Quantitative Trading Strategies

Statistical Arbitrage: Pair Trading
Pair Trading: Distance Model

A Simple Approach
Stocks from the Same Industry

- Reduce market risk, especially in bear market.
  - Stocks from the same industry are likely to be subject to the same systematic risk.

- Give some theoretical unpinning to pairs trading.
  - Stocks from the same industry are likely to be driven by the same fundamental factors (common trends).
Z Transform and Normalized Price

The normalized value of the price $x$ is

$$z = \frac{x - \bar{x}}{\sigma_x}$$

- $z$ is the normalized value of $x$
- $\bar{x}$ is the average of $x$
- $\sigma_x$ is the standard deviation of $x$

Modified normalized price

$$z = \frac{x - M(\bar{x})}{S(\sigma_x)}$$

- $M, S$ are “proprietary” functions to forecast the future average price and future standard deviation.
Finding Pair by Distance

The co-movement of stocks in a pair is measured by distance, which is the sum of squared differences between the two normalized price series.

\[
d = \sum_{i=1}^{T} (x_i - y_i)^2
\]

where \(x_i, y_i\) are the normalized prices via the Z transform.

Choose a pair of stocks among a collection with the smallest distance, \(d\).
Advantages of the Distance Model

- Model free
- No mis-specification
- No mis-estimation

- Does not guarantee stationarity.
- Cannot predict the convergence time (expected holding period).
- Ignores the dynamic nature of the spread process, essentially assumes that the price level distance is static through time.
Pair Trading: Co-Integration Model

More Rigorous Method
Stationary Time Series Model

- Mean and variance are constant at each time $t$.
- The covariance between $r_t$ and $r_{t-j}$ depends only on lag $j$.

Stock return is assumed to be stationary.
Random walk is non-stationary.

What’s the difference between stock return and price?

Stock price is assumed to be non-stationary.
White Noise

A stochastic time series \( \{w_1, w_2, \ldots, w_t\} \) is independent white noise if \( w_t \) is an independent and identically distributed (iid) variable with mean 0 and variance \( \sigma^2 \) at all time \( t \).

\[ w_t \sim \text{iid}(0, \sigma^2) \]

A special case is **Gaussian white noise**, where each \( w_t \) is independent and has a normal distribution at all time \( t \).

\[ w_t \sim N(0, \sigma^2) \]
Random Walk

- With $y_0$ being a constant, the random walk $y_t$ is essentially a sum of all the white noise realizations up to the current time.
  \[ y_t = y_0 + w_1 + w_2 + \ldots + w_t = y_{t-1} + w_t \]

- Equivalently, $y_t - y_{t-1} = w_t$, i.e. change of $y$ is white noise.

- This is a zero drift random walk.

- What is the expected value or mean of $y_t$?

- What is the variance of $y_t$?
How are Prices and Returns Related?

- Simple return from time $t-1$ to $t$:
  \[ R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \]

- So the price ratio is
  \[ \frac{P_t}{P_{t-1}} = 1 + R_t \]

- Define log price: $p_t = \ln (P_t)$.

- Change in log price from time $t-1$ to $t$ is simple return.
  \[ \Delta p_t = \ln(P_t) - \ln(P_{t-1}) = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(1 + R_t) \approx R_t \]

- So, change in log price is return
  \[ p_t - p_{t-1} \approx R_t \]
Log Price and Return

- If the log price is random walk, then the return is white noise.

- What is the meaning of random walk?

The Essence of Pair Trading

- Basically, pair trading requires the trader to trade an equal amount in security A of price $P_t$ and security B of price $Q_t$, i.e.,

$$P_t = \beta Q_t$$

- Beta $\beta$ is a number that makes the dollar amount equal on both sides.

- Take the log on both sides,

$$\ln(P_t) = \ln(Q_t) + \ln(\beta)$$

at a fixed time $t$. 
The Essence of Pair Trading

But at different $t$, the log price difference (spread) is a random variable $u_t$ plus a constant $\ln(\beta)$

$$\ln(P_t) - \ln(Q_t) = u_t + \ln(\beta)$$

The key of pair trading is to find a pair of securities A and B such that $u_t$ is white noise.
Why?

