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Mining direct antagonistic communities in signed social networks



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ABSTRACT

Social networks provide a wealth of data to study relationship dynamics among people. Most social networks such as Epinions and Facebook allow users to declare trusts or friendships with other users. Some of them also allow users to declare distrusts or negative relationships. When both positive and negative links co-exist in a network, some interesting community structures can be studied. In this work, we mine Direct Antagonistic Communities (DACs) within such signed networks. Each DAC consists of two sub-communities with positive relationships among members of each sub-community, and negative relationships among members of the other sub-community. Identifying direct antagonistic communities is an important step to understand the nature of the formation, dissolution, and evolution of such communities. Knowledge about antagonistic communities allows us to better understand and explain behaviors of users in the communities.

Identifying DACs from a large signed network is however challenging as various combinations of user sets, which is very large in number, need to be checked. We propose an efficient data mining solution that leverages the properties of DACs, and combines the identification of strongest connected components and bi-clique mining. We have experimented our approach on synthetic, myGamma, and Epinions datasets to showcase the efficiency and utility of our proposed approach. We show that we can mine DACs in less than 15 min from a signed network of myGamma, which is a mobile social networking site, consisting of 600,000 members and 8 million links. An investigation on the behavior of users participating in DACs shows that antagonism significantly affects the way people behave and interact with one another.

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1. Introduction

Each of us forms positive and negative relationships with others. With these relationships, communities are formed. At times, due to the nature of human interactions, some communities exhibit antagonistic behaviors among their members. Examples of such communities are many including social groups that hold differing opinions on topics such as industrialism *vs.* conservation, and formal organizations that are direct competitors in a market.

Several researchers have studied the nature of antagonistic communities (Dasgupta, 2009; Dasgupta & Kanbur, 2007; Denrell, 2005; Giles & Evans, 1986; Labovitz & Hagedorn, 1975; Tolsma, Jochem, Graaf, & Quillian, 2009). It is well known that

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individual level antagonism exists in any social networks. However, the existences of well "organized" form of antagonism between sub-groups should be detected in an early stage because they are potentially detrimental to the productivity and harmony of the community. Moreover, identification of antagonistic communities in the social networks could potentially open a path to further study the structure of the antagonistic community, the evolution of the antagonistic community, etc.

Antagonistic communities on the other hand are not always bad. In some cases, they are actually welcomed. In massively multiple player on-line games, gamers are expected to form groups to fight with gamers that do not belong to the same group. Here, a game's success depends very much on antagonistic behaviors among different gamer groups. The more antagonistic the groups are, the more fun and challenging the game would be. Also, by expressing common negative relationships with members from another group, users belonging to the same group may form stronger bonds among themselves. This increases loyalty within the group and improves the survivability of the group in the long run.

With the advent of Web 2.0, on-line social networking sites and forums have gained much popularity. In these venues, people declare their network of friends. Some venues also allow members to explicitly declare negative relationships with other members. Abundant information on positive and negative relationships which is unavailable to many past studies in sociology is now available to be analyzed. We therefore leverage these large scale information to shed light on antagonistic communities in Web 2.0 social networks.

In this study, our goal is to discover Direct Antagonistic Communities (DACs) automatically from explicit positive and negative relationships among people in a social network. We define a direct antagonistic community to be a pair of sub-communities with each sub-community having members with positive relationships with one another while having negative relationships with members of the other sub-community. Investigating mined direct antagonistic communities potentially enriches our understanding of antagonistic social interactions. Mined DACs could also be leveraged for various applications, e.g., in the monitoring and prevention of social conflicts, in improving friend recommendation systems, in designing better marketing or product survey strategies, etc. Mined antagonistic communities could be used to identify the existence of two sub-communities that are large and are antagonistic with each other. Friend recommendation could be improved by not recommending two potential friends from opposite sides of an antagonistic community. Also, when views are to be solicited from a network of users, the knowledge of antagonistic communities will help to select a fair subset of users so as to obtain a balanced set of views. An extended discussion of potential applications of DACs is provided in Section 6.

Identifying all direct antagonistic communities from a large signed network however is challenging. Many combinations of two sets of people, which are two sets of nodes in the network, need to be considered and checked for antagonism. In a large signed network, the number of such combinations is very large. To tackle this challenge, we design a graph mining algorithm to mine antagonistic communities by leveraging past work on computing strongest connected components and mining maximal bi-cliques.

We experiment with Epinions trust-distrust dataset and friend-foe information from myGamma, a mobile (i.e., mobile phone based) social network. We observe that antagonistic communities exist in both networks. We also find that members from opposing sub-communities in an antagonistic community tend to behave differently (e.g., give poorer ratings, have different group affiliations, etc.) compared with members within the same sub-community. This confirms that direct antagonistic relationships between two sub-communities affect their behavioral patterns. The contributions of this work are as follows:

- 1. We propose a new problem of mining antagonistic communities from signed social networks.²
- 2. We build a novel algorithm by using existing building blocks that have been shown to scale to large datasets hence enabling our approach to scale too.
- 3. We experiment our approach to extract antagonistic communities from real signed social networks.
- 4. We present more in-depth analysis and explore behaviors between two opposing sub-communities that have negative relationships with each other.

The structure of this paper is as follows. Section 2 describes related work. Section 3 describes some preliminary definitions. Section 4 describes our approach. Experiments are presented in Section 5. Section 6 discusses some interesting points and issues. We conclude and describe future work in Section 7.

2. Related work

Community finding is a key problem in social network analysis and it has been extensively studied (Cai, Shao, He, Yan, & Han, 2005; Girvan & Newman, 2002; Katz, 1953; Leicht & Newman, 2008; Newman, 2004; White, Harary, Sobel, & Becker,

² This article is an extended version of our short paper (six pages) appearing in the 20th ACM Conference on Information and Knowledge Management (CIKM 2011) entitled: Mining Direct Antagonistic Communities in Explicit Trust Network (Lo, Surian, Zhang, & Lim, 2011). We extend the conference paper in the following ways: (1) we include additional background materials and related work, (2) we describe our proposed and baseline solutions in more details through the inclusion of more detailed descriptions and examples, (3) we present our new experiments on additional case studies on Epinions and myGamma datasets enriched with additional inferred negative edges (see Section 5), (4) we present the results of a set of new efficacy experiments on the real datasets to investigate the differences between users in the same side and those in the opposing sides of antagonistic communities (see Section 5), and (5) we add a new discussion section.

2001;Yang, Cheung, & Liu, 2007). The traditional way of detecting community structure in a network is by performing hierarchical partitioning (Johnson, 1967; Ravasz, Somera, Mongru, Oltvai, & Barabasi, 2002; Scott, 2000). Girvan and Newman (2002) introduced a new algorithm to mine communities from networks based on edge betweenness. Newman (2004) proposed a modularity-based algorithm. Different from the above studies that focus on partitioning a network into cohesive communities, in this work, our goal is to extract subgraphs within the network which exhibit strong antagonism.

