The Black-Scholes Hedging Game

1. Description

The Black-Scholes Hedging Game is an educational game devised to help the student appreciate the significance of delta-hedging in the Black-Scholes Theory of Option Pricing.
In brief, the student plays the role of an option-dealer who has to balance his portfolio so that it remains risk-neutral by delta-hedging some options that have been traded.
The game ends with a summary report that assesses the performance of the student against prescriptions of the theory.

2. Learning Objectives

The following points in the Black-Scholes Theory are touched upon in the game:

- The volatility of the underlying stock impacts in a pivotal manner what the theory says about the price of an option and how one should risk-manage it.
- In the theory, vanilla option prices and their greeks (risk numbers) are expressible in formulas. Hence, these quantities are relatively easy to compute as soon as some computing tool is available.
- The significance of the price of an option in the theory is that it is the cost to the dealer for managing the risk of being exposed to the underlying stock price fluctuations upon the trading of the option.
- Delta-hedging is what the theory prescribes one to do in order to risk-manage the open option position.
- In the ideal case, the P&L of the dealer's portfolio should be zero throughout the life of a traded option as delta-hedging is meant to maintain it risk-neutral. In practice, this cannot be so and the discrepancy from being zero is the hedging error.

The game helps the student strengthen his understanding of these points.

3. Accessing the Game

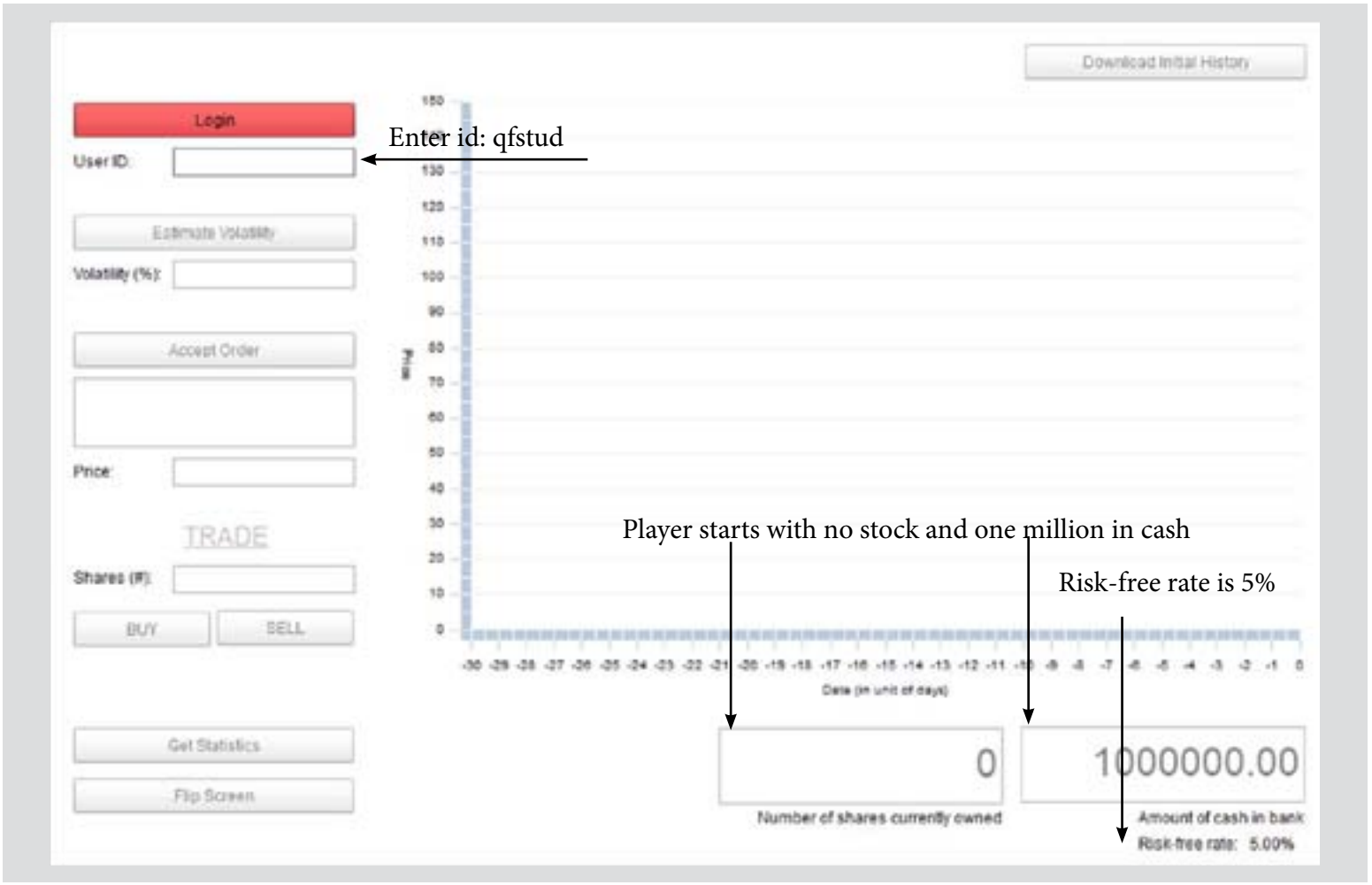
The game may be accessed at this link: http://qfuserver1.smu.edu.sg/djangosite/qfproj/test/6/

