Framework for Performance Analysis of Channel-aware Wireless Schedulers

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Abstract

Although many wireless channel-state dependent (CSD) schedulers have been proposed recently, their contributions lie in the design of the scheduling mechanism to meet some performance objectives. However, these objectives are often first-order statistics such as average or worst-case delay, which are insufficient to characterize the scheduler's performance. In this paper, we propose a matrix formulation to derive the delay probability density function for CSD schedulers over a Markovian wireless channel. Our analysis is then used to determine the admissibility of a wireless scheduler in terms of a minimum throughput requirement and a real-time QoS requirement. In addition, we evaluate the buffer size requirement of the wireless receiver and highlight the trade-off between buffer size requirements and channel efficiency.

1 Introduction

We consider the scenario as depicted in Fig. 1, where fixed-size packets from K flows (assumed to be always backlogged) have to be transmitted to K corresponding wireless receivers via a wireless media. The wireless scheduler allocates channel access in terms of fixed-size time slots corresponding to the transmission time of one packet. The design of the wireless scheduling mechanism is important for:

(a) Wireless application development, since it determines the Quality of Service(QoS), such as throughput and delay guarantees, as well as the *fairness* level that can be supported, and

(b) Wireless receiver design, since it determines the buffer requirement at each wireless receiver, which is limited due to size and processing power constraints of portable wireless devices.

Most of the prior work on performance analysis of wireless schedulers have dealt with (a), while little attention has been focused on (b). Hence, we focus on a performance analysis framework that enables us to study the trade-offs between buffer, QoS and fairness requirements in wireless scheduler design.

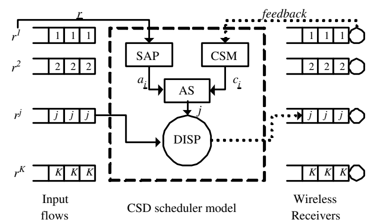


Figure 1: Wireless scheduling scenario

1.1 Related Work

In [1], the authors studied the delay performance of a simple ARQ error control strategy for communications over a bursty channel for a *single* flow. In [2], the author investigated the characteristics and traffic effects of variable-rate communication servers. However, the scheduling policy considered is not *channel-aware* since the channel is assumed to be location-independent. Channel-awareness is considered in the resource allocation problem in [3], where the authors characterized the stability properties of the system and proposed an optimal allocation policy that maximizes throughput and minimizes delay. However, the results apply only for uncorrelated channels, which is an impractical assumption for wireless channels.

Since wireless receivers are typically sufficiently separated spatially, it is reasonable to assume that the channel state of different flows are independent. Hence, it is highly likely that at least one flow with an error-free channel exists at any instant. The notion of channel-state dependence (CSD) or awareness was introduced in [4] to exploit this observation to improve the *channel efficiency* of wireline schedulers when deployed in a wireless media. A comprehensive survey of CSD schedulers can be found in [5, 6, 7, 8], where the contribution lies in the design of the wireless scheduling mechanism to achieve QoS and fairness.

Although the QoS performance has been analyzed and compared amongst recently proposed CSD schedulers [6], the metrics used are first order, e.g., average and worst-case Head-of-Line (HOL) packet delay, which are inadequate to characterize each scheduler's QoS capability. In fact, the evaluation of the second moment of delay is necessary to compute the required wireless receiver buffer size to maintain an acceptable packet dropping ratio [9].

1.2 Contributions of This Paper

Hence, we adopt a stochastic analysis approach (similar to [1]) to derive the probability density function (pdf) of the HOL packet delay for a CSD scheduler under a Markovian wireless channel. In [10], we verified that the stationary HOL packet delay pdf exists under a *homogeneous* scheduling scenario. In this paper, we extend the analysis and propose a performance analysis framework for more general scheduling scenarios. This enables useful performance metrics to be derived and hence represents a more complete characterization of the scheduler's performance.

The rest of the paper is organized as follows: In Section 2, we define the scheduling scenario. We then outline our matrix formulation to derive the HOL packet delay pdf of CSD schedulers in Section 3. Based on our analysis, we present some numerical results comparing the performance of variants of CSD schedulers in Section 4. Concluding remarks and possible extensions to the formulation are presented in Section 5.