Vuong et al. (2008) investigated content deletion between any two Wiki-pedia users as a form of disputes between them, and developed models to determine the degrees of controversy of users and articles using the dispute information. Different from Vuong et al.'s work, we mine direct antagonistic communities from signed networks.

Social network researchers have conducted a number of studies in signed networks (i.e., networks with both positive and negative relationships) in order to give more understanding about interactions between people in a community. The social balance theory developed for signed networks identifies triads with all positive relationships and triads with only one positive and two negative relationships as the balanced structure constructs (Easley & Kleinberg, 2010). Triads of other forms are known to be unbalanced. Doreian and Mrvar (2009) studied how to partition signed social network. Yang and Liu (2007) mined communities in signed networks using a heuristic clustering approach. In this work, we capture a set of people that are linked together by positive relationships and they also consistently oppose a common set of "enemies" in online social networks.

There are also several researchers from sociology, economics, and psychology communities, who have studied intergroup antagonism (Dasgupta, 2009; Dasgupta & Kanbur, 2007; Denrell, 2005; Giles & Evans, 1986; Labovitz & Hagedorn, 1975; Tolsma et al., 2009). This body of work however has not been widely validated on large online social networks.

We build our algorithm on the top of a pattern mining algorithm (i.e., to mine for maximal bi-cliques). Several algorithms have been proposed for mining association rules (Agrawal & Srikant, 1994; Wang, Han, & Pei, 2003), frequent sequences (Wang & Han, 2004; Yan, Han, & Afhar, 2003), frequent repetitive sequences (Ding, Lo, Han, & Khoo, 2009; Lo, Khoo, & Liu, 2007), frequent graphs (Yan & Han, 2002), etc.

The enumeration of bi-cliques from graph data has been studied before. Included in this body of work are the work by Alexe et al. (2004), Eppstein (1994), Makino and Uno (2004), Li, Liu, Li, and Wong (2007). The work by Li et al. (2007), to the best of our knowledge, is the latest in the series. They proposed a mapping between maximal bi-clique mining to frequent item-set mining problem. We make use of their translation and extend their technique to mine for Direct Antagonistic Communities (DACs). A DAC is not a bi-clique, but some constraints within a DAC, in particular negative relationships could be mapped to the problem of finding bi-cliques within a dataset. We use the specific nature of DACs to prune additional nodes. The resultant technique scales well to mine from large real networks at low minimum size thresholds.

The closest to our work is the work by Zhang, Lo, and Lim (2010) which proposed an approach to mine for antagonistic communities based on rating data. If two sets of users rate the same set of products differently most of the time, they would be mined as antagonistic communities. Zhang et al. (2010) focused on indirect antagonistic communities in rating networks where two sub-communities have conflicting ratings on some commonly rated objects. Different from Zhang et al. (2010)'s work, in this work we mine for antagonistic communities based on *explicit* signed relationships. We believe *explicit* signed relationships are more reliable than common or differing ratings. A user could have a positive relationship with another user although they might have a different "taste" on some common items of interest. Similarly, a user could still have a negative relationship with another user sharing the same "taste" on some common items of interest. Due to the different nature of the problem, there is a need to develop a new algorithm to mine for *direct* antagonistic communities from *signed* networks. The approach proposed in (Zhang et al., 2010) takes as input a transaction database where each transaction corresponds to an item that is rated, and every element in a transaction corresponds to a user and his/her rating for the item. Zhang et al. (2010) employs a similar algorithm as level-wise association pattern mining where apriori property is used to prune search space. Antagonistic communities of smaller sizes are generated first; based on these, communities of larger sizes are subsequently constructed in a level-by-level manner. In this work, we take as input a graph rather than a transaction database. We perform various pruning based on properties governing positive and negative edges of a direct antagonistic community. We design a graph mining algorithm that leverages past work on extracting strongest connected components and mining maximal bi-cliques.

3. Preliminaries and problem definition

In this section, we first describe preliminary concepts and definitions on graphs and frequent pattern mining. We then formalize some new definitions and our problem statement.

3.1. Preliminaries

Some standard definitions of graph, strongly connected sub-graph, strongly connected component, and bi-clique are given in Definitions 1–4 respectively.

Definition 1 (*Graph*). A graph is composed of a set of nodes and edges and is denoted as G = (N, E). An edge is a mapping from one node to another node.

Definition 2 (*Strongly Connected Sub-graph*). A strongly connected sub-graph (SCS) is a sub-graph G' in a larger graph G where: For each node n' in G', there exists a series of edges in G' connecting n' to every other node in G'.

Definition 3 (*Strongly Connected Component*). A strongly connected component (SCC) is a strongly connected sub-graph that is maximal in size.

An example of a strongly connected component (SCC) is shown in Fig. 1a. Note that the rightmost three nodes (i.e., V2, V4 and V5) form a strongly connected sub-graph (SCS) but do not form an SCC.

Definition 4 (Bi-Cliques). A bi-clique is a graph whose nodes could be decomposed of two sets of nodes where:

- 1. There are no edges among the nodes in each set
- 2. Each node is connected to every node in the other set.

We denote a bi-clique as (L,R), where L and R are the two sets of nodes having the characteristics described above. An example of a bi-clique is shown in Fig. 1b. Next, we describe some preliminary definitions of transaction database, mapping function, item-set, frequent item-set and closed pattern in Definitions 5–8 respectively. These terms are commonly used in frequent item-set mining first proposed by Agrawal and Srikant (1994).

Definition 5 (*Transaction DB and Mapping Function*). A transaction is a set of items from a domain D. A transaction database DB consists of a bag of transactions. Let map (S) be a mapping between a set of items S to the set of the identifiers of the transactions in the DB containing S.

Definition 6 (*Item-set Pattern*). An item-set pattern is a set of items. Consider a transaction database DB. The support of an item-set pattern *P*, is the number of transactions in DB that are super sets of *P*. The support of *P* is denoted as sup (*P*).

Definition 7 (*Frequent Item-set*). An item-set P is a frequent item-set with respect to a transaction database DB and a minimum support threshold min_{sup} if $sup(P) > min_{sup}$.

Definition 8 (*Closed Pattern*). An item-set *P* is a closed pattern, if *P* is frequent and there is no *P'* where $P' \supseteq P$ and sup(P') = sup(P).

An example of a transaction database is shown in Table 1. From this table, we could see that the item-set {A,B,C} is supported by two transactions namely T1 and T2. The support of the item-set is 2. Considering a minimum support of 2, the item-set is frequent. However, since there exists a larger item-set {A,B,C,D} with the same support, the item-set {A,B,C} is not closed. Item-set {A,B,C,D} however is closed.

3.2. Definitions and problem statement

We take as an input a network of users expressing positive and negative relationships among themselves. We refer to this network as a signed network defined in Definition 9.



