Evaluation of Time-Varying Availability in Multi-Echelon Inventory System under Combat Damage

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***Abstract* – The models for multi-echelon inventory systems in existing literatures predominantly address failures due to reliability in peacetime. In wartime or even peacetime operational scenarios, unexpected combat damage can cause a large number of systems to be heavily damaged, to the extent that they become irreparable. In this paper, we study a multi-echelon spare parts support system under combat damage, discuss the replenishment policy and propose an approximate method to evaluate the time-varying system performance – operational availability considering the effect of passivation. Experiments show our model works well and efficiently against simulation.**

Keywords – Multi-Echelon, Passivation, Combat Damage, Time-Varying, Inventory, Military application

I. INTRODUCTION

The analysis and optimization of multi-echelon reparable inventory models go back to METRIC developed by Sherbrooke [17]. As military systems such as aircrafts, ships or tanks with complex hierarchical product structures break down because the underlying components (line replace units or LRUs) are either worn out over time and/or damaged during usage, to insure continuity of operations, an ample supply of spare parts must allocated to provide maintenance for damaged components. However, there is a cost tradeoff since spares, especially LRUs are expensive, consume space and become obsolete over time. It is more difficult in reality to plan for spares according to time-varying demands since the utilization rate and hence demand rate varies over time due to seasonality. Apart from military application, these inventory models are also of paramount importance for industries or services with heavy utilization of equipment, such as continuous chemical or petrochemical process and mass transit systems [5].

To our knowledge, in almost all existing literatures, it is assumed that the demand for LRUs arises only as a result of reliability, such that the components will become obsolete or worn out over time and/or damaged during usage in training in peacetime. Furthermore based on the assumption of reliability, the demand rate is usually supposed to follow a Poisson or non-stationary Poisson distribution.

Contrary to demand behaviors in peacetime, during wartime, components and even the entire military system itself are subject to damage in combat, causing the failure of components to occur intensively at certain time points, especially the end of certain missions. Under combat damage, more than one component may be damaged simultaneously and most of these failures are irreparable. In this paper, we will present a generalized multi-echelon single-indenture model involving a mix of repairable items due to reliability and irreparable items due to combat damage.

The model for multi-echelon repairable items, both in the literature and in practical applications, is predominantly based on METRIC developed by Sherbrooke [17], which is being used extensively in the military world. In the past a few years, many extensions are developed to relax some of the constraints imposed on METRIC. For example, Graves [8] uses a negative binomial distribution instead of assuming Poisson demands by introducing variance. Díaz and Fu [5], Alfredsson [1, 2], Zijm and Avsar [21] extend the model to handle limited repair capacity. A commercial software OPUS developed by Systecon AB [15, 16] is an offshoot of Alfredsson's model. Jung [14], Slay et al. [20] present models with time-varying demands by simulation and analytical methods respectively. Lau *et al.* [12] recently propose a model by combining limited repair capacity with time-varying demands. They also study the effect of passivation (i.e. repairable items are switched off) on system time-varying availability [11].

Our contribution in this paper is an approximate method for evaluating system availability considering both repairable items due to reliability as well as irreparable items due to combat damage. The rest of the paper is organized as follows. In section 2, we define the problem by describing the spare parts support system structure, maintenance workflow, and replenishment policy of reliability and the notion of combat damage. Assumptions are made so as to simplify the model. In section 3, a mathematical model is developed to compute the time-varying EBO analytically, which is converted into time-varying operational availability introduced in section 4. Experimental results are presented in section 5 followed by conclusion in section 6.

II. PROBLEM DEFINITION

In this paper, we address a two-echelon support structure of inventory system model. Our work can be extended easily to a multi-echelon context, since the underlying principles are the same. In our environment, one depot supports a number of sites called bases where identical military systems are deployed, each of which is composed of multiple LRUs which are assumed to be connected in series.

