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Evaluation of time-varying availability in multi-echelon spare parts systems with passivation

Hoong Chuin Lau [a,*], Huawei Song [a], Chuen Teck See [b], Siew Yen Cheng [b]

[a] *The Logistics Institute—Asia Pacific, National University of Singapore, Singapore 119260*

[b] *Defense Science and Technology Agency, Singapore Ministry of Defense, Singapore 109679*

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Abstract

The popular models for repairable item inventory, both in the literature as well as practical applications, assume that the demands for items are independent of the number of working systems. However this assumption can introduce a serious underestimation of availability when the number of working systems is small, the failure rate is high or the repair time is long. In this paper, we study a multi-echelon repairable item inventory system under the phenomenon of *passivation*, i.e. serviceable items are passivated ("switched off") upon system failure. This work is motivated by corrective maintenance of high-cost technical equipment in the miltary. We propose an efficient approximation model to compute time-varying availability. Experiments show that our analytical model agrees well with Monte Carlo simulation.

Keywords: Inventory; Maintenance; Multi-echelon; Passivation

1. Introduction

Miltary systems such as aircrafts, ships or tanks are expensive and have complex structures that break down because the underlying components (line replacable units or LRUs) are either worn out over time and/or damaged during usage. One way to achieve high-operational readiness (or availability) is to acquire enough spare parts to provide immediate replacement of damaged components. However, since spares are costly, consume space and become obsolete over time, there is a need to tradeoff the cost of spares with availability. Logistics planners in the miltary often need to plan for spares according to time-varying

* Corresponding author. Tel.: +65 9767 2913; fax: +65 6872 3072.
E-mail address: lauhc@comp.nus.edu.sg (H.C. Lau).

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demands, since the utilization rate varies over time. This is known generally as the spares provisioning problem for corrective maintenance.

In almost all existing literature, it is assumed that the demand for LRUs does not depend on the number of working systems, which means that the LRUs within a system fail independently of each other. However in many situations, it is observed that when an LRU fails, it will affect the demand of the other LRUs within the same system. When a system fails, the failed LRU is transported to a repair shop and all the remaining system LRUs are switched off to maximise component life, which implies that there is no demand of those LRUs until the system has been restored. In other words, the system failure rate equals 0 during repair (as explained in [4]). This phenomenon is called *passivation*.

While the assumption of independent demands produces good analytical results for most problems, availability is seriously understated in scenarios when the number of working systems is small, the failure rate high and repair time long. For example, in [13], EBO (expected backorder) is overestimated by 37.3% without passivation, which cause the availability to be underestimated. In this paper, we study the effect of passivation on system availability in these settings. We are concerned with a multi-echelon single-indenture repairable item inventory model. In this paper, we use the term ''technical system'' to generally denote a miltary equipment such as an aircraft, ship or tank.

This paper is organized as follows. In Section 2, we describe the logistic system structure (based on assumptions that are also widely accepted in the literature) and the different demand scenarios. A literature survey is provided in Section 3. In Section 4, we develop a mathematical model for the system in terms of a single-item. In Section 5, we derive the equations which are used in Section 4 based on a dynamic form of Palm's theorem. Section 6 shows how to extend the analysis to multiple items so as to compute availability under passivation of a system that comprises a number of items. In Section 7, some experiments are presented to illustrate the effect on availability when passivation is considered. Section 8 concludes the paper.

2. Preliminaries

2.1. Logistic support structure

The literature typically discusses a 2-echelon support structure for illustration except [14]. Although the underlying principles are the same, the computational gap between 2-echelon and more than 2-echelons is quite substantial. We hence show our approach using 3-echelon structure as an example (see Fig. 1). Our approach can be extended easily to 4 echelons and beyond.

In the above example, one depot supports a number of repair sites called intermediate site which supports a number of units where the technical systems are deployed. All the technical systems are identical and each technical system is composed of multiple LRUs that are connected in series.

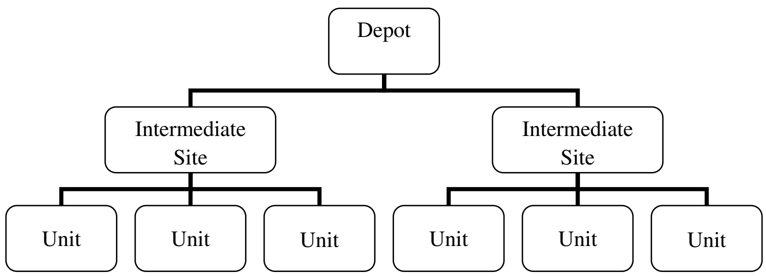

Fig. 1. 3-Echelon structure.

Depending on the nature of fault, the repair will occur on-site immediately if the fault can be rectified at the unit. Otherwise it will be sent to the intermediate site and an order is placed by the unit to be supplied from the intermediate site. If the LRU cannot be repaired at the intermediate site, it will be sent to the depot and an order is placed by the intermediate site. At the unit, the failed LRU will be removed and replaced by a good component should one be available, and the system becomes serviceable after a short delay, the time to remove and replace the failed LRU. Otherwise, a *backorder* is generated and the failed system has to wait for a spare part to arrive. When repair is completed, the working LRU will be sent to its originating support site or unit to function as spares. In either case, the organization does so by supplying a serviceable item for a failed item on a *one-for-one* basis. Underlying assumptions are presented as follows, most of which are also accepted in the literature.