Fig. 1. (a) A strongly connected component and (b) a bi-clique.

Table 1

A sample transaction DB.	
TID	Itemset
T1	{A, B, C, D, E}
T2	$\{A, B, C, D, E\}$
T3	{A, C, D}
T4	{E, F, K}

Definition 9 (*Signed Network*). A signed network is a graph whose nodes represent individuals and edges represent positive or negative relationships among them. The edges are directed and labeled as either: positive (P) or negative (N). The nodes are labeled with the identifiers of respective individuals. A signed network could then be denoted as $G = (N, E, N_L, E_L)$ where N, E, N_L , and E_L correspond to the nodes, the edges, a mapping from nodes to labels, and a mapping from edges to labels respectively.

Our goal is to mine a set of antagonistic communities with two sub-communities/groups, where members of each sub-community express explicit positive relationships among themselves and have negative relationships with members of the opposing sub-community. We refer to these communities as direct antagonistic communities defined in Definition 10.

Definition 10 (*Direct Antagonistic Community*). A Direct Antagonistic Community (DAC) is composed of two sub-communities L and R. L and R are both SCSs with respect to the directed positive edges. Furthermore L and R form a bi-clique considering bi-directional negative edges. We denote it as [L, R].

An example of such Direct Antagonistic Community (DAC) is shown in Fig. 2. We are interested in DACs obeying a minimum size requirement, i.e., $|L| \ge min_size$ and $|R| \ge min_size$. We refer to such DACs as *significant* DACs. In Fig. 2, the DAC example is significant if the minimum size threshold is set at 2; it would not be significant if the minimum size threshold is set at 3.



Fig. 2. A Direct antagonistic community.



Fig. 3. Example sub-bi-clique.

We next introduce the concept of *redundant* DACs, but first we need to describe *sub-bi-clique* operation. This is defined in Definition 11. As an example, Fig. 3 shows a sub-bi-clique of the bi-clique shown in Fig. 1b.

Definition 11 (*Sub-Bi-Clique*). Consider a bi-clique C = (L, R). We define a sub-bi-clique of *C*, as a bi-clique C' = (L', R') where either $L' \subseteq L$ and $R' \subset R$, or $L' \subset L$ and $R' \subseteq R$. A sub-bi-clique of a bi-clique is a bi-clique.

All sub-bi-cliques of a significant DAC are potentially significant DACs. Thus to prevent an explosion on the number of DACs, we mine only a compact representation of DACs. Given a set of mined DACs, we define redundant ones based on Definition 12. Only non-redundant DACs would be mined.

Definition 12 (*Redundant DAC*). Consider a set of DACs *ASET*. One DAC $a \in ASET$ is deemed as *redundant* iff there exists another DAC $a' \in ASET$, where a is a sub-bi-clique of a'.

Based on the above concepts and definitions, our problem is defined as follows:

Problem Definition. Given a signed network and a minimum size threshold *min_size*, find all non-redundant and significant DACs.

4. Mining antagonistic communities

In this section, we describe some properties of Direct Antagonistic Communities (DACs) and present our algorithm to mine them.

4.1. Properties

We use three properties in our mining algorithm outlined below. First, Property 1 describes a rule governing a node's membership to a significant DAC.

Property 1 (Membership). Consider a node n in graph G, if n is not a part of any SCSs of size min_size, n could not be a part of any significant DACs.

Proof. From Definition 10, each sub-community in a DAC must be an SCS of size at least *min_size*. Hence, such a node n could not be a part of any DACs.

Next, Property 2 describes the relationship between a strongly connected sub-graph and a strongly connected component in a signed network.

Property 2 (SCS and SCC). Every Strongly Connected Sub-graph (SCS) must be a part of a Strongly Connected Component (SCC).

Proof. From Definitions 2 and 3, an SCS could either be an SCC, or there is a super-graph of the SCS which is an SCC. We now define a new operation to convert a graph to a transaction database.

Definition 13 (*Graph to Transaction DB*). The GTD operation converts a graph *G* to a transaction database DB by creating a new set of transactions $t = \{n' | (n,n') \in G \cdot Edges\}$ for each node *n* in *G* and affixing the identifier of *n* to *t*. We denote this operation applied to a graph *G* as *GTD*(*G*).

Fig. 4 illustrates the GTD operation to derive a transaction database. With this operation, the duality between bi-cliques and closed patterns is established by Property 3.

Property 3 (Bi-cliques and Patterns: Duality). Consider a graph *G* and a transaction database GTD(G). The set of all bi-cliques corresponds to the set { $(c, map(c))|c \in CLS$ } where CLS is the set of all closed patterns in GTD(G).

Proof. The above property has been proven by Li et al. (2007). \Box



Fig. 4. Graph to transaction DB operation.

4.2. Proposed algorithm

A Direct Antagonistic Community (DAC) has two basic requirements based on the positive and negative relationships. On one hand, each sub-community must form a strongly connected component based on positive relationships. On the other hand, members of one sub-community must have negative relationships with all members of the other community. To mine for DACs, we perform the following steps:

- 1. Project input signed network G, to a graph G_+ keeping only positive edges in graph G.
- 2. Extract SCCs from G_+ of size more than the minimum support size threshold min_size . These are candidate sub-communities of DACs. Nodes that are not part of at least one SCC with size of at least min_size could not be part of any DAC (see Properties 1 and 2). We keep the set of nodes $N_+ = \{n|n \text{ is a node in the identified SCCs}\}$.
- 3. Project the input signed network G, to a graph G_{-} keeping only nodes in N_{+} and bi-directional negative edges.
- 4. Mine the set of maximal bi-cliques BCQ from G_{-} leveraging Property 3 and utilizing closed itemset mining algorithm.
- 5. For each bi-clique (L, R) in BCQ containing a set of nodes n_b^L and n_b^R for L and R respectively, project the input signed network G, to graphs $G_{n_b^L}$ and $G_{n_b^R}$ keeping only nodes in n_b^L and n_b^R respectively and their corresponding positive edges. Find SCCs SCC^L and SCC^R from the projected networks $G_{n_b^L}$ and $G_{n_b^R}$ respectively whose sizes are at least *min_size*. We keep the set of DACs $\{[l, r]| l \in SCC^L \land r \in SCC^R\}$.
- 6. Eliminate redundant DACs. There could still be redundant DACs at the end of step 5. This is the case even after we mine for maximal bi-cliques at step 4, as the DACs are sub-bi-cliques of these maximal bi-cliques. We iterate through the set of DACs generated at step 5 and remove redundant ones based on Definition 12.

As a running example, we consider the graph shown in Fig. 5a. The graph contains positive edges (denoted as solid arrows) and bi-directional negative edges (denoted as dashed bi-directional arrows).

