + W_k(s)}_{V(s)}$, under the assumption that $Z_1(s), Z_2(s), \cdots, Z_N(s), W_1(s), W_2(s), \cdots, W_N(s)$ are conditionally independent as well as $Z_k(s)$ and $W_k(s)$.

Corollary 1. *The Moment Generating Function* $M_{Y(s)}(t)$ *is given by:*

$$M_{Y(s)}(t) = \prod_{k=1}^{N} \left(1 + \sum_{m=1}^{\infty} \frac{(\sqrt{P_0} t)^m}{m!} R^{-\frac{\alpha}{2}m} \frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}\right) e^{\frac{1}{2}\sigma_w^2 t^2}.$$

Proof: See Appendix D. ■

In order to derive the Gram-Charlier series expansion, we need to find the cumulants denoted as κ. In Lemma 1, we express the cumulants.

Lemma 1. *The first four cumulants of* $Y(s) | \mathbf{X}_s, \mathcal{H}_1$, κ_i, $i = 1, 2, 3, 4$ *are given by:*

$$\kappa_1 = \sum_{k=1}^{N} \sqrt{P_0} \bar{Z}_{k1},$$

$$\kappa_2 = \sum_{k=1}^{N} P_0 \left(\bar{Z}_{k2} - \bar{Z}_{k1}^2\right) + \sigma_W^2,$$

$$\kappa_3 = \sum_{k=1}^{N} \sqrt{P_0}^3 \left(\bar{Z}_{k3} - 3\bar{Z}_{k2}\bar{Z}_{k1} + 2\bar{Z}_{k1}^3\right),$$

$$\kappa_4 = \sum_{k=1}^{N} P_0^2 \left(\bar{Z}_{k4} - 4\bar{Z}_{k3}\bar{Z}_{k1} - 3\bar{Z}_{k2}^2 + 12\bar{Z}_{k2}\bar{Z}_{k1}^2 - 6\bar{Z}_{k1}^4\right).$$

where $\bar{Z}_{ki}$, $i = 1, 2, 3, 4$ *are the* i^{th} *moment of* $Z_k(s)$ *presented in Theorem 2.*

Proof: See Appendix E. ■

Now that we have derived the four cumulants, we use the Gram-Charlier series expansion in (4) to approximate $p\left(Y(s) | \mathbf{X}_s, \mathcal{H}_1\right)$. Finally, the Event Detection algorithm under Homogenous HFBPP is presented in Algorithm 1.

$$
\begin{aligned}
M_{Z_k(s)|\mathbf{X}_s,\mathcal{H}_1}(t) &= \mathbb{E}_{f(Z_k(s))}\left[\exp^{tZ_k(s)}\right] \\
&= \int \frac{4}{R\alpha}\frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})}\beta(\frac{z_k(s)^{-4/\alpha}}{R^2};k+\frac{1}{2},N-k+1)z_k(s)^{-2/\alpha-1}\exp^{tz_k(s)}\,\mathrm{d}z_k(s)
\end{aligned} \tag{5}
$$

Algorithm 1 Event Detection in Sensor Networks with Random Deployment according to Homogenous FBPP

Input: $Y(s)$, γ, N, R, α, σ_{w_k}
Output: Binary decision ($\mathcal{H}_1$, $\mathcal{H}_0$)

1) Calculate $f(Z_k(s))$ according to Theorem 1.
2) Calculate cumulants κ_i according the Lemma 1.
3) Evaluate the Gram-Charlier series expansion in (4) to find $p\left(Y\left(s\right)|\mathbf{X}_s,\mathcal{H}_1\right)$.
4) Calculate $\Lambda(Y(s))$ via (2) and compare to the threshold γ.

IV. Simulation Results

In this section, we present the detection performance of our algorithm via Monte Carlo simulations. The simulations setting are as follows: the additive noise is assumed i.i.d Gaussian distributed at each sensor. The results are obtained from $50,000$ realizations for a given parameter set of $N, \sigma_w, P_0, R, \alpha$.