In this paper, we make the following assumptions:

1. There are infinite repair resources, i.e. a failed system can be repaired at once.
2. All LRUs are repairable at the depot, i.e. there is no irrepairable item.
3. Continuous resupply, i.e. an LRU can be sent up or down the echelon immediately at any time. The transport time for each item between two sites is a constant.
4. FCFS (first come first serve) replenishment policy.
5. The remove-and-replace time for each item follows an exponential distribution.
6. The repair time for each item follows an exponential distribution.
7. No lateral supply, i.e. no supply or shipment across sites within the same echelon.

2.2. Demands

The item demand rate is determined by the mean time between failures MTBF (i.e. the expected value of time duration between two consecutive failures) and the utilization rate UR (i.e. the usage rate of the item).

In the stationary-demand problem, we assume the utilization rate for each item is identical over the entire time horizon. Given the number of technical systems deployed at unit Nsys, the following formula is conventionally used to compute the demand rate DR:

$$\mathrm{DR} = \frac{\mathrm{UR}}{\mathrm{MTBF}} \times \mathrm{Nsys}. \tag{1}$$

In the case of time-varying demand, we assume the utilization rate for each item is a time-dependent piecewise constant function, i.e. the demand for an LRU is given by a non-stationary Poisson process. Given the time-varying utilization rate $\mathrm{UR}(t)$, the following formula is adopted to compute the time-varying demand rate $\mathrm{DR}(t)$:

$$\mathrm{DR}(t) = \frac{\mathrm{UR}(t)}{\mathrm{MTBF}} \times \mathrm{Nsys}. \tag{2}$$

2.3. Passivation

Due to the effect of passivation, the actual demand rate is dependent on the number of working systems, which changes over time. Hence, even for the stationary problem, the actual demand rate varies with time. Henceforth in this paper, we will only consider the time-varying demands problem under passivation.

Clearly, by setting $\mathrm{UR}(t) \equiv \mathrm{UR}$ for all t, we can easily handle stationary demands problem as well. Let $\mathrm{Nsys}(t)$ be the number of available technical systems at time t, the actual time-varying demand rate is computed as follows:

$$\mathrm{DR}(t) = \frac{\mathrm{UR}(t)}{\mathrm{MTBF}} \times \mathrm{Nsys}(t). \tag{3}$$

The computation of $\mathrm{Nsys}(t)$ will be discussed in Section 4.

Since the demand for an LRU is given by a time-dependent Poisson process, the objective function is inevitably time-dependent. In this paper, we use the conventional EBO (expected backorder) and Ao (operational availability) as our objective functions. Instead of computing EBO and Ao under steady state presented in the literature, we will compute EBO and Ao at each time point, thereby capturing the time-varying behavior of the objective functions. Aside from this, we will show how to derive Ao from EBO under the time-varying scenario.

The key result of this paper is an evaluation scheme that, given an allocation of spares at time 0 for each site in the multi-echelon support structure, efficiently computes EBO and Ao at each time point over a given time horizon, taking into consideration non-stationary demands and the effects of passivation.

3. Literature survey

METRIC (multi-echelon technique for recoverable item control) is a pioneer study for multi-echelon, single-indenture and multi-repairable-item optimization models presented by Sherbrooke in [18]. In METRIC, one central repair site (depot) supports multiple bases where aircrafts are allocated. It assumes that there are infinite repair resources at the depot and the failures at bases are Poisson processes. Sherbrooke provides an optimization procedure for METRIC by employing marginal analysis.

METRIC is only an approximation and during its implementation, it was found that expected number of backorders was underestimated. In [8], Graves proposes an approach to use negative binomial distribution instead of Poisson by introducing variance. This is because the variance-to-mean ratio should be 1 under Poisson distribution, but it is usually greater than 1 in practice. Graves produces some test cases and finds his method achieves higher accuracy than METRIC.

The models with limited repair facility have been studied recently because the assumption of infinite repair facility is unrealistic in industrial applications. In [6], Díaz and Fu develop a multi-echelon, single-indenture model, considering limited repair facilities (servers) at the depot where all failed LRUs are repaired. They provide an aggregation–disaggregation approach, trying to calculate the first two moments of per-class number in queue and repair under steady state. Unfortunately the variance of per-class number in queue and repair is derived only for single-server multi-class queuing model due to analytical complexity.

In [1,2], Alfredsson proposes OPRAL, a model for optimum spare allocation as well as repair facility allocation. This model is an offshoot of the commercial software OPUS developed by Systecon AB [15,16]. In his model, it is assumed that each failed LRU requires only one repair resource. Different LRUs may share a common repair resource. This assumption implies that LRUs can be partitioned into resource groups, each of which contains the LRUs that require a particular resource. Therefore, the queue in a resource group at the repair facility is modelled as $M/M/s$ so that the expected waiting time for an available resource can be calculated.

