EVALUATION OF MOMENT RISK

CAN THE SHARPE RATIO MAKE THE CUT?

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Outline of the Talk

- Motivation: Inference on Financial Risk
- Measures of Risk in Financial Returns
- Risk Adjusted Performance Measures
- Testing Moments of Return Distribution
- Smooth Moment Risk Statistics
- Empirical Example: Testing Market Neutrality
- Future Directions

How to measure and compare financial risk?

- Financial risk assessment and inference based on parametric measures like the *Sharpe Ratio* and Mean-Variance analysis ignore higher order moments of the return distribution, and possibly a non-linear structure (Agarwal and Naik, 2004, see Fama, 1970 or Campbell, Lo and MacKinlay, 1997, for review).
- Optimal portfolio choice based on Sharpe ratios are inherently dependent on the *normality* assumption of the return distribution besides *independence* (Sharpe, 1966, 1994, Jobson and Korkie, 1980, Memmel, 2003).
- Such tests are not strictly valid for financial data that are *leptokurtic*, and for time series that show *persistence* in volatility (e.g. stocks and mutual funds) or in levels (e.g. hedge funds, see Getmansky, 2004, Getmansky, Lo and Makarov, 2003).
- Resampling based tests on robust measures of (Studentized) Sharpe ratio can address leptokurtosis and *HAC*-type estimators address dependent structure (Andrews, 1991, Ledoit and Wolf, 2008).

Drawbacks of traditional measures for risk analysis.

- "...for certain applications the Sharpe ratio is not the most appropriate performance measure; e.g. when returns are far from normally distributed or autocorrelated..." (Ledoit and Wolf, 2008, p. 851, see Getmansky, 2004)
- Bootstrap-based methods might not capture the true dependent structure of the return distribution that can be obtained by a reasonably "close" parametric specification or for certain limited dependent variable distributions (see Hall, P., Horowitz, J., L. and Jing, B., Y., 1995).
- Tests based purely on the function of the first two moments like the Sharpe ratio fail to account for restrictions or differences in higher order moments jointly besides estimation error of the Sharpe ratios.
- For financial risk assessment Sharpe ratios are estimated based on past data to forecast distribution of future risk adjusted returns.

How do you map the uncertainty in future returns?

- A graphical test of Density Forecast Evaluation using probability integral transforms discussed by in Diebold, Gunther and Tay (1998) was formalized analytically in Ghosh and Bera (2006) as a variant of Neyman's smooth test for parametric models.
- They explicitly looked at the dependent structure of the model besides the fat tails to explore model selection issues along with testing.
- It has been empirically observed that although financial returns data of stocks and mutual funds do rarely show persistence or autocorrelation in levels, but they do often show persistence in higher order moments like volatility.
- Hedge funds and private equity funds tend to show some persistence in levels as well (Lo, 2001, Brooks and Kat 2002; Agarwal, V., and N.Y. Naik, 2004, Malkiel and Saha 2005; Getmansky, 2004, Getmansky, Lo, and Makarov, 2003, Kalpan and Schoar 2005; Ghosh, 2008).

Fixing ideas about the Return Distribution

- If we have the return data given by $R_1, R_2, ..., R_T$ then the population Sharpe ratio is

$$SR = \left(\mu_R - R_f\right) / \sigma_R \tag{1}$$

  where μ_R, σ_R^2 and R_f are the population mean, population variance of the Return distribution and the existing risk free rate, respectively.

- The corresponding sample counterpart or the estimated Sharpe ratio is

$$\widehat{SR} = \left(\hat{\mu}_R - R_f\right) / \hat{\sigma}_R \tag{2}$$

  where $\hat{\mu}_R = \frac{1}{T}\sum_{t=1}^{T} R_t$ and $\hat{\sigma}_R^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \hat{\mu}_R\right)^2$ are the unbiased sample mean and variance estimates.

Inference on first two moments of the Return Distribution

- We observe that if we assume the data to be independent and identically normally distributed then we can test the hypothesis $H_0 : \mu_R = R_f$ against $H_1 : \mu_R \neq R_f$, the test statistic is

$$tstat = \frac{(\hat{\mu}_R - R_f)}{\hat{\sigma}_R / \sqrt{T}} = \sqrt{T}\frac{(\hat{\mu}_R - R_f)}{\hat{\sigma}_R} = \sqrt{T}\widehat{SR}$$

where $\hat{\sigma}_R^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \hat{\mu}_R)^2$ is an unbiased estimator of the population variance.
- Incidentally, the distribution of $\widehat{SR} = (\hat{\mu}_R - R_f)/\hat{\sigma}_R = \frac{tstat}{\sqrt{T}}$ is nothing new, in fact, it was first proposed by Student (1908) himself, and only later Fisher (1925) formulated the test statistic and defined the Student's t distribution with $(T-1)$ degrees of freedom.
- However, this test is crucially dependent on the parametric assumption that the underlying distribution is normal, and that the data in independently and identically distributed.

Objectives and Contributions

- We explore the an incomplete list of competing *risk adjusted performance* (RAP) measures.
- We explore the probability distribution of a proposed measure of risk adjusted returns when estimated from a return distribution based on the smooth test methodology.
- We propose a test that is robust to violations of the *iid* assumptions under general conditions, and test them jointly.
- Our proposed score test that would address the leptokurtic and time series dependent structure not explicitly addressed in previous literature (see Leung and Wong, 2007, Ledoit and Wolf, 2008).
- We look at the hedge fund indices and test for equity market neutrality and sensitivity to the market and global hedge fund indices (Diez de Los Rios and Garcia, 2008; Patton, 2009).
- We also compare the nature of other hedge fund strategies based on the proposed *smooth moment risk* measures (SMR) incorporating dependence.

Why Risk Measure Matters to an Investor?

- Alternative investments like hedge funds suffer from severe information asymmetry as they are usually not under the purview of regulatory bodies like the Association of Investment Management and Research (AIMR) and compliance with AIMR-Portfolio Presentation Standards (AIMR-PMS) and more recently instituted Global Investment Performance Standards (GIPS) aimed to protect against predatory practices.
- Since Alfred Winslow Jones formed the first hedge fund in 1949, he managed to operate in almost complete secrecy for 17 years.
- Nearly 50 years later LTCM (Long Term Capital Management) whose spectacular collapse and bailout brought the attention back to Hedge Fund operational secrecy and risk measures (Lhabitant 2006).
- However, we are reminded the need for performance and monitoring after Bernie Madoff's hedge fund, Ascot Partners turned out to be a 50 billion dollar Ponnzi scheme in 2008 or Raj Rajaratnam's 7 billion dollar Galleon fund collapse before insider trading conviction in 2011.

How do we measure Risk?

