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Rank Chasing: Do fractiles of returns affect mutual fund flow?

Abstract

Tax liability influences investors' decision of investment choice. It is well-established that return chasing behavior among investors introduce a non-linearity in the performance-flow relationship. This non-linearity could be addressed in a robust way if we look at the ranks rather than merely risk-adjusted returns as measures of performance. We explore the distributional effects of the empirical distribution function (EDF) of risk-adjusted returns on mutual fund flow using fractile (or rank) regression methods. We use smooth tests for both the unadjusted inflow distribution as well as after conditioning for risk adjusted returns to illustrate how higher order moments are changed as a result of investors' tax exposure and tax regulations.

JEL Classification: G11, G23, C14

Keywords: mutual fund inflow, fractile graphical analysis, non-parametric regression, quantile regression, rank based test, after-tax returns

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1 Background and Motivation

It is well documented that return chasing behavior among risk-averse investors might introduce a non-linearity (convexity) in the performance-flow relationship (see Chevalier and Ellison, 1997; Sirri and Tufano, 1998). In the absence of a more tangible measure of genuine stock-picking ability, active mutual fund managers' performance has often been evaluated by risk adjusted excess returns over passive benchmarks (see Ippolito, 1992). In the short run there is some evidence that return based performance of skilled mutual fund managers do persist, however these results could be artifacts of similar (for example, style based or momentum) trading strategies employed in stock picking (Gruber 1996, Zheng 1999). More recent literature have suggested in line with standard models in corporate finance if there is a perfectly inelastic supply of capital to manage in a company a high ability manager will allocate it to different profitable investment opportunities. So it is rational to think about mutual fund inflow as a measure of performance of the fund manager. It is also rational to assume that rents that are being extracted due to higher stock-picking ability is not necessarily being passed onto the investor but kept as compensation by the fund managers. This implies among other things a higher ability fund manager might be able to extract such rents initially, however, as the fund grows a decreasing returns to scale sets in, and any such excess return evaporates rapidly. In a competitive market of mutual funds for individual uninformed investors this implies that the marginal investor has close to zero return to investment in an efficient capital market (Berk and Green, 2004). While this suggests a possible path to reconcile the empirical anomaly in the the finance literature "...flows are responsive to performance and performance that is not persistent." (Berk and Green, 2004 p 1272) with rationality of individual uninformed investors, it however throws little light on the nature of the non-linearity of the flow-performance relationship. It has also been suggested in the literature that higher cash flow volatility and fund performance might have a negative relationship (see Edelen, 1999, Rakowski, 2002). These findings among also corroborates a behavioral hypotheses that mutual inflow distribution convey sentiments about stocks (Frazenni and Lamont, 2005). However, this monotonic non-linearity in flow-performance relationship could be handled in a more parsimonious, and statistically robust method if we look at the ranks rather than merely risk-adjusted returns as measures of performance.

Tax liability on both mutual fund income distribution as well as on unrealized (and realized) capital gains exposure influence individual investor's decision on taxable investment accounts. Averages of mutual fund flow distributions are different before and after taxes with past year returns as covariate.(Bergstresser and Poterba, 2002). We want to investigate how these distributions are different when we control for the fractiles or normalized ranks of returns. Bergstresser and Poterba (2002) also documents that mutual funds with heavily taxed returns have lower subsequent inflows compared to ones with lower tax burdens. Our objective is to see if there is evidence in the inflow distributions to show whether higher moments including volatility or other shape parameters of the inflow distributions are affected by tax exposure.

We use *Fractile Regression* techniques (see Bera and Ghosh, 2005 and references therein) where we condition on the ranks of the covariate to explain changes in mutual fund flow. Thereafter, we use smooth tests for both the unadjusted inflow distribution as well as distribution after conditioning for risk adjusted returns to illustrate how higher order moments are changed due to investors' tax exposure. In Section 2 we discuss the model we use for the paper. In subsections 2.1 and 2.2 we discuss some relevant non-parametric and semiparametric regression techniques, and the smooth test of comparing distributions, respectively. In Section 3, we evaluate the components of mutual fund inflow distributions for mutual funds with high and low tax exposure. In Section 4, we conclude. In Appendix

A, we briefly discuss the sample size selection method we have adopted using Bera, Ghosh and Xiao (2005).

2 The Model and the Methodology

Our objective in this paper is to look at the age-old problem of the effect of covariates on distributions. Linear regression has always been the cornerstone of such an analysis where we investigate at the effects of the x-variables or covariates on the response variable y. Consider the Sharpe-Lintner CAPM model (following Ippolito, 1992)

$$R_t - R_{Ft} = \alpha + \beta\left(R_{Mt} - R_{Ft}\right) + \varepsilon_t \tag{1}$$

where R_t is the fund's rate of return in year t, R_{Ft} is a risk-free return in year t, R_{Mt} is the return on some broad market portfolio like S&P 500 Composite index in year t, and ε_t is an error term with mean zero. We can define the abnormal returns adjusted for market risk in the fund in some year t as

$$P_t = \left(R_t - R_{Ft}\right) - \beta\left(R_{Mt} - R_{Ft}\right) = \alpha + \varepsilon_t \tag{2}$$

as a measure of fund manager's performance. It has been pointed out that such a measure of performance will not only have a substantial noise component but will have significant serial correlation or persistence owing to similar ability (or lack thereof) of fund managers (Ippolito, 1992). For systematic market risk $\beta = 1$, the risk adjusted excess return measure becomes (Chevalier and Ellison, 1997)

$$P_t = R_t - R_{Mt} = \alpha + \varepsilon_t. \tag{3}$$

For P_t to be valid measure of performance we need to assume that α is fixed conditional on some manager or fund specific covariates, with the error term ε_t (referred to as "measurement error" in Econometrics). In particular, P_t is stochastic with an unknown distribution, related to that of ε_t.

Bergstressor and Poterba (2002) claims new money inflow is a more interesting measure of net inflow into a mutual fund that doesn't contain the reinvested dividend and capital gains 90% of which get reinvested, we define mutual fund inflow as

$$I_t = \frac{Assets_t}{Assets_{t-1}} - \frac{NAV_t}{NAV_{t-1}} \tag{4}$$

that measures the net growth rate of the assets of the fund over the growth rate of the intrinsic value of the stock given by the year end net asset value (NAV).

One possible model to estimate the relationship between flow and abnormal performance would be a partially linear model (similar to Chevalier and Ellison, 1997)

$$I_{it} = \gamma_0 + \gamma_1 g\left(P_{it-1}\right) + \mathbf{X}_{it}'\boldsymbol{\beta} + Y_t'\delta + \nu_{it} \tag{5}$$

where I_{it} is the net new money inflow for the i^{th} fund in the t^{th} year, $g\left(.\right)$ is a function on the perceived performance measure like risk adjusted excess or abnormal returns, $\mathbf{X}_{it}$ is a vector of covariates for the i^{th} fund included in the information set Ω_t at time t, Y_t is a vector of year specific variables, γ_0, γ_1, $\boldsymbol{\beta}$ and δ are coefficients, and finally, ν_{it} is an error term with mean (or median) zero. It must be noted that model in (5) is notoriously difficult to estimate due to two possible problems. First, the error term might be correlated with the independent variables - problem often times referred to as "endogeneity" in Econometrics; second, the estimation of the functional form $g\left(.\right)$ of the risk adjusted return would suffer from a "curse of dimensionality" even when it is identified to some scale.