First, in Fig. 2 we present an example of the Gram-Chalier series expansion approximation of $p\left(Y\left(s\right)|\mathbf{X}_s,\mathcal{H}_1\right)$. The result presents that the Monte Carlo density estimation and the Gram-Chalier series expansion are in perfect agreement. This shows the effectiveness and accuracy of our approach. We note that similar results were obtained for a large number of parameters.

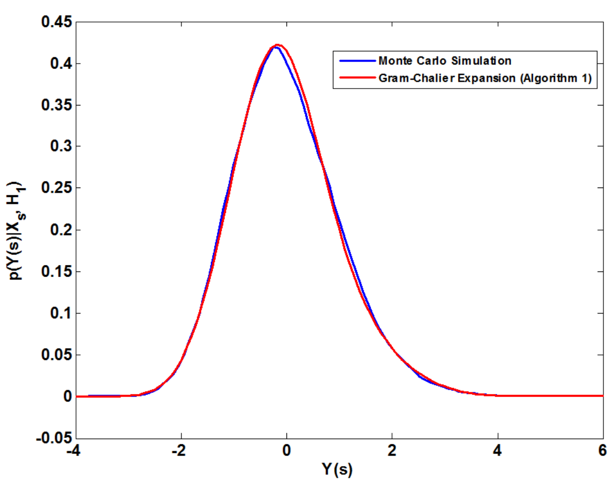

Fig. 2. Probability density estimation of $p\left(Y\left(s\right)|\mathbf{X}_s,\mathcal{H}_1\right)$ using Monte Carlo simulation and Gram-Chalier series expansion ($R=100, N=50, \alpha=2, \sigma_w=0.01, P_0=1$).

We now present the detection performance of the algorithms via Receiver Operating characteristics (ROC) for different configurations: Gaussian Noise variance $\sigma_w = \{0.005, 0.01, 0.015, 0.02, 0.025, 0.03\}$, source power $P_0 = \{1,2,3,4,5\}$, field radius $R = \{50,60,70,80,90,100\}$ and propagation loss coefficient $\alpha = \{2, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6\}$.

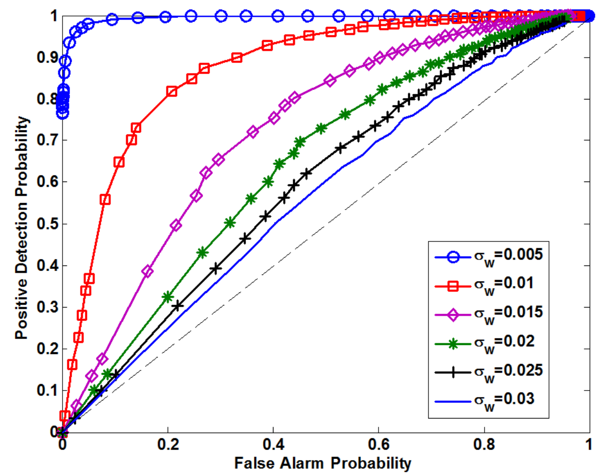

Fig. 3. ROC performance under different configurations of σ_w $\sigma_w \in \{0.005, 0.01, 0.015, 0.02, 0.025, 0.03\}, (R=100, \alpha=2, N=50, P_0=1)$

Fig. 3 presents ROC results for various vaues of the noise variance σ_w. This result clearly shows the effect the additive noise has on the ROC performance.

