Risk based Optimization for Improving Emergency Medical Systems

Sandhya Saisubramanian and **Pradeep Varakantham** and **Hoong Chuin Lau**
School of Information Systems, Singapore Management University
80 Stamford Road, Singapore 178902
{sandhyas, pradeepv, hclau}@smu.edu.sg

Abstract

In emergency medical systems, arriving at the incident location a few seconds early can save a human life. Thus, this paper is motivated by the need to reduce the response time – time taken to arrive at the incident location after receiving the emergency call – of Emergency Response Vehicles, ERVs (ex: ambulances, fire rescue vehicles) for as many requests as possible. We expect to achieve this primarily by positioning the "right" number of ERVs at the "right" places and at the "right" times. Given the exponentially large action space (with respect to number of ERVs and their placement) and the stochasticity in location and timing of emergency incidents, this problem is computationally challenging. To that end, our contributions building on existing data-driven approaches are three fold:

1. Based on real world evaluation metrics, we provide a risk based optimization criterion to learn from past incident data. Instead of minimizing expected response time, we minimize the largest value of response time such that the risk of finding requests that have a higher value is bounded (ex: Only 10% of requests should have a response time greater than 8 minutes).
2. We develop a mixed integer linear optimization formulation to learn and compute an allocation from a set of input requests while considering the risk criterion.
3. To allow for "live" reallocation of ambulances, we provide a decomposition method based on Lagrangian Relaxation to significantly reduce the run-time of the optimization formulation.

Finally, we provide an exhaustive evaluation on real-world datasets from two asian cities that demonstrates the improvement provided by our approach over current practice and the best known approach from literature.

Introduction

Emergency Medical Systems (EMS) – which aim to provide timely care to victims of sudden and unforeseen accidents, injuries or illnesses – are an integral part of the health care system. Saving a few seconds in responding to an incident can save a human life and due to this reason, many major cities around the world invest heavily in building emergency response systems that can reduce response time of ERVs. Increasing the fleet size of ERVs continuously to meet the varying demand and performance benchmarks is not practically feasible and hence research in EMS has focussed on optimizing the utilization of existing emergency vehicles. In this paper, we specifically focus on ambulances. However, the formulations and mechanisms developed are applicable to other emergency response settings. Figure 1 provides the sequence of events in responding to an emergency request.

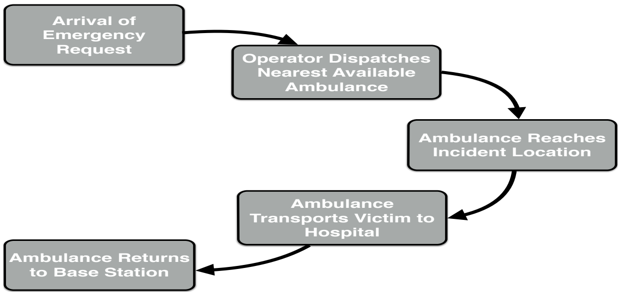

Arrival of Emergency Request
Operator Dispatches Nearest Available Ambulance
Ambulance Reaches Incident Location
Ambulance Transports Victim to Hospital
Ambulance Returns to Base Station

Figure 1: Process Flow

The significance of optimization in EMS is best illustrated by the rich research history it bears. The existing research has specifically focussed on the twin problem areas of: ambulance allocation and reallocation to base locations and ambulance dispatch to incidents. A survey of the existing approaches (**?**) highlights that much of the previous work in EMS has been on dispatch of ambulances from the base locations (**?**; **?**; **?**). While dispatch is an important research problem, given the practical difficulty of ascertaining the criticality of an emergency request over phone, in many EMS systems, ambulance dispatch procedure is fixed wherein the requests are served in the order of their arrival and from the nearest base location that has an idle ambulance. Furthermore, dispatch problem can only be simplified with a good allocation of ambulance to locations. Hence, our focus in this paper is on the first problem area of EMS – ERV allocation and reallocation to the base locations.