One way of dealing with this model is to do a two step procedure. In the first step, we find the residual of Inflow that cannot be predicted by fund-year specific variables other than the current and past performance measures like risk adjusted returns. We can use either parametric (linear regression) or semiparametric (rank or quantile regression) to do the first step. At the second step, we can estimate the following model

$$AI_{it} = \gamma_0' + \gamma_1' g\left(P_{it-1}\right) + u_{it} \tag{6}$$

where AI_{it} is the inflow variable adjusted for the fund-year specific covariates, γ_0', γ_1' are slope and intercept coefficients, and u_{it} is an error term with mean (or median) zero. Although the model in (6) does not solve the endogeneity in the original model but does reduce the curse of dimensionality that is commonly afflicts non-parametric regression problems.

The problem we are trying to address is not directly related to endogeneity, but the other aspect of the story missed by simple linear regression. It is very likely that funds with higher risk-adjusted excess return might have a different rate of adjusted inflow as compared to the one with lower risk adjusted return. Linear regression fails to capture this "differential" treatment of the covariates or in particular "fractiles or ranks of the covariates." So instead of looking at regression of AI on P, we should be looking at the regression of AI, possibly grouped according to the ranks of P, on P. So, we can answer the question for the bottom 10% of mutual funds what is the effect of one percent increase in ranks of returns all else remaining the same. We discuss the basic nonparametric regression methods and smooth test techniques used in this paper in the following subsections.

2.1 Nonparametric Regression Method

In non-parametric (Kernel-based) regression analysis we consider $Y_i \sim N\left(m\left(x_i\right), \sigma^2\right)$, $i = 1, 2, ..., n$, where conditional mean function $m(.)$ satisfies some regularity or smoothness conditions. Broadly, we can define the Nadaraya-Watson type location or regression estimator with the smoothing kernel $K(.)$ and bandwidth h as

$$\hat{m}_{NW}\left(x_o\right) = \arg\min_{\beta_o \in \mathcal{R}} \sum_{i=1}^{n} \left(y_i - \beta_o\right)^2 K\left(\frac{x - x_i}{h}\right) = \sum_{i=1}^{n} W_{in}^{NW}(x)\, y_i \tag{7}$$

We can think of replacing x_i by a monotonic rank-score of x_i, and use the weighted least squares type method as well. "Bandwidth" can be defined either in terms of actual width (kernel type) or the number of observations (nearest neighbor type). In nearest neighbor type regression estimate we replace x by the empirical distribution function $F_n(x)$ in Equation (7) to get (Altman 1992, Stute 1984)

$$\hat{m}_{NN}\left(x_o\right) = \arg\min_{\beta_o \in \mathcal{R}} \sum_{i=1}^{n} \left(y_i - \beta_o\right)^2 K\left(\frac{F_n(x) - F_n\left(x_i\right)}{h_n}\right) = \sum_{i=1}^{n} W_{in}^{NN}(x)\, y_i. \tag{8}$$

The major advantage that k-nearest neighbor type estimator has over the traditional kernel based estimator is that the former only depends on the ranks of $X_1, X_2, ..., X_n$. Hence, if $F(x)$ is continuous the problem gets transformed to much more tractable problem of estimating a regression function at $F\left(x_0\right)$ with the X-sample being uniformly distributed over $[0, 1]$. Its convergence properties in mean square has also been studied by Yang (1981). Stute (1984) showed that k-Nearest Neighbor type estimates are asymptotically normal if $E\left[Y^2\right] < \infty$, is much weaker than the conditions needed for existence of the Nadaraya-Watson type regression estimates like existence of the PDF $f(.)$ of X and that $E\left|Y\right|^3 < \infty$ (Schuster 1972).

In quantile regression, we look at the regression counterpart of univariate τ^{th} quantile of the dependent variable y is defined as

$$\hat{\alpha}(\tau) = \arg\min_{a \in \mathcal{R}} \sum_{i=1}^{n} \rho_\tau (y_i - a), \tag{9}$$

where $\rho_t(u) = (\tau - I(u < 0))\,u$ is often referred to as the check function.

The τ^{th} Regression Quantile of Y on X (Koenker and Bassett, 1978)

$$\hat{\beta}(\tau) = \arg\min_{b \in \mathcal{R}^p} \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^T b\right). \tag{10}$$

It should be noted that quantile regression controls for the quantiles of the y variable, and not of the original covariate i.e. the x variable.

To motivate fractile regression let's think of a linear regression function of Y on $X = x$ as

$$m(x) = E[Y|X = x] \tag{11}$$

Let $F(x)$ is the marginal cumulative distribution function (CDF) of X with a density function (PDF) $f(x)$.

We can show that the regression function is invariant under a strictly monotonic transformation of the covariate X to its probability integral transform (PIT), $F(x)$,

$$\begin{aligned} r(u) &= E[Y|F(X) = u] \\ &\Rightarrow r(u) = E\left[Y|X = F^{-1}(u)\right] = m\left(F^{-1}(u)\right). \end{aligned} \tag{12}$$

The partial regression coefficients of $r(u)$ is given by

$$\frac{\partial r(u)}{\partial u} = \frac{\partial m}{\partial x} . \frac{\partial F^{-1}(u)}{\partial u} = m'(x)\frac{1}{f(x)}, \tag{13}$$

where we divide the non-parametric regression coefficients by the density function evaluated at x. One interpretation of that could be the regression coefficients are weighted less where the density of the covariate is low. So if we know the density function $f(x)$, we can recover the original partial regression coefficients $m'(x)$.

2.2 Smooth Test for Comparing Distributions

For performing this test of comparison of distributions of we use the two sample version of smooth test procedure as proposed in Bera, Ghosh and Xiao (2004). Neyman's smooth test for $\mathbf{H}_0$: $\mathbf{F} = \mathbf{F}_0$. was for the one sample case with completely specified distribution under null hypothesis $H_0 : f(x)$ is the true PDF (for review see Bera and Ghosh, 2001). This is equivalent to testing $H_0 : y = F(x) = \int_{-\infty}^{x} f(u)\,du \sim U(0,1)$ .Neyman considered the following smooth alternative to the uniform density:

$$h(y) = C(\theta)\exp\left[\sum_{j=1}^{k}\theta_j \pi_j(y)\right] \tag{14}$$

$\pi_j(.)$ are orthogonal normalized Legendre polynomials. For $H_0 : \theta_1 = \theta_2 = \cdots = \theta_k = 0$ has a test statistic

$$\Psi_k^2 = \sum_{j=1}^{k}\frac{1}{n}\left[\sum_{i=1}^{n}\pi_j(y_i)\right]^2 \sim \chi_k^2(0) \text{ under } H_0.$$

If we take the problem of testing $H_0 : F = G$. We need to modify the original smooth test since both F and G are unknown. If $F(.)$ were known, we can construct a new random variable $Z_j = F(Y_j)$, $j = 1, 2, ..., m$.

The CDF of Z is given by

$$\begin{aligned} H(z) &= \Pr(Z \leq z) = \Pr(F(Y) \leq z) \\ &= G\left(F^{-1}(z)\right) = G(Q(z)) \end{aligned}$$

where $Q(z) = F^{-1}(z)$ is the quantile function of Z.