4.2.1. Pruning by positive edges: steps 1 and 2

First, we prune candidate nodes based on positive relationships. Negative edges are removed from the projected graph. Based on this graph, our goal is to throw away nodes which are not part of any large enough networks whose nodes are connected with one another via positive edges. The number of edges a node has follows power law, i.e., most nodes are not connected to any other nodes. Hence, a large number of nodes could be removed from consideration.

To realize this goal, we employ Tarjan's algorithm (Tarjan, 1972), that could compute maximal SCCs by a single depth-first search pass on the network containing only positive edges. Hence, it is scalable as the runtime cost is linear to the size of the network. We extract nodes that are parts of maximal SCCs with sizes $\geq min_size$.

Example 1. Consider *min_size* threshold being set to 2. From the example signed network described in Fig. 5a, there are two SCCs. The first SCC consists of nodes in the set {V1,V2}. The second consists of nodes in the set {V4,V5,V6,V7}. The two SCCs are drawn in Fig. 5a. We only retain nodes in the set {V1,V2,V4,V5,V6,V7} to be considered in the next step as these are nodes that are parts of SCCs.

4.2.2. Pruning by negative edges: steps 3 and 4

At these steps, we focus on *strong* (i.e., bi-directional) negative relationships. We project the input signed network, by removing positive edges and non bi-directional negative edges. Two sub-communities in a direct antagonistic community must form a bi-clique with respect to the bi-directional negative edges.

To realize the goal, we adapt a recent algorithm by Li et al. (2007) that extracts maximal bi-cliques from a graph following Property 3. The algorithm would return all maximal bi-cliques from the input network containing only bi-directional negative edges.

Example 2. The projected network consisting of nodes in the set {V1,V2,V4,V5,V6,V7} and bi-directional negative edges is shown in Fig. 6a. This network can be converted to a transaction database shown in Table 2. Mining for closed patterns from this transaction database would result in the set of patterns: {{V1,V2},{V4,V5,V6}}. This corresponds to one maximal bi-clique with $L = {V1,V2}$ and $R = {V4,V5,V6}$.

4.2.3. Formation of DACs: step 5

Each maximal bi-clique mined at step 4 is not necessarily a DAC as each of the two sets in the bi-clique does not necessarily form an SCS with respect to positive edges. Thus, a maximal bi-clique could map to 0 or more DACs.

Following Definition 11, every sub-bi-clique of a bi-clique is a bi-clique and hence it satisfies the requirement of negative relationships among members of opposing sub-communities. Hence, we could extract sub-bi-cliques SBQ from each bi-clique in which each of the two sets of nodes forms an SCS of size larger than *min_size*.

To realize this, we process each bi-clique BCQ identified in step 4. For each of the two sets of nodes in BCQ, i.e., BCQ, L and BCQ, R, we find SCSs on a projected network containing nodes in BCQ, L/BCQ, R and positive edges among them. These oper-



(b) Fig. 5. Running example: (a) signed network and (b) two SCCs.



Fig. 6. Running example: (a) projected network with only negative edges and (b) resultant projected network and a mined DAC (marked by red solid circles).

Table 2Running example: transaction DB.

TID	Item-set
V1	{V4,V5,V6}
V2	{V4, V5, V6}
V4	{V1,V2}
V5	{V1,V2}
V6	{V1,V2}
V7	0

ations would result in two sets of SCSs. Pairing one SCS from one set with another from the other set, would form a DAC which could then be included in the final result.

Example 3. Consider the maximal bi-clique with $L = \{V1, V2\}$ and $R = \{V4, V5, V6\}$. Fig. 6b shows a projected network with nodes appearing in the maximal bi-clique and both positive and negative edges. From this projected network, we could identify a DAC consisting of two opposing sets of nodes which are $\{V1, V2\}$ and $\{V4, V5\}$. This DAC is marked by the solid red circles in the figure.

4.2.4. Removal of redundant DACs: step 6

Usually, there are no or few redundant DACs left at the end of step 5. The running example is one of such cases. However, there exist corner cases where redundant DACs are present. This is the case as mined DACs are sub-bi-cliques of the maximal bi-cliques mined at steps 3 and 4. We remove redundant DACs by analysing the list of DACs mined at step 5 and detect for redundancies based on Definition 12. We do this by comparing each DAC with every other larger DAC mined at step 5; each of these comparisons simply involves checking for subset relations among sub-communities of two DACs which is sufficient to decide whether one is a sub-bi-clique of the other.

A signed network showing the corner case where redundant DACs exist after step 5 is shown in Fig. 7. After steps 1 and 2 of the algorithm, the SCCs of this signed network are shown in Fig. 8. The projected network containing only negative edges shown in Fig. 9 is produced after step 3 of the algorithm. After steps 4 and 5 of the algorithm, the maximal bi-cliques and the DACs found are shown in Fig. 10. We find two maximal bi-cliques as shown in Fig. 10a and b. The DACs corresponding to these two bi-cliques are indicated by the red solid circles. As the bi-clique in Fig. 10a is a sub-bi-clique of that in Fig. 10b, the DAC in Fig. 10a is redundant and needs to be removed.

4.2.5. Pseudocode

Our algorithm's pseudo-code is shown in Algorithm 1. At line 1, we perform graph projection and extract a network containing only positive edges from the input signed network. At lines 2–3, we perform SCC computation and extract nodes that participate in an SCC of size at least *min_size*. At lines 4, we extract a network containing only bi-directional negative edges from the input signed network. This network is then converted to a special transaction database, and bi-cliques are identified via a closed item-set mining algorithm (lines 5–6). At lines 7–16, we check each bi-clique if it corresponds to zero, one, or more direct antagonistic communities of size at least *min_size*. At lines 10–11, we construct SCCs



Fig. 7. Corner case: input signed network.



Fig. 8. Corner case: two SCCs.



Fig. 9. Corner case: projected network containing only negative edges.



Fig. 10. Corner case: two maximal bi-cliques and their corresponding DACs.

from the left and right set of nodes of each bi-cliques considering only positive edges. We remove SCCs with size less than *min_size* at line 12. These SCCs are then composed to form direct antagonistic communities (lines 13–15). A non-redundancy check is performed at line 16.