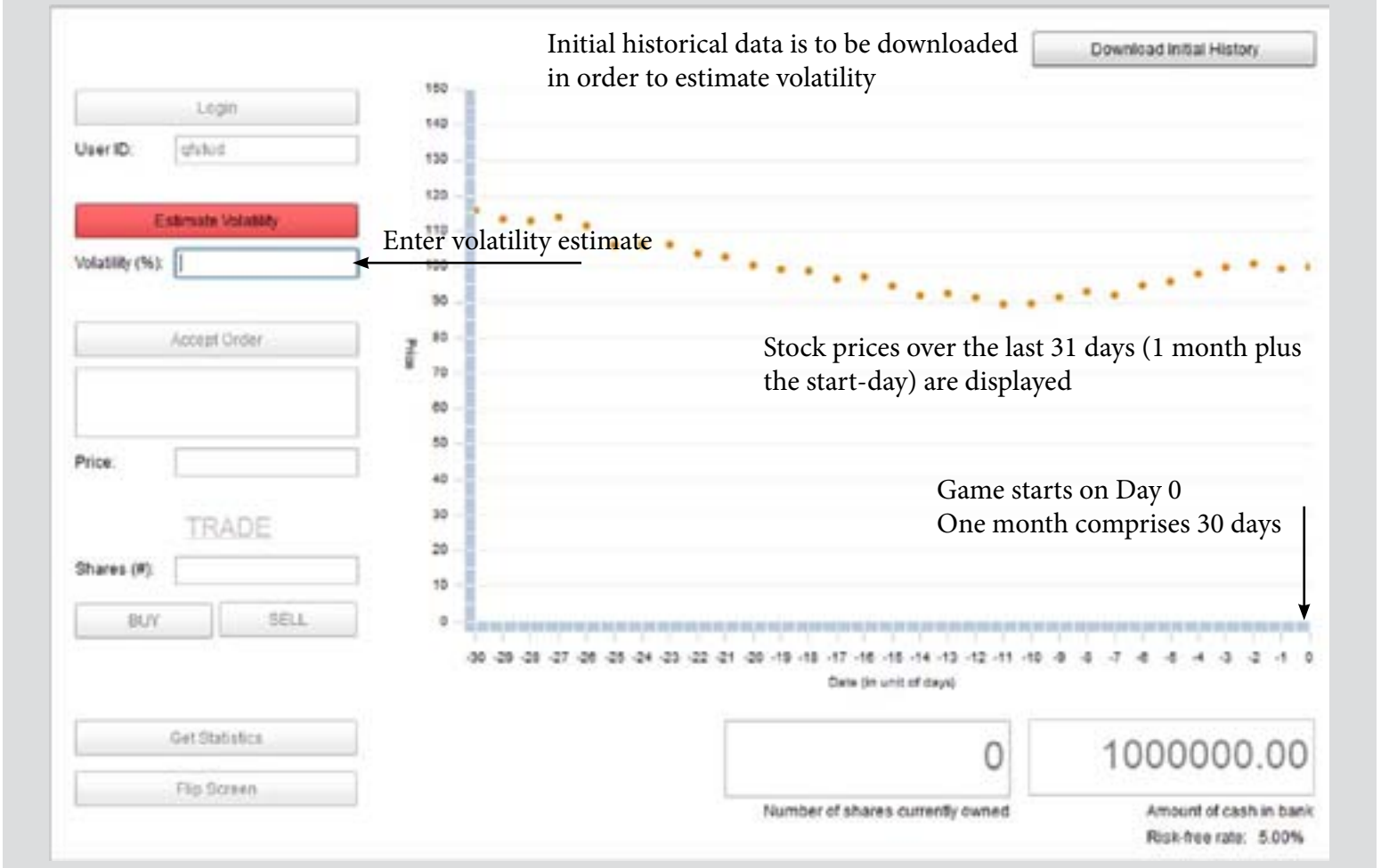
4. Assumptions

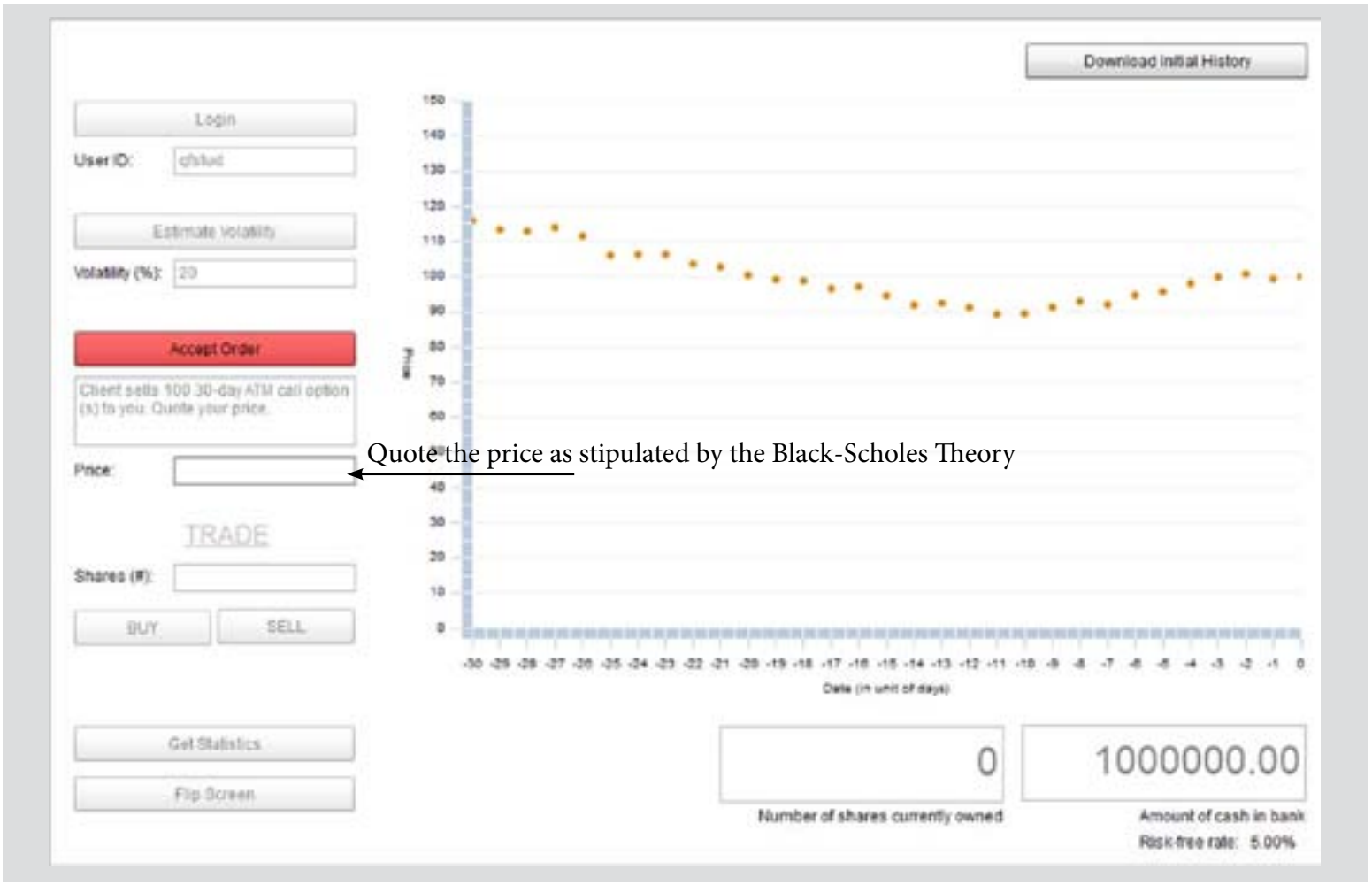
The player has an initial wealth of $1,000,000. The risk-free rate is 5%. There are 30 days in a month.