The PDF of Z is given by

$$\begin{aligned} h(z) &= \frac{d}{dz} H(z) = g\left(F^{-1}(z)\right) \frac{d}{dz} F^{-1}(z) \\ &= g\left(F^{-1}(z)\right) \frac{1}{f\left(F^{-1}(z)\right)} \\ &= \frac{g(Q(z))}{f(Q(z))}, \quad 0 < z < 1. \end{aligned} \tag{15}$$

The main problem of comparing two distributions is to find a suitable measure of distance or norm between two distribution functions, i.e. to say, for any $x \in (-\infty, \infty)$,

$$\|G(x) - F(x)\|$$

If a density function exists over the support of F and G, then for any $t \in (0,1)$ this problem to be equivalent to the distance

$$\left|G \circ F^{-1}(t) - t\right|.$$

Under $H_0 : G = F$, $G \circ F^{-1}(t) = t$. In fact, the $h(z)$ in (15) is the corresponding PDF for the distribution function $G \circ F^{-1}$ defined over $(0,1)$. The PDF $h(z)$ is a ratio of two densities; and itself is a valid density function. Therefore, we will call it the *Ratio Density Function (RDF)* (Bera, Ghosh and Xiao, 2005).

When $H_0 : F = G$ is true (i.e. $f = g$) then from (15), $h(z) = \frac{g(Q(x))}{f(Q(x))} = 1, 0 < z < 1$. Z has the *Uniform* density in $(0, 1)$. That means irrespective of what F and G are, the two-sample testing problems can be converted into testing only *one kind of hypothesis*; namely, *uniformity* of a transformed random variable.

For the two sample case with unknown F and G the Smooth test statistic is

$$\begin{aligned}
\Psi_k^2 &= \sum_{l=1}^{k} u_l^2, \; u_l = \frac{1}{\sqrt{m}} \sum_{j=1}^{m} \pi_l(z_j), l = 1, 2, ..., k \\
z_j &= F(y_j) = \int_{-\infty}^{y_j} f(\omega) \, d\omega, \; j = 1, 2, ..., m.
\end{aligned}$$

Under $H_0 : F = G, \Psi_k^2 \xrightarrow{D} \chi_k^2$.

The test has k components. Each component provides information regarding specific departures from $H_0 : F = G$.

However, in practice $F(.)$ is unknown. We use the Empirical Distribution Function (EDF),

$$\begin{aligned}
F_n(x) &= \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq x), \qquad \hat{z}_j = F_n(y_j) \\
\hat{\Psi}_k^2 &= \sum_{l=1}^{k} \frac{1}{m} \left[\sum_{j=1}^{m} \pi_l(\hat{z}_j) \right]^2
\end{aligned}$$

The following two theorems [for proof and details see Bera, Ghosh and Xiao (2004)] provide some restrictions on relative sample sizes for consistent asymptotic χ^2 distribution of the test statistic, and also to minimize size distortion of the two sample smooth test of comparing two distributions.

Theorem 2 *If* $\frac{m \log \log n}{n} \to 0$ *as* $m, n \to \infty$ *then* $\hat{\Psi}_k^2 - \Psi_k^2 = o_p(1)$.

Proof. *See Bera, Ghosh, Xiao (2005)* ■

Theorem 3 *The optimal relative magnitude of m and n for minimum size distortion is given by* $m = O\left(\sqrt{n}\right).$

Proof. *See Bera, Ghosh, Xiao (2005)* ■

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3 Comparing Inflow Distributions

We consider US domestic equity mutual funds data for January Releases from Morningstar Principia database with some conditions (see Bergstresser and Poterba, 2002 for filters) from 1996-2006 where the data corresponds to December 31 the year before (Bergstresser and Poterba, 2002). The main reason for selecting such a dataset is to reduce problems with survivorship bias caused by merger or extinction of non-performing mutual funds. Bergstresser and Poterba (2002) find that after-tax returns do indeed have more influence on cash inflows to mutual funds, however they did not test whether higher order moments of the inflow distribution are affected by after-tax returns. Figures 1-4 depict that mutual fund inflow distributions over all the years of data 1995-2005 are indeed quite different in potential (using potential tax liability, distribution yield and potential capital gains exposure) and actual tax burden based on the difference between pre-tax and after tax returns. The striking common feature of all these kernel density estimates is that they show not just different means but also volatilities and possibly different higher order moments between the flows.

If we consider the inflow distributions with high (potential tax liability more than zero) and low tax liabilities (potential tax liability less than or equal to zero) where potential tax liability is calculated by adding dividend yield and potential (both realized and unre-

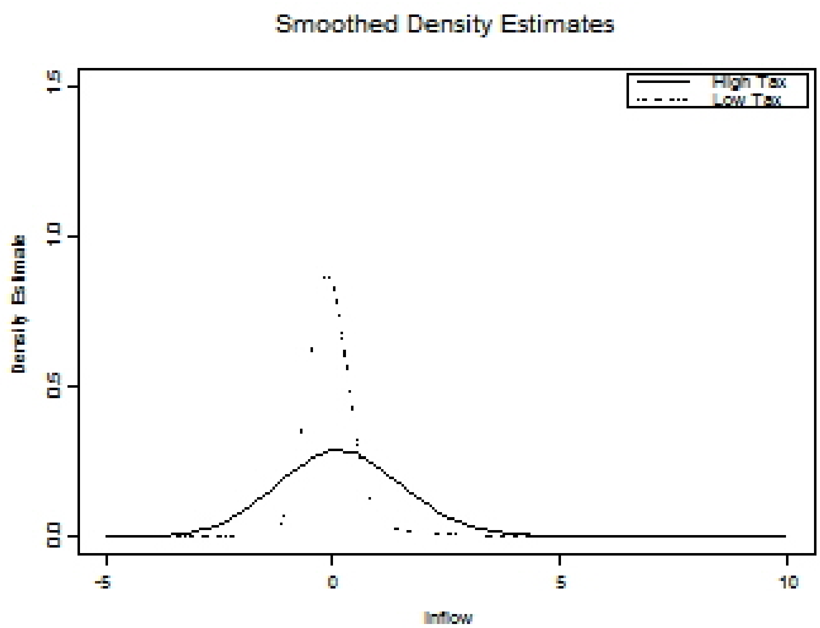


Figure 1: Inflow distributions with high and low potential tax liabilities.

