Schooling, Political Participation, and the Economy

(Online Supplementary Appendix: Not for Publication)

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Abstract

In this online appendix, we present the proofs for the propositions in the model developed at the end of the paper, "Schooling, Political Participation, and the Economy".

Proof of Proposition 1

Proof. The existence of a solution to the effort-allocation problem is guaranteed by the fact that the maximand (P7), where P indicates a numbered equation from the model set up in the paper, is a continuous function over the compact simplex defined by (P8) and the non-negativity constraints. Now, substitute $x = H - h_A - h_M - h_S$ into (P7). Treating this as an unconstrained maximization problem, the first-order conditions with respect to h_A, h_M, and h_S jointly imply that:

$$\alpha h_A^{\alpha-1} T^{1-\alpha} = \mu p_M A_M h_M^{\mu-1} K^{1-\mu} = \sigma p_S A_S h_S^{\sigma-1} S^{1-\sigma}, \tag{1}$$

and also that:

$$(1-\tau(X))\, \alpha h_A^{\alpha-1} T^{1-\alpha} = -\tau^{'}(X) \left[h_A^{\alpha} T^{1-\alpha} + \frac{\alpha}{\mu} h_A^{\alpha-1} T^{1-\alpha} h_M + \frac{\alpha}{\sigma} h_A^{\alpha-1} T^{1-\alpha} h_S \right],$$

This last equation can be rewritten as:

$$-\frac{1-\tau(X)}{\tau^{'}(X)} = \left[\frac{1}{\alpha} h_A + \frac{1}{\mu} h_M + \frac{1}{\sigma} h_S \right]. \tag{2}$$

The assumption that $\tau'(0) \longrightarrow -\infty$ and the Cobb-Douglas production functions (which satisfy a similar Inada condition), ensure that the non-negativity constraints do not bind in practice, since the infinite

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marginal product in a neighborhood of zero guarantees that it is optimal to allocate a positive amount of effort to every activity. Thus, the first-order conditions above from the unconstrained maximization problem also pin down the solution to the constrained problem.

Differentiating (1) yields:

$$\frac{dh_A}{dH} = \frac{1-\mu}{1-\alpha}\frac{h_A}{h_M}\frac{dh_M}{dH} = \frac{1-\sigma}{1-\alpha}\frac{h_A}{h_S}\frac{dh_S}{dH}, \tag{3}$$

while differentiating (2) yields:

$$\Theta\frac{dx}{dH} = \frac{1}{\alpha}\frac{dh_A}{dH} + \frac{1}{\mu}\frac{dh_M}{dH} + \frac{1}{\sigma}\frac{dh_S}{dH}, \tag{4}$$

where $\Theta \equiv [1 + \frac{1-\tau}{\tau'}\frac{\tau''}{\tau'}]$. Since $\tau' < 0$ and $\tau'' > 0$, this implies that $\Theta \geq 0$. Finally, differentiating the budget constraint (P8) yields:

$$\frac{dh_A}{dH} + \frac{dh_M}{dH} + \frac{dh_S}{dH} + \frac{dx}{dH} = 1. \tag{5}$$

Based on (3), we know that $\frac{dh_A}{dH}$, $\frac{dh_M}{dH}$ and $\frac{dh_S}{dH}$ share the same sign (since $0 < \alpha, \mu, \sigma < 1$). In addition, (4) implies that $\frac{dx}{dH}$ also shares this same sign because $\Theta \geq 0$. It immediately follows from (5) that this sign has to be positive. ■