Algorithm 1. Mine direct antagonistic communities

Input: *G*: Signed network; *min_size*: Minimum size threshold; **Output** Direct antagonistic communities with each opposing sub-community's size $\ge min_{size}$; 1: Let *Result* = {}; 2: Let G_{+} = Projected network of *G* containing only positive edges; 3: Let SCCList = Get maximal SCCs from the graph G_t by running (Tarjan, 1972); 4: Let $N_+ = \{n | n \in s1 \land s1 \in SCCList \land |s1| \ge min_size\};$ 5: Let G_{-} = Projected network of G containing only nodes in N_{+} and bi-directional negative edges; 6: Let $T_{-} = \text{GTD}(G_{-})$; 7: Let *CP* = Mine for closed item-sets from T_{-} with minimum support = *min_size*; 8: for each p in CP do 9: **if** $|p| \ge min_size$ **then** 10: Let BC = Form bi-clique (p, map(p)); Let *LT* = Construct SCCs from nodes in *BC.L* wrt. positive edges; 11: Let *RT* = Construct SCCs from nodes in *BC.R* wrt. positive edges: 12: 13: Remove SCCs from *LT* and *RT* with size < *min_size*; 14: **for each** pair $l \in LT$ and $r \in RT$ **do** 15: Create a new DAC *adc* from *l* and *r*: 16: Add adc to Result: 17: end for 18: end if 19: end for 20: Remove redundant DACs from Result; 21: return Result;

4.3. Baseline algorithm

The search space of all possible DACs is the set of all possible combinations of members in the signed network. As a baseline algorithm, we perform a standard depth-first search traversal of this search space. The pseudocode of the baseline algorithm is shown in Algorithms 2 and 3. To limit the size of the search space, in addition to the signed network and *min_size* threshold, the baseline algorithm takes an additional input which is the maximum size (*max_size*) threshold.

We start with sets of members of sizes one (Algorithm 2, line 1) and then grow each of these sets by adding additional members one by one (Algorithm 2, lines 2–4). At each step of the process, we check if each set is a DAC that satisfies the *min_size* and *max_size* thresholds (Algorithm 3, lines 1–3). This is similar to traversing the search space of all frequent item-sets in association rule mining, c.f., Zaki (2000). Unfortunately, apriori property that holds for frequent item-set mining does not apply here; although a set is not a DAC, its extension (i.e., by adding additional members) might be a DAC. If the *max_size* threshold has not been reached, we continue to grow the current set of members by adding one more new member to the set (Algorithm 3, lines 4–9). We repeat this process by recursively calling the procedure *Grow* (Algorithm 3, line 7).

Algorithm 2. Baseline algorithm: main

Input: *G*: Signed network; *min_size*: Minimum size threshold; *max_size*: Maximum size threshold;
Output: Direct antagonistic communities satisfying *min_size* and *max_size* thresholds;
1: Let *M*_{All} = Get the set of all nodes or members in *G*;
2: for each member *m* in *M*_{All} do
3: *Grow*(*G*,*min_size*,*max_size*,{*m*},*M*_{All});
4: end for

Algorithm 3. Baseline algorithm: grow procedure

Input: *G*: Signed network; *min_size*: Minimum size threshold; *max_size*: Maximum size threshold; *ms*: A set of members; *M_{All}*: All members in *G*

Output: Direct antagonistic communities containing nodes in *ms* satisfying *min_size* and *max_size* thresholds;

- 1: **if** |*ms*| is a DAC *d* satisfying *min_size* and *max_size* **then**
- 2: **Output** *d*
- 3: end if
- 4: if |ms| + 1 still satisfies max_size then
- 5: **for each** $m' \in M_{All}$ but $\notin ms$ **then**
- 6: Let $ms' = ms \bigcup \{m'\}$
- 7: Grow(G, min_size, max_size, ms', M_{All});
- 8: end for
- 9: end if

At line 1 of Algorithm 3, we check whether a set of members is a DAC of size no less than the *min_size* threshold. To do so, the following steps are performed:

- 1. We run Tarjan algorithm and generate SCCs.
- 2. We check if there are only two SCCs and both of them are of size no less than the *min_size* threshold.
- 3. We check if every member of an SCC has negative edges with all members of the opposite SCC.

5. Experiments and analysis

To evaluate the scalability and efficacy of our approach, we experiment with both synthetic and real datasets. We build our own synthetic signed network generator that generates networks of different sizes for scalability test. The efficacy test is performed on two real datasets, a signed network from Epinions³ and another signed network from myGamma mobile social network site.⁴ We use the Epinions data collected by Massa and Avesani (2006). myGamma dataset is provided by our industry partner BuzzCity. myGamma contains a friendship network where users could declare two kinds of relationships explicitly: friend or foe. The Epinions and myGamma datasets have 131,828 and 629,086 user nodes respectively, and about 841 K and 8.1 M edges respectively. myGamma has roughly 4.8 times more nodes and 9.6 times more edges compared with Epinions. As not all users have edges to other users, we remove those without any edge. As shown in Table 3, in the two real datasets, more than 90% of the edges are positive ones.

Since the original myGamma and Epinions datasets lack negative edges, we also enrich the datasets with pseudo negative edges. Our goal is to add pseudo negative edges in a principled manner that allows us to find more interesting DACs while not compromising the integrity of the network. For every pair of nodes, V1 and V2, at every negative edge's end point, we try to add negative edges from friends of V1 to V2 and vice versa. We add negative edges based on the idea that a common enemy of one's friends is likely to be one's enemy. Fig. 11 illustrates an example where a pseudo negative edge is added. We want to decide whether we should add a pseudo negative edge between X and Y or not. If the percentage of people having negative edge is added. In order to determine an appropriate threshold value, given an edge exists between X and Y, from the real datasets, we calculate the probability of the edge being a negative edge for every percentage of people who have negative edges for 0–100% of people who have negative relationships with Y, among those whom X has positive relationships with Y, apositive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with. Fig. 12 shows the probability of adding the negative edges for 0–100% of people who have negative relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relationships with Y, among those whom X has positive relatio

Table 3 shows the numbers of nodes and edges for all datasets. The enriched Epinions dataset has more than twice as many negative edges than that in the original dataset, while the enriched myGamma dataset has more than 200,000 negative edges than the number of negative edges in the original dataset.⁵

We run our algorithm on an Intel (R) Core (TM) 2 2.13 GHz PC with 2 GB of RAM running 32-bit Windows XP, Service Pack 3. The algorithm is written in Visual C#.Net.

5.1. Scalability experiment using synthetic datasets

We develop a synthetic signed network generator that uses three input parameters, i.e., |N| (i.e., the number of nodes in the graph), |E| (i.e., the number of edges in the graph), and τ (i.e., the proportion of negative edges). By varying the three

³ http://www.epinions.com/.

⁴ http://www.buzzcity.com/f/mygamma.

Table 3			
Real and	enriched	datasets:	statistics.

Network	#Nodes	#+ve edges	#-ve edges	#Bi-directional -ve edges
Epinions	131,828	717,667	123,705	2,383
		$(\sim 85.3\%)$	$(\sim 14.7\%)$	$(\sim 0.28\%)$
Epinions (enriched)	131,828	717,667	135,109	8,085
		(~ 84.16%)	(~ 15.84%)	$(\sim 0.95\%)$
myGamma	629,086	7,563,927	540,083	57,074
		(~ 93.34%)	$(\sim 6.66\%)$	$(\sim 0.7\%)$
myGamma (enriched)	629,086	7,563,927	1,794,553	684,309
		$(\sim 80.82\%)$	(~ 19.18%)	(~ 7.31%)



Fig. 11. Pseudo negative edge enrichment.