As far as inventory systems on time-varying demands are concerned, there are two influential works. In [12], Jung presents a methodology for a repairable inventory system with time-varying demand by implementing discrete event simulation. In [21], Slay et al. propose an aircraft sustainability model that can handle time-varying demand rates but infinite repair resources. The failure at the base is given by a non-stationary Poisson process whose mean value varies with time. It investigates the objective function and spare allocation only at specific times of interest.

4. Mathematical model

4.1. Notations

In this section, we present a mathematical model for the system in terms of a single-item. This model is applicable to each item of the system consisting of a number of items. In Section 6, we will show how to compute the time-varying availability of the multi-item system under passivation by combining the performances of all items it has. We adopt and extend the notations of those in [1]. As shown in Fig. 1, there is one depot supporting multiple intermediate sites. We use 0 to denote the depot and index the intermediate sites by i, $i = 1,\ldots,I$. Each intermediate site supports multiple units which are indexed by u, $u = 1,\ldots,U$. And we use $\mathscr{U}_i \subset \{1,\ldots,U\}$ to denote the units supported by intermediate site i. Given any two site i and j, we will use $i = \rho(j)$ to denote the relationship that site i supports site j.

As done in [1], the types of LRU are indexed by k, $k = 1,\ldots,K$. Other notations which are consistent with those used in OPUS and Dyna-METRIC [9,10,15,16] include

Input variables

- T the length of planning horizon;
- MTBF_k mean time between failures of LRU k;
- TAT^U_{uk} mean repair time of LRU k at unit u;
- TAT^I_{ik} mean repair time of LRU k at intermediate site i;
- TAT_{0k} mean repair time of LRU k at the depot;
- TPT^U_{uk} transport time of LRU k between unit u and its supporting intermediate site;
- TPT^I_{ik} transport time of LRU k between intermediate site i and the depot;
- MTTR_{uk} mean time to remove and replace of LRU k at unit u;
- NRTS^U_{uk} the probability that LRU k cannot be repaired at unit u;
- NRTS^I_{ik} the probability that LRU k cannot be repaired at intermediate site i;
- Nsys_u number of technical systems deployed at unit u;
- QPM_k quantity of LRU k that technical system has;
- $\mathrm{UR}(t)$ utilization rate at time t;
- s^U_{uk} number of spares of LRU k at unit u;
- s^I_{ik} number of spares of LRU k at intermediate site i;
- s_{0k} number of spares of LRU k at the depot.

Intermediate variables

- $\mathrm{DR}^U_{uk}(t)$ incoming demand rate of LRU k at unit u at time t;
- $\mathrm{DR}^I_{ik}(t)$ incoming demand rate of LRU k at intermediate site i at time t;
- $\mathrm{DR}_{0k}(t)$ incoming demand rate of LRU k at the depot at time t;
- $\lambda^U_{uk}(t)$ effective demand rate of LRU k at unit u at time t;
- $\lambda^I_{ik}(t)$ effective demand rate of LRU k at intermediate site i at time t;
- $\lambda_{0k}(t)$ effective demand rate of LRU k at the depot at time t, which is equal to the incoming demand rate at the depot in our case;
- $\mathrm{EBO}^U_{uk}(t)$ EBO of LRU k at unit u at time t;
- $\mathrm{EBO}^I_{ik}(t)$ EBO of LRU k at intermediate site i at time t;
- $\mathrm{EBO}_{0k}(t)$ EBO of LRU k at the depot at time t.

Decision variable
$\mathrm{Ao}_u(t)$ operational availability of the systems at unit u at time t.

4.2. Time-varying EBO function

With considering passivation, we first divide the time horizon into n periods, which are indexed by t, $t = 1, \ldots, n$, so that (a) the utilization rate in each period is constant and (b) the number of "up" technical systems can be regarded as constant, not varying as time in each period.

It is obvious that $\mathrm{EBO}(0) = 0$ for all stock positions and $\mathrm{Ao}(0) = 100\%$. For $t(\geqslant 1)$, the incoming demand rate of LRU k at unit u at time t with considering passivation is

$$\mathrm{DR}^U_{uk}(t) = \frac{\mathrm{UR}(t)}{\mathrm{MTBF}_k} \times \mathrm{QPM}_k \times \mathrm{Nsys}_u \times \mathrm{Ao}_u(t-1). \tag{4}$$

So,

$$\lambda^U_{uk}(t) = (1 - \mathrm{NRTS}^U_{uk})\mathrm{DR}^U_{uk}(t), \tag{5}$$

$$\mathrm{DR}^I_{ik}(t) = \sum_{u \in \mathscr{U}_i} \mathrm{NRTS}^U_{uk} \times \mathrm{DR}^U_{uk}(t), \tag{6}$$

$$\lambda^I_{ik}(t) = (1 - \mathrm{NRTS}^I_{ik})\mathrm{DR}^I_{ik}(t), \tag{7}$$

$$\lambda_{0k}(t) = \mathrm{DR}_{0k}(t) = \sum_{i=1}^{I} \mathrm{NRTS}^I_{ik} \times \mathrm{DR}^I_{ik}(t). \tag{8}$$

The following are intermediate variables for the purpose of computation.