Proof of Proposition 2

Proof. Substituting (3) into (4) and (5), we obtain:

$$\frac{dx}{dH} = \frac{1}{1+\Theta\Lambda}, \tag{6}$$

where $\Lambda \equiv \left[\frac{\frac{1}{1-\alpha}h_A+\frac{1}{1-\mu}h_M+\frac{1}{1-\sigma}h_S}{\frac{1}{\alpha}\frac{1}{1-\alpha}h_A+\frac{1}{\mu}\frac{1}{1-\mu}h_M+\frac{1}{\sigma}\frac{1}{1-\sigma}h_S}\right]$. It follows that the sign of $\frac{d^2x}{dHdT}$ depends on $\frac{d\Theta}{dT}$ and $\frac{d\Lambda}{dT}$. Our functional form assumption on $\tau(X)$ simplifies the problem as Θ is a positive constant (equal to $\frac{1}{\sigma_x}$). We can thus conclude that the sign of $\frac{d^2x}{dHdT}$ will be the opposite of the sign of $\frac{d\Lambda}{dT}$.

Now, note that:

$$\begin{aligned}\frac{d\Lambda}{dT} \quad \propto \quad & \left[\left(\frac{1}{\mu}-\frac{1}{\alpha}\right)\frac{1}{1-\mu}h_M + \left(\frac{1}{\sigma}-\frac{1}{\alpha}\right)\frac{1}{1-\sigma}h_S\right]\frac{1}{1-\alpha}\frac{dh_A}{dT} & (7)\\ & + \left[\left(\frac{1}{\alpha}-\frac{1}{\mu}\right)\frac{1}{1-\alpha}h_A + \left(\frac{1}{\sigma}-\frac{1}{\mu}\right)\frac{1}{1-\sigma}h_S\right]\frac{1}{1-\mu}\frac{dh_M}{dT} \\ & + \left[\left(\frac{1}{\alpha}-\frac{1}{\sigma}\right)\frac{1}{1-\alpha}h_A + \left(\frac{1}{\mu}-\frac{1}{\sigma}\right)\frac{1}{1-\mu}h_M\right]\frac{1}{1-\sigma}\frac{dh_S}{dT}. & (8)\end{aligned}$$

where $\propto$ denotes equality up to a positive multiplicative constant. In the proof of Proposition 3, we will show that $\frac{dh_A}{dT} > 0$, $\frac{dh_M}{dT} < 0$, and $\frac{dh_S}{dT} < 0$. Given this and the parameter assumption $0 < \alpha < \mu < \sigma < 1$, all of the terms in the expression above are negative, except for $\left(\frac{1}{\sigma}-\frac{1}{\mu}\right)\frac{1}{1-\sigma}h_S\frac{1}{1-\mu}\frac{dh_M}{dT}$. However, collecting the two terms involving $\left(\frac{1}{\sigma}-\frac{1}{\mu}\right)$ and using (16), we obtain:

$$\left(\frac{1}{\sigma}-\frac{1}{\mu}\right)\frac{1}{(1-\sigma)(1-\mu)}\left[h_S\frac{dh_M}{dT} - h_M\frac{dh_S}{dT}\right] = \left(\frac{1}{\sigma}-\frac{1}{\mu}\right)\frac{\mu-\sigma}{(1-\sigma)(1-\mu)}h_M\frac{dh_S}{dT} < 0. \tag{9}$$

It follows that $\frac{d\Lambda}{dT} < 0$, and hence $\frac{d^2x}{dHdT} > 0$.

A similar approach signs the cross-derivative with respect to H and S. We have an expression for $\frac{d\Lambda}{dS}$ that mirrors (7), the only difference being that T is replaced by S. The proof of Proposition 3 yields expressions for $\frac{dh_A}{dS}$, $\frac{dh_M}{dS}$, and $\frac{dh_S}{dS}$, and we plug these into the expression for $\frac{d\Lambda}{dS}$. Upon simplification, this yields:

$$\begin{aligned}\frac{d\Lambda}{dS} \quad &\propto \quad -\left(\frac{1}{1-\alpha}\right)^2\left[\left(\frac{1}{\mu}-\frac{1}{\alpha}\right)\frac{1}{1-\mu}h_M+\left(\frac{1}{\sigma}-\frac{1}{\alpha}\right)\frac{1}{1-\sigma}h_S\right]h_A \\ &\quad -\left(\frac{1}{1-\mu}\right)^2\left[\left(\frac{1}{\alpha}-\frac{1}{\mu}\right)\frac{1}{1-\alpha}h_A+\left(\frac{1}{\sigma}-\frac{1}{\mu}\right)\frac{1}{1-\sigma}h_S\right]h_M \\ &\quad +\frac{1}{1-\sigma}\left[\left(\frac{1}{\alpha}-\frac{1}{\sigma}\right)\frac{1}{1-\alpha}h_A+\left(\frac{1}{\mu}-\frac{1}{\sigma}\right)\frac{1}{1-\mu}h_M\right]\left[\frac{\frac{1}{\alpha}+\Theta}{\frac{1}{\sigma}+\Theta}\frac{h_A}{1-\alpha}+\frac{\frac{1}{\mu}+\Theta}{\frac{1}{\sigma}+\Theta}\frac{h_M}{1-\mu}\right] \qquad (10)\end{aligned}$$

The terms in (10) involving h_Ah_S and h_Mh_S are all unambiguously positive. Moreover, we can collect all terms in h_Ah_M, which are proportional (up to a positive multiplicative constant) to:

$$\begin{aligned}&\frac{1}{1-\alpha}\left(\frac{1}{\alpha}-\frac{1}{\mu}\right)-\frac{1}{1-\mu}\left(\frac{1}{\alpha}-\frac{1}{\mu}\right)+\frac{1}{1-\sigma}\left[\left(\frac{1}{\mu}-\frac{1}{\sigma}\right)\frac{\frac{1}{\alpha}+\Theta}{\frac{1}{\sigma}+\Theta}+\left(\frac{1}{\alpha}-\frac{1}{\sigma}\right)\frac{\frac{1}{\mu}+\Theta}{\frac{1}{\sigma}+\Theta}\right] \\ > \quad &\frac{1}{1-\alpha}\left(\frac{1}{\alpha}-\frac{1}{\mu}\right)-\frac{1}{1-\mu}\left(\frac{1}{\alpha}-\frac{1}{\mu}\right)+\frac{1}{1-\sigma}\left[\left(\frac{1}{\mu}-\frac{1}{\sigma}\right)+\left(\frac{1}{\alpha}-\frac{1}{\sigma}\right)\right] \\ > \quad &0.\end{aligned}$$

This last inequality follows from the restriction $0 < \alpha < \mu < \sigma < 1$, which in turn ensures that:

$$\frac{1}{1-\mu}\left(\frac{1}{\alpha}-\frac{1}{\mu}\right) < \frac{1}{1-\sigma}\left(\frac{1}{\alpha}-\frac{1}{\sigma}\right). \qquad (11)$$

Thus, we can conclude that $\frac{d\Lambda}{dS} > 0$, from which it follows that $\frac{d^2x}{dHdS} < 0$.

We can repeat the same exercise for $\frac{d\Lambda}{dK}$. This will yield an analogous equation to (7) with T replaced by K, in which the term in $\frac{dh_A}{dK}$ is positive, but the term in $\frac{dh_S}{dK}$ is negative, and the term in $\frac{dh_M}{dK}$ is again of an ambiguous sign. It turns out that this configuration implies an ambiguous sign for $\frac{d^2x}{dHdK}$. ■

Proof of Corollary 3

Proof. From (6), the behavior of $\frac{d^2x}{dHd\sigma_x}$ will depend on that of $\frac{d\Theta}{d\sigma_x}$ and $\frac{d\Lambda}{d\sigma_x}$. It is easy to show that $\frac{d\Theta}{d\sigma_x} < 0$, so $\frac{d^2x}{dHd\sigma_x}$ will be positive if $\frac{d\Lambda}{d\sigma_x} < 0$. To show this, start by differentiating (P8) and (1) with respect to σ_x, which gives us expressions analogous to (5) and (3), namely:

$$\frac{dh_A}{d\sigma_x} = \frac{1-\mu}{1-\alpha}\frac{h_A}{h_M}\frac{dh_M}{d\sigma_x} = \frac{1-\sigma}{1-\alpha}\frac{h_A}{h_S}\frac{dh_S}{d\sigma_x}, \qquad (12)$$

$$\frac{dh_A}{d\sigma_x}+\frac{dh_M}{d\sigma_x}+\frac{dh_S}{d\sigma_x}+\frac{dx}{d\sigma_x} = 0. \qquad (13)$$