In this paper, we focus on ambulance allocation and dynamic reallocation for the entire ambulance fleet and not for an individual ambulance (**?**). To facilitate the detection of patterns and avoid heuristic worst case planning (**?**), we employ a data-driven approach, similar to the work by (**?**). Ironically, even though target metrics and actual evaluation of

EMS are based on the notion of bounded risk (?; ?; ?; ?; ?; ?), researchers have focussed on optimizing other metrics such as expected response time (?; ?). We address this discrepancy by considering the risk based optimization criterion and that is also the main distinction from existing work.

In addition to the greedy mechanism[1] by (?), existing work has provided heuristic search based mechanisms for computing allocation (?) based on modelling as a coverage problem. On the contrary, we focus on a scalable approximation approach with guarantees on learned allocation to learn from past data. To that end, our main contribution is an optimization formulation that provides novel insights on enforcing resource capacity constraints for a continuous time line using scalable linear constraints. Furthermore, by employing intuitions from Sample Average Approximation (?), we enforce the risk constraint using discrete variables and a linear constraint. In order to achieve scalability that will assist in live decision support, we also develop a partition technique based on Lagrangian relaxation.

We evaluate the effectiveness of our techniques on real-world ambulance data sets from two large cities in Asia. The experimental results show that in addition to outperforming the best known approach in the literature (?), our approach is able to reduce the response time by at least a few minutes over current practice.

Motivation: Ambulance Allocation

Formally, ambulance allocation problem for a given set of requests is given by the tuple:

$$\langle \mathcal{R}, \mathcal{B}, \mathcal{A}, \mathbf{T}, C, \alpha \rangle$$

The set of emergency requests is given by $\mathcal{R}$ and any emergency request, r in $\mathcal{R}$ is defined as the tuple $\langle t, s, h \rangle$, where t is the arrival time, s is the source location of the emergency and h is the nearest hospital to serve the request. The set of base locations is given by $\mathcal{B}$ and the set of ambulances is given by $\mathcal{A}$. $\mathbf{T}$ is a two dimensional vector providing the time taken to travel between any two locations in the city and this information is typically obtained from mapping services like google maps. More specifically, T_{l_1,l_2} is the time taken to travel from l_1 to l_2. C_l provides the maximum number of ambulances that can be stationed at base location l.

The goal is to compute an allocation, **a** of ambulances to base locations, that has the least value of response time, such that the percentage of requests exceeding that response time is less than the tunable input paramater, α. We refer to this response time as the α-response time. Since a request can be served from more than one base locations, the MILP formulation to determine the "right" allocation of ambulances to base locations such that the α-response time is reduced, becomes challenging.

Variables/ Constants	Definition
a_l	Number of ambulances at l
a_l^r	Number of ambulances at l when r arrives
y_l^r	Binary variable that indicates if request r is served by an ambulance from l.
$F_l^{q,r}$	Binary constant indicating whether an ambulance at l, if deployed to serve request q, will be available to serve the next request r

Table 1: Notation

MILP formulation for Ambulance Allocation

We now provide a Mixed Integer Linear Program (MILP) formulation for ambulance allocation. A detailed discussion on major assumptions made in the formulation and how they can be relaxed are presented in the latter section. Table 1 provides the variables and constants that are employed in the formulation.

The goal of the linear formulation is to minimize a response time value δ by addressing the following challenges:

1. **Ensuring that the probability of exceeding δ is bounded by α**: This is a chance constraint:

$$Pr(\delta^r \geq \delta) \leq \alpha$$

where δ^r is the response time for request r and probability is computed over the set of requests $\mathcal{R}$.

Insight 1 *Since we have a discrete set of requests, we can use the technique adopted in Sample Average Approximation (SAA) (?; ?) to provide an equivalent set of linear constraints as follows:*

$$z^r \in \{0,1\}, z^r \geq \frac{\delta^r - \delta}{M}, \qquad \forall r \in \mathcal{R} \qquad (1)$$

$$\frac{\sum_r z^r}{|\mathcal{R}|} \leq \alpha \qquad (2)$$

It should be noted that the variable z^r is set to 1 if δ^r is greater than δ and 0 otherwise. Therefore, sum of all z^r variables divided by total number of requests gives the percentage and hence should be less than α.