- Risk as a concept is often individual or target specific, application or theory specific, uncertainty or risk aversion specific, and measures of risk also reflects such dichotomies.
- This however leads to conflicts in ranking of portfolios by measures of riskiness, as the measures are often non-affine or non-linear transformation, or sometimes not even functions of each other.
- In general, return of an individual asset in period t is composed of two parts gains and losses (Bernardo and Ledoit, 2000, Lhabitant, 2006). So,

$$R_t = G_t I\left\{R_t \geq 0\right\} - L_t I\left\{R_t < 0\right\}, \quad (3)$$

where G_t and L_t are absolute values of gains and losses made by the fund in period t, respectively, and $I\left\{A\right\}$ is an indicator function that takes a value 1 when A has occurred.

More Measures of Risk

- The *Gain-to-loss ratio* is average gains over average losses, $\bar{G}/\bar{L}$, is commonly used by fund managers. This measure however is noninformative about the riskiness, or frequency of gains or losses.
- *Mean Absolute Deviation (MAD) from Mean Return*

$$MAD = E\left|R_t - \bar{R}\right| = \frac{1}{T}\sum_{t=1}^{T}\left|R_t - \bar{R}\right|. \tag{4}$$

- *Sample variance*

$$\hat{\sigma}_R^2 = E\left(R_t - \bar{R}\right)^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \bar{R}\right)^2,$$

or it's positive square root $\hat{\sigma}_R$ termed as standard deviation which is *sufficient* with normal errors.
- However, we need to perform tests of normality (e.g. Jarque-Bera, 1983) of returns before determining the riskiness of the portfolio with such measures.

Should risk measures be symmetric?

- The case for an asymmetric treatment of positive and negative returns have solid foundations from the standpoints of economic and statistical theory, empirical evidence besides behavioral finance.
- Alternative investment like hedge funds use dynamic trading strategies that are often asymmetric like stop losses, actively managed leverage and options trading (Lhabitant 2006).
- Individual risk averse investors and institutions aspires to adopt investment strategies that essentially limit their downside risk be it from a benchmark or an average return.
- Statistical inference based on normality fail to differentiate the risk profile of individuals or institutions who have divergent higher order moments or will have very low or no power against such divergence.

Downside or asymmetric risk measures

- *Semi-variance* (or *semi-deviation*). Suppose we have a prespecified benchmark or target rate R^*,

$$Downside\ risk = \frac{1}{T}\sqrt{\sum_{t=1}^{T} d_t^2 I\left\{d_t < 0\right\}},$$

  where $d_t = R_t - R^*$ and $I\left\{d_t < 0\right\} = 1$ if $d_t < 0; = 0$ otherwise.
- When we replace R^* by the mean return we get the semi-deviation or *below-mean standard deviation* (Markowitz, 1959).
- If R^* is replaced by a moving target like the treasury bill rate (risk free rate) or the returns to a benchmark like S&P 500, we get a *below-target semi-deviation* often of interest to institutional investors.

Other measures of asymmetric risk-I

1. The *downside frequency* or the frequency of occurrence below a target R^* (i.e., $\sum_{t=1}^{T} I\{d_t < 0\}$);
2. The *gain standard deviation* $\sqrt{\frac{1}{T_G - 1}\sum_{t=1}^{T_G}\left(G_t - \bar{G}\right)^2}$, if $T_G = \sum_{t=1}^{T} I\{R_t \geq 0\}$, $T_G + T_L = T$;

Shortfall probability is defined with the target R^* as

$$\widehat{Risk} = P\left(\widehat{R_t < R^*}\right) = \frac{1}{T}\sum_{t=1}^{T} I\{d_t < 0\}$$
$$= \frac{downside\ frequncy}{T}.$$

Other measures of asymmetric risk-II

1. *Value-at-Risk* is defined at the maximum amount of capital that one can lose over a period of time say one month at a certain confidence level, say $100(p)\%$.

$$\widehat{VaR_p} = \underset{R}{Min}\left\{R : P\left(\widehat{R_t \leq R}\right) \geq p\right\} = \underset{R}{Min}\left\{R : \sum_{t=1}^{T} I\left\{(R_t - R) < 0\right\} \geq \right.$$

If the original return distribution is normal, it is simply $VaR_p = \mu_R + \xi_p \sigma_R$.

2. Any period to period drop can be taken as a *drawdown statistic* during a holding period, however, a maximal loss in percentage terms over a period (highest minus the lowest) is called the *maximum drawdown*. Maximum drawdown is really the range of percentage returns over a period of time.

$$\begin{aligned} Max.drawdown = \max\{&\max\left(G_t\right) + \max\left(L_t\right), \\ &\max(G_t) - \min(G_t), \max(L_t) - \min\left(L_t\right)\}. \end{aligned} \quad (5)$$

Risk Adjusted Performance Measures (RAPM)

- Sharpe (1966) introduced the ratio, "excess return per unit of volatility" that has stood the test of time, defined by

$$SR_P = \frac{\mu_P - R_f}{\sigma_P}.$$

- The attractiveness of the Sharpe ratio stems from the "leverage" invariant measure, all funds with different portfolio weights would have the same Sharpe ratio.
- Sharpe Ratio is *not related to the market index* (and hence the systematic risk) which might not be well defined (Roll, 1977).
- Sharpe (1994) generalized the definition to a benchmark portfolio return R_B,

$$\text{Information Ratio}_P = \frac{\mu_P - R_B}{TE_P} = \frac{\mu_P - R_B}{\sigma\left(R_P - R_B\right)},$$

where $TE_P = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(R_{Pt} - R_{Bt}\right)^2}$ is the *tracking error*.

Inference on Sharpe Ratio-I

- Sharpe ratio has been used to test between two portfolio using the method suggested by Jobson and Korkie (1981) who tested $H_0 : SR_1 = SR_2$ vs $H_1 : SR_1 \neq SR_2$ and used

$$Z = \frac{\sigma_1 \mu_2 - \sigma_2 \mu_1}{\sqrt{\theta}} \xrightarrow{d} N(0, 1),$$

where the asymptotic variance of the numerator is

$$\theta = \frac{1}{T} \left[2\sigma_1^2 \sigma_1^2 - 2\sigma_1 \sigma_2 \sigma_{12} + \frac{1}{2} (\mu_1 \sigma_2)^2 + \frac{1}{2} (\mu_2 \sigma_1)^2 - \frac{\mu_1 \mu_2}{\sigma_1 \sigma_2} \sigma_{12}^2 \right].$$

This however gives an asymptotic distribution that has low power for small samples, as Jorion (1985) noted at 5% level the power could be as low as 15%.

- One of the main problems in the test proposed by Jobson and Korkie (1981) is the assumption of normality that is entirely justified in financial asset returns.

Inference on Sharpe Ratio-II

- Gibbons et. al. (1989) suggested a test

$$W = \left[\frac{\sqrt{1+SR_2^2}}{\sqrt{1+SR_1^2}}\right]^2 - 1 \equiv \psi^2 - 1,$$

  for *ex-ante* portfolio efficiency using maximum Sharpe ratio (SR_2) for the effect of additional assets to the universe where SR_1 is the Sharpe ratio of the portfolio. This would have a Wishart distribution.
- A more tractable statistic is given by

$$F = \frac{T\left(T+N-1\right))}{N(T-2)} W \sim F_{N,T-N-1},$$

  under the null hypothesis where T is the number of returns observed and N is the number of assets originally present (Morrison, 1976).
- Lo (2002) finds that tests based on the Sharpe ratio crucially depend on the *iid* normality assumptions.

