A Wide Coverage Positioning System (WPS) for Underwater Localization

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***Abstract*—Underwater localization is challenging as its efficacy is affected by propagation delays, motion-induced doppler shift, phase and amplitude fluctuations, multipath interference etc that are inherent in underwater acoustic channels. In this paper, we consider a recently proposed Underwater Positioning Scheme, which offers unique localization only in a *finite* region. We quantify the conditions for unique localization and propose a variant that offers unique localization with high probability regardless of the reference and unknown node deployment. We demonstrate the trade-offs between both schemes in terms of localizability space, localization latency and energy consumption.**

I. Introduction

Underwater acoustic sensor networks (UASNs) are envisaged to fulfill the needs of a multitude of applications such as early warning systems for natural disasters, ecosystem monitoring and military surveillance. The data derived from UASNs is typically interpreted with reference to a sensor's location, e.g., reporting an event occurrence, tracking a moving object or monitoring a region's physical conditions.

Location discovery for underwater sensors is non-trivial in the oceanic medium as its efficacy is impacted by propagation delays, motion-induced doppler shift, phase and amplitude fluctuations, multipath interference etc. Moreover, since GPS signals do not propagate through water, underwater nodes need to rely on position references for localization, obtained either through spatial (multiple, fixed references) or time diversity (single, mobile reference). Since underwater acoustic devices are expensive and deployment is costly [1], we expect UASNs to be deployed for long durations.

Underwater Positioning System (UPS) [2] is a promising underwater localization scheme as it (i) requires no time synchronization, (ii) provides silent positioning, (iii) has low computation overhead and (iv) exhibits low positioning error. However, it suffers from the following drawbacks: (i) it relies on reactive beaconing from a fixed set of reference nodes; (ii) its feasible space is finite, i.e., *blind spots* exist where nodes within coverage area cannot be uniquely localized, and (iii) it assumes that the reference nodes cover (in terms of communication range) the entire network, thus limiting the area of interest. In [3], we illustrated the impact of (i) under realistic underwater channel conditions and proposed an Enhanced UPS scheme (E-UPS) to address this deficiency. In this paper, we propose a Wide Coverage Positioning System (WPS) to address the limitations of feasible space with UPS. The constraints imposed by finite communication range are not the focus of this paper, and can be overcome with multi-stage [4] and hierarchical localization [5], [6].

The paper is organised as follows: we review related works on underwater localization in Section II. We describe the generalized UPS algorithm, highlight the issue of finite feasibility region in the original UPS algorithm, and how we address this in WPS in Section III and IV respectively. We compare their performance numerically in Section V and provide some concluding remarks and directions for future research in Section VI.

II. Related Work

While earlier underwater localization techniques have been classified as infrastructure-based vs infrastructure-less [7], recent techniques can be further categorized as single-stage [2], [8]–[10] vs hierarchical / multi-stage [4]–[6].

In infrastructure-based localization, the location of a node is estimated by exchanging beaconing signals with reference nodes that are deployed on surface buoys (localized using GPS) or at pre-determined locations on the seabed, where d + 1 references are needed to localize in d-dimensional space. In [4], the authors propose a purely distributed localization framework that transforms the 3D underwater positioning problem into its 2D counterpart. In [5], the authors divide the localization process into anchor node and ordinary node localization and propose a distributed localization scheme that integrates 3D Euclidean distance estimation with a recursive location estimation method. This method is enhanced in [6] by introducing mobility prediction based on the predictable mobility patterns of underwater objects.

Infrastructure-less localization is usually implemented by using mobile beacon(s). In [8], the authors propose Dive-and-rise beacons that get their coordinates from GPS while floating above water, and broadcast their positions while sinking and rising. The multi-stage extension of this approach for large-scale networks is given in [11]. The need for synchronization amongst nodes with the above approaches is eradicated with AUV-aided localization using omnidirectional [9] or directional antennae [10].

III. Generalized UPS : **UPS**(N)

In this section, we begin by generalizing the concept of UPS [2] to N reference nodes, where our goal is to determine the location (x,y,z) of a sensor node s, given the location (x_j, y_j, z_j) of reference node R_j, $1 \leq j \leq N$. We denote the scheme by **UPS**(N), and let d_{sj} and d_{ij} be the Euclidean distance between s and R_j and R_i and R_j respectively.