$\mathrm{P}^U_{uk}(t)$ random variable representing number of LRU k in the pipeline of unit u at time t;
$\mathrm{P}^I_{ik}(t)$ random variable representing number of LRU k in the pipeline of intermediate site i at time t;
$\mathrm{P}_{0k}(t)$ random variable representing number of LRU k in the pipeline of the depot at time t;
$\mathrm{RP}^U_{uk}(t)$ random variable representing number of LRU k in the repair pipeline of unit u at time t;
$\mathrm{RP}^I_{ik}(t)$ random variable representing number of LRU k in the repair pipeline of intermediate site i at time t;
$\mathrm{RP}_{0k}(t)$ random variable representing number of LRU k in the repair pipeline of the depot at time t;
$\mathrm{OSP}^U_{uk}(t)$ random variable representing number of LRU k in the order-and-ship pipeline to unit u at time t;
$\mathrm{OSP}^I_{ik}(t)$ random variable representing number of LRU k in the order-and-ship pipeline to intermediate site i at time t;
$f^U_{uk}(t)$ fraction of LRU k at unit u contributing to the EBO at its supporting site;
$f^I_{ik}(t)$ fraction of LRU k at intermediate site i contributing to the EBO at the depot.

In addition, we will use $\mathrm{EBO}(s|\lambda)$ to denote EBO given stock level s when the mean pipeline is λ. Following standard probability, this quantity is computed as $\sum_{x>s}(x-s)Pr\{X = x\}$ where X is the pipeline random variable with mean $E[X] = \lambda$.

Then we have

$$\mathrm{EBO}_{0k}(t) = \mathrm{EBO}_{0k}(s_{0k}|E[\mathrm{P}_{0k}(t)]) = \sum_{x > s_{0k}} (x - s_{0k})Pr\{\mathrm{P}_{0k}(t) = x\}, \tag{9}$$

where $E[\mathrm{P}_{0k}(t)] = E[\mathrm{RP}_{0k}(t)]$.

$$\mathrm{EBO}^{I}_{ik}(t) = \mathrm{EBO}^{I}_{ik}(s^{I}_{ik}|E[\mathrm{P}^{I}_{ik}(t)]), \tag{10}$$

where

$$E[\mathrm{P}^{I}_{ik}(t)] = E[\mathrm{RP}^{I}_{ik}(t)] + E[\mathrm{OSP}^{I}_{ik}(t)] + f^{I}_{ik}(t - \mathrm{TPT}^{I}_{ik})\mathrm{EBO}_{0k}(t - \mathrm{TPT}^{I}_{ik}), \tag{11}$$

$$\mathrm{EBO}^{U}_{uk}(t) = \mathrm{EBO}^{U}_{uk}(s^{U}_{uk}|E[\mathrm{P}^{U}_{uk}(t)]), \tag{12}$$

where

$$E[\mathrm{P}^{U}_{uk}(t)] = E[\mathrm{RP}^{U}_{uk}(t)] + E[\mathrm{OSP}^{U}_{uk}(t)] + f^{U}_{uk}(t - \mathrm{TPT}^{U}_{uk})\mathrm{EBO}^{I}_{ik}(t - \mathrm{TPT}^{U}_{uk}). \tag{13}$$

In the following section, we will provide the details on how the above formulae can be computed and implemented.

5. Derivation of intermediate variables

Under the assumption that the number of items in the pipeline follows a Poisson distribution, the expected number of demands in the pipeline at time t can be computed by a dynamic form of Palm's theorem. Carrillo [5] presents a generalization of Palm's theorem by relaxing the input process and service time distribution assumptions.

Theorem 1 (Carrillo [5]). *Suppose we have non-homogeneous Poisson input with intensity function $\lambda(t) \geqslant 0$ for $t \geqslant 0$, $\lambda(t) = 0$ otherwise, and non-stationary service distribution G. Then, the number of arrivals undergoing service at time t has a Poisson distribution with mean*

$$\Lambda(t) = \int_0^t (1 - G(s,t))\lambda(s)\,\mathrm{d}s, \tag{14}$$

where the random service time Y at time t has the distribution $P[Y \leqslant y] = G(t, t + y)$.

From the assumption of constant transport time, it is easy to know that the number of LRUs in the order-and-ship pipeline follows a Poisson distribution by Theorem 1 and we can get its mean value as follows:

$$E[\mathrm{OSP}^{I}_{ik}(t)] = \int_{t-\mathrm{TPT}^{I}_{ik}}^{t} \mathrm{NRTS}^{I}_{ik} \times \mathrm{DR}^{I}_{ik}(s)\,\mathrm{d}s, \tag{15}$$

$$E[\mathrm{OSP}^{U}_{uk}(t)] = \int_{t-\mathrm{TPT}^{U}_{uk}}^{t} \mathrm{NRTS}^{U}_{uk} \times \mathrm{DR}^{U}_{uk}(s)\,\mathrm{d}s. \tag{16}$$

According to the assumption that the transport time is constant whereas repair time is exponentially distributed, we have to consider the two processes as a whole to compute the expected pipelines. We use repair pipeline to indicate the number of LRUs in retrograde process and repair service. Based on Theorem 1, we assume the repair time X is exponentially distributed with mean $1/\mu = \mathrm{TAT}$ and the transport time is L, which is constant. So the service time is $Y = X + L(Y \geqslant L)$ and the service distribution is

$$G(s,t) = Pr\{Y \leqslant t - s\} = Pr\{X + L \leqslant t - s\} = Pr\{X \leqslant t - s - L\} = 1 - \mathrm{e}^{-\mu(t-s-L)^{+}}.$$