Fig. 12. Probability of adding negative edges based on common enemy.

parameters, networks of different sizes (i.e., |N| and |E|) and composition of positive and negative edges (i.e., τ) can be generated. The synthetic network generation process follows the steps outlined in Algorithm 4.

Algorithm 4. Generate a synthetic signed network

```
Input: |N|: Number of nodes; |E|: Number of edges; \tau: Proportion of negative edges;

Output: Synthetic Signed Network (N, E);

1: Initialize N and E to be empty;

2: Create |N| nodes and assign to N unique node labels;

3: for each node n in N do

4: Create \frac{|E|}{|N|} edges from n to other nodes randomly and assign them to E;

5: Randomly assign \tau \cdot |E| edges from E as negative edges;

6: Assign the remaining (1 - \tau)|E| edges from E as positive edges;

7: end for

8: return (N, E);
```

Junning time for synthetic network. Dataset Proposed algorithm (s) Baseline algorithm			
(a)	0.297	372.930 s	
(b)	0.937	2985.169 s	
(c)	127.333	>24 h	
(d)	360.771	>24 h	
(e)	812.989	>24 h	

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Table 5

Time and |DACs|: Epinions and myGamma datasets.

min_size	Epinions		myGamma	
	Time (s)	DACs	Time (s)	DACs
1	14.8	1449	844.727	50,455
2	9.5	50	82.958	50
3	7.7	15	78.397	4
4	6.9	4	76.95	0
5	6.7	0	81.164	0

Five synthetic signed networks have been created with these parameters: (a) |N| = 100, |E| = 6000, $\tau = 0.4$; (b) |N| = 150, $|E| = 6,000,000, \tau = 0.1$. We mine DACs from these synthetic signed networks with min_size set to 2 using our proposed algorithm and our baseline algorithm (with maximum DAC size threshold set at 5).⁶ We show the running time for each synthetic signed network using our proposed algorithm and our baseline algorithm in Table 4.

From Table 4, we show that our approach is able to efficiently process a large network of 1 million nodes using a low min_size threshold within 15 min. We also observe that the running time increases with the network size. This is particularly due to the computing overhead of mining bi-cliques in the algorithm. We also notice that our proposed approach is more than 1000 times faster than the baseline approach. The baseline approach is not able to complete within one day for the larger signed networks (i.e., networks (c)-(e)). This result shows the power of our proposed pruning strategies.

5.2. Scalability and efficacy experiments using real datasets

We first describe our scalability experiments and then proceed to discuss our efficacy experiments that aim to distill information from mined DACs.

5.2.1. Scalability experiments

The running times for mining DACs on Epinions and myGamma datasets, including the enriched datasets at various minimum size thresholds using our proposed approach are shown in Table 5. Our baseline approach is not able to complete for these datasets. We show the number of direct antagonistic communities mined along with the time needed to mine them.

Table 5 shows the results on the original Epinions and myGamma datasets. As shown in the table, our algorithm completes extracting all DACs from the original Epinions dataset within 14.8 s. We do not find any large DACs with sub-community size ≥ 5 in the original datasets, as there is no antagonistic community found when the *min_size* threshold is set to 5. On the original myGamma dataset, the running time ranges from 81.164 s (for min_size = 5) to 844.727 s (for min_size = 1). We notice that there are more DACs mined from myGamma. However, interestingly proportion-wise, Epinions dataset has DACs of slightly larger sizes. myGamma does not have any DACs of sizes 5 and above.

The result on the enriched datasets can be seen on Table 6. As shown in the table, our algorithm mines more DACs from the enriched datasets. We find 1766 DACs from the enriched myGamma dataset with minimum size 2. The number of mined DACs is far larger than that mined from the original myGamma dataset, which is only 50. We also find many DACs with minimum size 5. The runtimes needed to mine DACs from the enriched datasets are comparable to the runtimes needed to mine DACs from the original datasets, except when we mine with the minimum size threshold set at 1.

5.2.2. Efficacy experiments

Our efficacy experiments on the Epinions and myGamma datasets focus on evaluating the interactions between members of the mined DACs. As DACs are pairs of opposing sub-communities, we examine interactions among members of the same sub-communities and contrast them with the interactions between members of opposing sub-communities. We expect the first type of interactions to be affirmative while the second type to be unfriendly.

⁶ We use this threshold as the largest size of maximal DACs reported by our approach in dataset (a) is 5.

Table 6

Time and |DACs|: enriched Epinions and myGamma datasets.

min_size	Epinions (enriched)		myGamma (enriched)	
	Time (s)	DACs	Time (s)	DACs
1	22.6	2122	60745.6	65,428
2	8	285	168.3	1766
3	7.6	139	129.5	437
4	7.3	66	125.9	204
5	7.2	22	119.6	118



Fig. 13. Rating scores: opposing vs allied user pairs.

5.2.2.1. Epinions dataset. For the Epinions dataset, we study if members from one sub-community in a DAC tend to give poor ratings to reviews made by the members from the opposing sub-community. The dataset contains a total of 1.2 M reviews and about 4.5 M ratings. Given a pair of members (u_i, u_j) , u_i may rate reviews written by u_j with ratings $\{r_{ij1}, \ldots, r_{ijk}\}$. We analyze the DACs mined using a minimum size threshold of 1. In Fig. 13, we show the normalized distribution of ratings of opposing user pairs and allied user pairs derived from DACs. For each DAC (U_s, U_t) , we derive the opposing user pairs as $\{(u_i, u_j) | u_i \in U_s\} \cup \{(u_i, u_j) | u_i, u_j \in U_t\}$. As shown in the Fig. 13, the ratings of opposing user pairs have significantly smaller values compared to those of allied user pairs. On the original Epinions dataset, the proportion of rating 5 among opposing user pairs $(\sim 60\%)$ is smaller than that of allied user pairs $(\sim 95\%)$. The proportion of rating 2 among opposing user pairs not involved in DACs as the line labeled "General" in Fig. 13. These "General" user pairs also have a small proportion of rating 2 compared to that of opposing user pairs. This shows that ratings between opposing sub-communities are usually low due to hostile relationships between the sub-communities, while ratings between the sub-communities are typically higher. The result from the enriched Epinions dataset follows the same trend as that from the original Epinions dataset.

Fig. 14 shows the number of ratings made by users on articles written by other users in the opposing sub-communities and within the same sub-communities in the mined DACs. It could be seen from the original and the enriched Epinions datasets that members of the same sub-communities tend to give more ratings than members of opposing sub-communities. This is an interesting result because the self selection behavior of users influences the number of ratings they give to other users. The smaller number of ratings of opposing user pairs suggests that opposing users are less likely to rate each other's reviews compared with allied users.