2 Scheduling Scenario

For simplicity of notations, for any discrete variable x_i^j, the superscript j and subscript i always correspond to the *flow* and *slot* indices respectively. In addition, we use $p_x(X)$, $E[x]$ and $Var[x]$ to denote the pdf, mean and variance of x respectively.

We consider the scheduling scenario as depicted in Fig. 1. Each flow j is characterized by an integer parameter, r^j, such that $\frac{r^j}{R}$ denotes the fraction of slots that should be allocated to that flow (i.e., fairness), where $R = \sum_{m=1}^{K} r^m$.

2.1 Wireless Channel Model

We consider a Two-State Markovian channel model where $c_i^j \in \{0,\ 1\}$ denotes the *per-flow* channel state variable. Such a model can be characterized by $(p_{c^j}(1|0), p_{c^j}(0|1))$, where

$$p_{c^j}(x|y) \quad = \quad \text{Prob}(c_i^j{=}\text{x} \mid c_{i-1}^j{=}\text{y})$$

Hence, $p_{c^j}(C)$ is given as follows:

$$p_{c^j}(C) \quad = \quad \begin{cases} \frac{p_{c^j}(0|1)}{p_{c^j}(1|0)+p_{c^j}(0|1)}, & C=0; \\ \frac{p_{c^j}(1|0)}{p_{c^j}(0|1)+p_{c^j}(1|0)}, & C=1. \end{cases}$$

We define the decimal equivalent of the binary sequence $c_i^K c_i^{K-1} \cdots c_i^1$ (denoted by $\overline{c}_i^K$) as the *ensemble* channel state variable, with state space given by $\{0, 1, 2, \cdots 2^K-1\}$. Therefore, the corresponding state transition probability matrix, $\underline{\underline{C}}^K$, is of dimensions $2^K \times 2^K$ and can be computed, for $K \geq 2$, using the following recurrence relation:

$$\underline{\underline{C}}^K \quad = \quad \begin{bmatrix} \underline{\underline{C}}^{K-1} \cdot p_{c^K}(0|0) & \underline{\underline{C}}^{K-1} \cdot p_{c^K}(1|0) \\ \underline{\underline{C}}^{K-1} \cdot p_{c^K}(0|1) & \underline{\underline{C}}^{K-1} \cdot p_{c^K}(1|1) \end{bmatrix}$$

where

$$\underline{\underline{C}}^1 \quad = \quad \begin{bmatrix} p_{c^1}(0|0) & p_{c^1}(1|0) \\ p_{c^1}(0|1) & p_{c^1}(1|1) \end{bmatrix}$$

If we define $\underline{f}_i = [p_{\overline{c}_i^K}(C)]_{C=0}^{2^K-1}$, then, for any $N{>}0$, we have:

$$\underline{f}_{i+N} \quad = \quad \underline{f}_i \times \prod_{u=1}^{N} \underline{\underline{C}}^K \tag{1}$$

2.2 CSD Scheduler Model

We consider a generic CSD scheduler model (similar to the one defined in [4]) that comprises a slot allocation policy (SAP), a channel status monitor (CSM), an arbitration scheme (AS) and a packet dispatcher (DISP), as shown in Fig. 1. The SAP allocates each slot i to flow a_i to achieve some performance in terms of QoS and buffer requirements in addition to fairness under error-free conditions. Loop schedulers of size R (i.e., $a_{i+R}{=}a_i$) are good choices since they guarantee fairness over any interval of length R and are simple to implement. In addition, they can also be designed to minimize the HOL delay variation [11].

The CSM maintains $\{c_{i-m}^j, m > 0\}_{j=1}^K$ and uses this information to predict the current channel state, $\hat{c}_i^{a_i}$. We consider one-step predictors where $\hat{c}_i^{a_i}$ is a function of $c_{i-1}^{a_i}$ only. If $\hat{c}_i^{a_i}{=}0$, the DISP dispatches the HOL packet of flow a_i for transmission; otherwise, the AS selects an alternative flow for transmission.