These imply that $\frac{dh_A}{d\sigma_x}$, $\frac{dh_M}{d\sigma_x}$ and $\frac{dh_S}{d\sigma_x}$ have the same sign, which has to be the opposite to that of $\frac{dx}{d\sigma_x}$. Differentiating (2) with respect to σ_x in turn yields the following (analogous to (4)):

$$\Theta\frac{dx}{d\sigma_x}+X\frac{d\Theta}{d\sigma_x} = \frac{1}{\alpha}\frac{dh_A}{d\sigma_x}+\frac{1}{\mu}\frac{dh_M}{d\sigma_x}+\frac{1}{\sigma}\frac{dh_S}{d\sigma_x}. \qquad (14)$$

This implies that $\frac{dh_A}{d\sigma_x}, \frac{dh_M}{d\sigma_x}, \frac{dh_S}{d\sigma_x} < 0$ and $\frac{dx}{d\sigma_x} > 0$. Quite intuitively, a greater human capital-intensity of political activity leads individuals to choose more political participation, and to devote less human capital to production.

The rest of the proof closely follows that of Proposition 2. Again, we have an expression for $\frac{d\Lambda}{d\sigma_x}$ that mirrors (7), with the only difference being that T is replaced by σ_x. We have three positive terms (the two on $\frac{dh_A}{d\sigma_x}$, plus one on $\frac{dh_M}{d\sigma_x}$), but collecting all terms that are multiplied respectively by $\left(\frac{1}{\mu} - \frac{1}{\alpha}\right)$, $\left(\frac{1}{\sigma} - \frac{1}{\alpha}\right)$, and $\left(\frac{1}{\sigma} - \frac{1}{\mu}\right)$ it is easy to show that these positive terms are dominated, so that $\frac{d\Lambda}{d\sigma_x} < 0$. This completes the proof. ■

Proof of Proposition 4

Proof. Part 1. We proceed in a similar fashion to the proof of Proposition 1, to set up a system of four equations in $\frac{dh_A}{dT}$, $\frac{dh_M}{dT}$, $\frac{dh_S}{dT}$, and $\frac{dx}{dT}$. To do so, we totally differentiate (P8), (1), and (2) with respect to T. The analogue of equation (3) is now:

$$\frac{dh_A}{dT} = \frac{h_A}{T} + \frac{1-\mu}{1-\alpha}\frac{h_A}{h_M}\frac{dh_M}{dT}, \tag{15}$$

$$\frac{1-\mu}{h_M}\frac{dh_M}{dT} = \frac{1-\sigma}{h_S}\frac{dh_S}{dT}. \tag{16}$$

Also, (4) remains unchanged, except that all derivatives with respect to H are replaced by derivatives with respect to T. Finally, the constraint (P8) now implies:

$$\frac{dh_A}{dT} + \frac{dh_M}{dT} + \frac{dh_S}{dT} + \frac{dx}{dT} = 0. \tag{17}$$