CAPM based RAPM-I

- CAPM model

$$E(R_P) = R_f + \beta [E(R_M) - R_f] \implies E(R_P) - R_f = \beta_P [E(R_M) - R_f],$$

  gives the securities market line (SML) where R_P and R_M are respectively the percentage returns on the portfolio P and on the market portfolio M, R_f denotes the riskfree rate, β_P is the beta of the portfolio P with respect to market portfolio M, and $E(.)$ denotes the expectation operator.
- The time-series *market model* that assigns *ex-post* excess return for individual asset i in time t is given in terms of risk premium as

$$R_{it} = \alpha_i + R_f + \beta_i (R_{Mt} - R_f) + \varepsilon_{it},$$

  where R_{it}, R_{Mt} and ε_{it} are the returns of individual asset and the market model in period t. For individual i, and α_i, β_i are individual firm specific effects and risk free rate R_f.

CAPM based RAPM-II

- According to the Sharpe-Lintner one factor CAPM model, while the standard deviation σ_P gives a measure of the total risk, the systematic risk is given by the regression slope coefficient β_P. Hence, while the Sharpe ratio gives a measure of the return with respect to unit volatility, a measure of the return for unit systematic risk ($\beta_P \neq 0$) is (Treynor, 1965; Treynor and Black, 1973)

$$\text{Treynor ratio}_P = \frac{\alpha_P}{\beta_P} = \frac{(R_P - R_f)}{\beta_P}.$$

- Treynor ratio is directly related to the CAPM slope β_P and is appropriate for a well diversified portfolio, hence will be affected by the critique that the market index might not be well defined (Roll, 1977).
- Srivastava and Essayyad (1994) proposed an extension of the Treynor ratio that combines beta's of different portfolio as a combined index that might be more efficient.

Modifications of the Sharpe Ratio

- The Double Sharpe Ratio was proposed to accommodate for the estimation error (Lo, 2002)

$$DSR_P = \frac{SR_P}{\sigma\left(SR_P\right)},$$

  where $\sigma\left(SR_P\right)$ is the bootstrap standard error of the Sharpe Ratio.
- Generalized Sharpe Ratio based on incremental VaR (Dowd, 2000) and similar method with the *benchmark VaR* (or BVaR) (Dembo 1997) has been proposed.
- It was noticed that both Sharpe and Information Ratio may lead to spurious ranking of mutual funds when excess returns are negative.
- To address this Israelson (2005) proposed the modified Sharpe ratio

$$SR_P^{\text{mod}} = \frac{\mu_P - R_f}{\sigma_P^{(\mu_P - R_f)/|\mu_P - R_f|}}.$$

Jensen's alpha

- Jensen's *alpha* for a portfolio P is defined as the abnormal return of the portfolio over and above the expected return under the CAPM model

$$\text{Jensen's } \alpha_P = R_P - E(R_P) = (R_P - R_f) - \beta_P (R_M - R_f),$$

  gives the difference between the observed and predicted risk premia (Jensen, 1968). We can perform statistical tests on Jensen's α using the standard t-tests assuming normality of the errors in the market model.
- Unlike the Sharpe and the Treynor ratio's Jensen's α can be expressed as an excess return and expressed in basis points, it also suffers from Roll's (1977) criticism as it depends on the market index.
- Money managers who practised market timing, Jensen's α might not be a good measure as it can turn negative and fails to address the manager's performance.

Jensen's alpha modified

- Modifications for varying beta as well as for higher moments of returns minus risk-free rate has been suggested (Treynor and Mazuy, 1966, Merton, 1981; Henriksson and Merton, 1981; Henriksson, 1984). This model was particularly useful to check market timing ability incorporating non-linearities in the CAPM framework (Jensen, 1972, Bhattacharya and Pfleiderer, 1983).
- There were other extensions of Jensen's α like Black's zero-beta model where there is no risk-free rate (Black, 1972), adjusting for the impact of taxes liabilities (Brennan, 1970), considering total risk σ_P as opposed to just market risk β_M (Elton and Gruber, 1995).
- However, the total risk measure called *Total Risk Alpha* along with Jensen's alpha can be manipulated using leverage, as opposed to Sharpe and Treynor ratios Jensen's α is not leverage invariant (Scholtz and Wilkens, 2005, Gressis, Philippatos and Vlahos, 1986).

Drawbacks of Model based RAPM-I

- One of the issues of all these three Sharpe Ratio, Jensen's alpha and Treynor Ratio is whether they will generate the same ranking of riskiness across funds or portfolios.
- For portfolios which are dominated by systematic risk compared to diversifiable non-systematic risk it is expected that the ranks of funds in terms of riskiness will give you similar rankings. However, in funds like hedge funds they are expected to generate very different rankings when the measure of risk is changed and the rankings will be similar only under very restrictive conditions (Lhabitatnt 2006, p. 467).
- CAPM is a single factor model where the only systematic risk is assumed to come from the market, this has been generalized to multi-factor models like the APT model.

Drawbacks of Model based RAPM-II

- There are some generalizations to the standard measures like extension of the Treynor ratio to a case of multifactor model by using orthonormal basis in the directions of risk (Hubner 2005).
- However, as discussed before, hedge funds are uniquely placed which focuses more on non-systematic or total risk, hence, Sharpe Ratios and generalizations discussed are more commonly used.

Can we interpret the standard measures of risk?

- Sharpe ratio gives the excess return from risk free rate per unit of volatility σ_P that is not well understood.
- M^2 measure was proposed to put all returns in excess of the risk free rate in terms of the same volatility, say the market or benchmark volatility σ_M (Modigliani and Modigliani, 1997; Modigliani, 1997).
- They suggested de-leveraging (or leveraging) using the risk free rate forming a portfolio P^* of the portfolio and treasury bills (with R_f and no volatility) to equate the Sharpe ratios, *i.e.*,

$$\frac{R_P - R_f}{\sigma_P} = \frac{R_{P^*} - R_f}{\sigma_M} \implies M^2 = R_{P^*} = \frac{\sigma_M}{\sigma_P}\left(R_P - R_f\right) - R_f,$$

  hence for this *risk-adjusted performance* (RAP) measure similar to Sharpe ratio the fund with the highest M^2 will have the highest return for any level of risk.
- The resulting ranking would be similar as Sharpe ratio of a portfolio on which M^2 is based is not affected by leverage with the risk free asset. Here the term σ_M/σ_P is called the *leverage factor*.

Interpretable measures of risk?