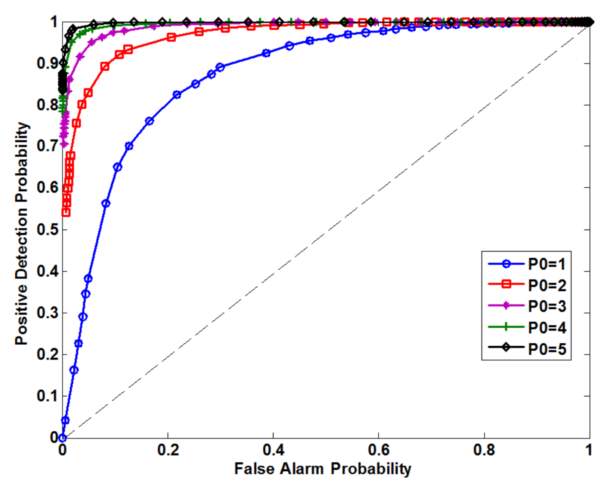

Fig. 4. ROC performance under different configurations of P_0, $P_0 \in \{1,2,3,4,5\}, (R=100, \alpha=2, \sigma_w=0.01, N=50)$

Fig. 4 presents ROC performance as a function of the transmitted power P_0. The result shows that with the increase of P_0, the performance of the algorithm increases. This is expected as high P_0 at the source will make it easier for the algorithm to distinguish between the two hypotheses.

Fig. 5 presents ROC results as the radius R of the deployed field varies. As expected, when the other parameters are fixed,

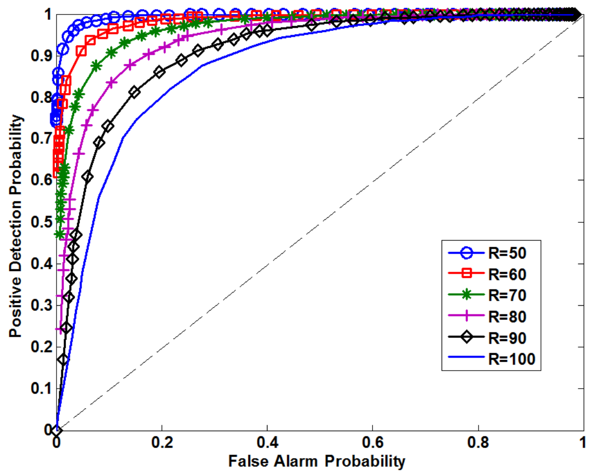

Fig. 5. ROC performance under different configurations of radius $R \in \{50, 60, 70, 80, 90, 100\}$, $(N = 50, \alpha = 2, \sigma_w = 0.01, P_0 = 1)$

the performance of algorithm improves with decreasingly R. This is because the larger R, the signals travel larger distances and as a result decay more, which makes it harder for GW to make a correct decision.

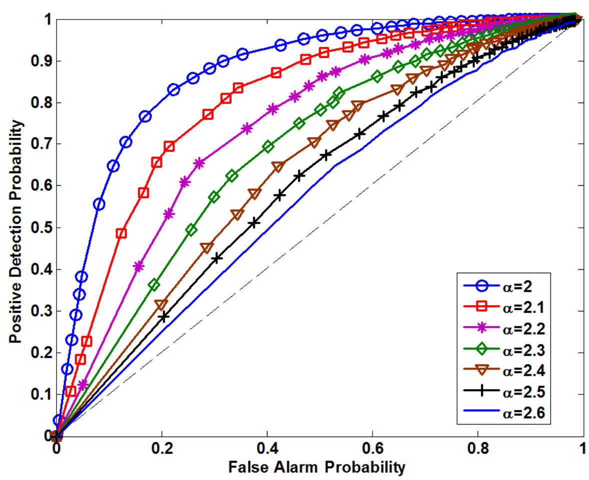

Fig. 6. ROC performance under different configurations of propagation loss coefficient $\alpha \in \{2, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6\}$, $(R = 100, N = 50, \sigma_w = 0.01, P_0 = 1)$

Fig. 6 presents ROC results as the radius propagation loss coefficient α varies. When the other parameters are fixed, the performance of algorithm improves with decreasingly α. This is because the smaller α, the stronger the signal the sensor observes, which makes it easier for GW to make a correct decision.