So

$$\Lambda(t) = \int_0^t \lambda(s)\mathrm{e}^{-\mu(t-s-L)^{+}}\,\mathrm{d}s. \tag{17}$$

When demand rate is constant i.e. $\lambda(t) \equiv \lambda$, and $t \to \infty$ which is the scenario in METRIC, we can show the result of mean pipeline is the same as METRIC as follows:

$$\Lambda = \lim_{t\to\infty} \int_{t-L}^{t} \lambda \, \mathrm{d}s + \lim_{t\to\infty} \int_{0}^{t-L} \lambda \mathrm{e}^{-\mu(t-s-L)} \, \mathrm{d}s = \lambda L + \lambda \mathrm{TAT} = \lambda(L + \mathrm{TAT}).$$

Therefore according to the above conclusion, we can compute the expected number of LRUs in the repair pipeline as follows:

$$E[\mathrm{RP}^U_{uk}(t)] = \int_0^t \lambda^U_{uk}(t) \mathrm{e}^{-\frac{1}{\mathrm{TAT}^U_{uk}}(t-s)} \, \mathrm{d}s, \tag{18}$$

$$E[\mathrm{RP}^I_{ik}(t)] = \sum_{u \in \mathscr{U}_i} \int_0^t (1 - \mathrm{NRTS}^I_{ik}) \mathrm{NRTS}^U_{uk} \mathrm{DR}^U_{uk}(s) \mathrm{e}^{-\frac{1}{\mathrm{TAT}^I_{ik}}(t-s-\mathrm{TPT}^U_{uk})^+} \, \mathrm{d}s, \tag{19}$$

$$E[\mathrm{RP}_{0k}(t)] = \sum_{u} \int_0^t \mathrm{NRTS}^I_{\rho(u)k} \mathrm{NRTS}^U_{uk} \mathrm{DR}^U_{uk}(s) \mathrm{e}^{-\frac{1}{\mathrm{TAT}_{0k}}(t-s-\mathrm{TPT}^I_{\rho(u)k}-\mathrm{TPT}^U_{uk})^+} \, \mathrm{d}s. \tag{20}$$

Now we will show how to distribute EBO at the supporting site to its supported sites. Since we assume FCFS replenishment policy, the waiting time for an available spare from the supporting site of all supported sites are the same. Therefore, we distribute EBO according to the proportion of demand rate. Given any two sites i, j that $i = \rho(j)$, we set $f_{jk}(t) = \frac{\mathrm{NRTS}_{jk} \times \mathrm{DR}_{jk}(t)}{\mathrm{DR}_{ik}(t)}$. Unfortunately, this direct approach is incorrect under passivation. This is because when an LRU fails, the whole technical system is down, which causes no demand of other LRUs on it. Hence, we need not compute the demand due to this system. However, it still contributes to the EBO since and as long as it is down. In order to compute the fraction, we introduce some variables.

- $\Lambda^U_{uk}(t)$ incoming demand rate of LRU k at unit u at time t without passivation;
- $\Lambda^I_{ik}(t)$ incoming demand rate of LRU k at intermediate site i at time t without passivation;
- $\Lambda_{0k}(t)$ incoming demand rate of LRU k at the depot at time t without passivation.

We have

$$\Lambda^U_{uk}(t) = \frac{\mathrm{UR}_u(t)}{\mathrm{MTBF}_k} \times \mathrm{QPM}_k \times \mathrm{Nsys}_u, \tag{21}$$

$$\Lambda^I_{ik}(t) = \sum_{u \in \mathscr{U}_i} \mathrm{NRTS}^U_{uk} \times \Lambda^U_{uk}(t), \tag{22}$$

$$\Lambda_{0k}(t) = \sum_{i=1}^{I} \mathrm{NRTS}^I_{ik} \times \Lambda^I_{ik}(t). \tag{23}$$

So, the fraction

$$f^I_{ik}(t) = \frac{\mathrm{NRTS}^I_{ik} \times \Lambda^I_{ik}(t)}{\Lambda_{0k}(t)}, \tag{24}$$

$$f^U_{uk}(t) = \frac{\mathrm{NRTS}^U_{uk} \times \Lambda^U_{uk}(t)}{\Lambda^I_{\rho(u)k}(t)}. \tag{25}$$

At last, we can compute EBO based on the stock level according to Eqs. (9)–(13).