- Scholtz and Wilkens (2005) suggests a measure that is a *market risk adjusted performance measure* (MRAP) that accounts for the market risk rather than total risk, similar to the Treynor Ratio.
- Muralidharan (2000) suggested the M^3 measure that corrects for the unaccounted for correlation in M^2. Lobosco (1999) developed the *Style RAP* (SRAP) and Muralidhar (2001) also developed the *SHARAD* measure is an extension of the M^3 measure that is adjusted for style specific investment benchmark (Sharpe, 1992). There were two further measures that were proposed GH1 and GH2 that also uses the leveraging-deleveraging approach of M^2 (Graham and Harvey, 1997).
- Similar in essence to the GH measures Cantaluppi and Hug (2000) proposed a measure of risk that is called the *efficiency ratio* that gives the best possible performance by a certain portfolio with respect to the efficient frontier.

Sharpe Ratio type Measures based on downside risk

- Define MAR as the *minimum acceptable return* and DD_P is the *downward deviation* below MAR, (Sortino and van der Meer, 1991, Sortino, van der Meer, Plantinga, 1999)

$$\text{Sortino Ratio}_P = \frac{R_P - MAR}{DD_P} = \frac{E\left(R_P\right) - MAR}{\sqrt{\frac{1}{T} \sum_{\substack{t=0 \\ R_{Pt}<MAR}}^{T} \left(R_{Pt} - MAR\right)^2}},$$

  which can be compared if the value of MAR is the same for the funds. Sortino and Price (1994) proposed the *Fouse Index* $=\mu_P - B\delta^2$ based on Expected Utility Theory where B is the degree of Risk Aversion and δ is downside risk with MAR.
- Sharpe ratio has strong foundation in the underlying theory of normality in mean-variance analysis, in particular, on the assumption of *independent and identically distributed* returns (Lo, 2002). Ziemba (2005) calculates the Sharpe Ratio with downside variance defined before as loss deviation σ_{x-}(divided by T-1),

Risk Measures based on downside risk-I

- When portfolios are non-normal standard mean-variance analysis do not it suffice to capture the risk distribution of the portfolio, and higher order moments like skewness and kurtosis need to be considered. If a three moment CAPM is assumed with a quadratic return process Hwang and Satchell (1998) proposed a new performance measure is proposed based on higher order moments.
- *Omega measure* is closely associated with downside risk, lower partial moments, gain-loss functions, breakdown of normality assumptions and need for higher order moments (Keating and Shadwick, 2002). It is simple to define as for certain MAR

$$\Omega\left(MAR\right)=\frac{\int_{MAR}^{b}\left(1-F\left(x\right)\right)dx}{\int_{a}^{MAR}F\left(x\right)dx},$$

defined on $\left(a,b\right)$ of possible returns and cumulative distribution function $F\left(.\right)$. The ranking based on the omega measure is expected to be different from Sharpe ratio, alphas and VaR.

Risk Measures based on downside risk-II

- The *Kappa measure* generalizes Sortino ratio and Omega measures (Kaplan and Knowles, 2004). Sterling ratio also considers drawdowns to measure risk defined as

$$\text{Sterling}_P = \frac{R_P - R_f}{\overline{drawdown}} \left(or = \frac{R_P - R_f}{\max.drawdown}, alternative \right),$$

where $\overline{drawdown}$ is the average of the "high" drawdowns during the period.

- Burke ratio looks at the average L_2-distance defined as the square root sum of squares of the drawdowns instead of the average or the maximum (Burke , 1994)

$$\text{Burke}_P = \frac{R_P - R_F}{\sqrt{\sum_{i=1}^{N} \left(drawdown_i\right)^2}}.$$

Testing moments and Sharpe Ratio

- Consider the following problem with the returns from two investment strategies say, R_{1t} and R_{2t}, $t = 1, 2, ..., T$.
- First we consider a strictly stationary distribution, hence, the covariance (and higher order moments) structure remain a bivariate distribution that is Ergodic with

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \text{ and } \sum = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}.$$

- To test $H_0 : SR_1 - SR_2 \equiv \frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma 2} = 0$ against $H_1 : SR_1 - SR_2 \equiv \frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma 2} \neq 0$, $g\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2\right)$ $= f\left(\mu_1, \mu_2, \mu_{12}, \mu_{22}\right) = f\left(,,\right)$, where μ_{ij} is the j^{th} raw moment of the i^{th} asset return distribution.
- For example, $\mu_{i1} = \mu_i, i = 1, 2$. The hypotheses becomes $H_0 : f\left(\mu_1, \mu_2, \mu_{12}, \mu_{22}\right) = 0$ vs. $H_0 : f\left(\mu_1, \mu_2, \mu_{12}, \mu_{22}\right) \neq 0$ where $f\left(\mu_1, \mu_2, \mu_{12}, \mu_{22}\right) = \frac{\mu_1}{\sqrt{\mu_{12} - \mu_1^2}} - \frac{\mu_2}{\sqrt{\mu_{22} - \mu_2^2}}$.

Asymptotic Test of Sharpe Ratio

- Under stationarity with the appropriate mixing conditions, existence of at least the fourth moment and normality, and a consistent estimator of the parameter vector we use the delta method to $\sqrt{T}\left(f\left(\hat{\theta}\right)-f\left(\theta\right)\right)\rightarrow N\left(0,\nabla' f\left(\theta\right)\Omega\nabla f\left(\theta\right)\right)$ where $\sqrt{T}\left(\hat{\theta}-\theta\right)\rightarrow N\left(0,\Omega\right)$ where Ω is an unknown symmetric positive semi definite matrix.
- Further, we can estimate Ω by a heteroscedasticity and autocorrelation consistent (HAC) estimator with an appropriate kernel like Bartlett kernel (Andrews, 1991, Andrews and Monahan, 1992, Newey and West, 1994).
- However, using the HAC estimator, for small or moderately big samples the inference the test have high size distortion, hence the true null hypothesis would be rejected too often (Andrews, 1991, Andrews and Monahan, 1992).

Moment based score test

- We propose a score test that will give at several advantages over Wald-type test that is commonly used.
- Unlike the Wald test it will be invariant to the specification of the different functional form.
- It will adjust for size distortion by appropriately controlling the same sizes and parameter estimation error in serially dependent structure like GARCH (see Ghosh and Bera, 2006).
- We will jointly test normality like the Jarque-Bera statistic which is also a ratio of excess skewness and kurtosis terms.
- Finally, the test will be an Locally Most Powerful Unbiased test and in general optimal test as it will be function of sample score statistics (Bera and Bilias, 2001).