Substituting $\frac{dx}{dT}$ from (17) into the new version of (4) yields:

$$\frac{dh_A}{dT} = \frac{1}{D_T}\left[\left(\frac{1}{\mu} + \Theta\right)\frac{1-\alpha}{1-\mu}\frac{h_M}{T} + \left(\frac{1}{\sigma} + \Theta\right)\frac{1-\alpha}{1-\sigma}\frac{h_S}{T}\right] > 0,$$

where we define $D_T \equiv \left(\frac{1}{\alpha} + \Theta\right) + \left(\frac{1}{\mu} + \Theta\right)\frac{1-\alpha}{1-\mu}\frac{h_M}{h_A} + \left(\frac{1}{\sigma} + \Theta\right)\frac{1-\alpha}{1-\sigma}\frac{h_S}{h_A}$ to keep notation simple. Substituting this into (15) in turn yields:

$$\frac{dh_M}{dT} = \frac{1}{D_T}\left[\frac{1-\alpha}{1-\mu}\frac{h_M}{h_A}\left[\left(\frac{1}{\mu} + \Theta\right)\frac{1-\alpha}{1-\mu}\frac{h_M}{T} + \left(\frac{1}{\sigma} + \Theta\right)\frac{1-\alpha}{1-\sigma}\frac{h_S}{T}\right] - D_T\frac{1-\alpha}{1-\mu}\frac{h_M}{T}\right],$$

which, with some straightforward manipulation, we can simplify as:

$$\frac{dh_M}{dT} = -\frac{1}{D_T}\frac{1-\alpha}{1-\mu}\frac{h_M}{T}\left[\frac{1}{\alpha} + \Theta\right] < 0.$$

Note that (16) now immediately implies that $\frac{dh_S}{dT} < 0$, since it must have the same sign as $\frac{dh_M}{dT}$. In fact:

$$\frac{dh_S}{dT} = -\frac{1}{D_T}\frac{1-\alpha}{1-\sigma}\frac{h_S}{T}\left[\frac{1}{\alpha} + \Theta\right] < 0.$$

Now we can substitute into (17) the expressions for $\frac{dh_A}{dT}$, $\frac{dh_M}{dT}$, and $\frac{dh_S}{dT}$ that we have just obtained. This yields:

$$\frac{dx}{dT} = -\frac{1}{D_T}\left[\frac{1-\alpha}{1-\mu}\frac{h_M}{T}\left(\frac{1}{\mu} - \frac{1}{\alpha}\right) + \frac{1-\alpha}{1-\sigma}\frac{h_S}{T}\left(\frac{1}{\sigma} - \frac{1}{\alpha}\right)\right].$$

Since $\frac{1}{\mu} - \frac{1}{\alpha} < 0$ and $\frac{1}{\sigma} - \frac{1}{\alpha} < 0$, we have $\frac{dx}{dT} > 0$.

Part 2. We proceed in an analogous fashion as in the proof of Part 1. All one has to note is that the role played by h_A in Part 1 is now played by h_M, and we should replace the parameters suitably as well; what used to be α is now μ, and vice versa. Once this is done, it is easy to check that $\frac{dh_M}{dK} > 0$ (just as $\frac{dh_A}{dT} > 0$), $\frac{dh_A}{dK} < 0$, and $\frac{dh_S}{dK} < 0$. We can also see why the sign of $\frac{dx}{dK}$ is ambiguous: $\frac{1}{\alpha} - \frac{1}{\mu} > 0$, while $\frac{1}{\sigma} - \frac{1}{\alpha} < 0$.

Part 3. A similar proof applies, with h_S and σ replacing h_M and μ respectively, in our proof of Part 2. It immediately follows that $\frac{dh_S}{dS} > 0$, $\frac{dh_A}{dS} < 0$, and $\frac{dh_M}{dS} < 0$. Now the sign of $\frac{dx}{dS}$ is negative, since $\frac{1}{\alpha} - \frac{1}{\sigma} > 0$, and $\frac{1}{\mu} - \frac{1}{\sigma} > 0$. ■

Proof of Proposition 5

Proof. Manipulating (1) yields:

$$h_M = \left(\frac{\mu}{\alpha} A_M p_M\right)^{\frac{1}{1-\mu}} \left(\frac{h_A}{T}\right)^{\frac{1-\alpha}{1-\mu}} K,$$
$$h_S = \left(\frac{\sigma}{\alpha} A_S p_S\right)^{\frac{1}{1-\sigma}} \left(\frac{h_A}{T}\right)^{\frac{1-\alpha}{1-\sigma}} S.$$