6. Conversion of EBO into Ao

In [20], Sherbrooke puts forward a variety of availabilities and corresponding formulas. Operational availability (Ao) is one of the most important availabilities, which is widely used in practice. In [20], the formula used is:

$$\mathrm{Ao} = \frac{\mathrm{MTBF}}{\mathrm{MTBF} + \mathrm{MTTR} + \mathrm{WT}} = \frac{1}{1 + \frac{\mathrm{MTTR}}{\mathrm{MTBF}} + \mathrm{EBO}}. \tag{26}$$

The following is one variation of the formula:

$$\mathrm{Ao} = \frac{1}{1 + \sum_k \left(\frac{\mathrm{EBO}_k}{\mathrm{Nsys}} + \frac{\mathrm{UR}}{\mathrm{MTBF}_k} \times \mathrm{QPM}_k \times \mathrm{MTTR}_k\right)}. \tag{27}$$

This formula suffers several limitations. First, we can only compute Ao under steady state by this formula. We cannot compute Ao in the transient periods even though the demand is given by constant Poisson process. In most cases such as ours, the demand is given by a non-stationary Poisson process, which may cause the system never to converge to steady state. Secondly, we need to compute the waiting time. While it is well known that the waiting time $\mathrm{WT} = \frac{\mathrm{EBO}}{\lambda}$ by Little's Law, it is not obvious how to compute waiting time under the non-stationary demands case. Experiments show that it does not readily translate to $\mathrm{WT}(t) = \frac{\mathrm{EBO}(t)}{\lambda(t)}$. In a recent paper [17], the authors provide a formula to compute Ao at time t based on an extension of [3]. Unfortunately their approach is very time-consuming.

6.1. Intuitive model

We first look at an intuitive model to compute Ao at any time under both stationary and non-stationary demand cases. The Ao discussed in this section is always within a given unit so that the subscript u is omitted. The proposed formula is a recursive formulation, defined as follows:

$$\mathrm{Ao}(t) = \frac{1}{1 + \sum_{k=1}^{K} \left(\frac{\mathrm{EBO}_k(t)}{\mathrm{Nsys}(t)} + \frac{\mathrm{UR}(t)}{\mathrm{MTBF}_k} \times \mathrm{QPM}_k \times \mathrm{MTTR}_k\right)}, \tag{28}$$

where $\mathrm{Nsys}(t) = \mathrm{Nsys} - \sum_{k=1}^{K} \mathrm{EBO}_k(t-1)$.

With this formulation, we can compute EBO and Ao at any unit at any time point t using values derived in time point $t-1$. This means that we can implement the computation iteratively from one period to the next. We will compare the results with those of simulation in the next section. Experimental results show our approach yields solutions that match simulation results very well within short computational time.

6.2. Proposed model

Although the intuitive model works well in general, it does not produce good results for the case when the MTTR is large (see Fig. 4). In the following, we propose a revised model to better approximate availability.

If the demand process is a simple Poisson process with mean λ and the service time is exponentially distributed with mean $1/\mu$, the time-varying availability is given by (see for example, [11])

$$A(t) = \frac{\mu}{\lambda + \mu} + \left(A(0) - \frac{\mu}{\lambda + \mu}\right)e^{-(\lambda+\mu)t}. \tag{29}$$

By setting $\mu = 1/\text{MTTR}$, the availability is maintenance availability (Am) [20]. We know $\lambda = \frac{\text{UR}}{\text{MTBF/QPM}}$, so we can compute maintenance availability at any time by Eq. (29). For the scenario that demand is given by a non-stationary Poisson process, we can show

$$\text{Am}(t_2) = \frac{\mu}{\lambda(t_2) + \mu} + \left(\text{Am}(t_1) - \frac{\mu}{\lambda(t_2) + \mu}\right)e^{-(\lambda(t_2)+\mu)(t_2-t_1)}, \tag{30}$$

where $\lambda(t_2) = \frac{\text{UR}(t_2)}{\text{MTBF/QPM}}$ and $\text{Am}(0) = 100\%$.

With this formulation, we can compute Am at any time point iteratively from one period to the next.

According to [20], the operational availability can be computed as the product of supply availability (As) and maintenance availability (Am) where

$$\text{Am} = \frac{1}{1 + \frac{\text{UR}}{\text{MTBF/QPM}} \times \text{MTTR}}. \tag{31}$$

We notice $\frac{\text{UR}}{\text{MTBF/QPM}} \times \text{MTTR}$ is just the right part of the denominator of Eq. (28). Replacing it with $\frac{1}{\text{Am}} - 1$, we can get the formula to compute operational availability at any time for both small MTTR and large MTTR under both stationary and non-stationary demand cases. The formula is defined as follows:

$$\text{Ao}(t) = \frac{1}{1 + \sum_{k=1}^{K}\left(\frac{\text{EBO}_k(t)}{\text{Nsys}(t)} + \frac{1}{\text{Am}_k(t)} - 1\right)}, \tag{32}$$

where $\text{Am}_k(t)$ can be computed according to Eq. (30). Experiment (see Fig. 5) shows our approach matches simulation results well.

7. Experimental results

In our experiments, we consider a multi-echelon problem where each technical system comprises more than 50 LRUs. The following two sets of experiments have been conducted:

1. Steady-state Ao computation given a single-spares allocation against METRIC model [16].
2. Time-varying demands against Monte-Carlo simulation [7].

The experiments were conducted on a Pentium III 1.2 GHz machine with 512 MB RAM. In all simulation models, we set the number of replications to be 1000 which is a fairly standard practice in simulation experimentation.

7.1. Steady-state Ao without passivation

First, we verify that our model also works under steady state without passivation. In these experiments, we consider a 5-system problem and compute their operational availability under steady state given a fixed spare allocation. We compare the results against the results obtained by METRIC by setting the probability distribution of the pipeline to be Poisson. The average run time per test case is 0.16 s.