2. **Computing available ambulances at each location l, at the point of arrival of request r:**

Insight 2 *Since the set of requests are known before hand, by computing binary constants, $F_l^{q,r}$, we can compute number of available ambulances using a linear constraint on the binary variables* **y** *as shown in Equation 3.*

$$a_l^r = a_l - \sum_{q \in \mathcal{R}, q.t < r.t} y_l^q \cdot F_l^{q,r}, \forall r \in R, l \in \mathcal{B} \qquad (3)$$

$$a_l^r \geq 0, \quad \forall r \in R, l \in \mathcal{B} \qquad (4)$$

$$a_l \leq C_l, \quad \forall l \in \mathcal{B} \qquad (5)$$

$$\sum_l a_l = |\mathcal{A}| \qquad (6)$$

[1] Unlike for expected response time objective (?), the learning problem for a given set of requests when optimizing with bounded risk notion is not sub-modular. Hence, greedy mechanism is also a heuristic and does not provide any quality guarantees.

For each location, constraint 3 computes the number of idle ambulances based on the number of ambulances still serving previous requests(second part of right hand side)[2]. Constraints 4,5,6 ensure number of ambulances is a whole number and is within the capacity of the location, while also preventing any violation of the fleet size.

3. **Preventing service of a request when there are no ambulances available**: This is achieved by using a logical constraint that sets y_l^r variables for all locations to be zero.

$$\sum_l y_l^r = min(1, \sum_l a_l^r), \qquad \forall r \in \mathcal{R} \quad (7)$$

4. **Computing response time given y variables**: We can only ensure that the response time has a lower bound using the following constraint. However, in conjunction with objective and bounded risk constraint, response time for every request is equivalent to the lower bound.

$$\delta^r \geq \sum_l (y_l^r \cdot \mathbf{T}_{l,r.s}) + (1 - \sum_l y_l^r) \cdot M \quad \forall r \in R \quad (8)$$

Constraints 1-8 along with an objective of "$\min_{\mathbf{a}} \delta$" form the complete MILP. Henceforth, we refer to the response time δ, as α-response time for the given set of requests, $\mathcal{R}$.

Partitioning the Request Set for Scalability

The computational complexity of the MILP introduced in the previous section increases rapidly with increase in the number of requests (Figure 6). To counter this computational complexity, we exploit the notion of independence among requests. A formal definition of dependence of requests is presented below using the binary vector, **F**.

Definition 1 *Two requests q and r are* ***dependent*** *if there exists at least one location l for which* $F_l^{q,r} = 1$.

The dependency between requests is with respect to being served by an ambulance from the same base location. We pursue a two step approach to identify and exploit independence between requests:

1. Identify the minimum set of requests, say $\mathcal{G}$ in the request set $\mathcal{R}$, that if removed can create ξ independent partition sets of the request set with $\cup_{k \leq |\xi|} \xi_k = \mathcal{R} \setminus \mathcal{G}$
2. Given the independent partitions and the new set of requests $\mathcal{R} \setminus \mathcal{G}$, we compute one ambulance allocation (and not one ambulance allocation for each partition) that works across the independent partitions by using the well known technique of Lagrangian dual decomposition(LDD) (**?**)[3].

For (1), in the case of dynamic reallocation of ambulances for a small interval of time (say 4 hours), we only consider requests for those 4 hours on multiple different days in creating the request set. Hence, all these requests over different days are naturally independent. In the more interesting case of static allocation, there may not exist an exact partition across days. Figure 2 provides a small example of requests (taken from data set) and their connectivity (obtained from **F**). In such cases, we adopt minimal separator algorithm (**?**) to provide the partitions and correspondingly get the number of requests to be removed. In the example of Figure 2, we had to remove 12% requests to obtain the two partitions. Even though we remove requests, it is possible to maintain the risk bound of the original problem and obtain a conservative estimate of the optimal α response time by increasing the risk bound to $\alpha + \epsilon$[4], where ϵ corresponds to the percentage of requests that are removed. This provides a conservative estimate, because we assume that all ϵ% of requests will have a response time higher than the optimal in the original.