V. Conclusion

We presented a new and novel event detection algorithm in sensor networks where the spatial deployment of the sensors is random. Our algorithm is based on the Gram-Charlier series expansion to approximate the intractable marginal likelihood under the alternative hypothesis and obtain the optimal likelihood ratio test. Through extensive simulations we demonstrated accuracy of the Gram-Charlier series and the detection performance of the algorithm under various scenarios.

Aacknowledgment

This work was carried out while Zhang was a postgraduate student at Nanyang Technological University sponsored by A*STAR's Graduate Scholarship (AGS) from September 2010 and August 2014.

Appendix A
Proof of Theorem 1

Proof: Let $Z_k(s) = \phi(r) = r^{-\alpha/2}$, $y \in (0, \infty)$.

According to [22], if $z = \phi(x)$ is differentiable, then the PDF of the random variable z is given by $g(z) = -f\left(\phi^{-1}(z)\right)\frac{d\phi^{-1}(z)}{\mathrm{d}z}$, if $\frac{\mathrm{d}\phi^{-1}(x)}{\mathrm{d}x} < 0$.
It is easy to see that for the transformation we consider, the derivative is negative, as follows: $\phi^{-1}(x) = x^{-2/\alpha}$. $\frac{d\phi^{-1}(x)}{\mathrm{d}x} = -\frac{2}{\alpha}x^{-2/\alpha-1} < 0$.

By applying the change of variable rule, we obtain: let $z_k = r^{-\alpha/2}$, $\phi(z_k) = z_k^{-\alpha/2}$. Then:

$$\phi^{-1}(z_k) = z_k^{-2/\alpha},\ \frac{\mathrm{d}\phi^{-1}(z_k)}{\mathrm{d}z_k} = -\frac{2}{\alpha}z_k^{-2/\alpha-1}$$

Therefore,

$$\begin{aligned}
f(Z_k(s)) &= -f(\phi^{-1}(Z_k(s)))\frac{\mathrm{d}\phi^{-1}(Z_k(s))}{\mathrm{d}Z_k(s)} \\
&= -C\beta\left(\frac{(Z_k(s)^{-2/\alpha})^2}{R^2}; k+\frac{1}{2}, N-k+1\right)\left(-\frac{2}{\alpha}\right)Z_k(s)^{-2/\alpha-1} \\
&= C\beta\left(\frac{Z_k(s)^{-4/\alpha}}{R^2}; k+\frac{1}{2}, N-k+1\right)z_k^{-2/\alpha-1}
\end{aligned}$$

where

$$C = -\frac{4}{R\alpha}\frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})}$$

■

Appendix B
Proof of Theorem 2

Proof: Let $x = \frac{Z_k(s)^{-4/\alpha}}{R^2}$, $Z_k(s) = (xR^2)^{-\frac{\alpha}{4}}$.

Let $C = -\frac{4}{R\alpha}\frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})}$. Then $\mathbb{E}[Z_k(s)^m]$ can be expressed in (7)

$$\begin{aligned}
C_2 &= \frac{4}{R\alpha}\frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})}\frac{R^{(1-\frac{\alpha}{2}m)(-\frac{\alpha}{4})}}{B(k+\frac{1}{2}, N-k+1)} \\
&= \frac{4}{R\alpha}\frac{\Gamma(k+\frac{1}{2})\Gamma(N+1)}{\Gamma(k)\Gamma(N+\frac{3}{2})}\frac{R^{(1-\frac{\alpha}{2}m)(-\frac{\alpha}{4})}\Gamma(N+1.5)}{\Gamma(k+0.5)\Gamma(N-k+1)} \\
&= R^{-\frac{\alpha}{2}m}\frac{\Gamma(N+1)}{\Gamma(k)\Gamma(N-k+1)}
\end{aligned}$$