Initially, node s sends a short beacon to wake up the reference nodes. Those that hear this beacon (i.e., that are within communication range of, and maintains a good communication link with, s) will respond with their ID, coordinates, the arrival time of s's wake-up beacon and the transmission time of the response beacon. Let R denote this set of reference nodes. We assume that each beaconing signal comprises a fixed-size packet, and the probability of packet transmission failure is p.

Node s then computes the beaconing sequence according to the order in which it receives the responses from R, notifies the reference nodes and starts the timer. Upon receiving the beaconing sequence, the reference nodes will execute the following procedures:

A. Step 1: Range Difference Computation

When R_1 (master node) receives the notification from s, it initiates a beacon signal at $t_1^{'}$. Let $t_{s,1}$, t_j, be the times when s and the reference nodes R_j, $j = \{2, \cdots, N\}$ receive R_1's signal. After some processing delay, δ_2, at time $t_2^{'}$, R_2 replies to R_1 with a beacon signal conveying $t_2^{'} - t_2 = \Delta t_2$ to s. The signal reaches s at $t_{s,2}$. After receiving beacon signals from R_1 and R_2, at time $t_3^{'}$, R_3 replies to R_1 with a beacon signal conveying information $t_3^{'} - t_3 = \Delta t_3$ to s. The signal reaches s at time $t_{s,3}$. In a similar way, R_j, $j = \{4, 5, \cdots, N\}$ will convey information Δt_j to s. Note, however, that for $j \geq 4$, R_j will transmit a beacon as long as it successfully receives the beacon from R_1 and R_{j-1}. This procedure is illustrated in Fig. 1.

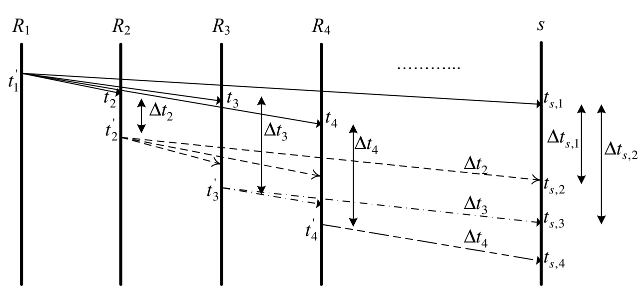

Fig. 1. Illustration of Generalised UPS.

Let v be the speed of sound underwater and let $\Delta t_{s,k} = t_{s,k+1} - t_{s,1}$, for $k = \{1, \cdots, N-1\}$. For $j = \{2, 3, \cdots, N\}$, we have

$$d_{1j} + d_{sj} - d_{s1} + v\Delta t_j = v\Delta t_{j-1},$$

which gives

$$d_{sj} = d_{s1} + k_{j-1}, \tag{1}$$

where $k_{j-1} = v\Delta t_{j-1} - v\Delta t_j - d_{1j}$.

B. Step 2: Location Computation

Expanding Eqn. (1), we obtain the following system of N equations with unknowns x, y, z and d_{s1}:

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = d_{s1}^2 \tag{2}$$

$$(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2 = (d_{s1} + k_{j-1})^2, \tag{3}$$

where $j = \{2, 3, \cdots, N\}$. Since we need at least 4 equations to solve for 4 unknowns, the necessary condition is $N \geq 4$.

C. Limitations of UPS(4)

Intuitively, the choice of N = 4 offers the best solution since it requires the fewest reference nodes (i.e., lowest infrastructure cost) - this reduces to the Underwater Positioning System [2], which we denote by **UPS**(4). Without loss of generality, let us assume that the four reference nodes are located at (0,0,0), (x_2,0,0), (x_3,y_3,0) and (x_4,y_4,z_4).

From Eqn. (2), and Eqn. (3) for $j = 2$, it follows that

$$x = A_x d_{s1} + B_x,$$

where

$$A_x = -\frac{k_1}{x_2}, \quad B_x = \frac{x_2^2 - k_1^2}{2x_2}.$$

From Eqn. (2), and Eqn. (3) for $j = 2, 3$, it follows that

$$y = A_y d_{s1} + B_y,$$

where

$$A_y = \frac{k_1}{x_2}\frac{x_3}{y_3} - \frac{k_2}{y_3}$$

$$B_y = \frac{x_3^2 + y_3^2 - x_2 x_3 + \frac{x_3 k_1^2}{x_2} - k_2^2}{2y_3}$$

and

$$z = A_z d_{s1} + B_z,$$

where

$$A_z = \frac{k_1}{x_2}\frac{x_4}{y_4} - \frac{k_3}{z_4} - \frac{y_4(\frac{k_1 x_3}{x_2} - k_2)}{y_3 z_4}$$