Because stationary process exhibits mean reverting behavior

- the process tends to remain near or tends to go back to the mean value after some time.
Stationary Noise of Mean Zero

- Separately $\ln(P_t)$ and $\ln(Q_t)$ are non-stationary.

- But when they are combined as a spread $S_t$ of log prices
  
  $$S_t = \ln(P_t) - \ln(Q_t)$$

  the result is a stationary time series of white noise $u_t$.

- The constant $\ln(\beta)$ serves as the average of the log spread

- What is the intuition?
What is the mean value?

\[ \ln \left( \frac{P_t}{\beta Q_t} \right) = u_t \]

Taking exponential on both sides, you get

\[ \frac{P_t}{\beta Q_t} = e^{u_t} \]

If \( u_t \) is normally distributed with mean 0 and variance \( \sigma^2 \), by stochastic calculus, the mean value is

\[ \mathbb{E} \left( \frac{P_t}{\beta Q_t} \right) = \mathbb{E} \left( e^{u_t} \right) = \exp \left( \frac{1}{2} \sigma^2 \right) \]
Insight!

Pair trading is really about the price ratio

\[
\frac{P_t}{\beta Q_t} = e^{u_t} \approx 1 + u_t + \frac{1}{2} u_t^2 + \ldots
\]

If \( u_t \) is normally distributed with mean 0 and variance \( \sigma^2 \), the expected value of the price ratio \( \frac{P_t}{\beta Q_t} \) is well approximated by \( 1 + \frac{1}{2} \sigma^2 \).

If \( u_t \) is normally distributed with mean 0 and variance \( \sigma^2 \), the variance of the price ratio \( \frac{P_t}{\beta Q_t} \) is well approximated by \( \sigma^2 + \frac{3}{2} \sigma^4 \).
Adjusted Hedge Ratio

Since the expected value of the price ratio $\frac{P_t}{\beta Q_t}$ is well approximated by $1 + \frac{1}{2} \sigma^2$, the expected spread is

$$P_t - \hat{\beta} \left(1 + \frac{1}{2} \hat{\sigma}^2\right) Q_t = 0$$

The hedge ratio should be adjusted to $\tilde{\beta}$ instead:

$$\tilde{\beta} = \hat{\beta} \left(1 + \frac{1}{2} \hat{\sigma}^2\right)$$

The spread $S_t$ in dollars is

$$S_t = P_t - \tilde{\beta} Q_t$$

and the mean spread price is zero.
At What Standard Deviation to Trade?

Since the variance of the price ratio $\frac{P_t}{\beta Q_t}$ is well approximated by $\sigma^2 + \frac{3}{2} \sigma^4$, i.e.,

$$\mathbb{V} \left( \frac{P_t}{\beta Q_t} \right) = \sigma^2 + \frac{3}{2} \sigma^4$$

the variance on the left hand side can be written as

$$\mathbb{V} \left( \frac{P_t}{\beta Q_t} \right) = \frac{\beta^2}{\beta^2} \mathbb{V} \left( \frac{P_t}{\beta Q_t} \right) = \frac{\beta^2}{\beta^2} \mathbb{V} \left( \frac{P_t}{\beta Q_t} - 1 \right)$$

$$= \frac{\beta^2}{\beta^2} \mathbb{V} \left( \frac{P_t - \beta Q_t}{\beta Q_t} \right)$$

$$\mathbb{V} \left( \frac{P_t - \beta Q_t}{\beta Q_t} \right) = \frac{\beta^2}{\beta^2} \left( \hat{\sigma}^2 + \frac{3}{2} \hat{\sigma}^4 \right)$$
Effective Standard Deviation and Spread Return

Define the **effective standard deviation**

\[
\omega = \sqrt{\frac{\beta^2}{\hat{\beta}^2}} \left( \hat{\sigma}^2 + \frac{3}{2} \hat{\sigma}^4 \right)
\]

- When the spread “return” \( \frac{P_t - \hat{\beta}Q_t}{\hat{\beta}Q_t} > 2\omega \), sell the spread \( S_t \). Wait for the mean reversion to 0 to happen.