The following tables give a summary of the results.

From Tables 1 and 2, we observe that our results are *exactly the same* as METRIC.

Table 1
Our result vs. METRIC (zero spare allocation)

Station	Ao-METRIC (%)	Ao-our model (%)	Error
System A	16.75	16.75	No error
System B	16.75	16.75	
System C	16.75	16.75	
System D	16.75	16.75	
System E	16.86	16.86	

Table 2
Our result vs. METRIC (spare allocation 1)

Station	Ao-METRIC (%)	Ao-our model (%)	Error
System A	61.08	61.08	No error
System B	61.12	61.12	
System C	61.12	61.12	
System D	61.85	61.85	
System E	55.08	55.08	

7.2. Ao under constant UR with passivation

Next, by using the test cases given in the previous section, we compute the time-varying availability with passivation. Since the METRIC model can only compute Ao under steady state without passivation, we benchmark our results instead with reliable simulation results. [1] The following table and chart give a summary of the results.

Fig. 2 shows that our results match simulation results very well within acceptable statistical errors. The Ao is about 72.85% when it goes into steady state under both simulation and our model, wheareas Ao under METRIC without passivation is 55.08%. From Table 3, we can find the availability is underestimated by about 20% for all systems without considering passivation.

We make a remark here on the choice of first-order method over second-order method. As discussed in [6], [8] and [20], first-order method (such as METRIC), which assumes that the number of items in the pipeline follows Poisson distribution, usually underestimates EBO (or overestimates Ao). Hence, second-order method (such as VARI-METRIC), based on negative binomial distribution, is used instead of Poisson distribution. However, from the table and figure shown above, we observe that when passivation is considered, the first-order method itself actually underestimates Ao instead of overestimating it. Moreover, the second-order method underestimates Ao more than the first-order method! In other words, we believe that the first-order method will generate better solutions than the second-order.

7.3. Time-varying demands

In this section, we verify the accuracy of modeling the number of items in the pipeline as a time-dependent Poisson distribution. In doing so, we effectively rule out the need for second-order approximation as proposed in [6], [8] and [19].

[1] The simulation results are obtained via a simulation software developed in-house at the Singapore Ministry of Defense which has been verified and adopted for use there.

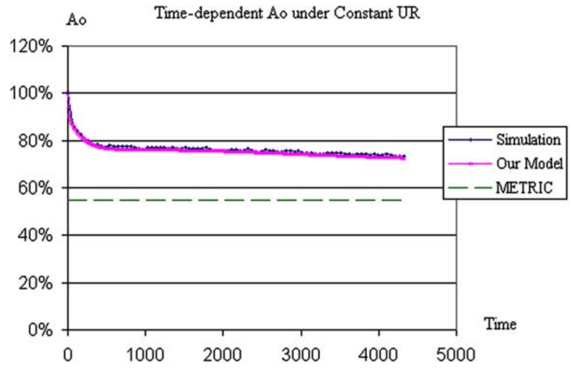

Fig. 2. The effect of passivation (System E given spare allocation 1).

Table 3
Ao with passivation vs. without passivation (spare allocation 1)

Station	Ao–passivation (%)	Ao–no passivation (%)	Error (%)
System A	76.50	61.08	20.16
System B	76.56	61.12	20.17
System C	76.56	61.12	20.17
System D	76.72	61.85	19.3
System E	72.85	55.08	24.39

In these experiments, we run the test cases with time-varying utilization rate (UR) and compare the results (Ao) against simulation results. The utilization rate is given as follows for all systems at all units: (time 0–1500), 0.08 (time 1500–2000), 0.6 (time 2000–2500), 0.3333 (time 2500–3000), 0.5 (time 3000–3500) and 0.25 (time 3500–4000) and the following chart gives a summary of the results.

From Fig. 3 we can see that our results match those produced by simulation with less than 0.1% absolute error on average. We also ran the test cases with other spare allocations and achieved results that match simulation results very well.

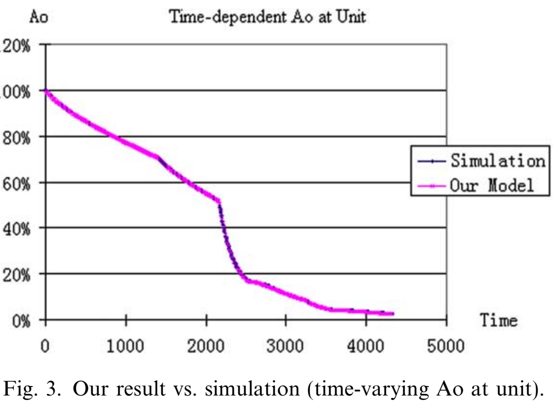

Fig. 3. Our result vs. simulation (time-varying Ao at unit).

7.4. The effect of passivation over varying availabilities

Next, we obtain some results using the previous test cases to study the effect of passivation over a range of availability values. The results are shown in Table 4.