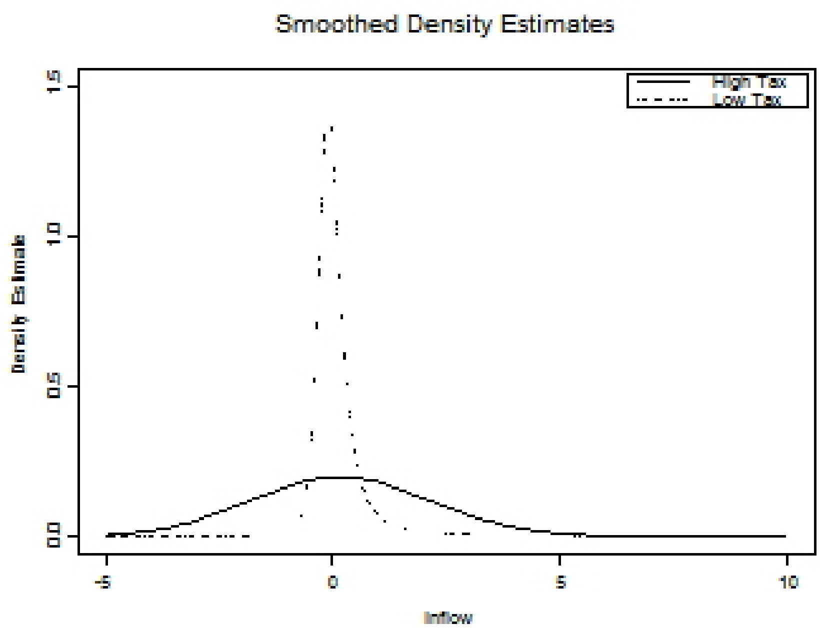


Figure 2: Inflow Distributions with high and low tax liabilities using Distribution Yield

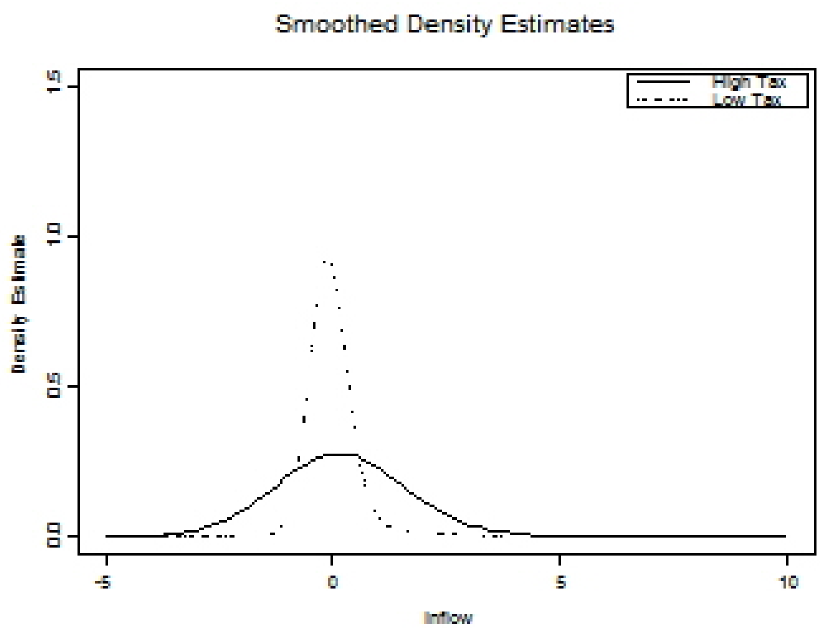


Figure 3: Inflow distribution with high and low Potential Capital Gains Exposure

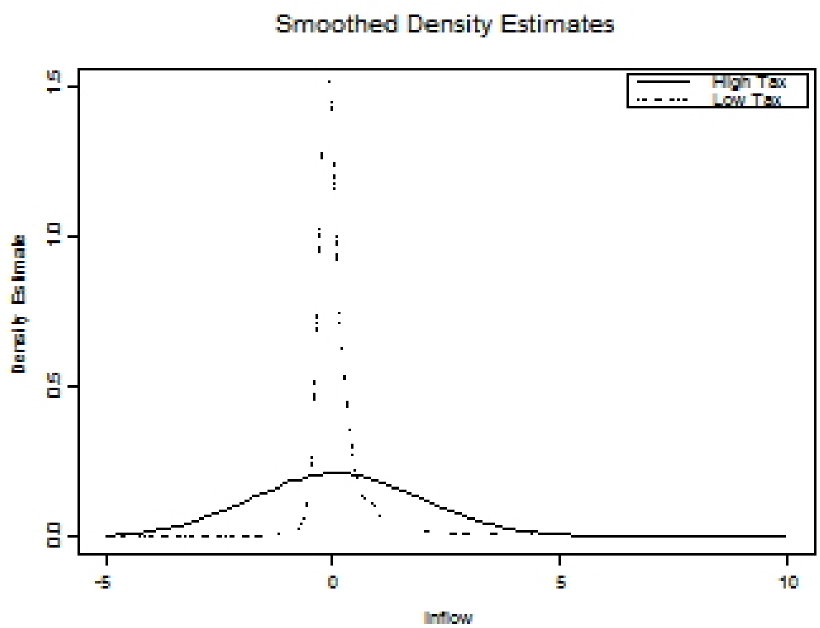


Figure 4: Inflow distributions with funds having high and low tax burdens

alized) capital gains exposure reported by Morningstar Corporation, we find that there are substantial differences in several moments of the distribution (see Table 1)

Variable	Obs.	Mean	Median	Std. Dev.	Min.	1^{st} Q.	3^{rd} Q.	Max.	coef. skew	excs.kurt
Low Tax	5106	0.3104	-0.0940	17.1107	-8.4933	-0.2116	0.0698	1216.114	70.2944	4989.486
High Tax	13624	1.0295	0.056	73.1684	-6.1652	-0.0926	0.3004	8510.708	115.5167	13426.642

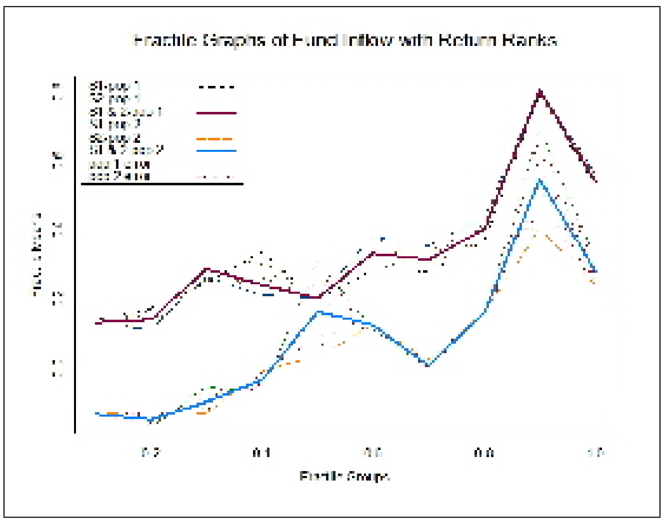
Table 1. Summary Statistic for Fund Inflows

Test	KS	CvM
Statistic (T*)	19.3117	171.9613
Critical Upper 0.1%	1.95	1.167

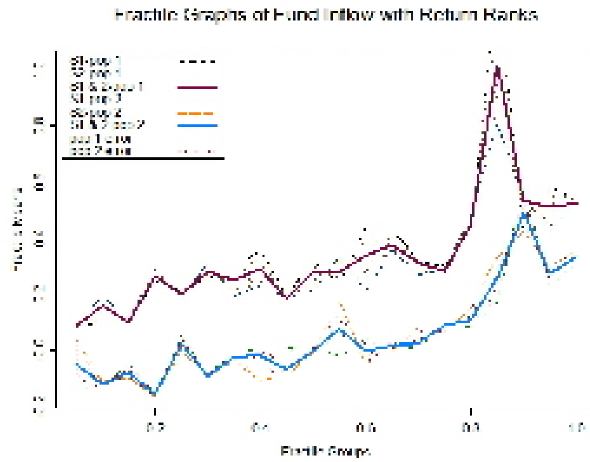
Table 2. Goodness-of-Fit Statistics based on the EDF.