3 Performance Analysis of Channel-State Dependent Schedulers

In this section, we outline a matrix formulation to derive $p_{n^f}(N)$, where n^f denotes the HOL packet delay of flow f. Let $S_{a_i}^f$ ($F_{a_i}^f$) denote a **S**uccessful (de**F**erred or **F**ailed) transmission of flow f in a slot allocated to flow a_i. The probability of occurrence of $S_{a_i}^f$ is determined by the AS, the values of $(\overline{c}_{i-1}^K, \overline{c}_i^K)$ and i. Conversely stated, given i and the AS, the occurrence of $S_{a_i}^f$ imposes a constraint on $[p_{\overline{c}_{i-1}^K}(C)]_{C=0}^{2^K-1}$

and $[p_{\overline{c}_i^K}(C)]_{C=0}^{2^K-1}$. Hence, we define the *constrained state transition matrix* for event $S^f_{a_i}$ as follows:

$$\underline{\underline{S}}^f_{a_i} = \underline{\underline{D}}_{i-1}(S^f_{a_i}) \times \underline{\underline{C}}^K \times \underline{\underline{D}}_i(S^f_{a_i})$$

where $\underline{\underline{D}}_x(S^f_{a_i})$ is a diagonal matrix such that the diagonal element of row m is the probability that $S^f_{a_i}$ will occur if $\overline{c}_x^K$=m-1. Since the events $S^f_{a_i}$ and $F^f_{a_i}$ are complementary,

$$\underline{\underline{S}}^f_{a_i} + \underline{\underline{F}}^f_{a_i} = \underline{\underline{C}}^K$$

Hence, $\underline{\underline{F}}^f_{a_i}$ can be evaluated from $\underline{\underline{S}}^f_{a_i}$ and $\underline{\underline{C}}^K$.

If we define the *constrained* pdf of the channel state as follows:

$$\underline{f}(E^f_{a_i}) = [\text{Prob}(\overline{c}_i^K = C, E^f_{a_i} \text{occurs})]_{C=0}^{2^K-1}$$

where $E \in \{S, F\}$. Then Eq. (1) can be written as follows *:

$$\underline{f}(\{E^f_{a_u}\}_{u=i}^{i+N}) = \underline{f}(E^f_{a_i}) \times \prod_{u=i+1}^{i+N} \underline{\underline{E}}^f_{a_u}$$

from which we have

$$\begin{aligned}\text{Prob}(\{E^f_{a_u}\}_{u=i}^{i+N} \text{ occurs} \mid i) &= \sum_{C=0}^{2^K-1} \underline{f}(\{E^f_{a_u}\}_{u=i}^{i+N}) \\ &= \underline{f}(E^f_{a_i}) \times \prod_{u=i+1}^{i+N} \underline{\underline{E}}^f_{a_u} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\end{aligned}$$

Un-conditioning on i, we have the following:

$$\text{Prob}(\{E^f_{a_u}\}_{u=i}^{i+N} \text{ occurs}) = \sum_{i=1}^{R} \underline{f}(E^f_{a_i}) \times \prod_{u=i+1}^{i+N} \underline{\underline{E}}^f_{a_u} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

If $\{E^f_{a_u}\}_{u=i}^{i+N} = \{S^f_{a_i}, \{F^f_{a_{i+u}}\}_{u=1}^{N-1}, S^f_{a_{i+N}}\}$, then $p_{n^f}(N)$ can be evaluated as follows:

$$p_{n^f}(N) = \sum_{i=1}^{R} \underline{f}(S^f_{a_i}) \times \prod_{u=i+1}^{i+N-1} \underline{\underline{F}}^f_{a_u} \times \underline{\underline{S}}^f_{a_{i+N}} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

where $\{\underline{f}(S^f_{a_i})\}_{i=1}^{R}$ can be evaluated based on a recurrence relation in terms of $\{\underline{\underline{S}}^f_{a_i}\}_{i=1}^{R}$, which can in turn be evaluated given the AS. Details of these evaluations can be found in [9].

*Note that the notation $\prod_a^b$ refers to a sequence of matrix products in the order a,a+1,a+2,$\cdots$ b.

4 Numerical Results

We shall present some numerical results for a homogeneous K-flow scheduling scenario, i.e., r^j=1 (rate-homogeneous) and $(p_{c^j}(1|0), p_{c^j}(0|1)) = (p_c(1|0), p_c(0|1))$ (channel-homogeneous) respectively for 1≤ $j \leq K$. In addition, we assume that (a) the channel is uncorrelated, i.e., $p_c(x|y) = p_c(x)$, (b) channel prediction is perfect, i.e., $\hat{c}_i^j = c_i^j$ and (c) the transmission of flow j is always (never) successful when c_i^j = 0(1).