When a military system breaks down due to a damaged LRU within it, depending on the nature of fault, repair will either occur on-site immediately if the fault can be rectified at the base; or it will be sent to the depot and an order is placed by the base to be supplied from the depot. At the base, the malfunctioned LRU will be removed and replaced by a good component should one be available, and the system becomes serviceable after a short delay, the time to remove and replace the failed LRU. Otherwise, a *backorder* is generated and the failed system has to wait for a spare part to arrive. When repair is completed, the working LRU will be sent to its originating support base to function as spares. In either case, the organization does so by supplying a serviceable item for a malfunction on a *one-for-one* basis. This policy is based on the fact that for low demand, high cost items such as LRUs, the EOQ tends to a size of one [5, 19].

Combat damage occurs at certain time points, especially at the end of certain missions. There are a number of combat damage types associated with different types of assault. Each combat damage type causes a different set of LRUs within a system to break down under a given probability distribution. These LRUs are irreparable and have to be discarded. However, spares are still required to remove and replace these failures in order to sustain a high availability for future missions. Another dimension of combat damage is attrition where the entire military systems are wiped off beyond repair. Hence, these systems also have to be discarded as a whole and consequently the total number of available systems will decrease over time assuming no external supply or procurement.

The spares allocated at the sites will be used to fulfill the demands of LRUs engendered by both reliability and combat damage. As discussed earlier, while one can assume that under reliability, a system is down due to failure of only *one* LRU, this assumption breaks down under combat damage. Hence, the standard FCFS (First Come First Serve) replenishment policy may no longer be good since a single LRU spare cannot be used to recover the down system. Consequently in this paper we propose the setting of priority to meet the demands caused by reliability, such that a spare will first be used to recover the system due to reliability (since it requires only that spare unit to become operational again)

In this paper, the following (fairly standard) assumptions are made:

1) There are infinite repair resources, i.e. a failed system can be repaired at once.
2) All repairable LRUs can be repaired at the depot, i.e. no external repair.
3) Continuous resupply, i.e. an LRU can be sent between the depot and bases immediately at any time. The transport time for each item is a constant.
4) The repair time of an item follows an exponential distribution.
5) The remove-and-replace time for each item follows an exponential distribution.
6) FCFS replenishment of a spare for reliability or combat damage respectively, but with priority over reliability.

Note that our problem is further complicated by (1) time-varying utilization rate and (2) the effect of passivation [11, 13]. These are included to model real life requirements as follows. (1) Military system usage typically varies from one mission to another and transit rapidly from peace to wartime. Hence, planners are interested to predict and plan for a short period, during which exercise/mission schedules (and hence utilization rates) are known. (2) When a system fails or damaged during mission, the down LRU is removed and all the remaining system LRUs are switched off until the system has been restored to maximize component life. This makes the demand varying not only over time, but also the derived availability of past time points, and consequently more difficult to predict analytically.

As with existing literatures, EBO (Expected Backorder) and Ao (operational availability) are both good system performance, here we will use Ao to denote the time-varying Ao.

III. Mathematical Model

A. Notations

We keep consistency with and extend the notations of those in [1, 9, 10]. We use j to index the sites, $j=0$ for the depot and $j = 1, \ldots, J$ for the bases. As done in [1, 11, 12], the types of LRUs are indexed by k, $k = 1, \ldots, K$. Other notations include:

Input Variables

T: length of planning horizon
$MTBF_k$: mean time between failures of LRU k
TAT_{0k}: mean repair time of LRU k at the depot
TAT_{jk}: mean repair time of LRU k at base j
OST_k: transport time for LRU k between the depot and base
$MTTR_{jk}$: mean time to remove and replace LRU k at base j
$NRTS_{jk}$: probability that LRU k cannot be repaired at base j
$Nsys_j$: number of military systems deployed at base j
QPM_k: quantity of LRU k that a military system has
$UR(t)$: utilization rate across all bases at time t
s_{0k}: number of spares of LRU k at the depot
s_{jk}: number of spares of LRU k at base j
t_q^{CD}: time when combat damage occurs, $q = 1, 2, \ldots, Q$
Na_{jq}: number of system attrition at base j at time t_q^{CD}
r: index of damage type, $r = 1, 2, \ldots, R$
M_{jqr}: number of system damaged at base j under damage type r at time t_q^{CD}
p_{kr}:probability that LRU k is damaged under damage type r