From Table 4, we observe that (1) the model without passivation always underestimates availability; and (2) the effect of passivation (i.e. the absolute value of relative error) will increase to a certain peak and then decrease as availability increases. The phenomenon can be explained as follows. As discussed before, availability is seriously underestimated when the number of working systems is small, the failure rate is high and/or the repair time is long. Hence, when the availability is high such as 95.21%, the number of working systems can be regarded as large (since the down systems can be restored quickly with enough spares) and hence the effect of passivation is not keenly felt. On the opposite extreme, when availability is low such as 16.78%, the spares are equal or near to zero, thus mitigating the effects of passivation. In all other cases, the number of working systems varies frequently as time and hence the effect of passivation is significantly felt.

We also measure the effect of passivation as parameters using the examples cited in [13]. Table 5 gives a summary of the results.

From Table 5, we observe that compared with the base case (Test 1), the effect of passivation will be large when the number of system is small (Test 2), the availability is low with few spares (Test 3), the repair time is long (Tests 5 & 6), and the failure rate is high (Test 8). Furthermore, we also notice that when the spares are zero, the effect of passivation is trivial (Tests 4 & 7).

7.5. The effect of large MTTR

Finally, we run test cases with large MTTR and compare results against simulation. In this case, MTTR is set to be 300 hour while the MTBF is 500. There are infinite spares and utilization rate varies with time at

Table 4
The effect of passivation as Ao

Ao–passivation (%)	Ao–no passivation (%)	Error (%)
16.78	16.76	−0.12
27.84	25.13	−9.73
38.85	35.90	−7.59
57.51	45.50	−20.88
65.02	56.09	−13.73
70.73	65.58	−7.28
79.60	75.47	−5.19
87.86	85.94	−2.19
95.21	95.03	−0.19

Table 5
The effect of passivation as parameters

Test	MTBF	TAT	Nsys	ORG	DSU	Ao–passivation (%)	Ao–no passivation (%)	Error (%)
1	40	30	2	0	3	70.41	65.14	−7.48
2	40	30	10	0	3	56.58	54.79	−3.16
3	40	30	2	1	6	94.95	94.20	−0.79
4	40	30	2	0	0	52.63	52.63	−0.00
5	40	7	2	0	3	86.47	86.28	−0.22
6	40	100	2	0	3	37.58	30.53	−18.76
7	40	100	2	0	0	27.40	27.40	−0.00
8	640	30	2	0	3	99.06	99.06	−0.00

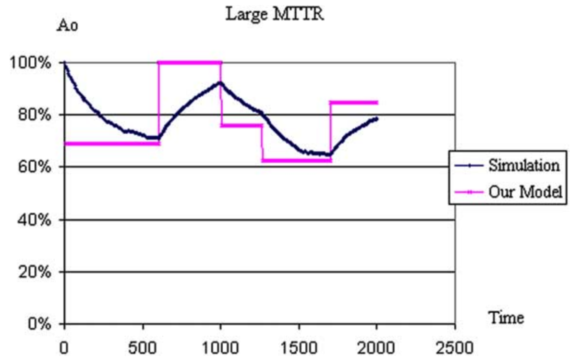

Fig. 4. Large MTTR by Eq. (28).

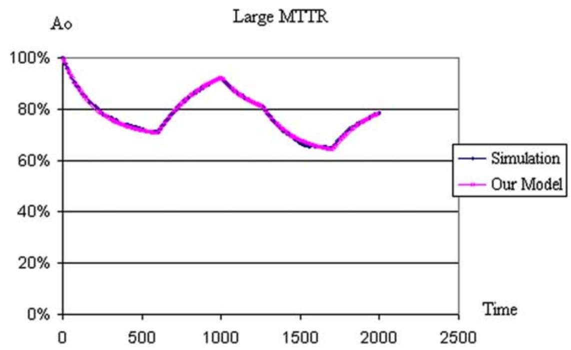

Fig. 5. Large MTTR by Eq. (32).

the unit as follows: 0.75 (time 0–500), 0 (time 500–1000), 0.5 (time 1000–1250), 1 (time 1250–1700) and 0.3 (time 1700–2000).

Fig. 4 shows that we cannot expect to obtain accurate results by applying Eq. (28). This is due to the effect of time-varying demands. On the other hand, by using the proposed Eq. (32), we overcome this problem and obtain accurate results that match simulation results very well as shown in Fig. 5. We also ran other test cases under other spare allocations and achieved accurate results as well.

8. Conclusion and future work

In this paper, we considered a multi-echelon single-indenture repairable item inventory system for technical corrective maintenance under passivation. We proposed an analytical approach to accurately compute time-varying EBO and operational availability. Our model is particularly relevant in the context of fast changing business/operating environment, where the assumption of constant demand-rate for inventory planning and optimization is no longer realistic. Experiments show that our analytical model is efficient and agrees well with Monte Carlo simulation.

In our work, we assume infinite repair resources such that failed systems can be repaired at once. We note in real-life that repair resources are not infinite since they are costly and consume space. What is challenging as future work is to relax the assumption of infinite repair resources. In practice we also notice failures sometimes arrive at units and the malfunctioning items are shipped from units to the maintenance depot, not individually, but in batches. Thus another interesting research is to handle this.

In our model, MTBF is assumed to be an item's inherent attribute, which is constant over the component's lifespan. We note in real-life that since items will become obsolete or worn-out over time, the probability of failure will be increasing. A natural extension is to hence apply our method to model the effect of wear-out.

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