Room for a moment based Score test

- Sharpe ratio and other measures crucially depend on the assumptions of normality and hence, linearity and symmetry, and independence (Lo, 2002, Getmansky, Lo and Makarov, 2003).
- Further more, the existence variation of higher order moments, and significant probability of extreme or "iceberg" risk further complicates the testing with Sharpe ratio alone (Bernardo and Ledoit, 2000, Brooks and Kat, 2002, Agarwal and Naik, 2004, Sharma, 2004, Malkiel and Saha, 2005, Diez de los Rios and Garcia, 2009).
- The need for a more robust test using measures like the Sharpe ratio has been highlighted in several papers (Ledoit and Wolf, 2008, Zakamouline and Koekebakker, 2009). It has also been noted that tests based on specific moments like the Sharpe ratio is prone to manipulation (Leland, 1999, Spurgin, 2001).
- Goetzmann, Ingersoll, Spiegel, Welch (2002) observes that "...the best static manipulated strategy has a truncated right tail and a fat left tail."

Advantages of the Smooth type score test

- A statistical inference framework that identifies the distributional differences among returns of funds, particularly in the directions of several moments.
- A joint test that identifies the nature of dependence structure of the return series that aids the testing, and hence estimation of moment based measures with minimal computational complexity.
- An inference framework that is robust to existence of higher moments on the return distribution ("iceberg risk" as defined by Osband, 2002).
- Finally, a test that limits the vagaries of simulation based inference due to issues with unspecified dependence structure and block length selection.

Smooth Moment Test and Issues

- GMM based method has been used to address most of these concerns except that it still suffers from the estimation of the variance covariance matrix (Lo, 2002, Getmansky, Lo and Makarov, 2003).
- Ledoit and Wolf's (2008) first procedure uses asymptotic inference with a HAC type robust covariance estimator (Andrews, 1991, Andrews and Manohan, 1992). Their second procedure address finite sample issues using a simulation based "studentized time series bootstrap."
- Our proposed smooth test framework addresses at least three of these concerns and partially address the fourth one.
- One main advantage of the procedure is the orthogonality of moment and dependence directions and the score test framework reduces the estimation complexity of the covariance matrix under the null.

Framework for the Smooth test for Dependent data

Let $(X_1, X_2, ..., X_n)$ has a joint probability density function (PDF) $g(x_1, x_2, ..., x_n)$. Define $\tilde{X}_1 = \{X_1\}$, $\tilde{X}_2 = \{X_2 | X_1 = x_1\}$, $\tilde{X}_3 = \{X_3 | X_2 = x_2, X_1 = x_1\}$, ..., $\tilde{X}_n = \{X_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2} \ \ ..., X_1 = x_1\}$. Then we have

$$g(x_1, x_2, ..., x_n) = f_{X_1}(x_1) f_{X_2|X_1}(x_2|x_1) \\ ... f_{X_n|X_{n-1}X_{n-2}...X_1}(x_n|x_{n-1}, x_{n-2}, ..., x_1).$$

The above result can immediately be seen using the Change of Variable theorem that gives

$$\begin{aligned} P(Y_i \le y_i, i = 1, 2, ..., n) &= \int_0^{y_1} \int_0^{y_2} ... \int_0^{y_n} f(x_1)\, dx_1 \\ &\quad ... f(x_n | x_1, ..., x_{n-1})\, dx_n \\ &= \int_0^{y_1} \int_0^{y_2} ... \int_0^{y_n} dt_1\, dt_2 ... dt_n \\ &= y_1 y_2 ... y_n. \end{aligned}$$

Theorem (Rosenblatt '52)

Let $(X_1, X_2, ..., X_n)$ be a random vector with absolutely continuous density function $f(x_1, x_2, ..., x_n)$. Then, if $F_i(.)$ denotes the distribution function of the i^{th} variable X_i, the n random variables defined by

$$Y_1 = F_1(X_1), Y_2 = F_2(X_2 | X_1 = x_1),$$
$$..., Y_n = F_n(X_n | X_1 = x_1, X_2 = x_2, ..., X_{n-1} = x_{n-1})$$

are IID $U(0,1)$.

Define $(Y_1, Y_2, ..., Y_n)$ as conditional CDF of $(X_1, X_2, ..., X_n)$, then the probability integral transforms (PIT) evaluated at $(x_1, x_2, ..., x_n)$,

$$Y_1 = F_{X_1}(x_1), ..., Y_n = F_{X_n | X_{n-1} X_{n-2} ... X_1}(x_n | x_{n-1}, x_{n-2}, ..., x_1)$$

are distributed as *IID* $U(0,1)$. Under null hypothesis H_0, $(Y_1, Y_2, ..., Y_n) = (U_1, U_2, ..., U_n)$ where $U_t \sim U(0,1)$, $t = 1, 2, ...n$, joint PDF is

$$h(y_1, y_2, ..., y_n | H_0) = h_1(y_1) ... h_n(y_n | y_{n-1}, y_{n-2}, ..., y_1) = 1$$

Smooth Test for Dependent Data

Under the alternative H_1, $Y_i^{\prime}s$ are neither uniformly distributed nor are they *IID*. Suppose the conditional density function of Y_t depends on p lag terms $(k \geq q)$,

$$h\left(y_t \middle| y_{t-1}, y_{t-2}, \ldots, y_1\right) = h\left(y_t \middle| y_{t-1}, y_{t-2}, \ldots, y_{t-p}\right)$$

$$= c\left(\theta, \phi\right) \exp\left[\begin{array}{c} \sum_{j=1}^{k} \theta_j \pi_j\left(y_t\right) \\ + \sum_{l=1}^{q} \phi_l \delta_l\left(y_t, y_{t-1}, \ldots, y_{t-p}\right) \end{array}\right]. \quad (6)$$

Feasible Smooth Test for Dependent Data

Theorem (Ghosh and Bera, '06)

If the conditional density function under the alternative hypothesis is given by equation (6) and $p = 1$, the augmented smooth test statistic is given by

$$\hat{\Psi}_k^2 = \begin{bmatrix} U'U + U'BEB'U - V'EB'U \\ -U'BEV + V'EV \end{bmatrix} = U'U + \left(V - B'U\right)' E \left(V - B'U\right)$$

has a central χ^2 distribution with $k + q$ degrees of freedom where U is a $k-$vector of components $u_j = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} \pi_j \left(y_t\right), j = 1, ..., k$, V is a $q-$vector of components $v_l = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} \delta_l \left(y_t, y_{t-1}\right), l = 1, ..., q$, $B = E\left[\pi\delta\right]$, $D = E[\delta\delta]$ are components of the information matrix and $E = \left(D - B'B\right)^{-1}$.