- When the spread “return” \( \frac{P_t - \hat{\beta}Q_t}{\hat{\beta}Q_t} < -2\omega \), buy the spread \( S_t \). Wait for the mean reversion to 0 to happen.
How to Estimate $\beta$?

- Start from

$$\ln(P_t) - \ln(Q_t) = u_t + \ln(\beta)$$

- Taking the expected value,

$$\mathbb{E}(\ln(P_t) - \ln(Q_t)) = \ln(\beta)$$

- Specifically, given $n$ observations,

$$\ln(\hat{\beta}) = \frac{1}{n} \sum_{t=1}^{n} \ln \left( \frac{P_t}{Q_t} \right)$$

- Then, $\hat{\beta} = \exp \left( \frac{1}{n} \sum_{t=1}^{n} \ln \left( \frac{P_t}{Q_t} \right) \right)$. 
How to Estimate the Variance of $u_t$?

Since,

$$\mathbb{V}(\ln(P_t) - \ln(Q_t)) = \mathbb{V}(u_t) = \sigma^2$$

So, given $n$ observations,

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{t=1}^{n} \left( \ln \left( \frac{P_t}{Q_t} \right) \right)^2 - \left( \ln(\hat{\beta}) \right)^2$$
Most Important Question

- How do we know whether $u_t$ is stationary?
- Standard $t$-test based on Student’s t distribution is not appropriate when dealing with non-stationary time series.

One of the solutions is

- Dickey-Fuller Test
Test for Stationarity: Intuition

If the series $u_t$ is stationary, then it has a tendency to return to a constant mean. Large values will tend to be followed by smaller values, and small values by larger values. Accordingly, the level of the series will be a significant predictor of next period's change, and will have a negative coefficient.

But if the series is non-stationary, then positive changes and negative changes will occur with probabilities that do not depend on the current level of the series.
Motivation for Dickey-Fuller Test

- Recall that random walk is
  \[ y_t - y_{t-1} = w_t \]

- A possible idea to test random walk is to introduce a coefficient \( \rho \) and write as
  \[ y_t = \rho y_{t-1} + w_t \]

- The null hypothesis is non-stationary, i.e., \( H_0: \rho = 1 \) and the alternative hypothesis is stationary, i.e., \( H_1: \rho < 1 \)

- Unfortunately, if \( \rho = 1 \), i.e., the case of unit root, the estimate of \( \rho \) is biased downwards. In addition, standard \( t \)-distribution is inappropriate.

- Dickey and Fuller provide a table to test the hypothesis
Dickey-Fuller Test

- Instead we use the Dickey-Fuller test

\[ \Delta y_t = \rho^* y_{t-1} + \omega_t \]

where \( \rho^* = \rho - 1 \)

- Dickey and Fuller provide a table of critical values to test the hypothesis:

\[ H_0: \rho^* = 0 \quad \text{non-stationary} \]
\[ H_1: \rho^* < 0 \quad \text{stationary} \]
Augmented Dickey-Fuller Test

- The Dickey-Fuller equation only tests for first order auto-correlation of $y_t$. If the order is higher, the test is invalid and the DF equation suffers from residual correlation.

- To counter this, lagged values of $\Delta y_t$ is added to the equation, giving rise to the augmented DF test.

- The purpose of the lags of $\Delta y_t$ is to ensure that the error $w_t$ is white noise.
  - Too few lags will leave autocorrelation in the error
  - Too many lags will reduce the power of the test statistic
Augmented Dickey-Fuller Test Model

\[ \Delta y_t = a + bt + \gamma y_{t-1} + \sum_{i=1}^{l-1} c_i \Delta y_{t-i} + \epsilon_t \]

- Control for past changes with \( l \) lags.
- Null hypothesis \( H_0: \gamma = 0. \) \((y_t \text{ non-stationary})\)
- \( a = 0, b = 0 \) corresponds to a random walk.
- \( b = 0 \) corresponds to a random walk with drift.