With rate-homogeneity, a trivial choice for the SAP is a Round-Robin scheduler. We consider two variants of AS: with *uniform* arbitration, (CSD_{UA} Scheduler), a flow m is *randomly* selected; with *prioritized* arbitration ($CSD_{PA}(P_h)$ Scheduler), *preference* for selection is given to flows whose next allocation (according to the SAP) is as *far* away as possible (bounded by P_h) from the current slot; a value of 0 restricts the transmission to the allocated flow alone while a value of P_{max} permits the selection of all flows.

4.1 Admissibility of Wireless Scheduler based on Efficiency Requirement

The throughput of each flow is given by the reciprocal of $E[n]$. Hence, the minimum overall throughput requirement, η_{min}, is related to $E[n]$ as follows:

$$E[n] \leq \frac{K}{\eta_{min}} \qquad (2)$$

We substitute the expressions of *E[n]* for each wireless scheduler [9] into Eq. (2) and illustrate the constraint graphically in Fig. 2(a) with η_{min}= 0.80.

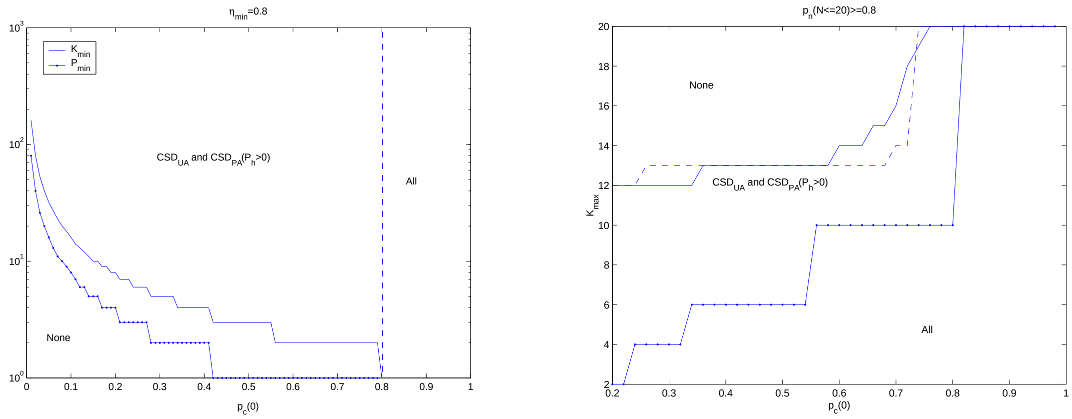
We can partition the operating region into three sub-regions. The region denoted by *All*, given by $p_c(0) \geq \eta_{min}$, indicates that all schedulers can be deployed for any K while satisfying the throughput constraint. On the other extreme, the region denoted by *None*, where $K < K_{min}$ for CSD_{UA} and $P_h < P_{min}$ for $CSD_{PA}(P_h)$, indicates that none of the schedulers can satisfy the throughput constraint. The remaining region stipulates the requirements on K and P_h for CSD schedulers to satisfy the requirement. Hence, given η_{min}, K and $p_c(0)$, we can determine which of the scheduler(s) are admissible with respect to η_{min}.

4.2 Admissibility of Wireless Scheduler for Real-Time Applications

For real-time applications, we can specify the QoS requirement as follows:

$$p_n(N > N_{max}) \leq \alpha$$

where N_{max} is the HOL packet delay bound and α is the tolerable delay violation probability. This imposes



(a) (b)

Figure 2: Operating Regions for various schedulers to satisfy (a) $\eta_{min} = 0.8$ and (b) $p_n(N \leq 20) \geq 0.8$

a constraint on the maximum number of flows, K_{max}, that can be supported for each scheduler for a given value of $p_c(0)$. We illustrate this constraint for $N_{max} = 20$ and $\alpha = 0.2$ in Fig. 2(b).

As with the efficiency constraint, we can partition the operating region into three sub-regions. All the schedulers can be deployed while satisfying the real-time QoS constraint as long as $(K, p_c(0))$ falls within the region denoted by *ALL*; on the other extreme, none of the schedulers can be deployed if $(K, p_c(0))$ falls within the region denoted by *NONE*. Only the CSD_{UA} or $CSD_{PA}(P_h > 0)$ can be deployed if $(K, p_c(0))$ falls within the remaining region. Hence, given $(K, p_c(0))$, we can determine which of the scheduler(s) are admissible with respect to the real-time QoS requirement.