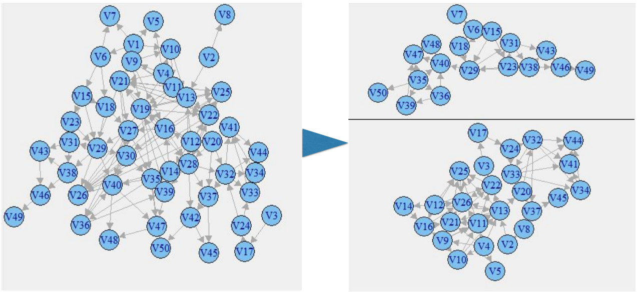


Figure 2: Partition of requests on a real request set

An updated optimization corresponding to partitions, ξ is:

$$\begin{aligned}
\min \quad & \sum_{k \leq |\xi|} \delta_k \\
s.t. \quad & Pr(\delta_k^r \geq \delta_k) \leq \alpha + \epsilon, && \forall r \in \xi_k, k \leq |\xi| \quad (9) \\
& a_{l,k}^r = a_{l,k} - \sum_{q \in \xi_k, q.t < r.t} y_{l,k}^q \cdot F_{l,k}^{r,q}, && \forall r, k, \forall l \in \mathcal{B} \quad (10) \\
& a_{l,k} \leq C_l, && \forall l \quad (11) \\
& \sum_l a_{l,k} = |\mathcal{A}|, && \forall k \quad (12) \\
& a_{l,k}^q \geq 0, && \forall r, l, k \quad (13) \\
& \sum_l y_{l,k}^r = min(1, \sum_l a_{l,k}^r), && \forall r, k \quad (14) \\
& \delta_k^r \geq \sum_l (y_{l,k}^r \cdot T_{l,r.s}) + (1 - \sum_l y_l^r) \cdot M, && \forall r, k \quad (15) \\
& a_{l,k} = n_l, \forall k, l && (16)
\end{aligned}$$

In the above optimization problem, we employ n_l to ensure all partitions have the same allocation of ambulances. Constraint 16 ensures that allocations across partitions are equal.

Dual Decomposition

Except for constraint 16, all the other constraints are defined on a specific partition. Hence, we obtain the Lagrangian by dualizing constraint 16:

$$\mathcal{L}(\lambda) = \min_{\mathbf{a}_k, \mathbf{n}} \sum_{k \leq |\xi|} \delta_k + \sum_{l,k} \lambda_{l,k} \cdot (a_{l,k} - n_l)$$

[2] Please refer to supplementary file for steps to compute F value.

[3] Please refer to the supplementary material for a background on Lagrangian dual decomposition.

[4] It should be noted that there may be no solution once the risk value is increased to account for the removed requests. In such a case, we can only get a heuristic approximation with this algorithm.

$$
\begin{aligned}
&\min\ \delta_k \\
&s.t.\ Pr(\delta_k^r \geq \delta_k) \leq \alpha + \epsilon, && \forall r \in \xi_k && (19)\\
&\quad a_{l,k}^r = a_{l,k} - \sum_{q \in \xi_k, q.t < r.t} y_{l,k}^q \cdot F_{l,k}^{r,q}, && \forall r, \forall l \in \mathcal{B} && (20)\\
&\quad a_{l,k} \leq C_l, && \forall l && (21)\\
&\quad \sum_l a_{l,k} = |\mathcal{A}| && && (22)\\
&\quad a_{l,k}^q \geq 0, && \forall r, l && (23)\\
&\quad \sum_l y_{l,k}^r = min(1, \sum_l a_{l,k}^r), && \forall r && (24)\\
&\quad \delta_k^r \geq \sum_l (y_{l,k}^r \cdot T_{l,r.s}) + (1 - \sum_l y_l^r) \cdot M, && \forall r && (25)
\end{aligned}
$$

Table 2: SOLVESLAVE(k)

or alternatively

$$= \min_{\mathbf{a}} \Big(\sum_{k \leq |\xi|} \Big[\delta_k + \sum_l \lambda_{l,k} \cdot a_{l,k}\Big]\Big) - \min_{\mathbf{n}} \sum_{k \leq |\xi|, l} \lambda_{k,l} \cdot n_l$$

Since n_l is unbounded, the overall optimization will be unbounded. The equivalent bounded optimization for calculating $\mathcal{L}(\lambda)$ is:

$$\min_{\mathbf{a}} \Big(\sum_{k \leq |\xi|} \Big[\delta_k + \sum_l \lambda_{l,k} \cdot a_{l,k}\Big]\Big)\ s.t. \sum_{k \leq |\xi|} \lambda_{k,l} = 0, \forall l \quad (17)$$

If we use an update rule for λ that ensures the constraint 17 is always satisfied, then $\mathcal{L}(\lambda)$ decomposes over the individual partitions. Table 2 provides the slave optimization problem corresponding to each partition k.