We would to reject $H_0 : F = G$ that the inflow distributions for high and low tax exposure are the same with the above tests but there is no indication of the nature of departure from H_0 using the traditional tests like Kolmogorov-Smirnov or Cramer-von Mises type tests (see Table 2).

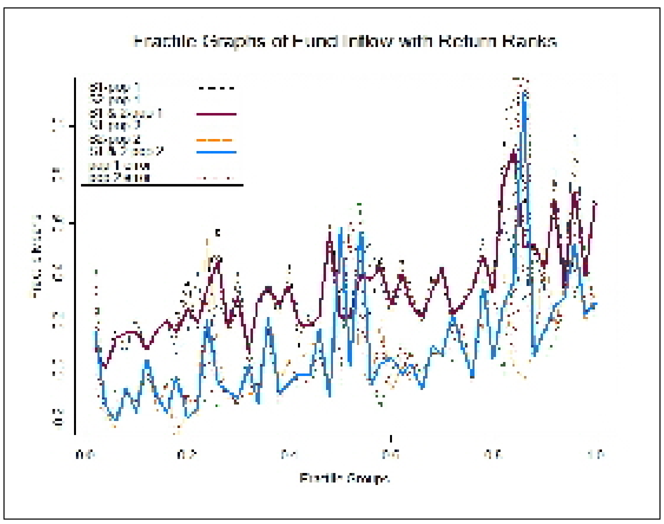
We use a modified version of fractile graphical analysis method (Mahalanobis, 1960, also see Bera and Ghosh, 2006) to test the overall distribution of inflow conditional on the ranks of the returns with high and low tax exposure. Figures (i),(ii) and (iii) representing the fractile graphs with number of fractile groups $g = 10, 20$ and 50 illustrates the difference between funds with high and low tax exposures. In the figures blue solid line (that is more or less above the red solid line) represents the mutual funds with higher tax exposure. The shaded area around the line represents the uncertainty that is the bootstrapped standard error at each fractile group mean. As we observe with higher number of fractile or rank groups of returns, the separation area between the two graphs is more fragmented. This also make it increasingly difficult to conclude whether the distributions are different overall. Hence we would need some more tangible analytical or simulation based hypothesis testing methodology to separate the two fractile graphs.



(i)10 Fractile Groups



(ii) 20 Fractile Groups



(iii)50 Fractile groups

Following the notation of Bera and Ghosh (2006), We divide the data into m groups of size g each i.e. $n = mg$. The group means of the variables ranked with respect to X are

$$u_i = \frac{1}{m} \sum_{r=(i-1)m+1}^{im} x_{(r)}, i = 1, 2, ..., g \tag{16}$$

$$v_i = \frac{1}{m} \sum_{r=(i-1)m+1}^{im} y_{[r]}, i = 1, 2, ..., g. \tag{17}$$

Samples $\left(x_1^1, y_1^1\right)$, $\left(x_2^1, y_2^1\right)$, ..., $\left(x_n^1, y_n^1\right)$ and $\left(x_1^2, y_1^2\right)$, $\left(x_2^2, y_2^2\right)$, ..., $\left(x_n^2, y_n^2\right)$, are independently drawn from population P^{12}.

Let G^1,G^2 and G^{12} be the plots of the g group means $\left(v_1^1, v_2^1, ..., v_g^1\right)$, $\left(v_1^2, v_2^2, ..., v_g^2\right)$ and $\left(v_1^{12}, v_2^{12}, ..., v_g^{12}\right)$ against the group ranks $1/g$ through 1. Also define, for population P^{34}, G^3,G^4 and G^{34} be the plots of the group means $\left(v_1^3, v_2^3, ..., v_g^3\right)$, $\left(v_1^4, v_2^4, ..., v_g^4\right)$ and $\left(v_1^{34}, v_2^{34}, ..., v_g^{34}\right)$ against the covariate group ranks. Define A_{12} be the error area bounded by fractile graphs G^1 and G^2 between the rank points of the covariate x, 1 and g; A_{34} be the error area bounded by graphs G^3 and G^4 between the rank points of the covariate x, 1 and g; and A_* be the separation area bounded between the combined graphs G^{12} and G^{34}.

One way of addressing the problem of the difference between two fractile graphs G^1 and G^2 is to look at a norm in a $g-$dimensional Euclidean space. The L_2-norm can be defined asone way of addressing the problem of the difference between two fractile graphs G^1 and G^2 is to look at a norm in a $g-$dimensional Euclidean space. The L_2-norm can be defined as

$$\begin{aligned} \triangle_{12} &= \left\|G^1 - G^2\right\| \\ &= \left\|\left(v_1^1 - v_1^2, v_2^1 - v_2^2, ..., v_g^1 - v_g^2\right)\right\| \\ &= \sqrt{w_{1(12)}^2 + w_{2(12)}^2 + ... + w_{g(12)}^2} \end{aligned} \tag{18}$$

Similarly, define $\Delta_{34} = \sqrt{w_{1(34)}^2 + w_{2(34)}^2 + ... + w_{g(34)}^2}$ between G^3 and G^4, and finally,

Δ_* between the combined graphs G^{12} and G^{34}.

Suppose, $B = ((b_{ij}))$ is a positive definite matrix like the covariance matrix, then a more general class of distance measure is

$$\Gamma_{12}^2 = \sum_{i=1}^{g}\sum_{j=1}^{g} w_{i(12)} w_{j(12)} b_{ij} = W_{(12)}^T B W_{(12)}. \tag{19}$$

Now, extending the result of Sethuraman (1961) with m fractile groups $m\Delta_{in}^2$ converges to a mixture of χ^2 variates where Δ_{in} is the error area of fractile graph $i = 1, 2$. If B is the inverse of the covariance matrix of W, $m\Gamma_{in}^2$ converges to χ^2 with g degrees of freedom. Furthermore if $m\Delta_{in}^2$,$i = 1, 2$ and $2m\Delta_{*n}^2$ are asymptotically independent,

$$\frac{2\Delta_{*n}^2}{\left(\Delta_{1n}^2 + \Delta_{2n}^2\right)} \rightarrow \text{Ratio of mixture of } \chi^2.$$

Similarly, for a suitable normalization matrix B, like the inverse of the bootstrapped variance covariance matrix,

$$\frac{2\Gamma_{*n}^2}{\left(\Gamma_{1n}^2 + \Gamma_{2n}^2\right)} \rightarrow F_{g,2g}.$$

We report the results of the individual and group F-tests in Table 2A, if we want to test the conditional fractile means jointly, we observe from the overall F-tests that the two fractile graphs are statistically different for different values of g. The overall F-test for fractile graphs helps us to compare the conditional means jointly. However, the overall F-test like the Kolmogorov-Smirnov and Cramer-von Mises tests do not give us any indication of the type of departures.