Intermediate Variables

$DR_{0k}(t)$: incoming demand rate of LRU k at depot at time t
$DR_{jk}(t)$: incoming demand rate of LRU k at base j at time t
$\lambda_{0k}(t)$: effective demand rate of LRU k at depot at time t
$\lambda_{jk}(t)$: effective demand rate of LRU k at base j at time t
$EBO_{0k}(t)$: EBO of LRU k at the depot at time t

$EBO_{jk}(t)$: EBO of LRU k at base j at time t

Decision Variables

$Ao_j(t)$: operational availability of the systems at base j at time t

B. Time-Varying EBO function

We first divide the time horizon into N periods indexed by n, n = 1, …, N, so that **(a)** the utilization rate in each period is constant, **(b)** the number of "up" military systems can be regarded as constant **(c)** the time when combat damages occur $\{t_1^{CD}, t_2^{CD}, \ldots, t_Q^{CD}\} \subseteq \{t_1, t_2, \ldots, t_N\}$ where t_n is the time at the end of period n and we set t_0 = 0.

Assume that $EBO(0) = 0$ for all stock positions and $Ao(0) = 100\%$. From [11], the incoming demand rate of LRU k at base j at time t considering passivation is

$$DR_{jk}(t_n) = \frac{UR(t_n)}{MTBF_k / QPM_k} \times Nsys_j \times Ao_j(t_{n-1}) \tag{1}$$

Hence,

$$\lambda_{jk}(t) = (1 - NRTS_{jk})DR_{jk}(t) \tag{2}$$

$$\lambda_{0k}(t) = DR_{0k}(t) = \sum_{j=1}^{J} NRTS_{jk} \times DR_{jk}(t) \tag{3}$$

The following intermediate variables are introduced for the purpose of computation.

$P_{0k}(t)$: random variable representing number of LRU k in the pipeline of the depot at time t

$P_{jk}(t)$: random variable representing number of LRU k in the pipeline of base j at time t

$RP_{0k}(t)$: random variable representing number of LRU k in the repair pipeline of depot at time t

$RP_{jk}(t)$: random variable representing number of LRU k in the repair pipeline of base j at time t

$OSP_{jk}(t)$: random variable representing number of LRU k in the order-and-ship pipeline to base j at time t

$f_{jk}(t)$: fraction of LRU k at base j contributing to the EBO at the depot

$cd_{jk}(t)$: number of combat damaged LRU k at base j at time t

$CD_{0k}(t)$: cumulative number of combat damaged LRU k at the depot by time t

$CD_{jk}(t)$: cumulative number of combat damaged LRU k at base j by time t

In addition, we will use $EBO(s|\lambda)$ to denote EBO given stock level s when the mean pipeline is λ. Following standard probability, this quantity is computed as $\sum_{x>s}(x-s)\Pr\{X=x\}$ where X is the pipeline random variable with mean λ.

Since the spares at sites will be used to fulfill the demands by both reliability and combat damage, the failures caused by combat damage will be credited into pipeline, which influences EBO based on stockage. Therefore from [11] we know with combat damage,

$$EBO_{0k}(t) = EBO(s_{0k} \mid E[P_{0k}(t)]) \tag{4}$$

and

$$EBO_{jk}(t) = EBO(s_{jk} \mid E[P_{jk}(t)]) \tag{5}$$

will still hold where

$$E[P_{jk}(t)] = E[RP_{jk}(t)] + E[OSP_{jk}(t)] + f_{jk}(t - OST_k) \cdot EBO_{0k}(t - OST_k) \tag{6}$$

But

$$E[P_{0k}(t)] = E[RP_{0k}(t)] + CD_{0k}(t) \tag{7}$$

In the following section, we will provide the details on how the above formulae can be computed and implemented.