$$
\begin{aligned}
\mathbb{E}[Z_k(s)^m] &= \int C\beta(x;k+\frac{1}{2},N-k+1)(xR^2)^{-\frac{\alpha}{4}(-\frac{2}{\alpha}-1)}(xR^2)^{-\alpha/4m}\frac{\mathrm{d}(xR^2)^{-\alpha/4}}{\mathrm{d}x}\mathrm{d}x \\
&= C(R^2)^{\frac{1}{2}+\alpha/4}(R^2)^{-4/\alpha m}(R^2)^{-\alpha/4}(-\alpha/4)\int \beta(x;k+\frac{1}{2},N-k+1)(x)^{\frac{1}{2}+\frac{\alpha}{4}}x^{-\alpha/4m-\alpha/4-1}\mathrm{d}x \\
&= CR^{2(\frac{1}{2}-\frac{\alpha}{4}m)(-\alpha/4)}\int \beta(x;k+\frac{1}{2},N-k+1)(x)^{\frac{1}{2}+\frac{\alpha}{4}}x^{-\alpha/4m-\alpha/4-1}\mathrm{d}x \\
&= C_1\int \beta(x;n+\frac{1}{2},N-k+1)x^{-\frac{1}{2}-\frac{\alpha}{4}m}\mathrm{d}x \\
&= C_1\frac{1}{B(k+\frac{1}{2},N-k+1)}\int_0^1 x^{k-\frac{1}{2}}(1-x)^{N-k}x^{-\frac{1}{2}-\frac{\alpha}{4}m}\mathrm{d}x \\
&= C_2\int_0^1 x^{k-1-\frac{\alpha}{4}m}(1-x)^{N-k}\mathrm{d}x \\
&= C_2B(k-\frac{\alpha}{4}m,N-k+1)
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\mathbb{E}[Z_k(s)^m] &= R^{-\frac{\alpha}{2}m}\frac{\Gamma(N+1)}{\Gamma(k)\Gamma(N-k+1)} \\
&\times \frac{\Gamma(k-\frac{\alpha}{4}m)\Gamma(N-k+1)}{\Gamma(N-\frac{\alpha}{4}m+1)} \\
&= R^{-\frac{\alpha}{2}m}\frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}
\end{aligned}
$$

■

APPENDIX C
PROOF OF THEOREM 3

Proof: The MGF for function $f(x)$ is given by the following series expansion:

$$
\begin{aligned}
&M_X(t) = \mathbb{E}\left[e^{tX}\right] \\
&= 1 + t\mathbb{E}\left[X\right] + \frac{t^2\mathbb{E}\left[X^2\right]}{2!} + \frac{t^3\mathbb{E}\left[X^3\right]}{3!} + \cdots + \frac{t^n\mathbb{E}\left[X^n\right]}{n!} \\
&= 1 + tm_1 + \frac{t^2m_2}{2!} + \frac{t^3m_3}{3!} + \cdots + \frac{t^nm_n}{n!} + \cdots .
\end{aligned}
$$

Using the result in Theorem 2 we can now express the MGF for $M_{Z_k(s)}\left(t\right)|\mathbf{X}_s,\mathcal{H}_1$ as follows

$$
M_{Z_k(s)}\left(t\right)|\mathbf{X}_s,\mathcal{H}_1 = 1 + \sum_{m=1}^{\infty}\frac{t^m}{m!}R^{-\frac{\alpha}{2}m}\frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}.
$$

■

APPENDIX D
PROOF OF COROLLARY 1

Proof:

Due to the independence of $Z_k(s)$ for $1 \le k \le N$, we have:

$$
\begin{aligned}
&M_V(t) = M_{Z_1}(t)M_{Z_2}(t)\cdots M_{Z_N}(t) \\
&= \prod_{k=1}^{N}\left(1 + \sum_{m=1}^{\infty}\frac{(\sqrt{P_0}t)^m}{m!}R^{-\frac{\alpha}{2}m}\frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}\right)
\end{aligned}
$$

Similarly, we find the Moment Generating Function for $y(n)|\mathcal{H}_1$ denoted as $M_Y(t)$.