Test statistic = \( \frac{\hat{\gamma}}{\sigma(\hat{\gamma})} \)

- The more negative the test statistic is, the more reason to reject \( H_0 \), implying \( y_t \) is more likely to be stationary.
Case Study 1: ACGB Bond Pair

- Data length = 4000
- Test statistic = -5.785
- ADF table:
  - 1% level: -3.961
  - 5% level: -3.412
  - 10% level: -3.128
Case Study 2: Tech Stocks Pair

- Test statistic = $-0.118$
Summary: Steps

- The starting model is $\ln(P_t) - \ln(Q_t) = u_t + \ln(\beta)$
- Find the mean of $\ln(P_t) - \ln(Q_t)$ to obtain the beta estimate $\hat{\beta}$.
- Obtain $u_t$ estimates by

$$\hat{u}_t = \ln(P_t) - \ln(Q_t) - \ln(\hat{\beta})$$

- Check whether $\hat{u}_t$ is stationary by running the augmented Dickey-Fuller Test
Summary: Trading the Pairs

- Given a collection of liquid assets, compute the pairwise co-integrating relationships.
- For each pair, validate stationarity by performing the augmented Dicker-Fuller test.
- For the strongly mean-reverting pairs, design trading strategies around them.
- The smaller the effective standard deviation is, the more frequent will the spread cross over 0 to the other side.
How Many Contracts to Trade?

Kelly’s Criterion
Gambler’s Ruin

- Even if your strategy has an edge, it is still possible to lose everything!

- Simple example: Your strategy wins 60% of the times. You bet all your funds. Back luck, the signal was wrong and so you lost all your funds. Game over.

- Position Sizing is crucial!
John Larry Kelly, Jr. (1923–1965)

A New Interpretation of Information Rate


“…the outcomes of a chance event on which bets are available at odds consistent with their probabilities (i.e., “fair” odds), a gambler can use the knowledge … to cause his money to grow exponentially. The exponential rate of growth of the gambler’s capital is equal to the rate of transmission of information over the channel.”

Kelly’s Criterion

How large should the size of each position be?

Kelly’s criterion answers this question by providing a technique that balances both risk and reward.

The Kelly position amount is the optimal amount for maximizing the expected equity growth.

Betting half the Kelly amount reduces equity volatility by 50%, but growth by only 25%.
The key idea is to achieve, on average, exponential growth in your equity $W$ from your initial equity $W_0$.

Starting with $W_0=\$25,000$

$$\mathbb{E}(W_t) = W_0 \exp(\mu t)$$
Setting up the Kelly Framework

- Let $p$ be the probability of winning, with an average gain of $g$ in percentages.
- The probability of losing is $1 - p$ and the average loss return is $l$ in percentages.
- Suppose the equity before the trade is $W_0$. Then the expected value of the equity $W_1$ after the trade from using a fraction $x$ of $W_0$ is

\[
\mathbb{E}(W_1) = W_0 (1 + gx)^p (1 - lx)^{1-p}
\]

- Since $W_0$ is known, it can be written as

\[
\mathbb{E} \left( \frac{W_1}{W_0} \right) = (1 + gx)^p (1 - lx)^{1-p}
\]
Expected Exponential Growth

Take the logarithm on both sides, we get

\[
\ln \left( \mathbb{E} \left( \frac{W_1}{W_0} \right) \right) = p \ln(1 + gx) + (1 - p) \ln(1 - lx) := f(x)
\]

In this way, we can achieve exponential growth of equity each time. On average, each time is

\[
\mathbb{E}(W_1) = W_0 \exp(f(x))
\]
How does $f(x)$ Look Like?

The average gain $g = 12\%$, average loss $l = 11\%$ of $W_0$. 

\[ p = 0.51 \]

\[ p = 0.5001 \]
So what should the fraction $x$ be for the growth to be optimal?

Differentiate $f(x)$ with respect to $x$, resulting in

$$\frac{df(x)}{dx} = \frac{pg}{1 + gx} - \frac{(1 - p)l}{1 - lx}$$

Solving the first-order condition, i.e., $\frac{df(x)}{dx} = 0$, results in the optimal value $x$:

$$x = p \frac{1}{l} - (1 - p) \frac{1}{g}$$
Application: Long First

- How to calculate the return from gain or loss?
- Notional amount of a futures contract = $mF_0$, where $m$ is the price multiplier (e.g. S$100 for SIMSCI futures).
- Trading futures is by margin. The broker will tell you the margin per contract. Suppose it is $Q$ dollars.
- The margin in percent is $\frac{Q}{mF_0} := a$.
- Your P&L for a long position = $m(F_1 - F_0)$ per contract.
- Return without margin = $\frac{m(F_1 - F_0)}{mF_0}$.
- Return with margin = $\frac{m(F_1 - F_0)}{amF_0} = \frac{m(F_1 - F_0)}{Q}$.
Numerical Example of Long First

Suppose you long 1 contract ABC futures, and make 12 ticks. Each tick is 0.1 index point or $20.