Update of Price Variables——————
In order to ensure that dualized constraint violations are minimized, the final optimization at the master is: $\max_\lambda \mathcal{L}(\lambda)$ and we solve this optimization using sub-gradient descent on λ. Combining the update required for sub-gradient descent and for satisfaction of constraint 17, we have the following update rule:

$$\lambda_{l,k}^{t+1} = \tilde{\lambda}_{l,k}^{t+1} - \sum_k \frac{\tilde{\lambda}_{l,k}^{t+1}}{\sum_k \tilde{\lambda}_{l,k}^{t+1}}\ \textbf{\&\&}\ \tilde{\lambda}_{l,k}^{t+1} = \lambda_{l,k}^t + \gamma^t \cdot a_{l,k} \quad (18)$$

Primal Extraction from Dual Solution, $\{\mathbf{A}_k\}_{k \leq |\xi|}$——
Let the dual solution values be $\{\mathbf{A}_k\}_{k \leq |\xi|}$, once all the slave optimization problems are solved. When the ambulance allocations across the partitions are not equal, the obtained dual solution is not a feasible primal solution. However, all the following derived allocations are feasible solutions given the dual solution: $\langle \mathbf{A}_1, \mathbf{A}_1, \cdots, \mathbf{A}_1 \rangle$, $\langle \mathbf{A}_2, \mathbf{A}_2, \cdots, \mathbf{A}_2 \rangle, \ldots$. Amongst all these solutions, we take the one which provides the least value of α-response time as the primal solution for the current iteration. α-response time for a given ambulance allocation, $\mathbf{A}_k$ for a partition q is obtained by replacing all occurrences of $a_{l,q}$ with $A_{l,k}$ in SOLVESLAVE(q);

Algorithm 1 provides the pseudo code for the overall procedure of Lagrangian dual decomposition. We stop our lagrangian approach when the duality gap (difference between the primal and dual value) falls below a tunable value,μ.

Algorithm 1: **SolveLDD()**

```
Initialize: $\lambda^0, it \leftarrow 0$ ;
repeat
    $\forall k : d_k, \mathbf{A}_k \leftarrow$ SOLVESLAVE($k$)
    Update prices according to Equation 18
    $p, \mathbf{A}_p \leftarrow$ EXTRACTPRIMAL ($\{\mathbf{A}_k\}_{k \leq |\xi|}$);
    $it \leftarrow it + 1$;
until $\left[p - \sum_{k \leq |\xi|} d_k\right] \leq \mu$ ;
return $p, \mathbf{A}_p$
```

Dynamic Reallocation

When there exists a strong pattern of requests coming through in a few hours, it is beneficial to alter the allocation computed from a large set of requests. For instance, on a wednesday or friday night, there is high chance of emergency events occuring near areas with clubs as there are large gatherings of people due to ladies' night or weekend. By using request sets for the next few hours from multiple days, SOLVELDD() can be used to obtain a new allocation. While such a modification is applicable, there are practical concerns: (a) only a few ambulances are available for allocation at that time step; and (b) ambulances could spend a significant time moving between base locations without serving requests.

Therefore, we consider reallocating ambulances by minimizing movements with respect to existing allocation of ambulances. This ensures that many ambulances do not remain unavailable for long while undergoing reallocation. We introduce a penalty parameter *p*, that is calculated as the difference between the orginal allocation and the planned reallocation. This penalty paramater helps to keep a check on drastic changes to the existing allocation and to ensure better α-response time with minimal movement of ambulances. Users can adjust the input parameter Γ which serves as upper bound for the penalty parameter *p*, to balance performance and reallocation of the fleet. We add the following constraints to achieve the reallocation with minimal movement. a_l denotes the number of ambulances at l, obtained by previous allocation and n_l denotes the number of ambulances at l after reallocation.