4.3 Comparison of wireless receiver buffer size requirements, B

Under high load conditions and assuming zero propagation delay in the wireless media, B can be approximated as follows [9]:

$$B \approx \frac{\left\lceil \frac{\ln \beta}{\ln(1 - \frac{1}{E[w]})} - 1 \right\rceil}{S}$$

where

$$E[w] = \frac{Var[n]}{2(E[n] - S)}$$

S is the constant wireless receiver service time, β is the acceptable packet dropping ratio, $\rho = \frac{S}{E[n]}$ and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. We plot B as a function of K for each scheduler for $\beta = 0.1$ with (a) $p_c(0) = 0.9$ and (b) $p_c(0) = 0.5$ in Fig. 3.

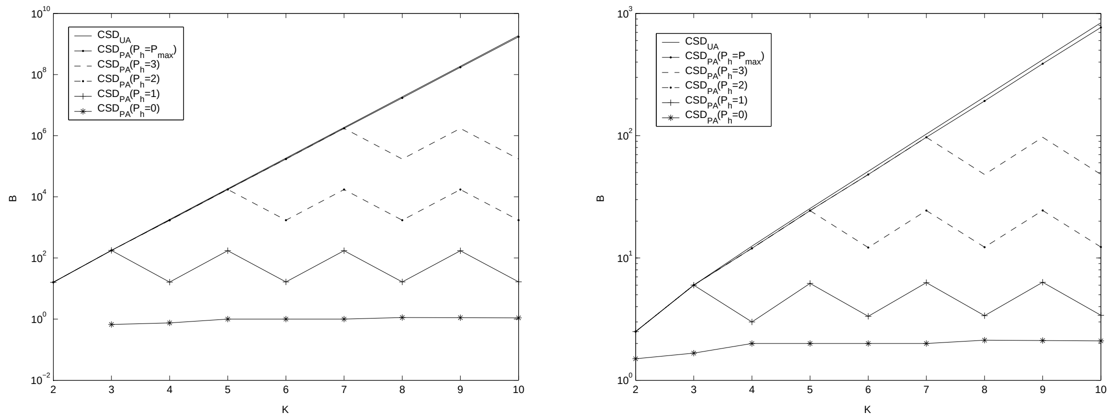
Although the CSD_{UA} and $CSD_{PA}(P_h = P_{max})$ achieve the same channel efficiency, the latter requires a marginally smaller buffer size to sustain the required packet loss rate. The buffer requirement increases as K increases. This is because the AS has a larger pool of eligible flows to choose from for transmission, and therefore, the delay variation is increased.

However, as P_h is reduced, the buffer requirement increases initially with K until $\frac{K}{2} = P_h$ for even K or $\frac{K-1}{2} = P_h$ for odd K. A further increase in K does not increase the pool of eligible flows for transmission and hence, the buffer requirement levels off. This 'levelling' off occurs at smaller values of K as P_h is reduced. However, this reduction in buffer requirement is traded-off with a corresponding reduction in channel efficiency.

5 Conclusions and Future Work

In this paper, we developed a matrix formulation to derive the packet delay probability density function for a generic channel-state dependent (CSD) scheduler operating under a Markovian wireless channel. The scheduling model can be abstracted in terms of the mechanism of error-free scheduling, the channel prediction scheme as well as the choice of the 'instantaneous' best flow (arbitration scheme) to transmit given the predicted channel information.

Our analysis can be used to derive useful performance metrics for CSD schedulers in addition to typical first-order metrics such as throughput guarantees. This is illustrated through numerical results, where we evaluated the admissibility of a wireless scheduler under an efficiency constraint as well as a real-time QoS constraint. In addition, we also evaluated the buffer size requirement at each wireless receiver, and high-



(a) (b)

Figure 3: Wireless Receiver Buffer Requirement, B, of various schedulers for (a) $p_c(0) = 0.9$ and (b) $p_c(0) = 0.5$

lighted the trade-offs with channel efficiency and QoS performance.

We have omitted the consideration of fairness in the numerical results since rate-homogeneous CSD schedulers are long-term fair. However, this no longer holds for rate-heterogeneous CSD scheduling. Although the fairness module [10] corrects for this unfairness, the analysis of such a model is complex for $K>2$. Hence, we are looking into alternative ways of unfairness correction in CSD schedulers and the possible trade-offs with other performance metrics.

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