The broker imposes a margin of $3,000 per contract.

The return is therefore $20 \times 12/$3,000 = 0.08 = 8\%.
Application: Short First

- Again, notional amount of a futures contract = $m F_0$, where $m$ is the price multiplier.
- Trading futures is by margin. The broker will tell you the margin per contract. Suppose it is $Q$ dollars.
- The margin in percent is $\frac{Q}{mF_0} := a$.
- Your P&L for a short position = $m (F_0 - F_1)$ per contract.
- Return without margin = $\frac{m(F_0 - F_1)}{mF_1}$.
- Return with margin = $\frac{m(F_0 - F_1)}{amF_1} = \frac{m(F_0 - F_1)}{Q F_1 F_0}$.
Numerical Example of Short First

- Short at $F_0 = 350.8$.
- Close the position at $F_1 = 350.0$.
- The return with margin is
  \[ \frac{20 \times 8}{(3000 \times 350/350.8)} = 5.35\% . \]
Difficulties

In futures trading, it is difficult to work with returns.

Reasons:

- Margin in dollars varies from broker to broker, so it is arbitrary.
- For back testing, historical margin is hard if not impossible to get.
Simplified Kelly’s Formula

Recall that the fraction of equity \( W_0 \) to take a position on:

\[
x = p \frac{1}{l} - (1 - p) \frac{1}{g}
\]

Multiply both sides by \( l \), then the fraction \( b \) of \( W_0 \) to bet each time is

\[
lx = p - (1 - p)r := b
\]

where \( r \) is the dollar loss to dollar gain ratio, i.e.,

\[
r = l/g.
\]

Intuitively, \( b \) is the fraction of \( W_0 \) that you can afford to lose.

The loss to gain ratio \( r \) must be less than one, of course.
How Should $p$ and $r$ be Estimated?

- Back-testing!
- Run your trading strategy on historical data.
- Compute the number of times you win and the number of times you lose.

$$p = \frac{\text{Number of wins}}{\text{Total number of executions}}$$

- The total number of executions is the sum of the numbers of wins and losses.
- The loss to gain ratio $r$ can be estimated by the average loss divided by average gain.
  - Better to use net loss and net gain that take costs into account
Practical Example

Suppose your margin deposited at the broker is such that you can trade at most 10 SIMSCI contracts.

If \( p = 0.6 \) and \( r = 0.5 \), then the maximum proportion according to the simplified Kelly’s criterion gives

\[
b = 0.6 - 0.5 \times (1 - 0.6) = 0.4 \text{ or } 40\%.
\]

So each time your trading strategy has a trading signal, commit no more than 4 lots.

The recommendation is not to trade 4 lots, but half of the number, i.e., 2 lots.
Another Practical Example

- Suppose your margin deposited at the broker (i.e., equity) is such that you can trade at most 10 SIMSCI contracts.

- If $p = 0.55$ and $r = 0.95$, then the maximum proportion according to Kelly’s criterion is

$$b = 0.55 - 0.95 \times (1 - 0.55) = 0.1225 \text{ or } 12.25\%.$$  

- So each time your trading strategy has a trading signal, commit no more than 1 lot.
Summary

- How many contracts to trade?
- Simplified Kelly’s Criterion: $b = p - (1 - p)r$

$$p = \frac{\text{Number of wins}}{\text{Number of wins} + \text{Number of losses}}$$

$$r = \frac{\text{Average Loss in Dollars}}{\text{Average Gain in Dollars}}$$

- When your equity increases (decreases) to some point, the number of contracts to trade increases (decreases)!
Risks in Quantitative Trading

How do you set your cut loss level?
What is Risk?

- Uncertainty that entails financial losses.