C. Derivation of Intermediate Variables

From [11], we know that the expected number of demands in the pipeline at time t can be computed by a dynamic form of Palm's theorem [4]. So

$$E[OSP_{jk}(t)] = \int_{(t-OST_k)^+}^{t} NRTS_{jk} \times DR_{jk}(s)ds \tag{8}$$

$$E[RP_{jk}(t)] = \int_0^t \lambda_{jk}(t) e^{-\frac{1}{TAT_{jk}}(t-s)} ds \tag{9}$$

$$E[RP_{0k}(t)] = \sum_{j=1}^{J} \int_0^t NRTS_{jk} DR_{jk}(s) e^{-\frac{1}{TAT_{0k}}(t-s-OST_k)^+} ds \tag{10}$$

Next we present how to compute the number of combat damaged LRUs. Since we assume the combat damaged LRUs are irreparable, and since the depot has to provide its stocks for these failures in order to recover the military system, those damaged LRUs can be regarded as though they are *always* in the pipeline, so that the number of combat damaged LRUs will be accumulated. Unlike demands due to reliability that arise over time, the failures due to combat damage take place at the *end* of missions *in a batch*. Hence, it is obvious that $CD_{jk}(0) = 0$ and for n (≥1):

$$CD_{jk}(t_n) = CD_{jk}(t_{n-1}) + cd_{jk}(t_n) \tag{11}$$

where the computation of $cd_{jk}(t)$ is as follows. If $t \notin \{t_1^{CD}, t_2^{CD}, \ldots, t_Q^{CD}\}$, $cd_{jk}(t) = 0$. Otherwise, there must exist a q^* so that $t_{q^*}^{CD} = t$ and we have

$$cd_{jk}(t) = \sum_{r=1}^{R} M_{jq^*r} \cdot p_{kr} \tag{12}$$

Finally the cumulative number of combat damaged LRUs at the depot by time t is given by

$$CD_{0k}(t) = \sum_{j=1}^{J} CD_{jk}(t) \tag{13}$$

Due to the effect of passivation, when an LRU fails, the whole military system is down, causing no further demands of other LRUs within it. While we need not compute the demand due to this system before it is recovered the down LRU *still* contributes to the EBO since and as long as it is down. Hence we compute fraction by using demand instead of demand rate

$$f_{jk}(t) = \frac{\int_{(t-TAT_{0k}-OST_k)^+}^{t} NRTS_{jk} \times DR_{jk}(s)ds + CD_{jk}(t)}{\int_{(t-TAT_{0k}-OST_k)^+}^{t} DR_{0k}(s)ds + CD_{0k}(t)} \tag{14}$$

Finally, using these formulae we can compute the expected number of items in the pipeline at both the depot and bases followed by EBO until the ultimate time-varying EBO for each LRU at each base, which is used to convert into our objective – time-varying availability – which is introduced in next section.

IV. CONVERSION OF EBO INTO AVAILABILITY

A. *Method of conversion*

In [3, 13, 15, 16], a number of methods to convert EBO into operational availability have been proposed, most of which work either under steady state or are not accurate for large *MTTR*. In [11], an efficient approach is designed to compute the time-varying operational availability at base *j* from EBO without combat damage:

$$A_j^{NCD}(t_n) = \frac{1}{1+\sum_{k=1}^{K}\left(\frac{EBO_{jk}(t_n)}{Nsys_j(t_n)} + \frac{1}{Am_{jk}(t_n)} - 1\right)} \quad (15)$$

where $Nsys_j(t_n) = Nsys_j - \sum_{k=1}^{K} EBO_{jk}(t_{n-1})$ and

$$Am_{jk}(t_n) = \frac{\mu_{jk}}{\lambda_{jk}(t_n)+\mu_{jk}} + [Am_{jk}(t_{n-1}) - \frac{\mu_{jk}}{\lambda_{jk}(t_n)+\mu_{jk}}]e^{-[\lambda_{jk}(t_n)+\mu_{jk}](t_n - t_{n-1})} \quad (16)$$

where μ_{jk} = 1/ $MTTR_{jk}$ and $Am_{jk}(0)$ = 100%. The experimental results in [11] show that the formula is accurate in computing operational availability at any time for both small and large *MTTR* under both stationary and nonstationary demand cases without combat damage.