5.2.2.2. myGamma dataset. For the myGamma dataset, we also study if the behavior of allied user pairs differs from that of the opposing user pairs. We analyze the DACs mined using a minimum size threshold of 2. As rating data is not available, we examine their group affiliations. myGamma users can form groups and each user can join multiple groups at the same time. Let G_k represents the set of groups which user u_k joins. We define the group overlap similarity of a pair of users u_i and u_j to be the Jaccard similarity of G_i and G_j :

$$gsim(u_i, u_j) = \frac{G_i \cap G_j}{G_i \cup G_j}$$

Note that gsim is symmetric, i.e., $gsim(u_i, u_j) = gsim(u_j, u_i)$.

(1)



Fig. 14. Number of ratings: opposing vs allied user pairs.



Fig. 15. Group overlap similarity: opposing vs allied user pairs.

Fig. 15 shows the distribution of group overlap similarities of opposing user pairs and allied user pairs in the mined DACs. On zero group overlap similarity, the relative number of opposing user pairs in the original and enriched myGamma dataset are greater than that of allied user pairs, while on group overlap similarity greater than zero, the relative number of allied user pairs in both the original and enriched myGamma datasets are greater than that of opposing user pairs.

We also perform hypothesis testing at 0.01 level of significance and we find that, for the Epinions dataset, the *distribution of rating scores, distribution of number of ratings*, and for the myGamma dataset, the *distribution of group overlaps*, among user pairs in opposing sub-communities in a DAC differ significantly from those in the same sub-community.

6. Discussion

In this section, we discuss the applicability of mined DACs, the scalability of our approach, and other interesting points and issues.

6.1. Applications of mined DACs

Employing our technique on a signed network would produce a set of direct antagonistic communities. These communities could be used for various purposes. The following paragraphs highlight three of the many potential applications which we leave as future work.

First, DACs could be used to enrich past studies on sociology on antagonistic behaviors of communities. Different from many sociology studies that investigate datasets of small sizes, we are able to analyze a large amount of data from Web 2.0. Mined DACs would be the first step to analyze questions such as: Why a sub-community opposes another? What are some patterns of antagonistic behaviors? How long do antagonistic behaviors persist? Do people change their antagonistic behaviors frequently? What are some common precursor events that trigger antagonistic behaviors?

Second, mining and monitoring DACs could have direct application in preventing unwanted escalated tensions among various factions either in a company or in a community. Differences in opinions is generally good however too much of it could create inefficiency in a company, or even violence, riot, or anarchy in a community.

Third, mining DACs could have potential applications in marketing. Increasing sales in a sub-community could mean reducing sales in the opposing sub-community. One could optimize a marketing strategy based on the DACs and other information, e.g., the purchasing power of different market segments/sub-communities.

6.2. Time complexity

Let us consider an input network or graph G = (N,E). As described in Section 4, our approach consists of several steps: pruning by positive edges, pruning by negative edges, formation of DACs, and removal of redundant DACs. The complexity of our algorithm is the sum of its parts. Let us consider the time complexity of each step. During pruning by positive edges, we run Tarjan's algorithm. The complexity of Tarjan's algorithm is O(|N| + |E|) (Tarjan, 1972). During pruning by negative edges, we run the maximal bi-clique mining algorithm by Li et al. (2007). The complexity of the maximal bi-clique mining algorithm is $O(|N| \times |E| \times |MB|)$, where |MB| is the number of maximal bi-clique and generate zero or more DACs. Typically each maximal bi-clique is small, and only a few DACs are constructed (if any) from it. Thus, let us consider the cost of constructing DACs from one maximal bi-clique to be a small constant *c*. Thus the time complexity of forming DACs is $O(|MB| \times c)$. During the removal of redundant DACs, we compare each DAC with other mined DACs. The time complexity of this step is thus $O(|DAC|^2)$, where |DAC| is the number of mined DACs before the non-redundancy removal step is performed. Typically there are only a small number of redundant DACs. Thus the overall time complexity is $O((|N| + |E|) + (|N| \times |E| \times |MB|) + (|DAC|^2)$. The above could be simplified to $O((|N| \times |E| \times |MB|) + (|DAC|^2)$. Since, $|DAC|^2$ is typically much smaller than $|N| \times |E| \times |MB|$, the time complexity could be further simplified to $O(|N| \times |E| \times |MB|)$. This is the complexity of the maximal bi-clique mining algorithm, which is the most expensive operation, as noted in our experiments.

6.3. Scalability and dealing with large signed networks

Although some parts of our algorithm (bi-clique extraction) could be expensive in the worst case, similar to other pattern mining algorithms in typical cases the process runs much faster. We have conducted a scalability experiments on synthetic networks containing up to 1 million nodes. We have also shown the scalability of our algorithm on a large real-life network extracted from myGamma (>600 k nodes, >7 m edges). On these datasets, we show that our algorithm is able to complete within 15 min. Our future work will investigate the scalability of our approach on yet larger networks with positive and negative edges.

6.4. Ground truth

Unfortunately as with many other studies on community mining ground truth is hard to be obtained. Still we conduct efficacy experiments to contrast the interactions of members in the same sub-community with those among members in opposing sub-communities. If the mined DACs are correct, we expect to see significant differences. In Section 5.2.2, we show that there are statistically significant differences between the rating scores and the number of ratings given between opposing and allied user pairs in Epinions dataset. We also show that opposing user pairs have lower group overlap similarities than allied user pairs in myGamma dataset.

6.5. Relaxing SCC constraint

In this paper, we require each of the two sub-communities in a DAC to be a strongly connected component (with respect to positive edges). It is also possible to relax this further to only require that only X% of the nodes are connected to other nodes in the sub-community or even to change the requirement to check for densely connected sub-communities. We leave these extensions for future work.

6.6. Comparisons to other baselines

To the best of our knowledge, this is the first work on mining direct antagonistic communities. Thus, in this work, we do not compare to other techniques as there is no existing comparable technique. The nearest work to ours is that by Zhang et al. (2010) which mine for two opposing groups of people who rate many items differently from rating datasets. In this work, we consider signed networks and mine for direct antagonistic communities considering the positive and negative edges in the networks. The problems are different and require different mining strategies. The two kinds of antagonistic communities are not directly comparable.

6.7. Volatility and evolution of antagonistic communities

In this work, we only consider a single snapshot of the signed network. It would be interesting to perform a longitudinal study and investigate the nature of antagonistic communities as they evolve over time. Are the communities stable or volatile? We plan to investigate this and other related questions in our future work.

7. Conclusion and future work

In this study, we analyze large signed networks to extract direct antagonistic communities (DACs). Within a DAC there are two opposing sub-communities. Members within a sub-community form a network of positive edges, while members from the opposing community have bi-directional negative relationships.