$$B_z = \frac{x_4^2 + y_4^2 + z_4^2 - x_2 x_4 + \frac{k_1^2 x_4}{x_2} - k_3^2 - \frac{y_4 x_3^2}{y_3}}{2z_4} + \frac{-y_3 y_4 + \frac{x_2 x_3 y_4}{y_3} - \frac{k_1^2 x_3 y_4}{x_2 y_3} + \frac{k_2^2 y_4}{y_3}}{2z_4}.$$

If we now replace in Eqn. (2) the expressions of x,y and z found above, we find that d_{s1} has to satisfy the following second degree equation:

$$d_{s1}^2(\Sigma_A - 1) + 2(A_x B_x + A_y B_y + A_z B_z)d_{s1} + \Sigma_B = 0, \tag{4}$$

where $\Sigma_A = A_x^2 + A_y^2 + A_z^2$ and $\Sigma_B = B_x^2 + B_y^2 + B_z^2$.

Denote by Δ the discriminant of Eqn. (4), where $\Delta > 0$, and let $d_{s1}^{'}$ and $d_{s1}^{"}$ be the two real solutions for d_{s1}. The uniqueness of d_{s1} depends on the value of Σ_A as follows:

$$\begin{aligned} d_{s1}^{'} = \quad & d_{s1}^{"}, \quad \Sigma_A = 1; \\ d_{s1}^{'} \cdot d_{s1}^{"} < \quad & 0, \quad \Sigma_A < 1; \\ d_{s1}^{'} \cdot d_{s1}^{"} > \quad & 0, \quad \Sigma_A > 1. \end{aligned} \tag{5}$$

The conditions stated in Eqn. (5) suggest that if $\Sigma_A > 1$ for a given deployment of reference nodes, and location of s, then node s *cannot* be uniquely localized since $d'_{s1}, d''_{s1} > 0$.

To illustrate the extent of this infeasible region, let us deploy $\{R_i\}_{i=1:4}$ at (0,0,0), (D,0,0), (0,D,0) and (0,0,D) respectively, where $\sqrt{3}D$ is the communication range of each node (including node s). We consider a 3-dimensional space, S, of size $D \times D \times D$ that contains the reference nodes as well as node s. The deployment is shown in Fig. 2.

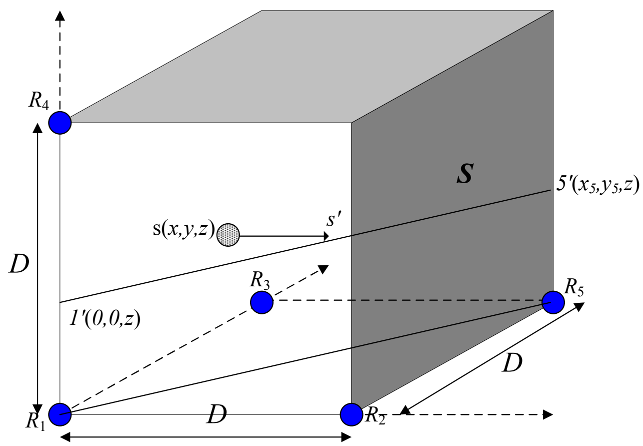

Fig. 2. Deployment of reference nodes and computation of TO_S.

We plot the feasible region for s (shaded), i.e., locations for which s can be uniquely localized in Fig. 3 for z=0, 10 and D, where D = 50. To quantify this, we plot the proportion of feasible region in S as a function of z in Fig. 4 for various values of D. We observe that up to 16% of the plane containing R_1, R_2 and R_3 is not localizable, which is quite significant.