Example 1: AR dependence

As an illustration of Theorem 2, let us now consider a very simple example of the smooth test for autocorrelation for

$$y_t - \mu = \rho \left(y_{t-1} - \mu\right) + \sigma_t \varepsilon_t \tag{7}$$

where $E\left(\varepsilon_t\right) = 0$, $V\left(\varepsilon_t\right) = 1$, $\sigma_t = \sigma$ and $a_1 = \frac{1}{\sqrt{12}}$. We define, if $m_1 = E\left(y_{t-1}\right)$,

$$\begin{aligned}\delta_1\left(y_t, y_{t-1}\right) &= \left(y_t - 0.5\right)\left(y_{t-1} - m_1\right) = \frac{1}{\sqrt{12}}\pi_1\left(y_t\right)\left(y_{t-1} - m_1\right) \quad (8)\\ &= a_1 \pi_1\left(y_t\right)\left(y_{t-1} - m_1\right).\end{aligned}$$

Then, we can denote
$v_1 = \frac{1}{\sqrt{n}}\sum_{t=1}^{n}\delta_1\left(y_t, y_{t-1}\right) = \frac{1}{\sqrt{n}}\sum_{t=1}^{n}\left(y_t - 0.5\right)\left(y_{t-1} - m_1\right)$.
Given information set $\Omega_t = \{y_{t-1}, y_{t-2}, \ldots\}$, defining
$\sigma^2 = E\left(y_{t-1} - m_1\right)^2$,

$$E\left[E\left[\int_0^1 \left(\left(y_t - 0.5\right)\left(y_{t-1} - m_1\right)\right)^2 dy_t \middle| \Omega_t\right]\right] = a_1^2 E\left[y_{t-1} - m_1\right]^2 = a_1^2\sigma^2.$$

Example 1: AR dependence (continued)

Hence, it follows that

$$\begin{aligned} E\left[\pi\delta\right] &= \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \end{pmatrix}' = B \\ E\left[\delta\delta\right] &= a_1^2 E\left[y_{t-1} - m_1\right]^2 = D, \end{aligned} \qquad (9)$$

which in turn gives the information matrix

$$\mathcal{I} = n \begin{bmatrix} 1 & 0'_{k-1} & 0 \\ 0_{k-1} & I_{k-1} & 0_{k-1} \\ 0 & 0'_{k-1} & a_1^2\sigma^2 \end{bmatrix} \qquad (10)$$

where I_p is the identity matrix of order p and 0_p is a p^{th} order vector of $0's$.

Example 1: AR dependence (continued)

In order to evaluate the inverse of the information matrix in (10) we use the following results:

$$\begin{aligned} D - B'B &= a_1^2 \left[E\left(y_{t-1}^2\right) - \left(E\left(y_{t-1}\right)\right)^2 \right] = a_1^2 \sigma^2, \\ U'BEB'U &= a_1^2 u_1^2 \mu^2 / \left(a_1^2 \sigma^2\right) = 0, \;\; V'EB'U = v_1 u_1 \mu / \left(a_1^2 \sigma^2\right) = 0, \\ V'EV &= v_1^2 / \left(a_1^2 \sigma^2\right). \end{aligned} \tag{11}$$

Hence, we have a correction term as an LM test for autocorrelation (Breusch, 1978)

$$\Psi_{k+1}^2 = \sum_{j=1}^{k} u_i^2 + \frac{1}{(a_1^2 \sigma^2)} \left[v_1^2 \right] = \sum_{j=1}^{k} u_i^2 + \frac{12\,(v_1)^2}{\sigma^2} \overset{a}{\sim} \chi_{k+1}^2 \Longrightarrow$$

$$12 \left(\frac{\sqrt{\frac{1}{n}} \sum_{t=2}^{n} (y_t - 0.5)(y_{t-1} - m_1)}{\sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (y_t - \overline{y})^2}} \right)^2 \overset{a}{\sim} \chi_1^2. \tag{12}$$

Example 2: ARCH(1)

ARCH (1) type alternative with volatility

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 \varepsilon_{t-1}^2 \tag{13}$$

For testing ARCH(1) dependence, define

$$\delta_2 \left(y_t, y_{t-1}\right) = \left(y_{t-1}^2 - m_2\right)\left(y_t^2 - \frac{1}{3}\right) = \left(y_{t-1}^2 - m_2\right)\left(a_1 \pi_1 \left(y_t\right) + a_2 \pi_2 \left(y_t\right.\right. \tag{14}$$

where $a_1 = \frac{1}{\sqrt{12}}$, $a_2 = \frac{1}{6\sqrt{5}}$, $a_3^2 = a_1^2 + a_2^2 = \frac{4}{45}$ and $m_j = E\left(y_{t-1}^j\right)$ for notational convenience.

The joint smooth test statistic incorporating an ARCH(1) type effect where $v_l = \frac{1}{\sqrt{n}} \sum_{t=1}^n \delta_l \left(y_t, y_{t-1}\right)$,

$$\hat{\Psi}_{k+1}^2 = \sum_{j=1}^k u_j^2 + \left(a_3^2 E\left(y_{t-1}^2 - m_2\right)^2\right)^{-1} \left[v_2\right]^2 \sim \chi_{k+1}^2 (0). \tag{15}$$

Smooth Total Moment Risk

- Further j^{th} order normalized Legendre polynomials are $\pi_0(y) = 1$, $\pi_1(y) = \sqrt{12}\left(y - \frac{1}{2}\right)$, $\pi_2(y) = \sqrt{5}\left(6\left(y - \frac{1}{2}\right)^2 - \frac{1}{2}\right)$, $\pi_3(y) = \sqrt{7}\left(20\left(y - \frac{1}{2}\right)^3 - 3\left(y - \frac{1}{2}\right)\right)$..
- The moments we are testing are in orthogonal directions of the normalized Legendre polynomials of the probability integral transform.
- Hence, we can define the smooth test statistic $\hat{\Psi}^2_{F,k}$ for each value of $k = 1, 2, 3, 4$ that provides the aggregated level of risk from each moment of the distribution upto that k as the *Smooth Total Moment Risk* ($STMR_F^{(k)}$) measure with respect to the benchmark distribution $F_t(.)$,

$$STMR_F^{(k)} = \hat{\Psi}^2_{F,k} = \sum_{j=1}^{k} u^2_{F,j} \sim \chi^2_k, \text{ where } u_{F,j} = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} \pi_j(y_t).$$

Smooth (Component) moment risk

- In particular, as we are interested in the amount of risk associated with i^{th} moment in the presence of higher order moments upto k, we define a new measure the i^{th} order *Smooth Moment Risk* $\left(SMR_{F,i}^{(k)}\right)$ with respect to $F\left(.\right)$ as

$$SMR_{F,i}^{(k)} = \frac{u_{F,i}^{2}}{\sum_{\substack{j=1\\ j\neq i}}^{k} u_{F,j}^{2} / \left(k-1\right)} \sim F_{1,k-1}$$

  has a central F distribution with 1 degree of freedom in numerator and $k-1$ degree of freedom in denominator asymptotically.
- For $k = 2$, this can give the overall risk associated with the first moment direction. For higher values of k, we can identify the levels of return risk from higher order moments.
- The main advantage of these smooth moment risk measures are they are themselves test statistic with tabulated asymptotic distributions.

Dependence Smooth Moment Risk

- These can be generalized to include different flexible dependent structures like AR(1) or ARMA(1,1) as discussed before, to get the *Dependence Smooth Total Moment Risk* ($DSTMR_F^{(k+q)}$) with benchmark distribution $F_t\left(.\right)$

$$DSTMR_F^{(k)} = \hat{\Upsilon}^2_{F,k} = U'U + \left(V - B'U\right)' E \left(V - B'U\right) \overset{a}{\sim} \chi^2_{k+q},$$

where U, V, B and E are as defined in Theorem 2 and proof.