- Many types of risk
  - Market risk
    - Position risk
    - FX risk
  - Model risk
    - Back testing bias risk
  - Liquidity risk
  - Credit risk
  - Algo trading risk
  - Operation risk
Algo Trading Risks

- Leaks that might arise from competitor’s efforts to reverse engineer an algorithm.

- Many algorithms lack the capacity to handle or respond to exceptional or rare events.
  - Thus, careful human supervision of algorithmic trading and other safeguards is crucial.
What happened?
Operation Risks

- Fat Finger: Click wrongly!
- Bandwidth congestion
- Connections lost
- Power failure
- Computer systems crash
- Exchange glitches

Always standby to call your broker to cancel orders!
Candle Stick Representation of 4 Prices

- Open (OL)
- Close
- High
- Low

- Open (HO)
- Close
- High
- Low
Practical Cut Loss Strategy for Intra-Day Direction Strategy

- Entry at opening and exit at closing
- The data must have Open, High, Low Close prices.
- Downside risk.
  - If the position is long at opening, then the downside is the quantum: \( \text{OL}_t = \text{Open}_t - \text{Low}_t \)
  - If the position is short at opening, then the downside is the quantum: \( \text{HO}_t = \text{High}_t - \text{Open}_t \)
- Gather all the \( \text{OL}_t \) for \( t = 1, 2, \ldots, T \).
- Likewise, collect all the \( \text{HO}_t \) for \( t = 1, 2, \ldots, T \).
Empirical Distribution of OL for Long First

- Mean = 2.04
- Standard Deviation = 1.65
- Median = 1.65
Empirical Distribution of HO for Short First

- Mean = 2.11
- Standard Deviation = 1.56

Median = 1.70

SIMSCI Futures
Four Possible Intra-Day Outcomes

- Median means that out of 100 attempts, there is a 50 percent chance that the stop loss is triggered.
- Let the probability of correct trading signal be $P_c$. So the probability of wrong trading signal is $1 - P_c$.
- Let the probability of a stop triggered be $P_t$. So the probability of no stop triggered is $1 - P_t$.
- Let $S$ be the stop relative to the entry price.
- Let $G$ ($L$) be the expected gain (loss) when the signal is correct.

<table>
<thead>
<tr>
<th></th>
<th>Signal Correct</th>
<th>Signal Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stop Triggered</td>
<td>$-P_t \times P_c \times S$</td>
<td>$-P_t \times (1-P_c) \times S$</td>
</tr>
<tr>
<td>Stop not Triggered</td>
<td>$(1-P_t) \times P_c \times G$</td>
<td>$-(1-P_t) \times (1-P_c) \times L$</td>
</tr>
</tbody>
</table>
The expected P&L is

\[
E(P&L) = -P_t P_c S - P_t (1-P_c) S \\
+ (1-P_t) P_c G - (1-P_t)(1-P_c) L \\
= -P_t S + (1-P_t) P_c G - (1-P_t)(1-P_c) L
\]

Let \( M \) be the largest of all OL and HO in the data.

The probability of a stop triggered should depend on \( S \).

Simple model for probability of trigger: \( P_t = 1 - S/M \).

To maximize the expected P&L, \( S \) must be

\[
S = \frac{1}{2} (M + (1 - P_c)L - P_c G)
\]
Flash Jump and Application Examples

- The largest swing $M$ may be too big due to a big "flash jump." It is more practical to use the 90-th percentile of all OL and HO in the data instead.

- Example 1: Long first. The 90-th percentile OL is 60 ticks. Suppose $P_c = 0.6$, $G = 25$ ticks, and $L = 28$ ticks. So the optimal cut loss amount $S$ for long first is
  
  $$S = (60 + 0.4 \times 28 - 0.6 \times 25)/2 = 28.1 \approx 28 \text{ ticks}.$$ 

  If the entry buy price is 360.0, then the stop is at 357.2.

- Example 2: Short first. The 90-th percentile HO is 57 ticks.

  $$S = (57 + 0.4 \times 28 - 0.6 \times 25)/2 = 26.6 \approx 27 \text{ ticks}.$$ 

  If the entry sell price is 360.0, then the stop is at 362.7.
Another Approach

Example: You enter into a long position of 1 contract of ABC @ 350.4.