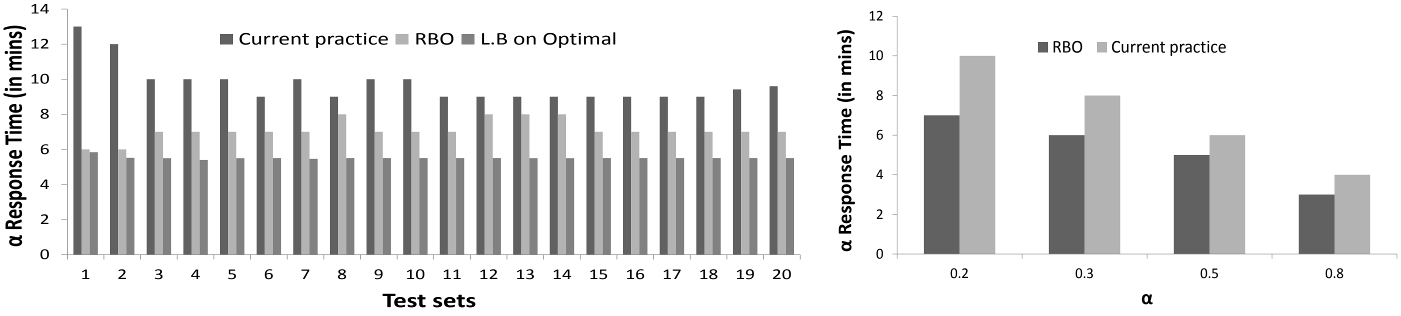
$$p = \sum_l (n_l - a_l); \quad p \leq \Gamma \qquad (26)$$

Experimental Results

We perform experiments on real world data by varying parameters to make the following performance comparisons:

- Risk based optimization (RBO) and the current practice (response time observed from the data).
- RBO and the best known approach for ambulance allocation in the literature(**?**).
- RBO with and without LDD.
- Dynamic reallocation and static allocation.

We experiment with two data sets, namely Dataset1 and Dataset2 obtained from two asian cities. Dataset2 is adopted



(a) Results on multiple test sets

(b) α response time comparison

Figure 3: RBO Vs Current practice

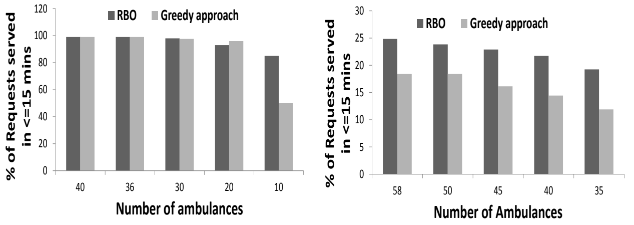
from (**?**). Both these datasets provide information about emergency requests over a sustained period of time (1 year). An emergency request consists of the time of request arrival, incident location and hospital location for the request. We also have information about number of ambulances, categories of ambulances, base locations etc.

The metrics employed for performance comparisons include the: α response time (α-rt), percentage of requests with response time less than 15 minutes[5] and run time. An $\alpha - rt = 8$ when $\alpha = 80\%$ indicates that at least 80% of the requests are served with response time $\leq$ 8 minutes. Given the number of cities that employ α response time as the performance indicator for emergency medical systems (**?**; **?**; **?**; **?**; **?**; **?**), we also use that as our main evaluation criterion. RBO (Risk Based Optimization) represents the approach introduced in this paper, "CP" refers to current practice and "Greedy" denotes the greedy approach (**?**).

In making a comparison, we always consider a training set and test set. Training sets are used to obtain an allocation and testing sets are used to evaluate such an allocation. To calculate the α response time on the test set using the obtained allocation, we use the standard dispatch technique, wherein a request that arrives first is serviced and by an available ambulance nearest to the incident location. Unless otherwise stated, the default settings of the experiments are $\alpha = 0.2$, fleet size of dataset1 = 40 and fleetsize of datatset2 = 58. The experiments involving RBO refer to RBO with LDD.