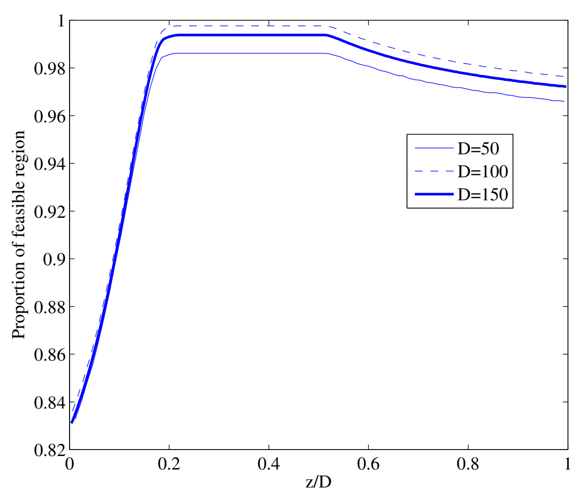

Fig. 4. Proportion of feasible region with **UPS**(4) [p=0].

The problem of infeasible regions has been pointed out in [2], where the authors claim that the correct position for s can be computed as long as it resides in the enclosed space by the four reference nodes, even when it is close to a reference node. According to our investigations, this is not true. For example, in Fig. 3, for z=10, we observe that infeasible locations exist.

D. *UPS(5)*

In this section, we address the limitations of **UPS**(4) by considering a fifth reference node, located at (x_5, y_5, z_5) in the space S. We denote this scheme as **UPS**(5).

Substituting for x,y,z found in Section III-C into Eqn. (3) for $j = 5$, we get:

$$d_{s1}^2(\Sigma_A - 1) + 2(\Sigma_{AB} - k_4)d_{s1} + \Sigma_{BB} - k_4^2 = 0, \quad (6)$$

where B_{aa} = B_a-a_5, $a \in \{x, y, z\}$ and

$$\begin{aligned} \Sigma_{BB} &= B_{xx}^2 + B_{yy}^2 + B_{zz}^2, \\ \Sigma_{AB} &= A_x B_{xx} + A_y B_{yy} + A_z B_{zz}. \end{aligned}$$

Eqn. (4) and (6) have the same solutions if the sum and product of the two solutions are identical. This happens under the following conditions:

$$\begin{aligned} \Sigma_{AB} - k_4 &= A_x B_x + A_y B_y + A_z B_z \\ \Sigma_{BB} - k_4^2 &= \Sigma_B. \end{aligned}$$

The above conditions can be rewritten as follows:

$$\begin{aligned} A_x x_5 + A_y y_5 + A_z z_5 &= k_4 \\ B_x x_5 + B_y y_5 + B_z z_5 &= \frac{x_5^2 + y_5^2 + z_5^2 - k_4^2}{2}. \end{aligned} \quad (7)$$

Suppose $\Sigma_A > 1$, i.e., s cannot be uniquely localized with **UPS**(4). Then, adding R_5 will not achieve unique localization only if the *additional* conditions in (7) are satisfied; otherwise, the value of d_{s1} is given by the solution of the following first degree equation:

$$2d_{s1}(\Sigma_A + k_4) - (d_{05}^2 - 2B_x x_5 - 2B_y y_5 - 2B_z z_5) + k_4^2 = 0, \quad (8)$$

where $d_{05}^2 = x_5^2 + y_5^2 + z_5^2$ and s *can* be uniquely localized with **UPS**(5).

IV. Wide Coverage Positioning System (WPS)

Although **UPS**(5) *achieves* unique localization to node s w.h.p compared to **UPS**(4), it may introduce additional latency and communication costs redundantly in cases where **UPS**(4) suffices. Accordingly, we propose a Wide Coverage Positioning (WPS) system that (i) relies on an infrastructure of 5 reference nodes but (ii) only utilizes beaconing from the fifth reference node when required.

In WPS, the reference nodes perform beaconing according to **UPS**(5), as described in Section III-A. Node s monitors the beacons received from the reference nodes. Upon receiving 4 beacons, it computes Σ_A and checks condition (5): if s cannot be uniquely localized, it will wait for the 5^{th} beacon and check condition (7) before declaring successful localization. The pseudo-code for node s is given in Algorithm 1.

Due to harsh underwater acoustic channel conditions, it is possible that s receives less than 4 beacons, in which case, a time-out will be triggered. The design of the time-out value, TO_S, is based on **UPS**(5) and is described next.