At what price should you put an intra-day stop order?

Answer: The formula is

\[ S = b \times \text{Standard Deviation} \]

The number of standard deviations is the intuitive meaning of \( b \), which is obtained from back-testing

- Suppose the standard deviation of OL and HO is 1.6, and \( b = 3/2 \).
- So the stop \( S = (3/2) \times 1.6 = 2.4 \)
- This is 24 ticks (=S$480) from 350.4.
Value at Risk Approach

- Typically, volatility is denoted by $\sigma$.  

$$\text{Stop Price} = (1 \pm b\sigma) \times \text{Entry Price}$$

- The stop price is analogous to value at risk

  For a given $b$, the volatility gives the maximum loss allowed for a position

  In case of extreme levels of volatility, for a given $b$, the value at risk of the strategy can increase dramatically
An Example

EURAUD (colored bar = Long, white bar = Short, half bar = Neutral)
What is Volatility, Really?

- Rate of stochastic vibration

- Volatility: Degree of vacillation in return over
  - A period of time, say 5 minutes
  - X number of trades
  - Y number of contracts traded

- Volatility: Coefficient diffusion over
  - A period of time, say 5 minutes
  - X number of trades
  - Y number of contracts traded
Suppose $H_t$ and $L_t$ are the high and low prices of trading day $t$.

Example

- $n = 5$ days
- $\sigma^2 = 0.9009$

$$\sigma^2 = \frac{1}{4 \ln(2)} \times \frac{1}{n} \sum_{t=1}^{n} \ln \left( \frac{H_t}{L_t} \right)^2$$
Example of Using VaR Approach

Example: You enter into a long position of 1 contract of ABC @ 350.4

The volatility is \( \sigma = \sqrt{0.9009} = 0.9492 \)

Suppose \( b = 0.005 \),

\[
\text{Stop Price} = (1 - 0.005 \sigma) \times \text{Entry Price} \\
= (1 - 0.005 \times 0.9492) \times 350.4 \\
= 348.7
\]

What is the stop price if you enter a short position of 1 contract of SIMSCI @ 350.4?
Different Faces of Volatility

- Historical volatility is estimated with past prices.

- Implied volatility is the forward looking volatility backed out from the option prices
  - Black-Scholes
  - Model-free: not using option pricing models

- Conditional volatility is the volatility forecast using, for example GARCH(1, 1) model.
An Example of Model-Free Volatility
Professional Trading as a Business
Prop Trading as a Business

- Trading is also a business.
- A business model is needed.
- No gun-slinging cowboys, but measured, stoic professionals.
- Risk budget, control, and optimization are very important
Funds Required

- **Capitals**
  - Margin deposited at futures broker, also called equity
  - Investment in computer systems, additional screens

- **Operation expenses**
  - Rentals for a desk space in the arcade
  - Monthly subscription to a trading software
  - Additional live feed
Most Important of All

- Repertoire of trading strategies, each capable of generating profits consistently
- Constant research into new trading strategies
- Well-defined risk management policies
- Quarterly review of risk management policies
- Don’t look at monthly statements of P&L, no company or hedge fund can make money every month after cost).

- But make it a practice to “report” to “someone” quarterly, just like any listed company.
Records Matter

- In all business, all transactions are properly recorded and documented.
- Similarly, it is crucial to keep records (see next slide) of your trades.
- Review the trades and examine how you can further improve on your execution and money management.
- **Who knows, these records may be useful when you start a hedge fund!**
### A Trading Record on 1st October 13