#Fractile Groups Under $H_0 : m_1(x) = m_2(x)$	Error Area High Tax $\Gamma_{12}^2 \sim \chi_g^2$	Error Area Low Tax $\Gamma_{34}^2 \sim \chi_g^2$	Area of Separation $\Gamma_*^2 \sim \chi_g^2$	Overall $\frac{2\Gamma_*^2}{(\Gamma_{12}^2+\Gamma_{34}^2)}$
$g = 10$	8.28 (0.60)	8.12 (0.62)	134.98*** (0.0)	16.46 (0.0
$g = 20$	25.31 (0.19)	28.07 (0.11)	167.55*** (0.0)	6.28 (0.0
$g = 50$	47.77 (0.56)	58.7 (0.19)	187.56*** (0.0)	3.52 (0.0

Table 2A. Asymptotic tests of normalized Error Area and the Area of Separation (Pvalues in parenthesis)

To determine the causes of departure from H_0, we apply the smooth test proposed by Bera, Ghosh and Xiao (2005), the results of the full sample test is given in Table 3.

Ψ_4^2	u_1^2	u_2^2	u_3^2	u_4^2
2095.366***	1629.2854***	331.3226***	0.6124	47.72***
(0.0)	(0.0)	(0.0)	(0.433887629)	(0.0)

Table 3. Smooth statistics and p-values (whole sample)

*** significant at 1%, ** significant at 5%, * significant at 10%.

As we discussed in Subsection 2.2 to reduce the effect of the relative sample sizes, we took a random sample of optimal size (10% of the sample size $m = 5106$, see APPENDIX A for details) 500 from the inflow distribution with lower tax exposure (the smaller sample size), and recomputed the smooth test statistics in Table 4 with the mutual fund inflows (see Figure 5). In particular we investigated the differences in the inflow distribution unadjusted for returns, then used the regression residuals from ols, median regression, fractile regression and finally, a median regression on the fractiles of risk adjusted returns. Since we are only looking at residuals for the purpose of our tests, we exclude the regression outputs that can be provided on request. We also tried non-linear specifications an lagged values of the risk adjusted returns according to Chevalier and Ellison (1997), Sirri and Tufano (1998), but the regression did not have any statistically significant higher order or lagged terms for our dataset.

Residuals with Returns	Ψ^2	u_1^2	u_2^2	u_3^2	u_4^2
Unadjusted	145.8494***	112.7042***	24.4997***	1.1872	1.9775
(Entire range, high, low tax)	(0.0)	(0.0)	(7.43×10^{-7})	(0.2759)	(0.1596)
OLS	6.6872	0.5455	0.7742	1.4052	3.9624**
(Entire range, high, low tax)	(0.1533)	(0.4602)	(0.3789)	(0.2359)	(0.0465)
Median Regression	107.403***	96.119***	9.0687***	0.3158	1.9
(Entire range, high, low tax)	(0.0)	(0.0)	(0.0026)	(0.5742)	(0.1682)
Fractile Regression	10.4964**	1.7897	0.5237	7.9711***	0.212
(Entire range, high, low tax)	(0.0328)	(0.181)	(0.4693)	(0.0048)	(0.6452)
Median-Fractile	101.8819***	92.4467***	6.8277***	0.8227	1.7848
(Entire range, high, low tax)	(0.0)	(0.0)	(0.009)	(0.3644)	(0.1816)

Table 4. Smooth statistic and p-values (sample $m = 500$).

*** significant at 1%, ** significant at 5%, * significant at 10%

A closer inspection of Table 4 reveals quite a few facets of the distribution of mutual fund inflows once adjusted for the covariate, in this case past years risk-adjusted returns. We also see that the type of regression we use to adjust for the effect of mutual fund returns does indeed make a difference in the distribution of inflows with high and low tax exposure. From the smooth test technique discussed in Section 2.2 and Bera, Ghosh and Ziao (2004), we observe that the unadjusted inflow distribution for mutual funds with high and low tax exposure differs significantly in the first ($u_1^2 = 112.7042$) and second $\left(u_2^2 = 24.4997\right)$ moment components. This essentially implies that the mutual fund inflow distribution is significantly different for funds with high and low tax exposures $\left(\Psi^2 = 145.8494\right)$. However, it has been an empirical regularity that in the absence of a comprehensive yet tangible measure of performance, uninformed taxable investors use past year's risk adjusted returns to evaluate performance of mutual funds (Ippolito, 1992, Patel, Zeckhauser and Hendricks, 1994, Chevalier and Ellison 1997, Sirri and Tufano, 1998).Past year's risk adjusted pretax returns are the most important factor in determining mutual fund inflows (regression results not shown here, please refer to Bergstresser and Poterba, 2002). Hence, to compare the explanatory power of high and low tax for determining mutual fund inflows, we need to adjust the inflow distribution for the variation in risk adjusted returns.

If we take ordinary least squares (OLS) residuals for the regression of mutual fund new money inflows on risk adjusted excess return, the distribution of inflows conditioned for risk adjusted returns in the high and low tax exposure groups are statistically identical (Overall $\Psi^2 = 6.6872$ with p-value=0.1533). On the basis of individual components, only the fourth moment is slightly significant ($u_4^2 = 3.9624$). This corroborates the conventional wisdom of "return chasing" behavior shown by individual uniformed investors. However, this does not explain how after-tax returns have any more predictive power over pre-tax returns. One possible justification of this phenomenon could be that US Securities and Exchange Commisson in 2001 mandated that all mutual funds must disclose their pre-tax as well as potential post-tax returns in their advertisements. Since a majority (nearly 12997 out of 18730) of our mutual funds-years are after 2001. Another possible explanation for this phenomenon is dependence within each fund-year group that might only show up in higher order moments (see Rakowski, 2002). We looked at the case $k = 6$, the overall model was still not significantly different between the two groups at 1% level ($\Psi^2 = 13.3183$), owing largely due to difference in the sixth moment ($u_6^2 = 5.99$). This result of course can also indicate the existence of outliers in the model.

One argument against OLS is the differential effect of a covariates like risk adjusted returns in different parts or quantiles of the inflow distribution cannpt be captured by OLS, and then there is also a significant effect of outliers in the data. In order to reduce the effect of outliers we can use Median Regression (essentially, 50% Quantile Regression proposed by Koenker and Bassett, 1978). We observe that the two adjusted distributions now only differ in the direction of the first and second moments (u_1^2 =96.119 and u_2^2 =9.0687), although the intensity is less than the slightly unadjusted case..The essential features of the distributional differences between inflow distributions are preserved even after we condition or adjust for risk adjusted return. The difference could be due to the difference in the risk premium (or location) and risk aversion (or scale) between the investors who invest into mutual funds with high and low tax exposure. This result could also be an artifact of the possibility that the distributions of returns are distinctly different between the mutual funds with low tax exposure and those with high tax exposure.

So, in order to make the two groups (high and low tax exposure) comparable we have to standardize the covariates. Hence, we look at the residuals using the fractile regression method without using any kernel type smoothing (Bera and Ghosh, 2005). The overall excess returns adjusted inflow distribution differs at 5% level between high and low tax exposures mainly in the directions of the third moments (u_3^2 =7.9711). This obviously points to an asymmetry in the measures of skewness in the two distributions.

If we condition for fractiles of the risk adjusted returns, and then look at the median regression residuals of inflow distribution, the results as expected points out in the direction of the first and second moments (u_1^2 =92.4467 and u_2^2 =6.8277), although the departure in the second moment is quite marginal. This method is obviously trying to reconcile the "return chasing" behavior of investors that has been well documented in the literature along with a natural tendency that after-tax returns distribution is truly different than that before taxes as Bergtresser and Poterba (2002) point out.