RBO vs Current practice——————

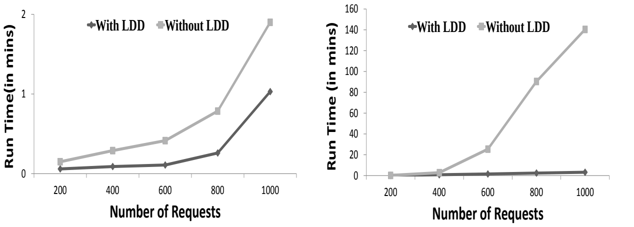
Figure 3(a) plots the α-rt for 20 different test sets from Datatset1, that were created from requests on different days. An allocation for RBO was computed from a training data set consisting of requests spread over a week. In addition to comparing with CP, we also compare with an approach, wherein we use RBO to compute an allocation on the test set requests. As it can be expected, this approach will provide a lower bound on the optimal α-rt, as test set requests are assumed to be known in advance. We observe that results from RBO are always better than CP, with the difference as much as 6 minutes in some occasions. Furthermore, results from RBO are typically closer to the lower bound on optimal α-rt than to the results from CP.



(a) Dataset1

(b) Dataset2

Figure 5: α response time $\leq$ 15 minutes



(a) Dataset1

(b) Dataset2

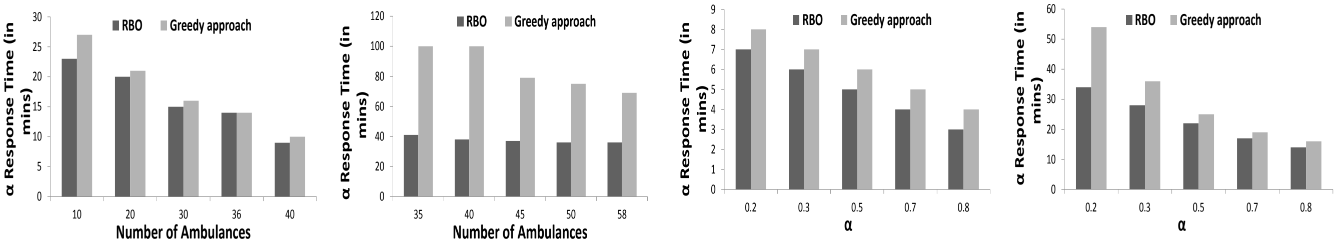
Figure 6: Run time comparison

Figure 3(b) presents the results of our approach on Dataset1 in comparison with current practice, when α is increased. On Y-axis, we provide the α-response time for both our approach and current practice. The α response time values reduce as α is increased for both approaches, since risk reduces as α is increased.The key result however is that RBO was able to provide lower α-response time value than CP for all values of α. In some instances, this difference was as high as 3 minutes. Since we did not have sufficient information to compare our approach with the current practice on dataset2, we restrict the comparison of our approach against the current practice to datatset1.

RBO vs Greedy——————

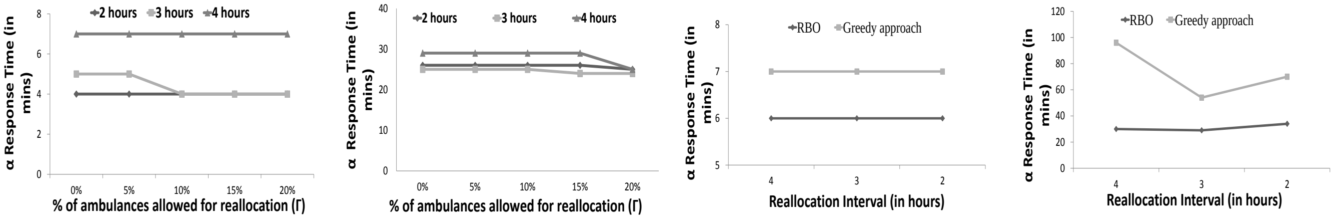
Figure 4 presents the results with respect to the α-rt metric on both the datasets. Figure 4(a)-(b) compare α-rt performance as fleet size is varied on both the datasets. Irrespective of the fleet size, we observe that RBO performs

[5] Fitness measure employed in (**?**)



(a) Dataset1: Effect of fleet size (b) Dataset2: Effect of fleet size (c) Dataset1: Effect of α (d) Dataset2: Effect of α