<table>
<thead>
<tr>
<th>Size</th>
<th>B/S</th>
<th>Symbol</th>
<th>Type</th>
<th>Limit Price</th>
<th>Duration</th>
<th>Avg Fill Price</th>
<th>Time</th>
<th>Order #</th>
<th>Last Fill Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.3</td>
<td>DAY</td>
<td>08:33:01</td>
<td></td>
<td>333194604</td>
<td>08:34:12</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.3</td>
<td>DAY</td>
<td>08:39:29</td>
<td></td>
<td>333184781</td>
<td>08:39:30</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.3</td>
<td>DAY</td>
<td>08:38:54</td>
<td></td>
<td>333165381</td>
<td>08:39:40</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.3</td>
<td>DAY</td>
<td>08:40:23</td>
<td></td>
<td>333194620</td>
<td>08:41:36</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.2</td>
<td>DAY</td>
<td>08:40:14</td>
<td></td>
<td>333175507</td>
<td>08:42:04</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.4</td>
<td>DAY</td>
<td>08:39:45</td>
<td></td>
<td>333165385</td>
<td>08:44:05</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.1</td>
<td>DAY</td>
<td>08:42:33</td>
<td></td>
<td>333214135</td>
<td>08:44:53</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.4</td>
<td>DAY</td>
<td>08:42:01</td>
<td></td>
<td>333204621</td>
<td>08:45:35</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.4</td>
<td>DAY</td>
<td>08:42:10</td>
<td></td>
<td>333194625</td>
<td>08:45:36</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.5</td>
<td>DAY</td>
<td>08:34:50</td>
<td></td>
<td>333204608</td>
<td>08:45:41</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.6</td>
<td>DAY</td>
<td>08:33:22</td>
<td></td>
<td>333175483</td>
<td>08:45:44</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.5</td>
<td>DAY</td>
<td>08:45:52</td>
<td></td>
<td>333165405</td>
<td>08:47:53</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.4</td>
<td>DAY</td>
<td>08:49:52</td>
<td></td>
<td>333165413</td>
<td>08:58:06</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>363.6</td>
<td>DAY</td>
<td>08:56:35</td>
<td></td>
<td>333214155</td>
<td>08:59:38</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>LMT</td>
<td>364.1</td>
<td>DAY</td>
<td>08:53:00</td>
<td></td>
<td>333204641</td>
<td>09:00:02</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>STL</td>
<td>364.4</td>
<td>DAY</td>
<td>09:07:42</td>
<td></td>
<td>333214185</td>
<td>09:09:04</td>
</tr>
<tr>
<td>1</td>
<td>Sell</td>
<td>ZGV3</td>
<td>STL</td>
<td>364.3</td>
<td>DAY</td>
<td>09:07:48</td>
<td></td>
<td>333175566</td>
<td>09:14:58</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>364.2</td>
<td>DAY</td>
<td>09:14:56</td>
<td></td>
<td>333214196</td>
<td>09:16:28</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>364.2</td>
<td>DAY</td>
<td>09:16:46</td>
<td></td>
<td>333165484</td>
<td>09:16:46</td>
</tr>
<tr>
<td>1</td>
<td>Buy</td>
<td>ZGV3</td>
<td>LMT</td>
<td>364.2</td>
<td>DAY</td>
<td>09:16:47</td>
<td></td>
<td>333204678</td>
<td>09:16:48</td>
</tr>
</tbody>
</table>
Practical Advice

- Stick to the collection of trading strategies and risk management policies slavishly.
- Never start trading after turning on the computer systems.
- Be mentally prepared at least 10 minutes before trading.
- Get a sense of what’s happening in the world – any breaking news and what the implications are.
- When in a bad run and becoming emotional, stop trading.
- Make sure that tomorrow you have capital to fight again.
Practical Tips

- Of course, make sure that you have sufficient collateral so that you won’t get a margin call.

- Margin calls tend to force a hasty and bad decision to cut the position, resulting in more than expected losses.

- Since it is a business, it is important to keep records of your filled orders, and analyze them to improve your trading skills.

- Keep a simple dairy of the major events and your trading.
Three Most Difficult Things for Retail Traders

- Creative in searching for new trading strategies
- Methodical in execution
- Responsible for own action: never blame others or luck and vent anger on someone

- First rule of trading is discipline.
- Second rule of trading is not to break the first rule.
Important!

- Know the trading software inside out.

- Know the futures products you trade well.
  - Time of opening
  - Time when order cannot be canceled
  - Last trading day

- When in doubt, don’t trade.

- A strong mental fortitude is crucial.
Final Advices

- Important to fully understand the products you trade
- There are dangers when many are following the same strategy
- Beware of hedgers becoming speculators
- Risk must be quantified and risk limits set
  - Exceeding risk limits not acceptable even when profits look good.
- Be diversified in derivatives, trading strategies, and also counterparties.
- Scenario analysis and stress testing are useful.