Since $W(s)$ is i.i.d. Gaussian distribution, the Moment Generating Function for $W(s)$ is:

$$
M_{W_s}(t) = \mathrm{e}^{\frac{1}{2}\sigma^2_{w(s)}t^2}
$$

where $\sigma^2_{w(s)} = \sum_{k=1}^{N}\sigma^2_{w_k(s)}$. Due to the independence of $V(s)$ and $W(s)$, we have:

$$
\begin{aligned}
&M_{Y(s)}(t) \\
&= M_{V(s)}(t)M_{W(s)}(t) \\
&= \prod_{k=1}^{N}\left(1 + \sum_{m=1}^{\infty}\frac{(\sqrt{P_0}t)^m}{m!}R^{-\frac{\alpha}{2}m}\frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}\right)\mathrm{e}^{\frac{1}{2}\sigma^2_wt^2}
\end{aligned}
$$

■

APPENDIX E
PROOF OF LEMMA 1

Proof: We first derive the cumulant generating function $g(t)$.

$$
\begin{aligned}
g(t) &= \log M_{Y(s)}(t) \\
&= \sum_{k=1}^{N}\log\left(1 + \sum_{m=1}^{\infty}\frac{(\sqrt{P_0}t)^m}{m!}R^{-\frac{\alpha}{2}m}h(k)\right) + \frac{1}{2}\sigma^2_wt^2,
\end{aligned}
$$

where

$$
h(k) := \frac{\Gamma(N+1)\Gamma(k-\frac{\alpha}{4}m)}{\Gamma(k)\Gamma(N-\frac{\alpha}{4}m+1)}.
$$

Next we obtain the first cumulant κ_4 as follows:

$$
\begin{aligned}
&\kappa_1 = \frac{\mathrm{d}g(t)}{\mathrm{d}t}|_{t=0} \\
&= \sum_{k=1}^{N}\frac{\frac{\sqrt{P_0}t^0}{1}R^{-\frac{\alpha}{2}}h(k) + \sum_{m=2}^{\infty}m\frac{(\sqrt{P_0})^mt^{m-1}}{m!}R^{-\frac{\alpha}{2}m}h(k)}{1 + \sum_{m=1}^{\infty}\frac{(\sqrt{P_0}t)^m}{m!}R^{-\frac{\alpha}{2}m}h(k)}|_{t=0} \\
&+ \sigma^2_wt|_{t=0} \\
&= \sum_{k=1}^{N}\sqrt{P_0}R^{-\frac{\alpha}{2}}h(k) \\
&= \sum_{k=1}^{N}\sqrt{P_0}\bar{Z}_{k1}.
\end{aligned}
$$

Similarly, by taking the second, third and forth derivative of $g(t)$. We next find the κ_2, κ_3 and κ_4.

$$\kappa_2 = \sum_{k=1}^{N} P_0 \left(\bar{Z}_{k2} - \bar{Z}_{k1}^2\right) + \sigma_W^2.$$

$$\kappa_3 = \sum_{k=1}^{N} \sqrt{P_0}^3 \left(\bar{Z}_{k3} - 3\bar{Z}_{k2}\bar{Z}_{k1} + 2\bar{Z}_{k1}^3\right).$$

$$\kappa_4 = \sum_{k=1}^{N} P_0^2 \left(\bar{Z}_{k4} - 4\bar{Z}_{k3}\bar{Z}_{k1} - 3\bar{Z}_{k2}^2 + 12\bar{Z}_{k2}\bar{Z}_{k1}^2 - 6\bar{Z}_{k1}^4\right),$$

where $\bar{Z}_{ki}$ for $i = 1, 2, 3, 4$ is the i^{th} moment of $Z_k(s)$ given in Theorem 2. ■

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