Figure 4: RBO Vs Greedy approach



(a) Dataset1: Effect of Γ (b) Dataset2: Effect of Γ (c) Dataset1: Effect of Reallocation Interval (d) Dataset2: Effect of Reallocation Interval

Figure 7: Dynamic Reallocation results

better than greedy. The difference is particularly significant on Dataset2. Figure 4(c)-(d) compare the α-rt performance as α is increased. We again observe that RBO performs better than Greedy. Previously, the metric employed in evaluating the "Greedy" technique of (**?**) is the maximum number of requests with a response time less than 15 minutes. Figure 5 shows the % of requests served within the metric by our approach in comparison with the greedy approach on both the datasets. As can observed from the graph, RBO outperforms greedy by atleast 20% on all instances of dataset2.

RBO with and without LDD——
From figure 6, we demonstrate the reduction in runtime when LDD is used in conjunction with the MILP. While the time taken by RBO with LDD increases linearly, the time taken by RBO without LDD increases exponentially for Dataset2. However, on Dataset1 for same number of requests in the training set,while LDD provides a difference, it is not significant. This is because the run time depends on how densely populated the requests are on the time line.

Dynamic Reallocation——
To allow for "live" reallocation of ambulances and to determine the significance of dynamic reallocation, we apply RBO with minimal movements constraints (constraint 26) on both datasets. By varying the upper bound Γ of the penalty parameter p , we observe the effect on the performance when a fraction of the fleet is used for reallocation.

Figures 7(a) and (b) provide the α-rt on Dataset1 and Datatset2 for different reallocation intervals and different percentage of ambulances available for reallocation (Γ). When $\Gamma = 0$, no ambulances are allowed for reallocation and hence this result gives us the result of existing allocation for that interval of time. We observe that as we increase the Γ value, we allow more ambulances to be reallocated and the α response time either reduces or remains the same as that of static allocation in most cases.From figure 7(c) and (d), we observe that our reallocation technique provides better results than the greedy reallocation apporach over the three reallocation intervals.

In some cases, dynamic reallocation did not improve α-rt (as shown in Figure 7) and in few other scenarios (not shown here), the α-rt with dynamic reallocation was worse than with the static allocation. Hence, identifying when and where dynamic reallocation is helpful and is an important problem for future work.

Discussion and Future Work

In this section, we discuss some of the important assumptions made in the paper. First, in the MILP formulation for ambulance allocation, we assumed that if there is no ambulance to serve a request when it arrives, then the optimization problem assigns a large value of response time or equivalently it is considered as not served. However, in practice, the dispatcher would wait for a grace time (ex: 5-10 minutes) before considering the request as unserved. This can be trivially implemented in our model by modifying the definition of $F_l^{q,r}$ to consider the grace time. That is to say, for a grace time of 5 minutes, $F_l^{q,r}$ is set to 0 if request q finishes within 5 minutes after arrival of request r.

Second, after serving a request, ambulances return to the respective base locations from where they were dispatched to serve the request. We can potentially allocate ambulances after the victim has been delivered to the hospital. Exploiting on the fly allocation may or may not be an opportunity depending on how the requests arrive. Understanding when and where on the fly allocation is useful, is an important problem and we intend to explore it in the future.

Third, in dynamic reallocation, it is advisable to not move the same ambulance repeatedly over two locations. Since we cannot identify individual ambulances in our model(as we consider homogenous fleet), this criteria cannot be explicitly handled by our model. However, it is possible to have a more specific penalty constraint, wherein ambulance movement from different base locations are penalised differently.

Finally, while we assume that all ambulances can cater to all requests, certain very specific requests such as cardiac emergencies will need ambulance with specialized cardiac support system. In such cases, allocation and dispatch of ambulances should be based on the type of request. To perform this, additional information about the type of illness should be considered in the definition of emergency request. With minor modifications to our MILP, we can address such specific constraints.

Acknowledgements

We thank the anonymous reviewers for their valuable feedback. This research is supported by the Singapore National Foundation under its International Research Centre @ Singapore Funding Initiative and administered by the IDM Programme Office, Media Development Authority (MDA).