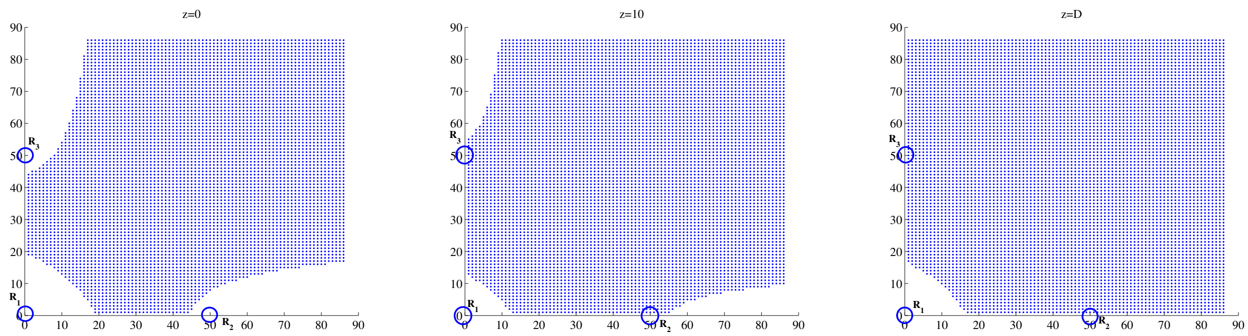

Fig. 3. Feasible region with **UPS**(4) for z=0 (left), z=10 (centre) and z=D (right) (D = 50).

Algorithm 1 WPS: Pseudocode for node s

```
procedure WPS(TO_S)
    t = 0
    rMSG = 0
    LOCALIZED = 0
    Start timer and notify beaconing sequence
    while t < TO_S & LOCALIZED==0 do
        if (receive new beacon) then
            rMSG += 1
        end if
        if (rMSG == 4 & Σ_A ≤ 1) | (rMSG == 5 & condition (7)==FALSE) then
            LOCALIZED=1
        end if
    end while
    if (LOCALIZED==0) then
        node s times-out
    end if
end procedure
```

A. Design of TO_S

Let t_p be the packet transmission time. The processing delays δ_j can be computed at s based on the time-stamps it receives in response to the wake-up beacon.

If all beacon transmissions are successful, the *maximum* localization time is given by:

$$T_0 = \tau_{s,1} + \sum_{i=1}^{4} \tau_{i,i+1} + \tau_{5,s} + \sum_{i=1}^{5} \delta_i + \delta_s + 6t_p,$$

where $\tau_{a,b}$ is the propagation delay incurred for sending a message from node a to node b. By projecting s onto the plane formed by the line joining nodes 1 and 5 and orthogonal to the x-y plane (denoted by s'), as shown in Fig. 2, and applying the triangular inequality, we obtain the following:

$$\begin{aligned} \tau_{1,s'} + \tau_{5,s'} &\leq \tau_{1,1'} + \tau_{1',5} \\ &= \frac{z}{c} + \sqrt{(\frac{z}{c})^2 + \tau_{1,5}^2}. \end{aligned}$$

Since $d_{1s} \leq d_{1s'} + d_{SS'}$ and $d_{5s} \leq d_{5s'} + d_{ss'}$, we have:

$$\tau_{1,s} + \tau_{5,s} \leq \frac{z}{c} + \sqrt{(\frac{z}{c})^2 + \tau_{1,5}^2} + 2\tau_{ss'}.$$

Since s' is constrained to lie on the line joining 1' and 5', we can write the following:

$$\tau_{ss'} \leq \tau_{3,5}.$$

Hence, we can express T_0 as follows:

$$T_0 \leq \sum_{i=1}^{4} \tau_{i,i+1} + \frac{z}{c} + \sqrt{(\frac{z}{c})^2 + \tau_{1,5}^2} + 2\tilde{\tau}_{15} + \Delta,$$

where

$$\Delta = \sum_{i=1}^{5} \delta_i + \delta_s + 6t_p.$$

Since s is constrained within space S, $z \leq Y$, i.e.,

$$\begin{aligned} T_O &\leq \sum_{i=1}^{4} \tau_{i,i+1} + \frac{Y}{c} + \sqrt{(\frac{Y}{c})^2 + \tau_{1,5}^2} + 2\tilde{\tau}_{15} + \Delta \\ &\equiv T_{min}. \end{aligned}$$

Hence, if s is unable to localize itself by T_{min}, it should trigger a time-out to re-initiate the localization procedure. Accordingly, we can set TO_S = T_{min}, i.e.,

$$TO_S = \sum_{i=1}^{4} \tau_{i,i+1} + \frac{Y}{c} + \sqrt{(\frac{Y}{c})^2 + \tau_{1,5}^2} + 2\tilde{\tau}_{15} + \Delta.$$

Since node s knows the location of all the reference nodes, it will be able to compute TO_S.