With combat damage, we must modify the above formulae. When computing EBO, we do not take into account attrition since the attrited systems are irreparable and hence do not require any spares and there are neither supply nor maintenance backorders for them. However, when we compute operational availability – the number of available systems, the attrited systems must be subtracted from the total number of deployed systems. Hence we will first modify the computation of $Nsys_j(t)$. Let $Nna_j(t)$ be the number of non-attrited systems at time *t* and we have $Nna_j(0) = Nsys_j$. So

$$Nsys_j(t_n) = Nna_j(t_n) - \sum_{k=1}^{K} EBO_{jk}(t_{n-1}) \quad (17)$$

and

$$Nna_j(t_n) = Nna_j(t_{n-1}) \quad \text{if } t_n \notin \{t_1^{CD}, t_2^{CD}, \ldots, t_Q^{CD}\} \quad (18)$$

Otherwise there must exist a q^* so that $t_{q^*}^{CD} = t_n$ and then

$$Nna_j(t_n) = Nna_j(t_{n-1}) - Na_{jq^*} \quad (19)$$

Secondly, by using Equation (15), the derived operational availability in fact is the availability of all systems excluding the attritions because they have been subtracted. Therefore, the final availability at time *t* will be

$$Ao_j(t) = \frac{Nna_j(t) \times A_j^{NCD}(t)}{Nsys_j} \quad (20)$$

Combining Equations (15) – (20), we can compute the time-varying operational availability with combat damage.

B. *Multiple failures by combat damage*

In our experiments, we found that the above method works well under the scenario that $\sum_{k=1}^{K} p_{kr} = 1$, for each *r*, i.e. the events that each LRU is damaged in combat are mutually exclusive. In case where the probabilities do not sum to 1, i.e. when there are multiple components failures and they are independent of each other in a single combat damage, the proposed formula gives an error of more than50%! This is not surprising since the underlying model is based on classical reliability theory which always assumes that one system is down due to one and only one LRU. Equation (15) is designed based on this assumption indicated by *EBO/Nsys*. Under combat damage however, this assumption no longer holds and (15) does not and cannot reflect this fact. To our knowledge, there is no literature dealing with evaluation of operational availability under such a scenario.

To preserve the model as far as possible, we propose a fix based on normalization handle. Under a given combat damage type, we normalize the probabilities that each LRU is damaged by so that the new summation of probabilities becomes 1 under combat damage. By this method, the deviation is reduced to around 15%, which we believe is the best improvement possible.

For benchmark purpose, we also implement Monte-Carlo simulation [6]. Compared with the analytical model, in order to obtain reliable performance estimates, we must conduct a large number of independent replications and compute the average number to reduce the statistical noise. Obviously, the larger the number of replications, the more accurately, but less efficiently we can obtain the simulation estimates.

V. EXPERIMENTAL RESULTS

In our experiments, we consider a multi-echelon problem where each military system is composed of more than 50 LRUs. The experiments were conducted on a Pentium III 1.2 GHz machine with 512MB RAM. In all simulation models, we set the number of replications to be 1000, which is a fairly standard practice in simulation experimentation. Note that the run time of our analytical approach is within seconds whereas that of simulation is about minutes or even hours for large-scale problems.

A. *Single failure by combat damage*

We compare the time-varying *Ao* by our analytical method against those by Monte-Carlo simulation [6].

First we run a small test case where one depot supports one base where 40 identical military systems are deployed, each of which is composed of seven LRUs. The utilization rate is given by a piecewise function as follows: 0.0486(0-1000hr), 0.0833(1000-2000hr), 0.5833(2000-3000hr). Combat damages occur at time 1000hr and 2000hr when 0.432 and 0.27 systems are attrited respectively*. There are 6 damage types. The number of systems damaged under every damage type at combat damage time is given in Table I.

TABLE I

SYSTEM DAMAGED UNDER DIFFERENT DAMAGE TYPE

Time	Type1	Type2	Type3	Type4	Type5	Type6
1000	1.368	1.368	1.368	1.368	1.368	1.368
2000	0.855	0.855	0.855	0.855	0.855	0.855

The probability that LRU 1 under damage type 1 is 1, LRU 2 under type 2 is 1, LRU 3 and LRU 4 under type 3

* Here we allow the number of systems not only in attrition but in damage as well not to be integer because this number is estimated based on the combination of probability and forecast.

are 0.5 respectively, LRU 5 under type 4 is 1, LRU 6 under type 5 is 1 and LRU 7 under type 6 is 1. The results comparing against simulation is shown in Fig. 1.