We propose a new framework to extract DACs efficiently from large networks. Our approach consists of several steps including: pruning of nodes not involved in any sizable network of positive edges, detection of bi-cliques with respect to bi-directional negative edges among two sets of users, formation of DACs from these bi-cliques, and detection of redundant DACs. Experiments have been conducted on both synthetic and real datasets. On the synthetic dataset our approach is able to scale to 1,000,000 nodes and 6,000,000 edges.

Zooming deeper into mined antagonistic communities, we notice that user pairs from opposing sub-communities tend to give lower and fewer ratings than those from the same sub-communities in the Epinions dataset. Members from opposing sub-communities tend to join fewer common groups than members in the same sub-communities in the myGamma dataset.

As future work, we plan to investigate the evolution of DACs as a social network changes over time. Also, we plan to investigate factors leading to the creation of antagonistic communities. Further experiments on effects of antagonistic communities and analysis of other signed social networks would be interesting. It would also be of interest to extend our mining algorithm to accommodate various possible extensions described in Section 6.

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References

Agrawal, R. & Srikant, R. (1994). Fast algorithms for mining association rules. In International conference on very large data bases (VLDB).

Alexe, G., Alexe, S., Crama, Y., Foldes, S., Hammer, P. L., & Simeone, B. (2004). Consensus algorithms for the generation of all maximal bicliques. *Discrete* Applied Mathematics, 145, 11–21.

Cai, D., Shao, Z., He, X., Yan, X., & Han, J. (2005). Community mining from multi-relational networks. In European conference on machine learning and principles and practice of knowledge discovery in databases (ECML/PKDD).

Dasgupta, I. (2009). 'Living' wage, class conflict and ethnic strife. Journal of Economic Behavior & Organization, 72(2), 750-765.

Dasgupta, I., & Kanbur, R. (2007). Community and class antagonism. Journal of Public Economics, 91(9), 1816–1842.

Denrell, J. (2005). Why most people disapprove of me: Experience sampling in impression formation. *Psychological Review*, 112(4), 951–978.

Ding, B., Lo, D., Han, J., & Khoo, S. -C. (2009). Efficient mining of closed repetitive gapped subsequences from a sequence database. In *IEEE international* conference on data engineering (*ICDE*).

Doreian, P., & Mrvar, A. (2009). Partitioning signed social networks. Social Networks., 31, 1-11.

Easley, D., & Kleinberg, J. (2010). Positive and negative relationships. In Networks, crowds, and markets: Reasoning about a highly connected world (pp. 119–152). Cambridge University Press.

Eppstein, D. (1994). Arboricity and bipartite subgraph listing algorithms. *Information Processing Letters*, *51*, 207–211.

Giles, M., & Evans, A. (1986). The power approach to intergroup hostility. The Journal of Conflict Resolution, 30(3), 469-486.

Girvan, M., & Newman, M. E. J. (2002). Community structure in social and biological networks. Proceedings of the National Academy of Sciences of the United States of America, 99(12), 7821–7826.

Johnson, S. C. (1967). Hierarchical clustering schemes. Psychometrika, 2, 241-254.

Katz, L. (1953). A new status index derived from sociometric analysis. Psychometrika, 18(1), 39-43.

Labovitz, S., & Hagedorn, R. (1975). A structural-behavioral theory of intergroup antagonism. Social Forces, 53(3), 444-448.

Leicht, E. A., & Newman, M. E. J. (2008). Community structure in directed networks. Physics Review Letter, 100(11), 118703.

Li, J., Liu, G., Li, H., & Wong, L. (2007). Maximal biclique subgraphs and closed pattern pairs of the adjacency matrix: A one-to-one correspondence and mining algorithms. *IEEE Transactions on Knowledge and Data Engineering (TKDE)*, *19*(12), 1625–1637.

Lo, D., Khoo, S. -C., & Liu, C. (2007). Efficient mining of iterative patterns for software specification discovery. ACM SIGKDD conference on knowledge discovery and data mining (KDD).

Lo, D., Surian, D., Zhang, K., & Lim, E. -P. (2011). Mining direct antagonistic communities in explicit trust networks. In ACM conference on information and knowledge management (CIKM) (pp. 1013–1018). Makino, K., & Uno, T. (2004). New algorithms for enumerating all maximal cliques. In 9th Scandinavian workshop on algorithm theory.

Massa, P., & Avesani, P. (2006). Trust-aware bootstrapping of recommender systems. In *Proc. of ECAI workshop on recommender systems* (pp. 29–33). Newman, M. E. J. (2004). Fast algorithm for detecting community structure in networks. *Physics Review*, *E*(69).

Ravasz, E., Somera, A. L., Mongru, D. A., Oltvai, Z. N., & Barabasi, A.-L. (2002). Hierarchical organization of modularity in metabolic networks. Science, 297(5586), 1551–1555.

Scott, J. (2000). Social network analysis: A handbook. Sage Publications.

Tarjan, R. (1972). Depth-first search and linear graph algorithms. SIAM Journal on Computing, 1(2), 146-160.

Tolsma Jochem Graaf, N. D., & Quillian, L. (2009). Does intergenerational social mobility affect antagonistic attitudes toward ethnic minorities? British Journal of Sociology, 60(2), 257–277.

Vuong, B. -Q., Lim, E. -P., Sun, A., Le, M. -T., Lauw, H., & Chang, K. (2008). On ranking controversies in wikipedia: Models and evaluation. In ACM international conference on web search and data mining (WSDM).

Wang, J., & Han, J. (2004). BIDE: Efficient mining of frequent closed sequences. In IEEE international conference on data engineering (ICDE).

Wang, J., Han, J., & Pei, J. (2003). Closet+: Searching for the best strategies for mining frequent closed itemsets. In ACM SIGKDD conference on knowledge discovery and data mining (KDD).

White, D. R., Harary, F., Sobel, M., & Becker, M. (2001). The cohesiveness of blocks in social networks: Node connectivity and conditional density. Sociological Methodology..

Yan, X., & Han, J. (2002). gSpan: Graph-based substructure pattern mining. In IEEE international conference on data mining (ICDM).

Yan, X., Han, J., & Afhar, R. (2003). CloSpan: Mining closed sequential patterns in large datasets. In SIAM conference on data mining (SDM).

Yang, B., Cheung, W., & Liu, J. (2007). Community mining from signed social networks. *IEEE Transactions on Knowledge and Data Engineering (TKDE)*, 19(10), 1333–1348.

Yang, B., & Liu, D.-Y. (2007). A heuristic clustering algorithm for mining communities in signed networks. Journal of Computer Science and Technology, 22, 320–328.

Zaki, M. (2000). Scalable algorithms for association mining. IEEE Transactions on Knowledge and Data Engineering (TKDE), 12, 372-390.

Zhang, K., Lo, D., & Lim, E. -P. (2010). Mining antagonistic communities from social networks. In Pacific-Asia conference on knowledge discovery and data mining (PAKDD).