- Similarly, the different dependence functions can be tested with the i^{th} *Dependence Smooth Moment Risk* ($DSMR_F^{(k+q)}$) (like the *Autocorrelation Smooth Moment Risk*, *Leverage Smooth Moment Risk*, *ARCH smooth moment risk* etc.) as

$$DSMR_{F,i}^{(k)} = \frac{Correction_{F,i}}{DSTMR_F^{(k)}/k} \overset{a}{\sim} F_{1,k},$$

where $Correction_{F,i}$ itself has a χ^2_1 distribution asymptotically.

Are Market Neutral fund truly Moment Neutral?

- We address the issue of distributional test of neutrality of equity hedge funds indices using a equity market neutral and other index fund provided in Diez de Los Rios and Garcia (2009).
- In particular, we want to compare the equity neutral fund index (C4 in their Table 1) with the global index they created.
- The data provided is monthly between Jan 1996 till March 2004 (99 observations).
- We would compare some standard risk measures and our smooth moment risk measures across the board. We wish to address the issue raised in Patton (2008) about whether Equity Neutral Funds are truly neutral with this index returns.

Smooth Moment Risk for Market Neutral Hedge Fund

F(.)	$STMR^{(4)}$	$\hat{u}_1^2$	$\hat{u}_2^2$	$\hat{u}_3^2$	$\hat{u}_4^2$	$SMR_2^{(4)}$	$DSTMR^{(6)}$
EDF	201.9^* (0.00)	4.8^+ (0.03)	104.6^* (0.00)	22.8^* (0.00)	69.6^* (0.00)	3.2 (0.17)	201.9^* (0.00)
ARMA(1,1)	194.1^* (0.00)	0.4 (0.52)	109^* (0.00)	2.1 (0.15)	82.6^* (0.00)	3.8 (0.14)	194.9^* (0.00)
MA(1)-t GARCH (1,1)	183.6^* (0.00)	0.4 (0.55)	105.9^* (0.00)	1.9 (0.17)	75.5^* (0.00)	4.1 (0.13)	183.8^* (0.00)
MA(1)-t-GJR-GARCH (1,1)	172.5^* (0.00)	0.5 (0.09)	102.2^* (0.00)	2.5 (0.12)	67.4^* (0.00)	4.4 (0.13)	172.6^* (0.00)

* *significant at* 1% *level*. $^+$ *significant at* 5% *level*.

Table 1.Smooth Moment Risk and components (p-values are in parenthesis).

Issues on Market and Moment Neutrality

- We estimate distribution of the global hedge fund database using the smooth test technique starting with the naive model with the empirical distribution function (EDF), then gradually increase the level of complexity (reported in Table 1).
- We observe that there is substantial difference of all the moments in particular, the second, third and fourth moments from the market index fund (here we are using the Value Weighted S&P 500 returns from WRDS database).
- We further update the model using an ARMA specification, but it gives the same qualitative results, although now only the second and fourth moment are significant $\left(u_1^2 = 0.42 \text{ or } u_2^2 = 109\right)$.
- We introduce conditional heteroscedasticity along with MA(1) term and *leverage effect* using GJR-GARCH model. The overall smooth total moment risk (STMR) declines slightly with higher level of complexity in the model, and is statistically distinguishable from the equity market index.

Findings on a test of market neutrality

- This implies that there is significant influence of higher order moment directions like skewness and kurtosis that affects the returns dispersion.
- If however we use only moment directions there will be overwhelming evidence that the second moment direction is strongly significant in determining Equity Neutral Hedge Fund index returns.
- So based on this evidence we cannot support the claim that Equity Neutral hedge Fund index seems to be fairly independent of the market risks both in returns and in volatility.
- We also calculate the augmented smooth test jointly for autoregressive and ARCH type errors that gives the dependent smooth total moment risk ($DSTMR^{(6)}$), which shows a very similar pattern as the $STMR^{(4)}$ and hence dependence across the moments does not seem to have an affect either.

Comparision of Moment Risk of Hedge Fund Indices

	STMR	$\hat{u}_1^2$	SMR$_1$	$\hat{u}_2^2$	SMR$_2$	AR	DSMR	Beta	Alpha
Under $H_0 \sim$	χ^2_4	χ^2_1	$F_{1,3}$	χ^2_1	$F_{1,3}$	χ^2_1	$F_{1,4}$	t_{97}	t_{97}
C1 Cnvrt. Arb	156.1^{+}	0.1	0.0	93.9^{+}	4.5	0.3	0.0	0.3	4.6
C2 Fxd.Inc.Arb.	176.4^{+}	0.5	0.0	98.8^{+}	3.8	0.9	0.0	0.1	2.7
C3 Evnt Driven	115.5^{+}	0.0	0.0	77.5^{+}	6.1	0.1	0.0	0.5	2.4
C4 Eqt. Neutral	183.6^{+}	0.4	0.0	105.9^{+}	4.1	0.1	0.0	0.1	3.3
C5 Lng-Shrt Eqt.	28.6^{+}	0.2	0.0	27.4^{+}	68.5^{+}	1.1	0.2	1.0	1.7
C6 Global Macro	83.8^{+}	0.8	0.0	59.7^{+}	7.5	0.7	0.0	0.1	2.1
C7 Emrgng Mkts.	5.04	0.9	0.7	1.4	1.2	3.1	2.4	1.7	-3.4
C8 Ded Shrt Bias	11.8*	3.5	1.3	5.8*	2.9	0.1	0.1	-1.8	7.8
C9 Mngd Fut.	32.4^{+}	1.0	0.1	28.6^{+}	22.6*	1.0	0.1	-0.1	2.8
C10 Fnd of Fnd	94.3^{+}	0.6	0.0	67.2^{+}	7.4	0.0	0.0	0.5	-0.6
Global Index	74.1^{+}	0.1	0.1	58.4^{+}	11.1*	0.0	0.0	0.7	0.3

Comparison of Risk of Hedge Fund Indices: Equity Neutral

- We use the MA(1)-t-GARCH(1,1) as a benchmark distribution of the market index (Value weighted returns), and evaluate all the 10 hedge fund indices. We evaluate how the market index affects Hedge Funds in our sample, in particular with respect to the Equity Market Neutral Index (Patton, 2008).
- We find Equity Market Neutral Funds to be quite strong in significance in smooth moment risk coming from all moment directions ($STMR = 183$).
- This does confirm the doubt about overall market neutrality of such funds (Patton 2009). If we look closely enough, none of the significance is coming in the direction of the return level $\left(\hat{u}_1^2\right)$ but mostly, from the second moment dispersion $\left(\hat{u}_2^2\right)$ except emerging market funds.
- We observe strong overall statistically significant difference or significant $STMR^{(k)}$ almost all hedge funds indices except for Emerging Markets and marginally for Dedicated Short funds.