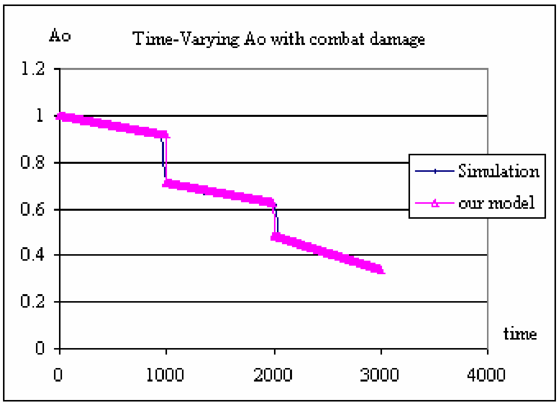

Fig. 1 Comparison of Time-Varying Ao between our model and simulation with combat damage.

From Fig. 1, we observe that our results match simulation results very well with short computational time. We also note during reliability, the availability decreases as time gently. However, there is a precipitation when combat damage occurs.

Next we run the test case in which there are three echelons. 100 military systems are allocated at each base, each of which comprises more than 50 LRUs. We allocate a number of spares at the sites. Utilization rate is given as 0(0-10), 1(10-18), 0(18-24), 1(24-34), 0(34-48). Combat damages occur at time 13, 18, 34. The results are shown in Fig. 2. Fig. 2 shows that our results are very close to simulation results. In addition, we find there is no precipitation in Fig. 2. This is because we allocate enough spares so that when combat damage happens, the damaged LRUs can be removed and replaced by stocks almost immediately. This indicates in fact the drop of availability when combat damage takes place can be overcome. Planners can allocate a certain number of spares to provide immediate replacement of damaged components at the beginning of or during missions based on the existing information and rational forecast.

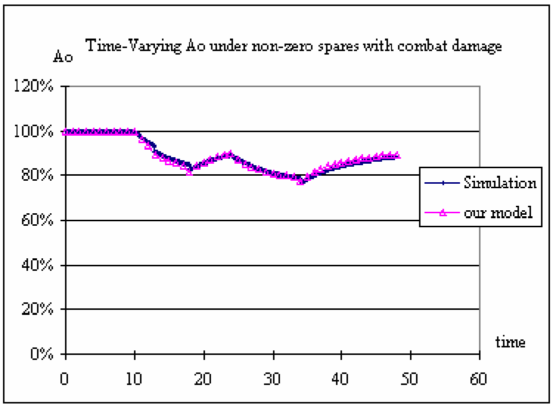

Fig. 2 Comparison of Time-Varying Ao between our model and simulation with combat damage under non-zero spares allocation.

B. Multiple failures by combat damage

Using the test cases in the previous section, we further set the attrition rate to be 0.6 and the number of systems damaged during mission to be 2. We derive three distinct test cases: one where there is both attrition and system damaged, one where there is only attrition, which is 2.6, and one where there are only 2.6 systems damaged.

First we run them without setting the priority, replenishing on the basis of FCFS regardless of the cause of demands. Fig. 3 gives a summary of the results. From Fig. 3, the availability under only attrition is even higher than those under both only system damage and both of them. In other words, to sustain a high availability, it is more preferable to attrite systems (although only some components within them are damaged) rather than repairing damaged components to restore the system. That is obviously preposterous to reality since military systems are always expensive. This demonstrates the importance of setting the priority to meet the failures caused by reliability. We run the test cases once more with priority and the results are shown in Fig. 4, which is observed to be consistent with commonsense practice.

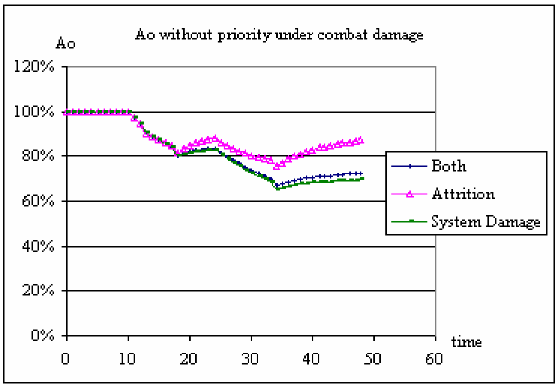

Fig. 3 Time-Varying Ao under combat damage without setting priority.