We have established that using smooth test techniques after adjusting for returns using fractile regression methods does indeed keep the facets of the original distribution unchanged while conditioning for risk adjusted excess returns. However, it remains to be tested whether the regulation from SEC that mandates mutual fund companies to report after-tax returns did indeed change the behavior of the mutual fund flow with respect to behavior of taxable investors. To answer that question, we investigate whether the mutual fund inflow distribution itself has changed before and after 2001 when SEC mandated mutual fund companies to disclose potential after-tax returns at the highest tax brackets in their advertisements (Figure 5). Inflow distribution before and after 2001 when

SEC required all mutual funds to disclose potential after tax returns seems to be substantially different in almost all the measures of central tendency and standard deviations. Goodness-of fit like Kolmogorov Smirnocv and Cramer-von Mises tests also indicates that the two distributions are distinctly different (Tables 5, 6). Figure 5 also confirms our belief that inflow distributions are different before and after 2001. However, we need to run a joint statistical test like the smooth test to objectively determine what are the directions of departure between unconditional and conditional inflow distributions before and after

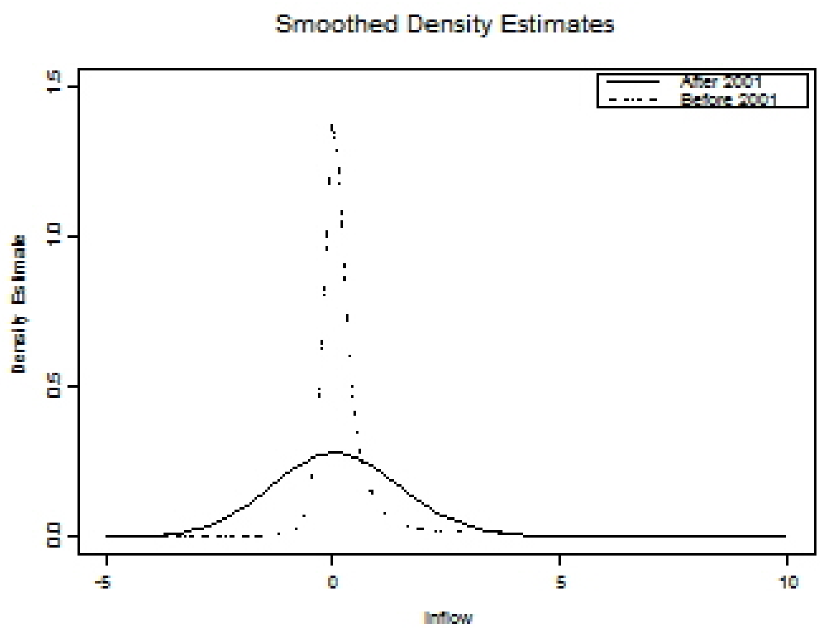


Figure 5: Inflow distributions before and after 2001

2001.

Variable	Obs.	Mean	Median	Std. Dev.	Min.	1^{st} Q.	3^{rd} Q.	Max.	coef. skew	excs.kurt
Before 2001	5733	0.3718	0.1071	3.4161	-6.1652	-0.0636	0.3629	236.9686	58.8373	4017.677
After 2001	12997	1.0371	-0.0243	75.6425	-8.4932	-0.1529	0.1784	8510.7078	109.9089	12326.272

Table 5. Summary Statistic for Fund Inflows before and after 2001

Test	KS	CvM
Statistic (T*)	16.4610	106.1507
Critical Upper 0.1%	1.95	1.167

Table 6. Goodness-of-Fit Statistics based on the EDF.

Residuals with Returns	Ψ^2	u_1^2	u_2^2	u_3^2	u_4^2
Unadjusted	103.6286***	68.4923***	18.3223***	13.7541***	3.0599*
(before and after 2001)	(0.0)	(0.0)	(1.86×10^{-5})	(2.08×10^{-4})	(0.0802)
Fractile Regression	489.7059***	423.0520***	63.1092***	3.5445*	0.00006
(before and after 2001)	(0.0)	(0.0)	(0.0)	(0.0597)	(0.9936)
Median-Fractile	218.4536***	170.3281***	17.6892***	15.0684**	15.3679**
(before and after 2001)	(0.0)	(0.0)	(2.6×10^{-5})	(1×10^{-4})	(8.8×10^{-5})
Unadjusted	17.1575***	0.1056	15.8711***	0.3866	0.7942
(After 2001-high, low tax)	(0.0018)	(0.7453)	(6×10^{-5})	(0.5341)	(0.3728)
Median-Fractile	5.9152	0.8718	4.328**	0.496	0.2194
(After 2001-high-low tax)	(0.2056)	(0.3505)	(0.0375)	(0.4813)	(0.6395)

Table 7. Smooth statistic and p-values (sample $m = 500$).

*** significant at 1%, ** significant at 5%, * significant at 10%

First, we want to test the inflow distributions before and after 2001. The unadjusted inflow distribution changed significantly in the direction of the first three moments ($u_1^2 = 68.49$, $u_2^2 = 18.3223$, $u_3^2 = 13.7541$) and marginally in the direction of the fourth moment $\left(u_4^2 = 3.08\right)$. The results remain essentially the same when we look at median regression on fractiles, although the departures between the two distributions are now substantially stronger. One way of reconciling this would be possible dependence in the data, or endogeneity of performance measure like excess return with the error term in our regression model. Estimating a structural equation non-parametrically or semiparametrically would be a better solutions. If we take fractile regression residuals our overall test is more strongly rejected $\left(\Psi^2 = 489.7059\right)$, however we observe less effect from higher order terms. It seems overwhelmingly likely there was phenomenal growth in the mutual fund industry as well changes in volatility and higher moments of the flow variable (Rakowski, 2002).

Finally, to evaluate the effect of the regulatory regime of disclosure of potential after-tax returns we restrict ourselves only to data after 2001. We want to investigate the explanatory power of after tax returns compared to pre-tax returns in explaining mutual fund inflow. The overall smooth test for unadjusted flow variable turns out to be significant $\left(\Psi^2 = 17.1575\right)$ at 1% level, however, this is almost purely driven by a difference in volatility measure or the second moment of the flow distribution $\left(u_2^2 = 15.8711\right)$. This discrepancy could be driven by the fact that lower tax funds are probably indexed funds which have less tax exposure for relatively low turnover. It is also possible, the lower tax mutual funds are the ones that are now substantially bigger and the fund managers have less time to spend on individual stock picking exercises that carefully or in other words diminishing returns have set in, so performance is going down (Berk and Green, 2004).

When we adjust inflows by conditioning for market risk, keeping the original flow distribution relatively unchanged by running a median regression on the fractiles of risk adjusted return, smooth test reveals that the after-tax and pre-tax distributions are statistically identical. (overall $\Psi^2 = 5.9152$ or p-value of 0.2056). The only component that is slightly significantly different is in the direction of volatility $\left(u_2^2 = 4.328\right)$, but with much reduced intensity than the unadjusted case. Hence, we can conclude that ranks of returns do indeed provide a parsimonious of conditioning the inflow distribution without disturbing the inherent ordering of inflow based on "rank chasing" behavior of uninformed taxable investors.