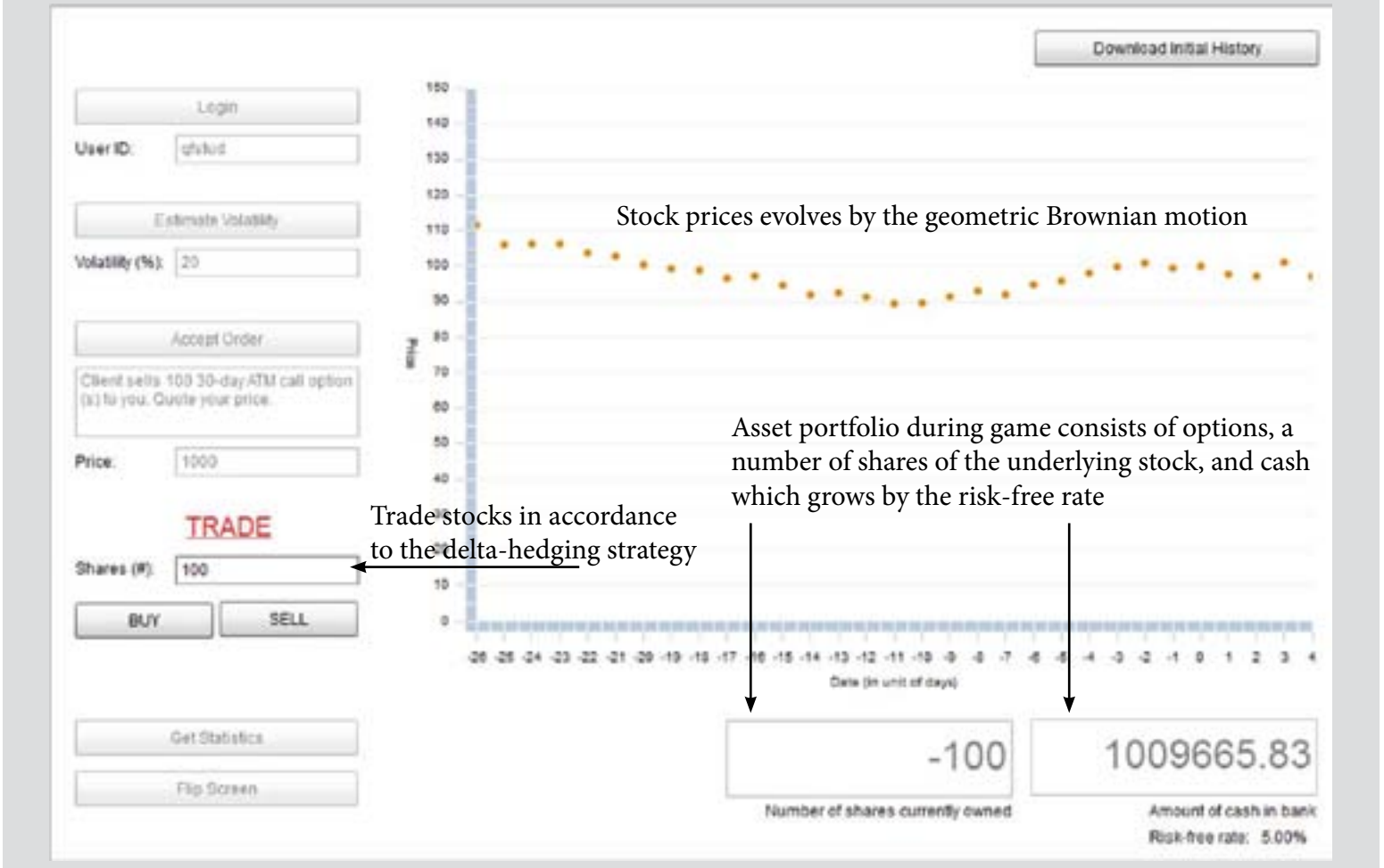
5. Game Screens and Actions

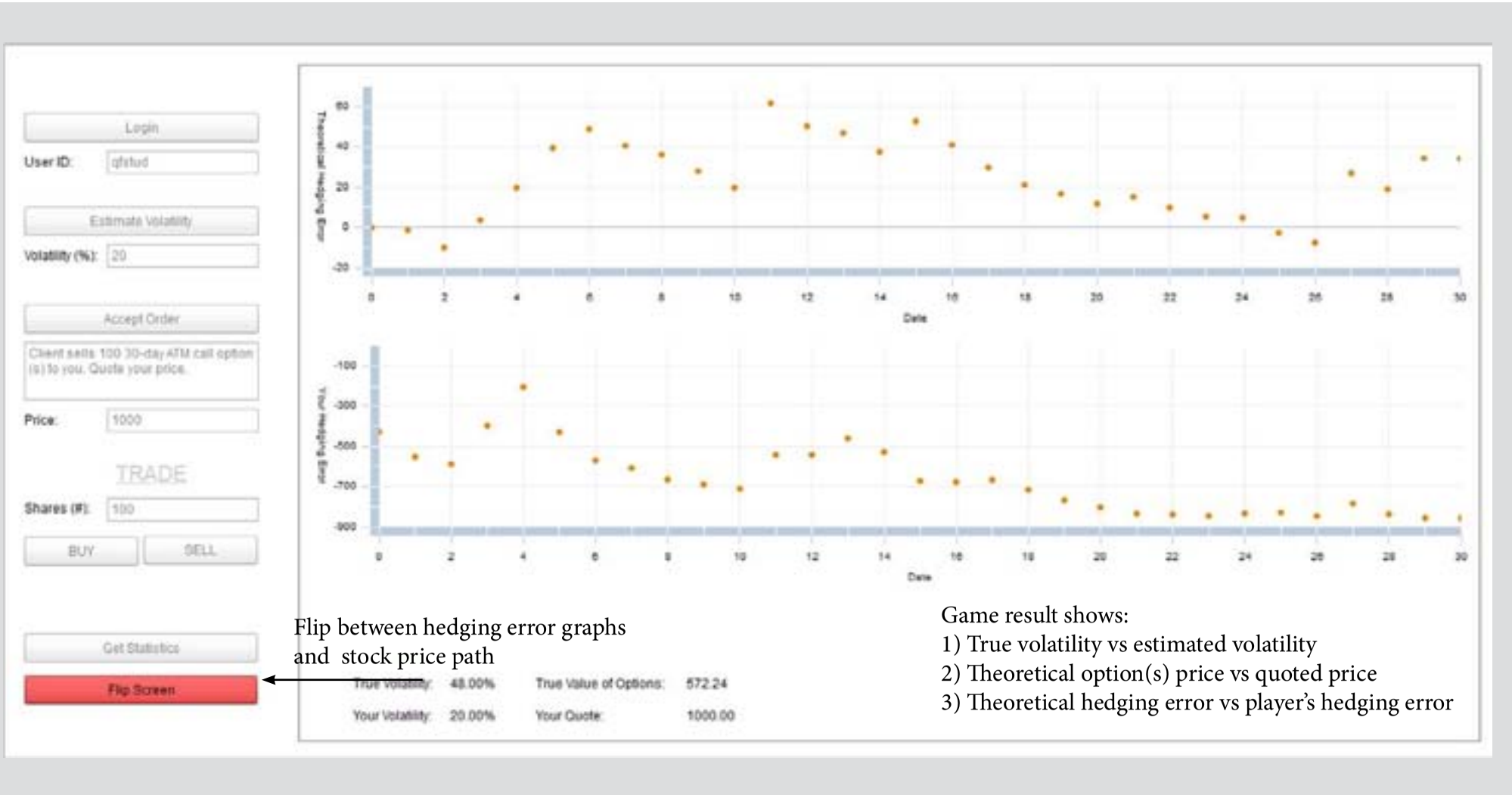
The screens below are shown in the same order as they would be encountered during the game.