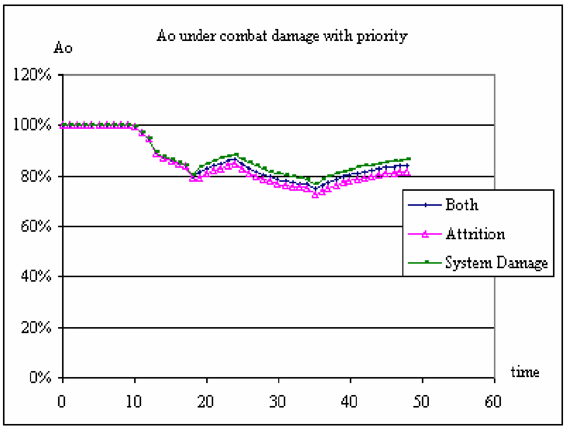

Fig. 4 Time-Varying Ao under combat damage with setting priority to the demands by reliability.

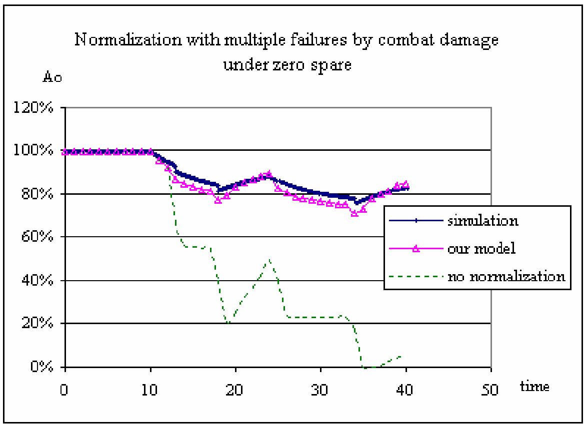

Fig. 5 Time-Varying Ao by normalization with multiple failures by combat damage under zero spares.

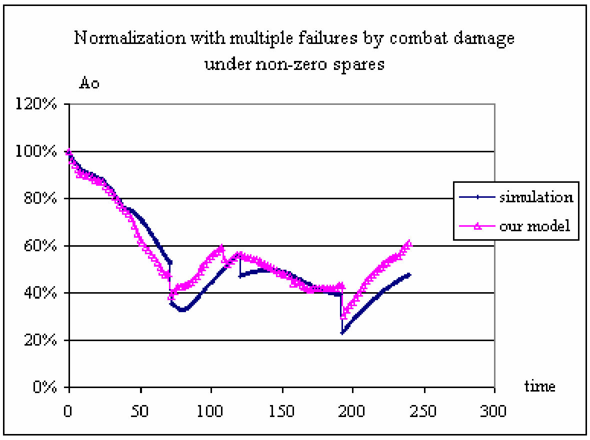

Fig. 6 Time-Varying Ao by normalization with multiple failures by combat damage under non-zero spares.

In Fig. 5, we experiment with multiple failures caused by combat damage and compare the results generated by simulation, our approximation with and without normalization. From the results, it can be seen how the fundamental weakness of Equation (15) to handle multiple failures can be fixed with normalization. We also run a test case under a given spare allocation and results in Fig. 6 show it achieves reasonably good approximate solutions.

VI. Conclusion and Further Research

In this paper, we consider a multi-echelon single-indenture inventory system that copes with combat damage occurring at certain time points. We proposed an approximate method to compute time-varying operational availability. Our model is particularly relevant in the context of both peacetime and wartime when heavy damages occur regularly during and after combat missions. Experimental results show our approach is accurate when the summation of component damage probabilities over all LRUs is 1 under each damage type and the replenishment policy. A challenging future work is to develop a more accurate analytical approach to model the scenario where the damage of LRUs is batched, i.e. dependent on each other, within a single combat damage. Another interesting research problem is to optimize the spare allocation in a time rolling horizon setting by reallocating the spares or replenishing from external provider based on the existing operational availability in response to dynamical changes and requirements with combat damage.

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