Imposing symmetry ($h_S = S$) on these two equations immediately implies:

$$h_A = \left(\frac{\alpha}{\sigma A_S p_S}\right)^{\frac{1}{1-\alpha}} T,$$
$$h_M = \left(\frac{\mu A_M p_M}{\sigma A_S p_S}\right)^{\frac{1}{1-\mu}} K.$$

Now we can use the functional form for $\tau(X)$, and equation (P8), to obtain:

$$\begin{aligned} & \frac{Nx}{\sigma_x} = \frac{1}{\alpha} h_A + \frac{1}{\mu} h_M + \frac{1}{\sigma} h_S \\ \Rightarrow \quad & \frac{N}{\sigma_x}\left[H - h_S - h_A - h_M\right] = \frac{1}{\alpha} h_A + \frac{1}{\mu} h_M + \frac{1}{\sigma} h_S \\ \Rightarrow \quad & h_S = \frac{N\sigma}{\sigma_x + N\sigma} H - \frac{\sigma}{\alpha}\frac{N\alpha + \sigma_x}{N\sigma + \sigma_x} h_A - \frac{\sigma}{\mu}\frac{N\mu + \sigma_x}{N\sigma + \sigma_x} h_M \end{aligned}$$

Substituting this into (7) yields:

$$x = \frac{\sigma_x}{\sigma_x + N\sigma} H + \frac{(\sigma - \alpha)\sigma_x}{\alpha(N\sigma + \sigma_x)} h_A + \frac{(\sigma - \mu)\sigma_x}{\mu(N\sigma + \sigma_x)} h_M$$

The expression for X follows immediately from the definition $X \equiv Nx$. ■

Proof of Proposition 6

Proof. Assuming an interior solution, the optimal amount of H and K from the ruler's standpoint must satisfy the first-order conditions:

$$\tilde{N} p_S A_S \left[\tau'(X)X + \tau(X)\right] = \frac{1}{F'_H(H)} \tag{18}$$

$$\tilde{N} p_S A_S \frac{\sigma - \mu}{\mu} \left(\frac{\mu A_M p_M}{\sigma A_S p_S}\right)^{\frac{1}{1-\mu}} \left[\tau'(X)X + \tau(X)\right] = \frac{1}{F'_K(K)}. \tag{19}$$

$$\tau'(X)X + \tau(X) = 0. \tag{20}$$

where $\tilde{N} = \frac{N^2\sigma}{N\sigma + \sigma_x}$ is a positive constant that depends only on model parameters. The first term in square brackets in (18) and (19), $\tau'(X)X$, captures the marginal "political" cost of providing citizens with an extra unit of human capital: It will increase political participation, and thus reduce the share that can be captured by the ruler. On the other hand, the second term, $\tau(X)$, represents the marginal benefit, which stems from the additional output that is generated, part of which goes to the ruler. The marginal benefit, net of the marginal political cost, has to equal the marginal cost of factor provision, which is foregone consumption.

We now characterize how the *ex ante* choice of H , which is implicitly defined by (20), will be affected by the key variables of the model. Define $G(X) \equiv \tau'(X)X + \tau(X)$. It is easy to show, using the implicit function theorem applied to (18) and the second-order conditions of the optimization problem, that for any variable or parameter of interest j, we will have the sign of $\frac{\partial H}{\partial j}$ being equal to the sign of $\frac{\partial G}{\partial j}$. Moreover, we have:

$$\frac{\partial G}{\partial j} = \left[\tau''(X)X + 2\tau'(X)\right]\frac{\partial X}{\partial j} = (1 + \sigma_x)\,\tau'(X)\frac{\partial X}{\partial j}. \tag{21}$$

From this, since $\tau'(X) < 0$, it immediately follows that any variable that increases aggregate political participation will lead to less investment in human capital by the ruler. In particular, (21) and (P13) immediately yield the result. ■