This analysis reveals how we can adjust for a covariate that might not be comparable across two regimes using the linear rank transformation like the Empirical Distribution

Function of the covariate, before comparing the distributions of the response variable across two regimes.

4 Conclusions and Future Research

Conditioning on normalized ranks or fractiles of the return distribution gives a natural way to control for ranks without losing the essential feature of ordering of the data. This enables us to compare across different regimes or years. This rank ordering of risk adjusted returns also induces an ordering on a *concomitant variable*, in this case mutual fund flow (sometimes also referred to as an *induced order statistics*, see David 1973, Bhattacharya, 1974, 1984, Yang, 1977). Moreover, ranking risk adjusted returns might provide a more parsimonious model than higher order terms to incorporate possible non-linearity or convexity of risk adjusted return in the flow-performance relationship (see Chevalier and Ellison, 1997, Sirri and Tufano, 1998). However, we should also keep in mind that ranking risk adjusted returns, and to then run standard t-tests for corresponding inflows might induce data-snooping biases (Lo and MacKinlay, 1990).

It has been well documented in the literature (Ippolito, 1992, Patel, Zeckhauser and Hendricks, 1994) for an average taxable uninformed investor in the absence of discerning ability to recognize fund managers stock-picking ability, past year's risk adjusted excess returns are used to evaluate performance (for exception to this rule see Gruber, 1996, Zheng 1990). However, empirically mutual fund flow and performance seem to have a non-linear, in particular, convex relationship (Sirri and Tufano, 1998). It has also been shown that there is some persistence in returns or lagged values of returns have been used to predict flow (Chevalier and Ellison, 1997). This non-linearity of the fund-flow relationship makes it necessary to search for a more robust measure of performance that can also be used to compare across different regimes like after-tax return reporting regulations setup by

SEC in 2001. Bergstresser and Poterba (2002) found that in that in the absence of such a regulatory disclosure regime, after-tax risk adjusted returns do indeed predict inflow better than pre-tax returns.

Our humble attempt in this paper is to apply a new methodology to compare inflow distributions across different regimes by conditioning on the fractiles of risk adjusted returns. Fractiles or normalized rank groups also give us a easily implimented non-parametric rank regression technique, that preserves the inherent ordering and distribution of the flow variable after conditioned for ranks of the risk adjusted excess returns. Using this technique, we demonstrated that after adjusting for risk adjusted returns in the inflow distribution, the OLS residuals of the two groups becomes overall statistically identically distributed. Although, when we increased the number of moment directions (from $k = 4$ to $k = 6$), we found somewhat significant deviations in the directions of the fourth and the sixth moments. This was probably caused by dependence in the data and/or presence of outliers.

We also established that there was a significant change in the inflow distribution before and after the establishment of mandatory after-tax returns reporting guidelines for mutual funds instituted by the US Securities and Exchange Commission in 2001. While this indicates possibly emergence of mutual funds as a dominant portfolio holding of the average, low net-worth taxable non-institutional investor (Bergstresser and Poterba, 2002), it might also indicate a sense of security in a transparent regime free of "window-dressing" or "tax-trading" concerns, besides freedom from potential capital gains overhang problems indicated in the literature. We found that although the unadjusted inflow distributions were still marginally different between high and low tax burdens (owing mainly to difference in volatility of inflow distribution), after conditioning for the risk adjusted return using fractile regression technique combined with median regression, the inflow distributions were statistically identical upto the first few moments.

As a future direction of work, a full fledged non-parametric (rank based) structural model should be estimated instead of parametric linear regressions, however the curse of dimensionality in models like that prvents elaborate modeling. For prediction purpose, however, parsimony is still the strongest tool at hand.

5 Appendix A (Sample Size Selection Bera, Ghosh and Xiao, 2005)

For finite sample, for each fixed n_2, we may divide the index set $\mathcal{N} = \{1, \ldots, n\}$ into two mutually exclusive and exhaustive (large) sets $\mathcal{N}_1$ and $\mathcal{N}_2$ with cardinalities n_1 and n_2, where $n_1 + n_2 = n$, and define the **training set**

$$\mathcal{Z}_1 = \{(X_j), j \in \mathcal{N}_1\}$$

and the testing set

$$\mathcal{Z}_2 = \{(X_j), j \in \mathcal{N}_2\}.$$

Then we can estimate $F(\cdot)$ using data $\mathcal{Z}_1$ and construct

$$F_{n_1}(X_i) = \frac{1}{n_1} \sum_{j \in \mathcal{N}_1} I(X_j \leq X_i), \text{ for } i \in \mathcal{N}_2.$$

$\mathcal{Z}_1$ and $\mathcal{Z}_2$ are from the same distribution F, $F(X_i)$ $(i \in \mathcal{N}_2)$ are uniformly distributed and $F_{n_1}(X_i)$ provides an estimator for the uniform distribution, we may compare it with the CDF of standard uniform, say, using some criterion function

$$\frac{1}{n_2} \sum_{i \in \mathcal{N}_2} d(F_{n_1}(X_i), U[0,1])$$

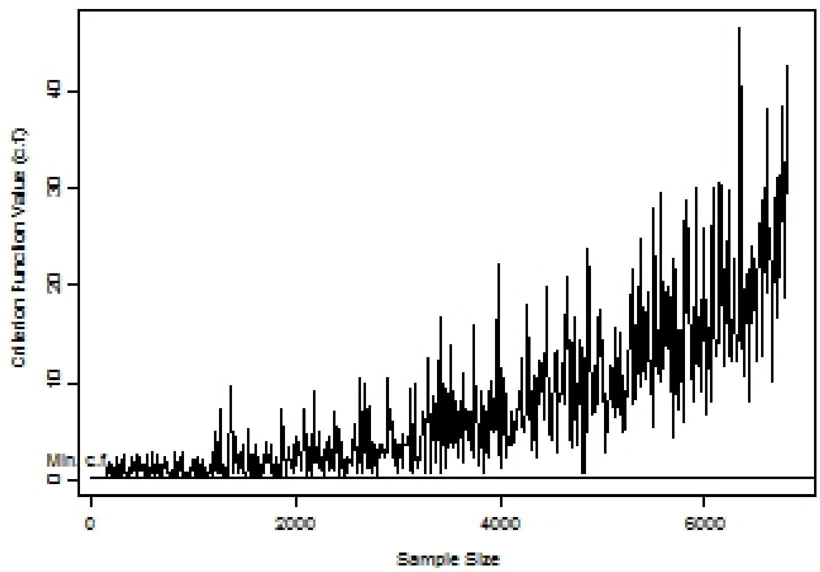


Figure 6: Plot of criterion function to choose finite sample size

and take average over R replications

$$\frac{1}{R}\sum_{r=1}^{R}\left[\frac{1}{n_2}\sum_{i\in\mathcal{N}_2} d(F_{n_1}^{r}(X_i), U[0,1])\right]$$

For each value of n_2, we can calculate the above criterion function. We may choose n_2 that minimizes the above criterion.

Finally, we choose

$$m = \frac{n_2}{n_1} \times n.$$

The above method may have applications in more general settings. This is a cross-validation type procedure to select sample size. In the above problem the criterion function is showed in Figure 6, $\frac{n_2}{n_1} = 12.37\%$. For convinience we chose 500, about 10% of the smallest sample size.

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