V. Numerical Results

In this section, we compare the performance of WPS against **UPS**(4) and **UPS**(5) in terms of the localization time, total number of transmissions until successful localization and probability of unique localization via simulations conducted using the Qualnet simulator [12]. Note that the localization time and number of transmissions are evaluated from the instant node s starts its timer until it is successfully (and not necessarily uniquely) localized.

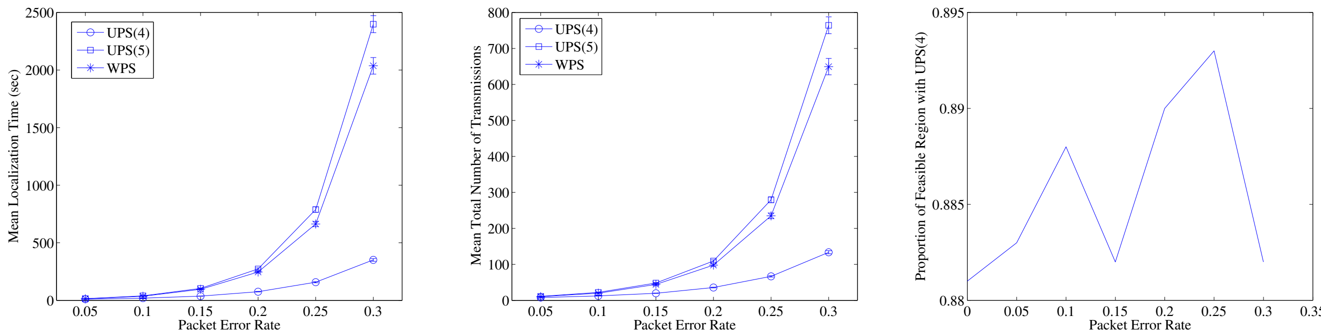

Fig. 5. Localization latency, communication costs and uniqueness: WPS vs **UPS**(4), **UPS**(5).

We assume that the reference nodes are deployed according to Figure 2, with $D = 1000$. In each simulation run, node s is deployed randomly within the space S. In addition, the processing delay at each node is fixed at δ=0.01, the beacon size is 256 bytes and the link rate is 5kbps. We vary the channel quality by considering p to be in the range [0.05:0.3] in steps of 0.05. For each parameter setting, we obtain the mean and 95% confidence interval over 1000 simulation runs. The results are plotted in Fig. 5.

As expected, the localization time and total number of transmissions increase with the packet error rate for each scheme. In addition, the performance obtained with WPS is bounded by the corresponding performance with **UPS**(4) and **UPS**(5). WPS achieves between 10 to 20% performance gain compared with **UPS**(5) as it reduces the redundancy provided by the fifth node when **UPS**(4) suffices to provide unique localization. Although its performance is significantly worse than **UPS**(4), it guarantees unique localization for the given deployment, while **UPS**(4) achieves unique localization around 89% of the time.

VI. Conclusions and Future Work

In this paper, we consider the problem of underwater localization. We generalize the recently proposed range-based Underwater Positioning System with an infrastructure of N reference nodes (**UPS**(N)), which relies on minimal transmission from the node to be localized and more importantly, is not based on the premise of synchronized clocks. While the original scheme (**UPS**(4)) requires minimal infrastructure for 3-D localization, it does not guarantee unique localization, i.e., there exist an infeasible region. We illustrate the extent of this infeasible region and quantify the conditions for unique localization. We show that, by introducing a fifth reference node, unique localization can be guaranteed with high probability using **UPS**(5).

Accordingly, we propose a Wide Coverage Positioning System (WPS) that (i) relies on an infrastructure of 5 reference nodes but (ii) only utilizes beaconing from the fifth reference node when required, so as to minimize the localization time. We show, via simulations, that WPS achieves better localization speed with lower communication costs than **UPS**(5). Although it performs worse than **UPS**(4), it guarantees unique localization, while the latter does so only 89% of the time.

For future work, we plan to combine WPS (that offers unique localization with high probability) with E-UPS [3], which improves the robustness of **UPS**(4) in harsh underwater channel conditions. In addition, we plan to consider a more realistic underwater acoustic channel model in future performance evaluations, and investigate the impact of larger values of N for different deployment of reference nodes.

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