Comparison of Risk of Hedge Fund Indices: Other Styles

- Convertible Arbitrage and Fixed income arbitrage from the index, particularly in the direction of the second moment.
- This does assure us that hedge funds indeed does "hedge" or change the variability of the return distribution compared to an equity fund.
- There is however a very strong influence on higher moment directions that causes the F-statistics in the form of both first and second Smooth moment risk (SMR) measures.
- They show that comparatively there is insignificant effect in the direction of the first risk moment (SMR_1) for all funds. Further, only Long-Short Equity that thrives on volatility, and Managed Future funds have a higher contribution of volatility compared to other moments (SMR_2).
- We also looked at the level of dependence in terms of autoregressive smooth moment risk ($DSMR^{(4)}$) and found no residual dependence in that direction.

Comparision across Risk Measures

- We also report the Sharpe-Lintner CAPM based measures like the Beta (β) and Jensen's alpha (α). As expected the Market Neutral Hedge Fund does show close to "Beta neutrality," as it is close to zero, but so is Global Macro and Fixed Income Arbitrage.
- The highest beta is for the Emerging Market fund that is really an international mutual fund, and the lowest one is on Dedicated Short Bias that thrives on betting against the market.
- From the smooth total moment risk standpoint (STMR), *Beta* does not replicate the same ordering. This is expected as beta is based on inherent normality assumption of CAPM that assumes away dispersion risk in higher order moments.

Concordance of RAPMs

- In fact, systematic risk from beta can be take to be the risk associated with market, hence those funds which play the market like emerging market and dedicated short are most sensitive, while equity neutral strategy is not.
- Higher Jensen's alpha also do not price higher order moments hence are not dependent on STMR.
- Using Spearman's rank correlation and Pearson's product moment correlation (not reported here) we see that STMR is negatively correlated with Beta, moderately correlated with alpha and quite strongly correlated with the Sharpe Ratio.

Risk Comparision of Global HF and Style Indices

	STMR	DSTMR	$\hat{u}_1^2$	SMR_1	$\hat{u}_2^2$	SMR_2	AR	DSMR	SR
Under $H_0 \sim$	χ_4^2	χ_6^2	χ_1^2	$F_{1,3}$	χ_1^2	$F_{1,3}$	χ_1^2	$F_{1,4}$	$T^{-0.5} t_T$
C1 Cnvrt. Arb	13.6^*	17.6^*	1.1	0.3	10.4^*	9.7	3.8	1.1	1.6
C2 Fxd.Inc.Arb.	33.3^*	34.8^*	0.4	0.0	12.0^*	1.7	0.6	0.1	0.7
C3 Evnt Driven	4.6	8.0	0.1	0.1	2.7	4.2	0.7	0.6	1.0
C4 Eqt. Neutral	6.3	7.2	5.1^+	12.4^+	0.3	0.1	0.0	0.0	1.3
C5 Lng-Shrt Eqt.	4.7	6.1	1.1	0.9	1.1	0.9	0.4	0.3	0.8
C6 Global Macro	5.9	8.3	2.3	2.0	3.4	4.2	1.1	0.7	0.4
C7 Emrgng Mkts.	5.6	6.9	2.2	2.0	0.2	0.1	0.6	0.4	0.5
C8 Ded Shrt Bias	8.8	12.9^*	2.2	1.0	4.2^+	2.8	2.5	1.1	-0.2
C9 Mngd Fut.	8.0	17.6^*	0.8	0.3	5.6^*	7.2	8.2^*	4.1	0.2
C10 Fnd of Fnd	2.8	4.6	1.0	1.7	0.0	0.0	0.1	0.1	0.5
Global Index									0.7

Table 3: HF Index with Global Index with R_f=3.75% ($^+$ *sig.at* 5%, * *sig.at* 1%)

Risk in Hedge Fund Styles Compared with Global HF Index-I

- We explore the relationship with the Global Hedge Fund Index (Diez de los Rios and Garcia, 2009).
- Table 3 also provides the Sharpe ratio for all the hedge fund indices (using $R_f = 3.775$, given in Table 1 of Diez de los Rios and Garcia 2009).
- Both the Arbitrage Funds (C1 and C2) shows a substantial risk exposure measured by STMR compared to the Global Hedge Fund Index.
- Both these are in the specific direction of volatility as shown in $\hat{u}_2^2$, however due to the presence of significant higher order moments their contribution measured by Smooth Moment Risk of first and second order are not statistically significant at 5%. This implies that the arbitrage funds probably strategize on opportunities that are possibly asymmetric, and in the tails of the return distribution.

Risk in Hedge Fund Styles Compared with Global HF Index-II

- Further, we find that event driven fund, long short equity and global macro shows very little dispersion in moment risk from the global index as they form a majority of the funds out there at that period.
- However, equity market neutral funds have a strong deviation in the direction of the first moment though overall it is similar to the global index. Short bias and Managed Futures funds shows affects of overall dependence and variation in volatility risk from global hedge fund.
- Fund of Funds is very similar and indistinguishable from the Global index. Sharpe ratio gives an indication of the level of risk assuming underlying normality.
- Hence funds that have higher order moment exposure like Arbitrage funds and dependence like Managed Futures and volatility dynamics like dedicated short are not adequately treated by the Sharpe Ratio.

Conclusions...

- Financial risk evaluation had attracted substantial attention of late both in the academic community and outside with the growing financial crisis that might have had its genesis in faulty methodology.
- Rampant use measures like Value-at-Risk as expressed by practitioners, consultants and field experts described in the New York Times as *Risk Mismanagement*, fails to prevent the effect of "*black swans*" or very rare events like market crashes or meltdown. There was a need for formal instruments that have well specified distributions.
- Our objective in this paper is to look at the instruments of risk assessment like the Sharpe Ratio that are commonly used and make it more robust in cases of extreme uncertainty or misinformation that leads to noisy data (see Garcia, Renault and Tsafack, 2005). One way of achieving that would be to account for the higher order moments of adjusted return distributions.

...and Future Directions

- Evaluate the effectiveness of the forecast models for risk management using out-of sample performance (see, Santos, 2008). Out-of-sample forecast evaluation risk adjusted return distributions using "in-sample" bootstrap confidence intervals might not be optimal.
- Further more, the commonly used risk measures like the Sharpe ratio or Value-at-Risk might not be "a coherent measure of risk" (Artzner et. al, 1999, Garcia, Renault and Tsafack, 2005). Distributional tests of Sharpe Ratio is still in its infancy particularly accommodating for higher order moments and dependence.
- Explore selection biases like survivorship and other non-linearities particularly for Private Equity and Hedge Fund data (Agarwal, and Naik, 2004,. Diez de los Rios and Garcia, 2005).
- Explore finite sample properties of the proposed test procedure in the presence of survivorship and other selection biases (Cakici and Chatterjee, 2008, Carlson and Steinman, 2008).