Download Initial History
Login
User ID:
Estimate Volatility
Volatility (%):
20
Accept Order
Client sells 100 30-day ATM call option (s) to you. Quote your price.
Price:
TRADE
Shares (#):
BUY
SELL
Get Statistics
Flip Screen
Quote the price as stipulated by the Black-Scholes Theory
Price
Date (in unit of days)
0
Number of shares currently owned
1000000.00
Amount of cash in bank
Risk-free rate: 5.00%


Download Initial History
Login
User ID:
Estimate Volatility
Volatility (%):
20
Accept Order
Client sells 100 30-day ATM call option (s) to you. Quote your price.
Price:
1000
TRADE
Shares (#):
100
BUY
SELL
Get Statistics
Flip Screen
Stock prices evolves by the geometric Brownian motion
Asset portfolio during game consists of options, a number of shares of the underlying stock, and cash which grows by the risk-free rate
Trade stocks in accordance to the delta-hedging strategy
Price
Date (in unit of days)
-100
Number of shares currently owned
1009665.83
Amount of cash in bank
Risk-free rate: 5.00%



Flip between hedging error graphs and stock price path

Game result shows:
1) True volatility vs estimated volatility
2) Theoretical option(s) price vs quoted price
3) Theoretical hedging error vs player's hedging error

6. What is Hedging Error?

The hedging error is the difference between the value of the asset portfolio and what it would have been if everything were held in cash. The graphs on the game result page depict this quantity computed for each day in the game.

In more detail, let the following symbols be defined:

Untraded cash value - Z_t
Portfolio cash value - C_t
Number of shares held - N_t
Stock price - S_t
Number of options held - M
Option value - V_t

The hedging error is this quantity: $MV_t + N_tS_t + C_t - Z_t$

Option value is computed with the Black-Scholes formula using the true volatility as generated by the game.

The number of shares held is Delta in the theoretical hedging error, while in "your" hedging error, it is the number that is shown on the game screen which changes according to the player's trades.

Portfolio cash value changes as the player trades stock and options and accumulates as the days progress according to the risk-free rate.

Untraded cash value grows by the risk free rate: over a day, it grows by a factor of (1 + rdt), where dt is taken to be 1/360.

7. Something's wrong with the game!

When an alert box appears or if you think something ought to happen but doesn't, one of the following points may tell you what could actually be happening:

- As the game is hosted on a remote server, network connectivity may be a factor.
- The login id is incorrect.
- Volatility and Price must be postive real numbers.
- Number of shares traded must be a positive integer.
- There is a timeout so that you may not be able to proceed with the game after a period of inaction. Try to restart the game.

If you think there might be something wrong with the game and this list hasn't been helpful to you, please drop me an email at: chonghuitan@smu.edu.sg